Event-Triggered Quantized Communication-Based Consensus in Multiagent Systems via Sliding Mode

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***Abstract*—To handle the common existing constraints, that is, limited energy supplies and limited communication bandwidth in multiagent systems (MASs), this article investigates the consensus problem in MASs with event-triggered communication (ETC) and state quantization. In order to compensate for the effect brought by mismatched disturbances, we also propose a novel multiple discontinuous sliding-mode surface, and the corresponding sliding-mode control law is constructed by considering the event-triggered and dynamic quantized mechanisms jointly. Under such a scheme, it is shown that the state trajectories of all the agents will be regulated to achieve consensus asymptotically and the Zeno behavior can be avoided completely. We further extend this work to self-triggered and periodic event-triggered cases. Particularly, in a periodic event-triggered approach, the new form of triggering conditions and upper bound of the sampling periods are provided explicitly. As a result, all agents can reach bounded consensus. Moreover, the upper bound of the consensus error can be arbitrarily adjusted by appropriately selecting parameters, and the periodic event-triggered case will be reduced to the event-triggered case when the bound approaches 0 (sampling periods approach 0 at the same time). A numerical example is illustrated to verify the effectiveness of the proposed algorithms.**

***Index Terms*—Distributed consensus, event-triggered communication (ETC), multiagent systems (MASs), sliding-mode control (SMC), state quantization.**

I. Introduction

THE PAST decade has witnessed the emergence of research on multiagent systems (MASs) due to the increasing applications of large-scale networks [1]–[5]. As a fundamental problem in the study of MASs, the consensus problem that aims at regulating all agents to reach an agreement has been attracting more attention. (See [6]–[10] and the references therein.)

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It should be noted that most of the existing works on the consensus problem are based on sampled-data control, which means that the transmission tasks between agents are executed periodically. However, in many scenarios, this may lead to a lot of wasted energy since sometimes there is no great fluctuation of sampled information and, therefore, the communication here is unnecessary. To counter this drawback, event-triggered communication (ETC) has been introduced to the research community and gained considerable attention [11]–[15]. The main idea of ETC is that the transmission tasks will be executed only after a certain predesigned condition is satisfied. This is to say, the transmission actions may be triggered aperiodically, and the predesigned triggering condition is thereby designed to ensure that the communication only takes place when it is necessary (to preserve the desired control performance). Since limited energy supplies is a common problem among MASs, such as the unmanned air vehicles system and multinode-networked systems, ETC has been shown to be greatly beneficial with respect to this consideration. Some works, for example, [16]–[28], have brought ETC to MASs and have proved its feasibility and superiority. However, it should be pointed out that the classical ETC assumes that the triggering conditions can be evaluated continuously along with the time elapsed, which is almost impossible in practice. Therefore, periodic ETC (PETC), which combines the advantages of sampled-data control and ETC, is more meaningful to be investigated from the digital implementation perspective. Unfortunately, only a little bit of literature has considered this issue [17], [25]–[28]. What is worse, only [26] and [28] present some results on asynchronous PETC, which works in a more practical way as each agent can determine its own sampling period using only local information.

Another concern comes from the fact that limited communication bandwidth is also a common constraint that existed in MASs. This constraint is also referred to as the limited transmission data rate. To avoid the imperfections, such as time delay induced by an overloaded transmission data rate, it is inevitable for the agents to communicate with each other with the discretized state information through a quantized mechanism with some sampling errors as the expense. Along this line, significant works have been made to handle the sampling errors induced by quantization while preserving desired control performance. (See [1], [7], [19], [20], [24], [25], [29]–[31], and the references therein.) Of them, it should be pointed out that [19], [20], [24], and [25] have considered the co-design of control algorithms under both the ETC and state quantization, which is a more general framework for the consensus problem in MASs from the practical implementation perspective. However, the main drawback of the aforementioned works is that only the bounded consensus can be achieved, with the exception of [24].

On the other hand, it is well known that the disturbances and parameter uncertainties widely existed in control systems, and the sliding-mode control (SMC) is an efficient tool to overcome them [32]–[36]. There is some literature that has combined SMC with the ETC and state quantization, for instance, [37] and [38]. Some interesting works also have been devoted to the SMC-based consensus problem due to the frequent existence of uncertain disturbances in MASs (see [39] and [40]). Motivated by these aforementioned considerations, in this article, we propose a framework for SMC-based consensus in MASs with ETC and state quantization. We first introduce a dynamic quantized mechanism that consists of a finite-level quantizer and encoder/decoder pair(s) for each agent, with which the communication bandwidth can be mostly saved. To mitigate the great waste of energy caused by unnecessary communication, we also propose an event-triggered mechanism to determine when it is necessary to transmit information. Then, by incorporating the integral SMC (ISMC) strategy with the event-triggered and dynamic quantized mechanisms, the objective that all agents reach an agreement can be achieved. The main contributions of this article can be summarized as follows.

1) We propose a novel multiple discontinuous sliding-mode surface that consists of a series of individual sliding-mode surfaces generated by events. With the co-design of the event-triggering condition, the quantizer, and the encoder/decoder pair(s), the constructed SMC law can drive the state trajectories of all agents to achieve consensus asymptotically despite mismatched disturbances. Furthermore, under such a scheme, the quantizer will never be saturated and the Zeno behavior, which bothers a lot of event-triggered controllers, can be avoided.
2) We also extend the proposed algorithm to the self-triggered and periodic event-triggered cases. Particularly, in the periodic event-triggered approach, we redesign the triggering condition and give a systematic way to determine the sampling period for each agent. The quantizer will never be saturated as well, and all agents that can realize bounded consensus with the error bound can be adjusted arbitrarily. Note that our periodic event-triggered strategy adopts asynchronous sampling, which means that each agent can determine its sampling interval using only local information.

The remainder of this article is organized as follows. Section II presents some preliminaries that are useful throughout this article and formulates the problem to be addressed in this article. Section III describes the main results of the algorithm design in this article. Section IV gives some extensions of the results in Section III. A numerical example is illustrated in Section V to verify the algorithms proposed in Sections III and IV, followed by the conclusion in Section VI.

Notations: $\|\cdot\|$ denotes the infinity-norm. $|a|$ represents the absolute value of a. $\mathbb{R}$ and $\mathbb{R}^n$ are sets of real numbers and n-dimensional real vectors, respectively. $\mathbb{Z}_{>0}$ is the set of positive integers and $\mathbb{Z}_{\geq 0}$ contains $\mathbb{Z}_{>0}$ and 0. $sgn(\cdot)$ is the sign function and $\lfloor\cdot\rfloor$ represents the floor function.

II. Preliminaries and Problem Formulation

A. Graph Theory

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ be the graph of MASs comprising N nodes with $\mathcal{V} = \{1, 2, \ldots, N\}$ and let $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denote the set of edges. We say $(j, i) \in \mathcal{E}$ if node j can receive the information broadcasted from node i and in this sense, node i can be called a neighbor of node j. We use $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$ to denote the set of neighbors of node i. Define a weighted adjacency matrix $\mathcal{A} = (a_{ij})_{N\times N}$ with $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. If a graph is undirected, then $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$; and if a graph is connected, then for $\forall i, j \in \mathcal{V}$, there exists at least one path that connects nodes i and j. Denote $\mathcal{D} = (d_{ij})_{N\times N}$ as the degree matrix, which is a diagonal matrix, and $d_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$. Also define the Laplacian matrix for graph $\mathcal{G}$ as $\mathcal{L} = \mathcal{D} - \mathcal{A} = (l_{ij})_{N\times N}$.

Assumption 1: The graph $\mathcal{G}$ is undirected and connected.

