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Mechanical and Aerospace Engineering

Stability Constrained Voltage Control in Distribution Grids

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Acknowledgements

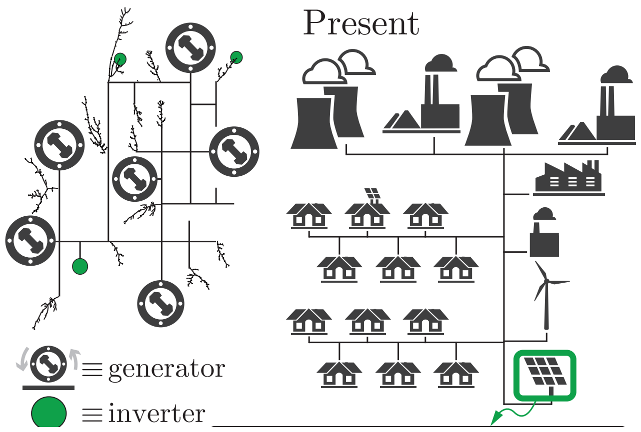
Collaborators:

- **Jorge Cortés**, Professor at UC San Diego
- **Yuanyuan Shi**, Assistant Professor at UC San Diego
- **Jie Feng**, PhD Candidate at UC San Diego
- **Guido Cavraro**, Senior Researcher at NREL
- **Manish K. Singh**, Assistant Professor at UW-Madison

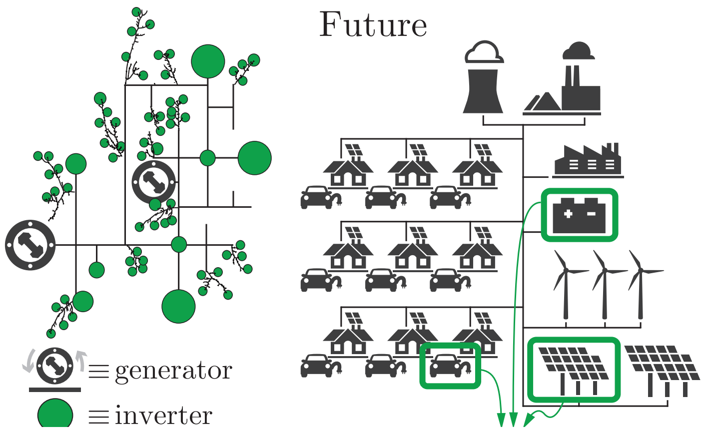
Related Papers:

1. Z. Yuan, G. Cavraro, M. K. Singh, and J. Cortés, "**Learning provably stable local Volt/Var controllers for efficient network operation**," *IEEE Transactions on Power Systems*, 39(1): 2066-2079, 2024.
2. Z. Yuan, G. Cavraro, and J. Cortés, "**Learning stable Volt/Var controllers in distribution grids**," *Big Data Application in Power Systems*, ed. R. Arghandeh and Y. Zhou, Elsevier, Oxford, UK, 2024, 2nd edition.
3. Z. Yuan, J. Feng, Y. Shi, and J. Cortés, "**Stability constrained voltage control in distribution grids with arbitrary communication infrastructures**," in *IEEE Conference on Decision and Control*, Milan, Italy, Dec. 2024. Submitted.

Motivations



Changing grids



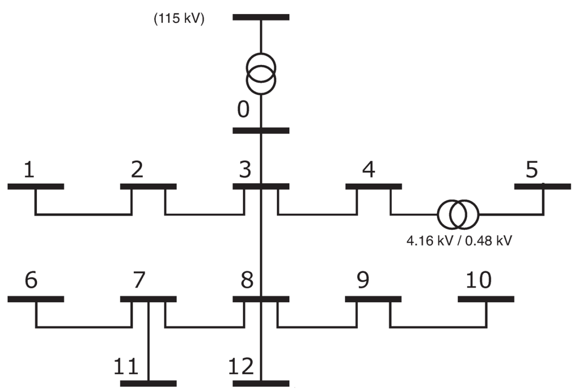
- Not sustainable
- Large rotational inertia as buffer
- Central & dispatchable generation
- Robust frequency/voltage control
- Slow & inflexible control

