

10.

Example: $A_n = \{x | 0 \leq x \leq \frac{1}{n}\}$ $n=1,2,\dots$

Prove that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n = 0$

Proof:

(1) Prove that $A_{n+1} \subset A_n$ for all n

Since $A_n = \{x | 0 \leq x \leq \frac{1}{n}\}$ $n=1,2,\dots$,

We have $A_{n+1} = \{x | 0 \leq x \leq \frac{1}{n+1}\}$, since $\frac{1}{n} > \frac{1}{n+1}$, so we have $A_{n+1} \subset A_n$ for all n

The statement has been proved.

(2) Prove that $\bigcap_{n=1}^{\infty} A_n = 0$

When $n \rightarrow \infty$, $\frac{1}{n} \rightarrow 0$, $A_n = \{x | 0 \leq x \leq \frac{1}{n}\} \rightarrow \{x | 0 \leq x \leq 0\}$

Since there is only one real number 0 in A_n when $n \rightarrow \infty$, according to the definition of intersection, $\bigcap_{n=1}^{\infty} A_n = 0$. The statement has been proved.