B. Quantization

We define a finite-level uniform quantizer $Q : \mathbb{R} \to \mathbb{R}$ as follows:

$$Q(z)\begin{cases} K\Delta, & z \geq \left(K+\frac{1}{2}\right)\Delta \\ \left\lfloor \frac{z}{\Delta}+\frac{1}{2}\right\rfloor\Delta, & 0\leq z < \left(K+\frac{1}{2}\right)\Delta \\ -Q(-z), & z<0 \end{cases} \tag{1}$$

where z is the input of the quantizer, $K \in \mathbb{Z}_{\geq 0}$, Δ represents the quantization interval, and the output levels of the quantizer are $2K + 1$. Note that $|Q(z) - z| \leq (\Delta/2)$ can be guaranteed whenever the quantizer is not saturated, that is, $|z| \leq (K + (1/2))\Delta$.

C. Event-Triggered Communication

Different from time-triggered communication, in ETC schemes, the transmission instants are determined by a predesigned triggering condition. This is to say, transmissions only occur at discrete-time instants when a specific event is satisfied, which we denote by $t_k, k \in \mathbb{Z}_{\geq 0}$, satisfying $0 = t_0 < t_1 < t_2 < \cdots$ Under such a scheme, unnecessary communications are avoided, thereby much energy can be saved. Generally speaking, ETC can be divided into the following.

1) Continuous Event-Triggered Communication: Without a specific definition, continuous ETC (CETC) is often known as ETC. A continuous event-triggered mechanism takes, for example, the form

$$t_0 = 0, t_{k+1} = \inf\{t > t_k \,|\, \|y(t_k) - y(t)\| \geq \varrho\} \tag{2}$$

where $y(t)$ denotes either the state measurement or the output measurement of the considered system, and ϱ is a predesigned function whether state dependent or time dependent. In this case, the triggering condition $\|y(t_k) - y(t)\| \geq \varrho$ is detected continuously along with the time elapsed. Once it is satisfied, the transmission is triggered; otherwise, there will be no transmission.

2) Periodic Event-Triggered Communication: Instead of monitoring the triggering condition continuously, in PETC, the triggering condition will only be evaluated periodically. The periodic event-triggered mechanisms are often of the form

$$t_0 = 0, t_{k+1} = \inf\{t = t_k + \kappa h \mid \|y(t_k) - y(t)\| \geq \varrho\} \quad (3)$$

where h is the sampling period and $\kappa \in \mathbb{Z}_{>0}$. It is worth pointing out that as monitoring the triggering condition continuously is almost impossible in practical embedded implementation, the study on PETC is somewhat meaningful for us to find the maximum-allowable sampling period (MASP) of CETC under which the considering system can still preserve the acceptable performance.

D. Problem Formulation

Consider the following continuous-time MAS, which consists of N agents:

$$\dot{x}_i(t) = u_i(t) + d_i(t), \ \ i \in \mathcal{V} \triangleq \{1, 2, \ldots, N\} \quad (4)$$

where $x_i \in \mathbb{R}^n$ is the state of agent i, $u_i \in \mathbb{R}^n$ is the control input of agent i, and $d_i \in \mathbb{R}^n$ represents the mismatched disturbance and it satisfies that $\|d_i(t)\| \leq d_i^*$.

The aim of this article is to propose an SMC-based algorithm to address the consensus problem in MASs. To take account of the limited communication resources, namely, limited energy supplies and limited communication bandwidth, both the ETC and state quantization need to be considered.

III. Event-Triggered Sliding-Mode-Based Consensus Algorithm Design

In this section, we will address the consensus problem for system (4) with ETC and state quantization. Especially, we propose the co-design of the event-triggered mechanism and dynamic quantized mechanism for each agent. Then, by incorporating the integral sliding-mode strategy, the asymptotical consensus can be achieved in the presence of mismatched disturbances.

To begin with, we give an example first to illustrate how ISMC helps system (4) reach consensus under the full local information, that is, no communication resource limitations.

Example 1: Consider the integral sliding-mode surface as

$$S_i(t) = x_i(t) - x_i(0) - \int_0^t u_i^{\text{nom}}(\tau)d\tau, \ \ i \in \mathcal{V} \quad (5)$$

where $u_i^{\text{nom}}(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}(x_i(t) - x_j(t))$. Then, the SMC law is constructed as

$$u_i(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}\big(x_i(t) - x_j(t)\big) - \lambda_i \text{sgn}(S_i(t)). \quad (6)$$

Consider the Lyapunov candidate $V(t) = \sum_{i=1}^N V_i(t) = \sum_{i=1}^N (1/2) S_i^T S_i$, taking the time derivative of both sides yields

$$\begin{aligned}\dot{V} = \sum_{i\in\mathcal{V}} \dot{V}_i(t) &= \sum_{i\in\mathcal{V}} S_i^T(t)(-\lambda_i \text{sgn}(S_i(t)) + d_i(t)) \\ &\leq \sum_{i\in\mathcal{V}} -\|S_i(t)\|\big(\lambda_i - d_i^*\big). \end{aligned} \quad (7)$$

It implies that if $\lambda_i > d_i^*$ for $i \in \mathcal{V}$ can be satisfied, then it can be guaranteed that $\dot{V}(t) = \sum_{i=1}^N \dot{V}_i(t) < 0$. Therefore, the state trajectory will be attracted to the region that $S_i(t) = \dot{S}_i(t) = 0$, and it indicates that $\dot{x}_i(t) = u_i^{\text{nom}}(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}(x_i(t) - x_j(t))$. As it has been discussed in [6], the consensus can be achieved asymptotically for MASs (4) in the presence of mismatched disturbances.

As we can see clearly, such a kind of the ISMC strategy can help the system (4) overcome the effect of mismatched disturbances. It is quite natural for us to study if it is still feasible for scenarios where the communication resources are limited, that is, the transmitted information will be quantized with finite precision and transmitted action will not be actuated periodically. Now, we are ready to show how the ISMC strategy works under the CETC and state quantization.

A. Algorithm Design

We use $t_p^i \in \{t_0^i, t_1^i, t_2^i, \ldots,\}$ to represent the triggering instants for agent i, and $t_{k_i}^i$ is the latest triggering instant at time instant t. Then, we shall introduce the encoder and decoder designs [41]. The encoder of agent i is designed as

$$x_i^e(t) = x_i^e\Big(t_{k_i}^i\Big) - \eta(t) Q\left(\frac{x_i^e\Big(t_{k_i}^i\Big) - x_i(t)}{\eta(t)}\right) \quad (8)$$

where $x_i^e(t) \in \mathbb{R}^n$ is the internal state of the encoder with $x_i^e(t_0^i) = 0$. Define $\omega_i(t) = [(x_i^e(t_{k_i}^i) - x_i(t))/\eta(t)]$, which is the information that will be quantized and then transmitted to the subscribers. $\eta(t)$ is a uniformly bounded positive function and will be designed later. In fact, $\eta(t)$ is an adjustable sensitive parameter that may influence the precision of quantization. The larger the $\eta(t)$, the larger the quantization range is, with the cost that quantization error goes larger as well. Note that the encode action will only take place at the triggering instants, that is, $\omega_i(t)$ will only be transmitted at $t = t_p^i, p \in \mathbb{Z}_{\geq 0}$.

For each agent j that subscribes, the agent i will receive the quantized information $Q(\omega_i)$ from agent i and use the following decoder to update the estimation of state x_i:

$$x_{j,i}^d\Big(t_{k_i+1}^i\Big) = x_{j,i}^d\Big(t_{k_i}^i\Big) - \eta\Big(t_{k_i+1}^i\Big) Q\Big(\omega_i\Big(t_{k_i+1}^i\Big)\Big) \quad (9)$$

where $x_{j,i}^d(t) \in \mathbb{R}^n$ is the estimation of $x_i(t)$ estimated by agent j under the ETC and state quantization, with $x_{j,i}^d(0) = 0$. The decode action will only take place when agent j receives a new message from its neighbors, that is, at the triggering instants $t = t_p^i, p \in \mathbb{Z}_{\geq 0}$. Therefore, agent j obtains $x_{j,i}^d(t_{k_i}^i)$ as the estimation of $x_i(t)$ in time period $[t_{k_i}^i, t_{k_i+1}^i)$. It is also worth pointing out that how many other agents the agent j subscribes, then how many decoders the agent j should equip.