- Sustainable
- Almost no energy storage
- Distributed & uncoordinated generation
- Fragile frequency/voltage control
- Fast & flexible control

DERs are *smart-enabled*: have **sensing**, **computation**, and **communication** capabilities, and flexibility in power electronic interface (ancillary services for control - feedback control for voltage regulation)

Volt/Var Control Problem in Distribution Grids

Distribution network:



DistFlow model:

$$P_{mn} - \sum_{(n,k)\in\mathcal{E}} P_{nk} = -\left(p_n + \Re(d_n)\right) + r_{mn}\frac{P_{mn}^2 + Q_{mn}^2}{v_m^2},$$

$$Q_{mn} - \sum_{(n,k)\in\mathcal{E}} Q_{nk} = -\left(q_n + \Im(d_n)\right) + x_{mn}\frac{P_{mn}^2 + Q_{mn}^2}{v_m^2},$$

To be controlled

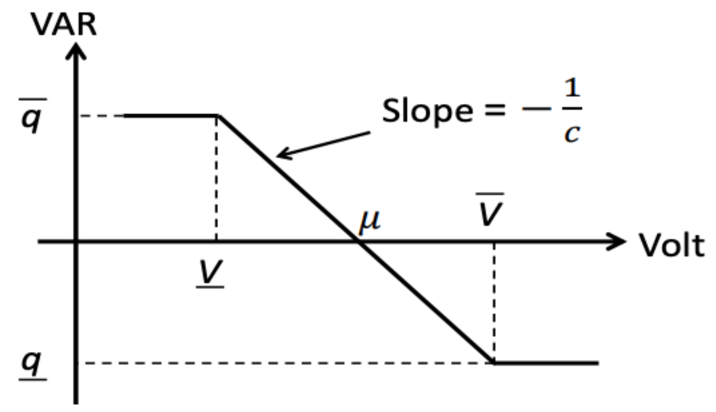
$$v_m^2 - v_n^2 = 2\left(r_{mn}P_{mn} + x_{mn}Q_{mn}\right) - \left(r_{mn}^2 + x_{mn}^2\right)\frac{P_{mn}^2 + Q_{mn}^2}{v_m^2}.$$

- **Centralized strategies** (Low, TCNS'14)
  - Compute power injections by solving OPF problem in *open-loop* fashion
  - Heavy communication & computation burden
- **Distributed strategies** (Bolognani *et al*., TAC'14; Qu & Li, TPWRS'19)
  - Distributed algorithms steer the network to OPF solutions in *closed-loop* fashion
  - Still needs a real-time communication network meeting strict requirements
- **Decentralized strategies** (Li *et al*., Allerton'14; Zhu & Liu, TPWRS'15)
  - DERs make decisions based only on local information in *closed-loop* fashion
  - Have intrinsic performance limitations, generally without optimality considerations

Increasing deployment of DERs in DGs
Scalability consideration

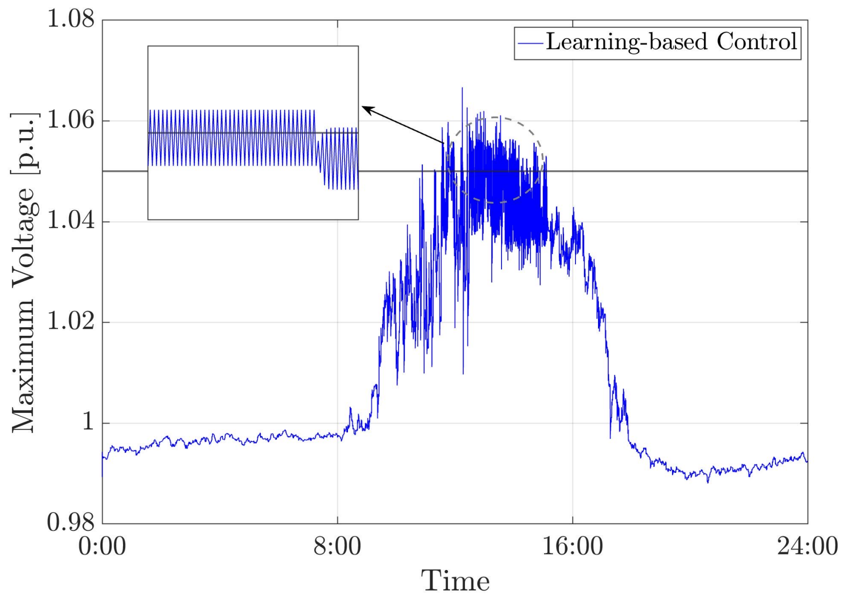
Classical Local Strategies

Droop Volt/Var Curve according to IEEE 1547.8 standard[1]



