

7.

Proof:

Prove by induction.

For $n=1$, $2^n=2=2^{n+1}-2$

Assume statement holds for n

For $n+1$

$$\begin{aligned} 2+2^2+2^3+\cdots+2^n+2^{n+1} &= 2^{n+1}-2+2^{n+1} \text{ (by the induction hypothesis)} \\ &= 2 \times 2^{n+1} - 2 = 2^{n+2} - 2 = 2^{(n+1)+1} - 2 \end{aligned}$$

So, the equality holds for $n+1$

The statement has been proved by induction.