Note that from (8) and (9), it is easy to observe that for every agent j that subscribes agent i, it has $x^d_{j,i}(t^i_{k_i}) = x^e_i(t^i_{k_i})$, which means that agent i also knows the estimation value of $x_i(t)$ obtained by agent j. Therefore, the triggering condition can be easily checked by the encoder in agent i through comparing $x_i(t)$ and $x^e_i(t^i_{k_i})$. Denote $e_i(t) = x^e_i(t^i_{k_i}) - x_i(t), t \in [t^i_{k_i}, t^i_{k_i+1})$ as the estimation error induced by ETC and state quantization. For each agent i, we choose the triggering conditions of the form

$$\Gamma_i(t) = \|e_i(t)\| - \theta(t) \tag{10}$$

where $\theta(t) = \gamma\eta(t)$, in which $\gamma \in ((\Delta/2), [((2K+1)\Delta)/2]]$. Once the triggering condition satisfies $\Gamma_i(t) \geq 0$, $\omega_i(t)$ will be transmitted to its subscribers in a quantized form. Otherwise, agent i will not broadcast the message. In this sense, the next triggering instant for agent i can be expressed as

$$t^i_{k_i+1} = \inf\left\{t \mid t > t^i_{k_i}, \Gamma_i(t) \geq 0\right\}. \tag{11}$$

Note that we acquiesce that the communication will be triggered at the initial instant, that is, $t = 0$. As $t = 0$, every agent i do not have the initial states of its neighbors, it is reasonable for the agents to communicate with its neighbors in order to obtain their initial information, which is essential for the following computation.

Before we present the main results of this section, some restrictions for the selection of $\eta(t)$ should be stated and summarized in the following assumption.

Assumption 2: $\eta(t)$ is continuously differentiable and satisfies that

$$\dot{\eta}(t) < 0, \lim_{t\to\infty} \eta(t) = 0, \eta(0) \geq \frac{\max_{i\in\mathcal{V}}\|x_i(0)\|}{\gamma}.$$

For each agent i, we propose a novel multiple discontinuous sliding-mode surface to be designed as

$$S^i_{k_i}(t) = x_i(t) - x_i\left(t^i_{k_i}\right) - \int_{t^i_{k_i}}^{t} u^{\mathrm{nom}}_i(\tau)d\tau, \;\; t \in \left[t^i_{k_i}, t^i_{k_i+1}\right). \tag{12}$$

Then, the SMC law under ETC and state quantization can be constructed as

$$u_i(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}\left(x^e_i\left(t^i_{k_i}\right) - x^e_j\left(t^j_{k_j}\right)\right) - \lambda_i \mathrm{sgn}\left(\hat{S}^i_{k_i}(t)\right) \tag{13}$$

for $t \in [t^i_{k_i}, t^i_{k_i+1})$, where $\hat{S}^i_{k_i}(t) = x_i(t) - x_i(t^i_{k_i}) + \int_{t^i_{k_i}}^{t}(\sum_{j\in\mathcal{N}_i} a_{ij}(x^e_i(t^i_{k_i}) - x^e_j(t^j_{k_j})))d\tau$. λ_i denotes the switching gain and will be designed later.

In summary, the quantizer (1), encoder (8), decoder (9), event-triggered mechanism (10), sliding-mode surface (12), and SMC law (13) form the basis of the event-triggered sliding-mode-based consensus algorithm, which is formally illustrated in Algorithm 1.

Remark 1: Here, we propose a novel multiple discontinuous sliding-mode surface. Observing (12), we can see that a new sliding-mode surface is generated at the triggering instant and the initial value of each individual sliding-mode surface, that is, $S^i_{k_i}(t^i_{k_i})$, equals to 0. In other words, the number of individual sliding-mode surfaces equals the number of triggering instants. This kind of multiple discontinuous sliding-mode surface is of great benefit such as improving the convergent rate since it naturally avoids the accumulation of error of the integral term along the time by eliminating it at the triggering instants.

Algorithm 1 Event-Triggered Sliding-Mode-Based Consensus Algorithm

Input: $x_i(0), i \in \mathcal{V}$
for each $i \in \mathcal{V}$ **do**
  if $\Gamma_i(t) \geq 0$ is satisfied **then**
    broadcast $\omega_i(t)$, construct the new sliding mode surface $S^i_{k_i}(t)$ and update control signal $u_i(t)$
  end if
  if $\omega_j(t)$ is received from some neighbor(s) $j \in \mathcal{N}_i$ **then**
    update $x_{i,j}(t^j_{k_j})$ through decoder and update control signal $u_i(t)$
  end if
  calculate $x_i(t)$ via (4)
end for
Output: $x_i(t), i \in \mathcal{V}$

Remark 2: Due to the ETC and state quantization, agent i may not have the nominal states of its neighbors. $\hat{S}^i_{k_i}(t)$ can be interpreted as the estimation of $S^i_{k_i}(t)$ under the incomplete information. Note that though agent i has the nominal state information of itself, we still use the estimated version, that is, $x^e_i(t^i_{k_i})$ to construct u_i for convenience in the following design.

B. Convergence Analysis

In this part, we will show that the algorithm proposed above will help the system (4) reach asymptotical consensus with ETC and state quantization. Before the main theorem is presented, two propositions are given first.

Proposition 1: Consider the encoder and decoder designed in (8) and (9) and the triggering condition (10), it can be ensured that $e_i(t) \leq \theta(t)$ for $t \in [0, +\infty)$ and the uniform quantizer (1) will not be saturated when transmitting $\omega_i(t^i_{k_i})$.

Proof: From (10), it can be known that $\|e_i(t)\| = \|x^e_i(t^i_{k_i}) - x_i(t)\| < \theta(t)$ for $t \in (0, +\infty)$. It indicates that

$$\|\omega_i(t)\| = \frac{\left\|x^e_i\left(t^i_{k_i}\right) - x_i(t)\right\|}{\eta(t)} < \frac{\theta(t)}{\eta(t)}. \tag{14}$$

Recall that $\theta(t) = \gamma\eta(t)$, it follows that:

$$\|\omega_i(t)\| < \gamma \leq \left(K + \frac{1}{2}\right)\Delta, t \in (0, +\infty). \tag{15}$$

Then, we consider the case that $t = 0$, according to Assumption 2, it can be seen clearly that

$$\|e_i(0)\| = \left\|x^e_i(0) - x_i(0)\right\| = \|x_i(0)\| \leq \gamma\eta(0) = \theta(0) \tag{16}$$

$$\|\omega_i(0)\| = \frac{\|e_i(0)\|}{\eta(0)} = \frac{\theta(0)}{\eta(0)} = \gamma \leq \left(K + \frac{1}{2}\right)\Delta. \tag{17}$$

Therefore, it can be concluded that $e_i(t) \leq \theta(t)$ for $t \in [0, +\infty)$ and the quantizer (1) will never be saturated. ■

Proposition 2: It can be guaranteed that $\mathrm{sgn}(\hat{S}^i_{k_i}(t)) = \mathrm{sgn}(S^i_{k_i}(t))$ when $t \in [t^i_{k_i}, t^i_{k_i+1})$ if the sliding surface $S^i_{k_i}(t)$

satisfies that $\|S^i_{k_i}(t)\| \geq \gamma\|l_i\|\chi\eta(t^i_{k_i})$, where χ denotes the maximum triggering interval in the entire process.

Proof: In order to give the proof, we shall give the relationship between $\hat{S}^i_{k_i}(t)$ and $S^i_{k_i}(t)$ first

$$\begin{aligned}
&\left\|\hat{S}^i_{k_i}(t) - S^i_{k_i}(t)\right\| \\
&\quad= \left\| x_i(t) - x_i\left(t^i_{k_i}\right) + \int_{t^i_{k_i}}^{t} \sum_{j\in\mathcal{N}_i} a_{ij}\left(x^e_i\left(t^i_{k_i}\right) - x^e_j\left(t^j_{k_j}\right)\right)d\tau \right. \\
&\qquad\left. - x_i(t) + x_i\left(t^i_{k_i}\right) - \int_{t^i_{k_i}}^{t} \sum_{j\in\mathcal{N}_i} a_{ij}\big(x_i(\tau) - x_j(\tau)\big)d\tau \right\| \\
&= \left\| \int_{t^i_{k_i}}^{t} \sum_{j\in\mathcal{N}_i} a_{ij}\big(e_i(\tau) - e_j(\tau)\big)d\tau \right\| \\
&= \int_{t^i_{k_i}}^{t} \|l_i e(\tau)\| d\tau \leq \gamma\|l_i\| \int_{t^i_{k_i}}^{t} \eta(\tau)d\tau
\end{aligned} \tag{18}$$

where $e(t) = [e_1(t), e_2(t), \ldots, e_N(t)]$, and l_i represents the ith row of the Laplacian matrix $\mathcal{L}$. By the definition of the triggering condition and the fact that $\dot{\theta}(t) < 0$ and $\lim_{t\to\infty}\theta(t) = 0$, we can know that $\max_{i\in\mathcal{V}} |t^i_{k_i+1} - t^i_{k_i}| < +\infty$. Denote $\chi = \max_{i\in\mathcal{V}} |t^i_{k_i+1} - t^i_{k_i}|$, then (18) can be transformed to

$$\left\|\hat{S}^i_{k_i}(t) - S^i_{k_i}(t)\right\| \leq \gamma\|l_i\|\chi\eta\left(t^i_{k_i}\right). \tag{19}$$

It indicates that $|\|S^i_{k_i}(t^i_{t_i})\| - \|S^i_{k_i}(t)\|| \leq \gamma\chi\|l_i\|\eta(t^i_{k_i})$. Hence, it can be concluded that $\mathrm{sgn}(\hat{S}^i_{k_i}(t)) = \mathrm{sgn}(S^i_{k_i}(t))$ will not be violated if $S^i_{k_i}(t)$ satisfies $\|S^i_{k_i}(t)\| \geq \gamma\|l_i\|\chi\eta(t^i_{k_i})$. This completes the proof. ■

Theorem 1: Consider the MAS (4) with the control input given in (13), the encoder and decoder given in (8) and (9), and the triggering condition in (11), under Assumptions 1 and 2, if we select $\lambda_i \geq \|l_i\|\gamma\eta(0) + d^*_i$, then all states of the MAS will reach consensus asymptotically.

Proof: Consider the Lyapunov candidate $V(t) = \sum_{i\in\mathcal{V}} V_i(t) = \sum_{i\in\mathcal{V}}(1/2){S^i_{k_i}}^T S^i_{k_i}$. For each $V_i(t) = (1/2){S^i_{k_i}}^T S^i_{k_i}$, taking the time derivative of both sides, for $t \in [t^i_{k_i}, t^i_{k_i+1})$, yields

$$\begin{aligned}
\dot{V}_i(t) &= {S^i_{k_i}}^T(t)\left(-\sum_{j\in\mathcal{N}_i} a_{ij}\left(x^e_i\left(t^i_{k_i}\right) - x^e_j\left(t^j_{k_j}\right)\right) \right. \\
&\qquad - \lambda_i \mathrm{sgn}\left(\hat{S}^i_{k_i}(t)\right) + d_i(t) \\
&\qquad \left. + \sum_{j\in\mathcal{N}_i} a_{ij}\big(x_i(t) - x_j(t)\big) \right) \\
&= {S^i_{k_i}}^T(t)\left(-\sum_{j\in\mathcal{N}_i} a_{ij}\big(e_i(t) - e_j(t)\big) \right. \\
&\qquad \left. - \lambda_i \mathrm{sgn}\left(\hat{S}^i_{k_i}(t)\right) + d_i(t) \right) \\
&= {S^i_{k_i}}^T(t)\left(-l_i e(t) - \lambda_i \mathrm{sgn}\left(\hat{S}^i_{k_i}(t)\right) + d_i(t) \right).
\end{aligned} \tag{20}$$

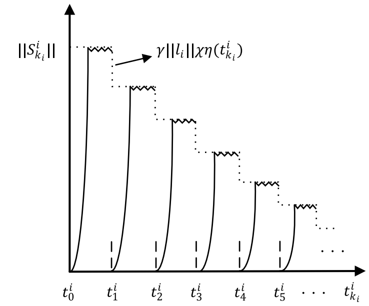


Fig. 1. Illustration of the trajectory of $\|S^i_{k_i}\|$ under Algorithm 1.

According to Proposition 2, when $\|S^i_{k_i}(t)\| \geq \gamma\|l_i\|\chi\eta(t^i_{k_i})$, (20) can be transformed to

$$\begin{aligned}
\dot{V}_i(t) &= -\lambda_i\left\|S^i_{k_i}(t)\right\| - {S^i_{k_i}}^T(t) l_i e(t) + {S^i_{k_i}}^T(t) d_i(t) \\
&\leq -\lambda_i\left\|S^i_{k_i}(t)\right\| + \|l_i\|\|e(t)\|\left\|{S^i_{k_i}}^T(t)\right\| + d^*_i\left\|{S^i_{k_i}}^T(t)\right\| \\
&= -\left\|S^i_{k_i}(t)\right\|\big(\lambda_i - \|l_i\|\|e(t)\| - d^*_i\big).
\end{aligned} \tag{21}$$

Recall Proposition 1, one can obtain that $\|e(t)\| \leq \theta(t)$. Hence, (21) can be rewritten as

$$\dot{V}_i(t) \leq -\left\|S^i_{k_i}(t)\right\|\big(\lambda_i - \|l_i\|\theta(t) - d^*_i\big). \tag{22}$$

Since $\lambda_i \geq \|l_i\|\gamma\eta(0) + d^*_i \geq \|l_i\|\theta(t) + d^*_i$, it can be concluded that $S^i_{k_i}(t)$ will be attracted into the region $\mathcal{R} = \{S^i_{k_i}(t) | \|S^i_{k_i}(t)\| < \gamma\|l_i\|\chi\eta(t^i_{k_i})\}$. Since $\lim_{t\to\infty}\eta(t) = 0$, it is clear that when $t \to +\infty$, $S^i_{k_i}(t)$ will be driven into the region $\mathcal{R}^* = \{S^i_{k_i}(t) | \|S^i_{k_i}(t)\| = 0\}$. At the same time, it can be known that $\dot{S}^i_{k_i} = 0$, which implies that $\lim_{t\to\infty}\dot{x}_i(t) = u^{\mathrm{nom}}_i(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}(x_i(t) - x_j(t))$. Similar to Example 1, asymptotical consensus can be achieved. This completes the proof. ■

Remark 3: To be more illustrative, Fig. 1 shows how the sliding-mode surface $S^i_{k_i}(t)$ be driven into the region $\mathcal{R}^* = \{S^i_{k_i}(t) | \|S^i_{k_i}(t)\| = 0\}$ under Algorithm 1.

C. Zeno Behavior Analysis

In this part, we use the following theorem to show that the Zeno behavior can be avoided.

Theorem 2: Consider the MAS (4) with the control input given in (13), the encoder and decoder given in (8) and (9), and the triggering condition in (11), under Theorem 1, we have

$$t^i_{k_i+1} - t^i_{k_i} > \epsilon, \;\; \epsilon > 0. \tag{23}$$

Proof: By definition $\forall t \in [t^i_{k_i}, t^i_{k_i+1})$, $e_i(t)$ can be expressed as

$$e_i(t) = e_i\left(t^i_{k_i}\right) + \int_{t^i_{k_i}}^{t} \dot{e}_i(\tau)d\tau = e_i\left(t^i_{k_i}\right) - \int_{t^i_{k_i}}^{t} \dot{x}_i(\tau)d\tau. \tag{24}$$

Since $\dot{x}_i = u_i(t) + d_i(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}(x^e_i(\tau) - x^e_j(\tau)) - \lambda_i \mathrm{sgn}(S_i(t^i_{k_i})) + d_i(t)$. Then, (24) can be rewritten as

$$\|e_i(t)\| = \left\| e_i\left(t^i_{k_i}\right) - \int_{t^i_{k_i}}^{t} \left(-\sum_{j\in\mathcal{N}_i} a_{ij}\left(x^e_i(\tau) - x^e_j(\tau)\right) \right.\right.$$

$$- \lambda_i \mathrm{sgn}\left(\hat{S}^i_{k_i}\left(t^i_{k_i}\right)\right) + d_i(\tau)\Bigg) d\tau \Bigg\|$$

$$\leq \left\| e_i(t^i_{k_i}) \right\| + \int_{t^i_{k_i}}^{t} \left(\left\| l_i x^e(\tau) \right\| + \lambda_i + d_i^* \right) d\tau \tag{25}$$

where $x^e(t) = [x^e_1(t), x^e_2(t), \ldots, x^e_N(t)]$. As it has been discussed in Theorem 1, it is guaranteed that $x_i(t)$ will be forced to satisfy $\|x_i(t) - x_i(t^i_{k_i}) - \int_{t_{k_i}}^{t} u_i^{\mathrm{nom}}(\tau)d\tau\| < \gamma \|l_i\| \chi \eta(t^i_{k_i})$. It follows that $\|x_i(t)\| < \|x_i(t^i_{k_i}) + \int_{t_{k_i}}^{t} u_i^{\mathrm{nom}}(\tau)d\tau\| + \gamma \|l_i\| \chi \eta(t^i_{k_i})$. Moreover, it should be noted that by the definition of $u_i^{\mathrm{nom}}(t)$, we can deduce that $\|x_i(t^i_{k_i}) + \int_{t^i_{k_i}}^{t} u_i^{\mathrm{nom}}(\tau)d\tau\| \leq \max_{i \in \mathcal{V}} \|x_i(t^i_{k_i})\| \leq \max_{i \in \mathcal{V}} \|x_i(0)\|$. On the other hand, the term $\gamma \|l_i\| \chi \eta(t^i_{k_i})$ becomes smaller with the time elapses. Thereby, it can be ensured that $\|x_i(t)\|$ is bounded when $t \in [t^i_{k_i}, t^i_{k_i+1})$. Similarly, by considering all the triggering periods, we could deduce that $\|x_i(t)\|$ is bounded all the time. Since $\|x^e_i(t)\| \leq \|x_i(t)\| + \|e_i(t)\|$ and $\|e_i(t)\| < \theta(t) \leq \theta(0)$ according to Proposition 1, it gives that $\|x^e_i(t)\|$ is also bounded. Going back to (25), it can be known consequently

$$\|e_i(t)\| \leq \left\| e_i\left(t^i_{k_i}\right) \right\| + \left(t - t^i_{k_i}\right)\left(\|l_i\| \left\|x^e(t)\right\| + \lambda_i + d_i^*\right). \tag{26}$$

By Proposition 1, the quantizer is not saturated, which indicates that $\|e_i(t^i_{k_i})\| \leq (1/2)\Delta\eta(t^i_{k_i})$. Recall the triggering condition, we then know that

$$t^i_{k_i+1} - t^i_{k_i} \geq \frac{\theta\left(t^i_{k_i+1}\right) - \frac{1}{2}\Delta\eta\left(t^i_{k_i}\right)}{\|l_i\|\|x^e(t)\| + \lambda_i + d_i^*}. \tag{27}$$

Also define

$$\zeta\left(t^i_{k_i}, t\right) = \frac{\theta(t) - \frac{1}{2}\Delta\eta\left(t^i_{k_i}\right)}{\|l_i\|\|x^e(t)\| + \lambda_i + d_i^*} \tag{28}$$

as we can see, since $\theta(t^i_{k_i}) = \gamma\eta(t^i_{k_i}) > (1/2)\Delta\eta(t^i_{k_i})$, it can be known easily that $\zeta(t^i_{k_i}, t^i_{k_i}) > 0$. As $\dot{\eta}(t) < 0$, we obtain $\dot{\theta}(t) < 0$, which further implies that $\dot{\zeta}(t^i_{k_i}, t^i_{k_i}) < 0$. Hence, there must exist a positive constant ϵ ensures that $\zeta(t^i_{k_i}, t^i_{k_i} + \epsilon) = 0$. Therefore, it can be concluded that $t^i_{k_i+1} - t^i_{k_i} \geq \epsilon$. This completes the proof. ∎

IV. Extensions to Self-Triggered and Periodic Event-Triggered Sliding-Mode-Based Consensus

In this section, we will show some extensions of the results in Section III. Particularly, we study the self-triggered and PETC cases with the quantizer and encoder/decoder pair(s) keep the same with before.

A. Self-Triggered Approach

In the self-triggered approach, the next triggering instant is determined by state information sampled in the last step instead of evaluating the triggering condition.

For each agent i, we design the triggering instants to be computed by

$$t^i_{k_i+1} = t^i_{k_i} + h_i\left(t^i_{k_i}\right) \tag{29}$$

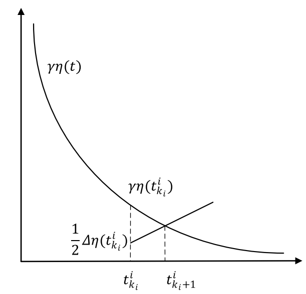


Fig. 2. Illustration that (30) always has a unique positive solution.

where $h_i(t^i_{k_i})$ is the solution to

$$\frac{1}{2}\Delta\eta\left(t^i_{k_i}\right) + h_i\left(t^i_{k_i}\right)\left(\left\|\sum_{j \in \mathcal{N}_i} a_{ij}\left(x^e_i\left(t^i_{k_i}\right) - x^e_j\left(t^j_{k_j}\right)\right)\right\| + \lambda_i + d_i^*\right)$$
$$= \gamma\eta\left(t^i_{k_i} + h_i\left(t^i_{k_i}\right)\right). \tag{30}$$

As we can see in Fig. 2, obviously, there always exists such h_i, which is the unique solution of (30) since $\gamma\eta(t^i_{k_i}) > (1/2)\Delta\eta(t^i_{k_i})$. It also indicates that the Zeno behavior is excluded automatically.

Proposition 3: Consider the encoder and decoder designed in (8) and (9) and the triggering instants designed in (29) and (30), it can be ensured that $e_i(t) \leq \theta(t)$ for $t \in [0, +\infty)$ and the uniform quantizer (1) will never be saturated when transmitting $\omega_i(t^i_{k_i})$.

Proof: By definition, for $t \in (t^i_{k_i}, t^i_{k_i+1})$, it can be known that

$$\|e_i(t)\| \leq \Bigg\| e\left(t^i_{k_i}\right) - \int_{t^i_{k_i}}^{t} \Bigg(-\sum_{j \in \mathcal{N}_i} a_{ij}\left(x^e_i\left(t^i_{k_i}\right) - x^e_j\left(t^j_{k_j}\right)\right)$$
$$- \lambda_i \mathrm{sgn}\left(\hat{S}^i_{k_i}\left(t^i_{k_i}\right)\right) + d_i(\tau)\Bigg) d\tau \Bigg\|$$
$$\leq \frac{1}{2}\Delta\eta\left(t^i_{k_i}\right) + \left(t - t^i_{k_i}\right)\Bigg(\left\|\sum_{j \in \mathcal{N}_i} a_{ij}\left(x^e_i\left(t^i_{k_i}\right) - x^e_j\left(t^j_{k_j}\right)\right)\right\|$$
$$+ \lambda_i + d_i^*\Bigg). \tag{31}$$

Obviously, the upper bound of $\|e_i(t)\|$ increases along the time, hence for $t = t^i_{k_i+1}$, $\|e_i(t)\|$ arrives the maximum point. Recalling (30), it can be obtained that $\|e_i(t)\| \leq \|e_i(t^i_{k_i+1})\| \leq \gamma\eta(t^i_{k_i+1}) = \theta(t^i_{k_i+1}) \leq \theta(t)$ for $t \in (t^i_{k_i}, t^i_{k_i+1})$. It also implies that

$$\|\omega_i(t)\| = \frac{\|e(t)\|}{\eta(t)} < \frac{\theta(t)}{\eta(t)} = \gamma \leq (K + \frac{1}{2})\Delta. \tag{32}$$

Then, we consider the case when $t = t^i_{k_i+1}$, it can be obtained that

$$\left\|\omega_i\left(t^i_{k_i+1}\right)\right\| = \frac{\left\|x^e_i\left(t^i_{k_i}\right) - x_i\left(t^i_{k_i+1}\right)\right\|}{\eta\left(t^i_{k_i+1}\right)} \leq \frac{\theta\left(t^i_{k_i+1}\right)}{\eta\left(t^i_{k_i+1}\right)}$$

$$\leq \gamma \leq \left(K+\frac{1}{2}\right)\Delta. \tag{33}$$

Finally, we consider the case that $t = 0$, according to Assumption 2, it follows that $\|e_i(0)\| = \|x_i(0)\| \leq \gamma\eta(0) = \theta(0)$, and therefore:

$$\|\omega_i(0)\| = \frac{\left\|x_i^e(0)-x_i(0)\right\|}{\eta(0)} = \frac{\|x_i(0)\|}{\eta(0)} \leq \gamma \leq \left(K+\frac{1}{2}\right)\Delta. \tag{34}$$

In summary, $\|e_i(t)\| < \theta(t)$ and $\|\omega_i(t)\| \leq (K+(1/2))\Delta$ can be guaranteed for $t \in [0,+\infty)$. This completes the proof. ■

We note that in Proposition 3, we assume that the neighbors of agent i do not send new information to agent i for $t \in (t^i_{k_i}, t^i_{k_i+1})$ so that (30) is solvable [$x_i^e(t^i_{k_i})$ and $x_j^e(t^j_{k_j})$ are constants in this situation]. However, the received information from the neighbors of agent i may be updated, that is, $x_j^e(t^j_{k_j})$ may vary during $t \in (t^i_{k_i}, t^i_{k_i+1})$. In this case, (30) should be adjusted online when computing, which can be briefly described as follows.

1) Compute all the temporary next triggering instants $t^i_{k_i+1}(\text{temp})$ for each agent i using (29) and (30).
2) Detect if there is any neighbor(s) of agent i will send new information to agent i during $t \in (t^i_{k_i}, t^i_{k_i+1}(\text{temp}))$, in other words, if there is any agent $j \in \mathcal{N}_i$ satisfies that $t^j_{k_j+1}(\text{temp}) \in (t^i_{k_i}, t^i_{k_i+1}(\text{temp}))$. [Here, we assume that the information of $t^j_{k_j+1}(\text{temp})$ will also be transmitted to the agent i simultaneously with $x_j^e(t^j_{k_j})$.]
3) If there is no agent $j \in \mathcal{N}_i$ satisfies $t^j_{k_j+1}(\text{temp}) \in (t^i_{k_i}, t^i_{k_i+1}(\text{temp}))$, then the exact $t^i_{k_i+1}$ can be obtained as $t^i_{k_i+1} = t^i_{k_i+1}(\text{temp})$.
4) If there exists such agent(s) $j \in \mathcal{N}_i$, then for these time instants $t^j_{k_j+1}(\text{temp})$ satisfies $t^j_{k_j+1}(\text{temp}) \in (t^i_{k_i}, t^i_{k_i+1}(\text{temp}))$, we select the one that is closest to $t^i_{k_i}$ and denote it by $\mathcal{T}$. Then, (30) should be adjusted to $(1/2)\Delta\eta(t^i_{k_i})+(\mathcal{T}-t^i_{k_i})(\|\sum_{j\in\mathcal{N}_i} a_{ij}(x_i^e(t^i_{k_i})-x_j^e(\mathcal{T}^-))\|+\lambda_i+d_i^*)+(t^i_{k_i+1}(\text{temp})-\mathcal{T})(\|\sum_{j\in\mathcal{N}_i} a_{ij}(x_i^e(t^i_{k_i})-x_j^e(\mathcal{T}^+))\|+\lambda_i+d_i^*) = \gamma\eta(t^i_{k_i+1}(\text{temp}))$.
5) Go back to step 1 and update $t^i_{k_i+1}(\text{temp})$ until the exact $t^i_{k_i+1}$ is obtained.

We assume that all the above-mentioned actions take place instantaneously. The proposed adjusted algorithm can also generate the conclusion of Proposition 3. The proof is straightforward and thus omitted here.

Corollary 1: Consider the MAS (4) with the control input given in (13), the encoder and decoder given in (8) and (9), and the triggering instants designed in (29) and (30), under Assumptions 1 and 2, if we select $\lambda_i \geq \|l_i\|\gamma\eta(0)+d_i^*$, then all states of the agents will reach consensus asymptotically.

Sketch of the Proof: Based on Proposition 3, the idea of the proof for the self-triggered approach is basically the same with the results in Section III (we refer the readers to the proof of Theorem 1 and the proof here is therefore omitted). ■

B. Periodic Event-Triggered Approach

In the periodic event-triggered approach, the triggering conditions are evaluated every fixed period, thus naturally excludes the Zeno behavior.

For each agent i, we redesign the triggering conditions of the form

$$\Gamma_i(t) = \|e_i(t)\| - \delta_i\theta(t) \tag{35}$$

where δ_i satisfies $0 < \delta_i < 1$. Denote h_i as the sampling period. In this sense, the next triggering instant for time instant t can be expressed as

$$t^i_{k_i+1} = \inf\left\{t = t^i_{k_i} + \kappa h_i \mid \Gamma_i(t) \geq 0 \cup t - t^i_{k_i} \geq \chi_i\right\} \tag{36}$$

where $\chi_i > 0$ is the upper bound of triggering interval, $\kappa \in \mathbb{Z}_{>0}$.

In this part, the selection of $\eta(t)$ is somewhat different from Section III. The new restrictions about $\eta(t)$ are stated in the following assumption.

Assumption 3: $\eta(t)$ is continuously differentiable and satisfies that

$$\dot{\eta}(t) \leq 0,\ \lim_{t\to\infty}\eta(t) = \mu,\ \frac{\eta(t+\rho)}{\eta(t)} \geq \sigma(\rho)$$
$$\eta(0) \geq \frac{\max_{i\in\mathcal{V}}\|x_i(0)\|}{\gamma}$$

where μ is a positive constant that is sufficiently small, $\rho > 0$, and $\sigma(\rho)$ is a positive function in terms of ρ satisfies that $0 < \sigma(\rho) < 1$.

Remark 4: The reason we redesign the event-triggered condition (10)–(35) is that when $e_i(t)$ reaches $\theta(t)$ right after a sampling instant, which we refer to as *the worst case*, then $e_i(t) \leq \theta(t)$ may not be guaranteed since the triggering condition will not be evaluated in the following sampling period h_i. Therefore, in order to ensure that $e_i(t) \leq \theta(t)$ even at the worst case, it is a natural idea to consider such δ_i satisfies $0 < \delta_i < 1$ to make allowances for the periodic samplings.

Remark 5: Notice that in (36) an additional constraint on the maximum time interval between two successful triggering instants is enforced, which means that agent i will not be without communication for more than χ_i units of time. This constraint comes from the fact that $\lim_{t\to\infty}\eta(t) \neq 0$, which results in the persistent quantization error. Hence, an upper bound of the triggering interval is necessary to suppress the accumulative error. Fortunately, χ_i can be designed arbitrarily large as long as it is a finite constant so it will not be a restrictive requirement.

Now, we are going to show the requirements of selecting h_i and δ_i

$$\sigma(h_i) \geq \beta_i \tag{37a}$$

$$h_i \leq \min\frac{(\beta_i-\delta_i)\theta\left(t^i_{k_i}\right)}{\sum_{j\in\mathcal{N}_i}\left\|a_{ij}\left(x_i^e\left(t^i_{k_i}\right)\right)-\left(x_j^e\left(t^j_{k_j}\right)\right)\right\|+\lambda_i+d_i^*} \tag{37b}$$

where β_i is a positive constant satisfies that $\delta_i < \beta_i < 1$. Note that the explicit value of the right-hand side of (37b) will be provided in the following corollary.

Proposition 4: Consider the encoder and decoder designed in (8) and (9), the sampling period h_i designed in (37), and the triggering condition (36), it can be ensured that $\|e_i(t)\| \leq \theta(t)$ for $t \in [0, +\infty)$ and the uniform quantizer (1) will never be saturated when transmitting $\omega_i(t_{k_i})$.

Proof: Consider the worst situation, that is, $e(t_{k_i}^{i\,+}) = \delta_i \theta(t_{k_i}^i)$, then it gives

$$\begin{aligned}\left\|e_i\left(t_{k_i}^i + h_i\right)\right\| &< \Bigg\|\delta_i\theta\left(t_{k_i}^i\right) - \int_{t_{k_i}^i}^{t_{k_i}^i + h_i} \\ &\quad \times \left(-\sum_{j\in\mathcal{N}_i} a_{ij}\left(x_i^e\left(t_{k_i}^i\right) - x_j^e\left(t_{k_j}^j\right)\right)\right. \\ &\quad \left. - \lambda_i \operatorname{sgn}\left(\hat{S}_{k_i}^i\left(t_{k_i}^i\right)\right) + d_i(\tau)\right) d\tau\Bigg\| \\ &\leq \delta_i\theta\left(t_{k_i}^i\right) + h_i\left(\left\|\sum_{j\in\mathcal{N}_i} a_{ij}\left(x_i^e\left(t_{k_i}^i\right) - x_j^e\left(t_{k_j}^j\right)\right)\right\|\right. \\ &\quad \left. + \lambda_i + d_i^*\right) \\ &\leq \delta_i\theta\left(t_{k_i}^i\right) + (\beta_i - \delta_i)\theta\left(t_{k_i}^i\right) \leq \theta\left(t_{k_i}^i + h_i\right).\end{aligned} \tag{38}$$

It indicates that even for the worst case, it is still ensured that $\|e_i(t)\| < \theta(t)$. Then, similar to Proposition 3, it can also be obtained that $\|\omega_i(t)\| \leq (K + (1/2))\Delta$ for $t \in [0, +\infty)$ under Assumption 3. This completes the proof. ■

Corollary 2: Consider the MAS (4) with the control input given in (13), the encoder and decoder given in (8) and (9), and the triggering condition (36), for suitable β_i, if the selected sampling period h_i and δ_i satisfy that $\sigma(h_i) \geq \beta_i$ and $h_i \leq [((\beta_i - \delta_i)\gamma\mu)/(\|l_i\|\ (\max_{i\in\mathcal{V}} \|x_i(0)\| + \gamma\|l_i\|\chi\eta(0) + \gamma\eta(0)) + \lambda_i + d_i^*)]$, where $\chi = \max_{i\in\mathcal{V}} \chi_i$. Then, under Assumptions 1 and 3, if we select $\lambda_i \geq \|l_i\|\gamma\eta(0) + d_i^*$, all states of the agents will reach bounded consensus and $\lim_{t\to\infty} \max_{i,j\in\mathcal{V}} \|x_i(t) - x_j(t)\| < 2\max_{i\in\mathcal{V}} \gamma\|l_i\|\chi_i\mu$.

Sketch of the Proof: Based on Proposition 4, using the same methods in Theorem 1, we can prove that $S_{k_i}^i(t)$ will be attracted into the region $\mathcal{R} = \{S_{k_i}^i(t) | \|S_{k_i}^i(t)\| < \gamma\|l_i\|\chi_i\eta(t_{k_i}^i)\}$ as well for the periodic event-triggered approach. As $\lim_{t\to\infty} \eta(t) = \mu$, it can be seen clearly that $\lim_{t\to\infty} \|S_{k_i}^i(t)\| = \gamma\|l_i\|\chi_i\mu$, which means that $\lim_{t\to\infty} \|x_i(t) - (x_i(t_{k_i}^i) + \int_{t_{k_i}^i}^t u_i^{\text{nom}}(\tau)d\tau)\| < \gamma\|l_i\|\chi_i\mu$. Note that $\lim_{t\to\infty}(x_i(t_{k_i}^i) + \int_{t_{k_i}^i}^t u_i^{\text{nom}}(\tau)d\tau)$ equals to the consensus state of agent i under the ideal input $u^{\text{nom}}(t)$. Therefore, it can be concluded that

$$\lim_{t\to\infty} \max_{i,j\in\mathcal{V}} \left\|x_i(t) - x_j(t)\right\| < 2\max_{i\in\mathcal{V}} \gamma\|l_i\|\chi_i\mu. \tag{39}$$

Now, we are going to give the lower bound of $[((\beta_i - \delta_i)\theta(t_{k_i}^i))/\ \ (\sum_{j\in\mathcal{N}_i} \|a_{ij}(x_i^e(t_{k_i}^i) - x_j^e(t_{k_j}^j))\| + \lambda_i + d_i^*)]$. As it has been discussed in the proof of Theorem 2, we know that $\|x_i(t)\| < \|x_i(t_{k_i}^i) + \int_{t_{k_i}}^t u_i^{\text{nom}}(\tau)d\tau\| + \gamma\|l_i\|\chi_i\eta(t_{k_i}^i)$, in which the first term on the right-hand side is bounded by $\max_{i\in\mathcal{V}} \|x_i(0)\|$. It follows that $\|x_i^e(t)\| \leq \|x_i(t)\| + \|e_i(t)\| \leq \max_{i\in\mathcal{V}} \|x_i(0)\| + \gamma\|l_i\|\chi_i\eta(0) + \gamma\eta(0)$. Then, observe that $\sum_{j\in\mathcal{N}_i} \|a_{ij}(x_i^e(t_{k_i}^i) - x_j^e(t_{k_j}^j))\| = \|l_i\|\|x^e(t)\|$, we obtain that $\sum_{j\in\mathcal{N}_i} \|a_{ij}(x_i^e(t_{k_i}^i) - x_j^e(t_{k_j}^j))\| + \lambda_i + d_i^* \leq \|l_i\|(\max_{i\in\mathcal{V}} \|x_i(0)\| + \gamma\|l_i\|\chi\eta(0) + \gamma\eta(0)) + \lambda_i + d_i^*$. On the other hand, $(\beta_i - \delta_i)\theta(t_{k_i}^i) \geq (\beta_i - \delta_i)\mu$. Therefore, we obtain the lower bound of $[((\beta_i - \delta_i)\theta(t_{k_i}^i))/\ \ (\sum_{j\in\mathcal{N}_i} \|a_{ij}(x_i^e(t_{k_i}^i) - x_j^e(t_{k_j}^j))\| + \lambda_i + d_i^*)]$, the upper bound of h_i in other words

$$h_i \leq \frac{(\beta_i - \delta_i)\gamma\mu}{\|l_i\|\left(\max\limits_{i\in\mathcal{V}}\|x_i(0)\| + \gamma\|l_i\|\chi\eta(0) + \gamma\eta(0)\right) + \lambda_i + d_i^*} \tag{40}$$

where $\chi = \max_{i\in\mathcal{V}} \chi_i$. This completes the proof. ■

Remark 6: It is clear to see that the parameters β_i, δ_i, χ, and μ may influence the upper bound of the h_i and the upper bound of steady-state error for consensus. Especially, when μ approaches to 0, the steady-state error for consensus (39) will approach to 0 as well, at the same time, h_i will be arbitrarily small. In this sense, the periodic event-triggered approach will be reduced to a continuous event-triggered approach.

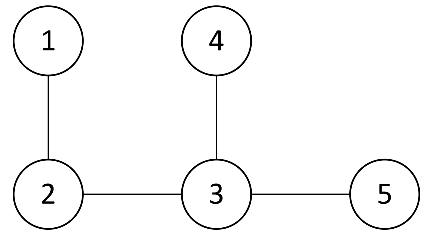


Fig. 3. Undirected graph with five agents.

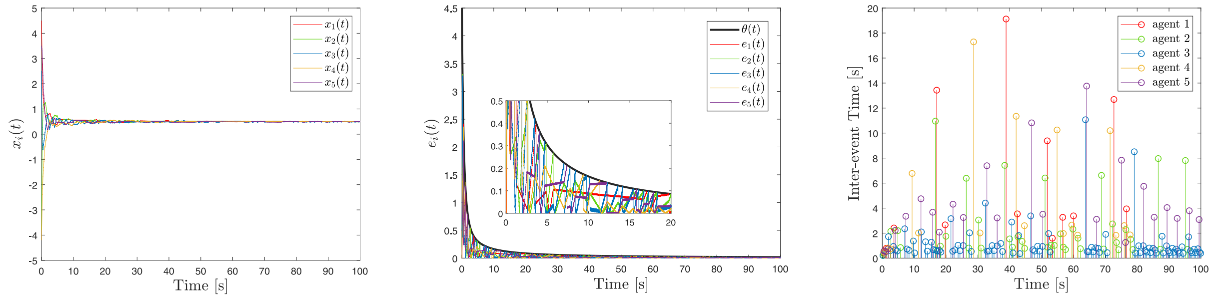
V. Numerical Example

In this section, we will verify the algorithms proposed in this article. Especially, we use a numerical MAS consists of five agents to verify both the event-triggered approach and the periodic event-triggered approach. We do not verify the self-triggered approach since it is ideologically the same with the event-triggered approach and the feasibility of the event-triggered approach naturally supports the self-triggered approach. The corresponding graph of the MAS is shown in Fig. 3. The Laplacian matrix can be expressed as

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 3 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$

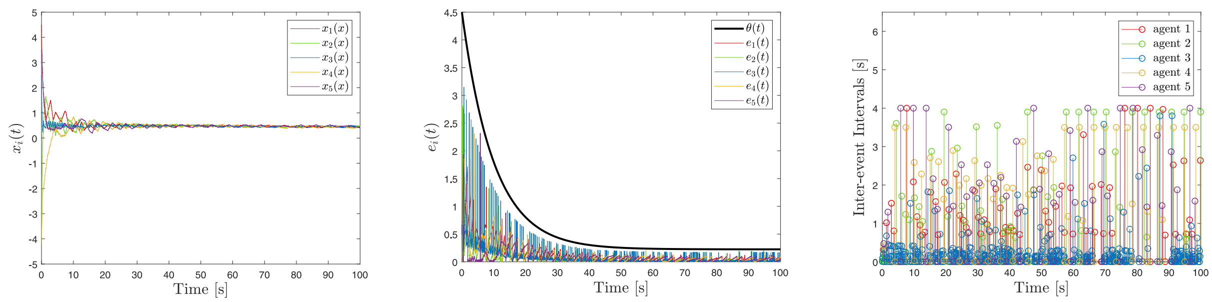
A. Event-Triggered Approach

We set the initial states $x_1(0) = 4.5$, $x_2(0) = -3.8$, $x_3(0) = 2.5$, $x_4(0) = -4.2$, and $x_5(0) = 3.5$, and mismatched disturbances $d_1(t) = 2\sin(t)$, $d_2(t) = \cos(t)$, $d_3(t) = \sin(2t) + e^{-0.1t}$, $d_4(t) = 2\cos(2t)$, $d_5(t) = 2e^{-0.2t}$, then it can be known



(a) (b) (c)

Fig. 4. (a) State trajectories, (b) error between $x_i(t)$ and $x_i(t^i_{k_i})$, and (c) interevent intervals of the five agents via the continuous event-triggered approach.



(a) (b) (c)

Fig. 5. (a) State trajectories, (b) error between $x_i(t)$ and $x_i(t^i_{k_i})$, and (c) interevent intervals of the five agents via the periodic event-triggered approach.

that $d_1^* = 2, d_2^* = 1, d_3^* = 2, d_4^* = 2$, and $d_5^* = 2$. We consider the quantizer in (1) with $K = 4$ and $\Delta = 1$. We choose $\eta(t) = [(0.4 + 0.6e^{-t})/(t+1)]$ in order to satisfy Assumption 2. Select $\gamma = (K + (1/2))\Delta = 4.5$ that implies $\theta(t) = (K + (1/2))\Delta\eta(t) = [(1.8 + 2.7e^{-t})/(t+1)]$. According to Theorem 1, we select $\lambda_1 = 6.5, \lambda_2 = 10, \lambda_3 = 15.5, \lambda_4 = 6.5$, and $\lambda_5 = 8.5$. Then, the simulation results are illustrated in Fig. 4. It is shown that the proposed algorithm is effective in regulating all the agents to reach consensus while releasing the communication burdens.

B. Periodic Event-Triggered Approach

We consider $\eta(t) = 0.05 + 0.95e^{-0.1t}$ in order to satisfy Assumption 3, it gives that $\mu = 0.05$ and $\sigma(\rho) = \min_{t\geq 0} e^{-0.1\rho} + [(0.05(1 - e^{-0.1\rho}))/(0.05 + 0.95e^{-0.1t})] = 0.05 + 0.95e^{-0.1\rho}$. Select $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0.95$, to ensure that (37a) is satisfied, we obtain that h_1, h_2, h_3, h_4, and h_5 should be less than 0.541. Then, consider $\delta_1 = 0.35, \delta_2 = 0.25, \delta_3 = 0.15, \delta_4 = 0.45$, and $\delta_5 = 0.35$ and $\chi_1 = 4, \chi_2 = 3.9, \chi_3 = 3.8, \chi_4 = 3.5$, and $\chi_5 = 4$, then according to Corollary 2, we obtain that $h_1 \leq 0.0038, h_2 \leq 0.0033, h_3 \leq 0.0021, h_4 \leq 0.0032$, and $h_5 \leq 0.0036$ need to be guaranteed. Therefore, we choose $h_1 = 0.0035, h_2 = 0.003, h_3 = 0.002, h_4 = 0.003$, and $h_5 = 0.0035$. $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, and λ_5 are selected as the same with the event-triggered approach. The simulation results are illustrated in Fig. 5. It indicates that the extension to the periodic event-triggered case of our algorithm is efficient as well and the error bound is small enough.

VI. Conclusion

In this article, we have addressed the consensus problem for MASs with communication constraints, namely, limited energy supplies and limited communication bandwidth. By jointly introducing the event-triggered and dynamic quantized mechanisms, we proposed a novel integral sliding mode controller that can drive the state trajectories of all the agents to achieve consensus asymptotically in the presence of mismatched disturbances while much unnecessary waste of energy and communication bandwidth can be avoided. It also has been proven that the Zeno behavior can be excluded automatically. We also extended this work to the self-triggered and periodic event-triggered cases. Especially, for the periodic event-triggered approach, we have discussed the systematical way to determine the new triggering conditions and upper bound of sampling intervals. It has been shown that all the agents can reach bounded consensus in this case and the bound can be adjusted by properly selecting associated parameters.

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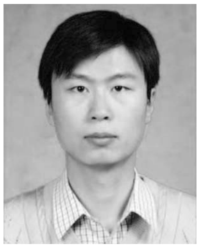

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