Cons: No **optimality** considerations in general

Machine learning looks attractive to reduce the gap with centralized/distributed strategies? How well it works?



An unstable example of learning-based controller

Observations: ML-based controllers can easily be unstable

Why? How should we avoid the instability issues in learning-based Volt/Var control?

1. H. Zhu and H. J. Liu, “Fast local voltage control under limited reactive power: Optimality and stability analysis,” *IEEE Transactions on Power Systems*, 31(5): 3794-3803, 2015.

Today's Perspective

Closed-loop system analysis using control-theoretical notions

$\Downarrow$

Find a class of controllers with performance guarantees (e.g., asymptotic stability, transient safety)

$\Downarrow$

Machine learning kicks in to find an optimal controller within this specific class

Increasing body of work on learning + control w/ performance guarantees

- **Frequency Control:** (Cui *et al*., TPS'23; Yuan *et al*., SCL'24; Jiang *et al*., OJCSYS'22; Sun *et al*., LCSS'23; Cui *et al*., NeurIPS'24 …)
- **Voltage Control:** (Cui *et al*., EPSR'22; Yuan *et al*., TPS'24; Yuan *et al*., LCSS'23; Feng *et al*., TCNS'24, Gupta *et al*., TSG'24 …)

Leveraging the model-based control-theoretical analysis to shape the way we use ML for control

Closed-Loop System Dynamics

DistFlow model:

$$P_{mn} - \sum_{(n,k)\in\mathcal{E}} P_{nk} = -\left(p_n + \Re(d_n)\right) + r_{mn}\frac{P_{mn}^2 + Q_{mn}^2}{v_m^2},$$

$$Q_{mn} - \sum_{(n,k)\in\mathcal{E}} Q_{nk} = -\left(q_n + \Im(d_n)\right) + x_{mn}\frac{P_{mn}^2 + Q_{mn}^2}{v_m^2},$$

$$v_m^2 - v_n^2 = 2\left(r_{mn}P_{mn} + x_{mn}Q_{mn}\right) - \left(r_{mn}^2 + x_{mn}^2\right)\frac{P_{mn}^2 + Q_{mn}^2}{v_m^2}$$

$$\approx 2(v_m - v_n)$$

$$\mathbf{v} = \mathbf{R}(\mathbf{p} + \Re(\mathbf{d})) + \mathbf{X}(\mathbf{q} + \Im(\mathbf{d})) + \mathbf{1}$$

$$\mathbf{v} = \mathbf{X}\mathbf{q} + \underbrace{\mathbf{R}\mathbf{p} + \mathbf{R}\Re(\mathbf{d}) + \mathbf{X}\Im(\mathbf{d}) + \mathbf{1}}_{\hat{\mathbf{v}}}$$

PD matrices $\mathbf{X}$ and $\mathbf{R}$

Consider the local control scheme:

$$q_n(t+1) = (1-\epsilon)q_n(t) + \epsilon\phi_n(v_n), \ \epsilon \in [0,1]$$

ϕ_n: Volt/Var curves to be designed, also called *equilibrium functions*

$$\mathbf{q}(t+1) = (1-\epsilon)\mathbf{q}(t) + \epsilon\boldsymbol{\phi}(\mathbf{v}(t))$$
$$\mathbf{v}(t+1) = \mathbf{X}\mathbf{q}(t+1) + \hat{\mathbf{v}}$$

Constant

Note: We assume the load/generation are fixed since the local control algorithms are acting on a time scale that is faster than the load/generation variability

The case of only a subset of buses are controllable …

Global Asymptotic Stability

If the following conditions hold

(C1) The function ϕ_n is Lipschitz and **monotonically decreasing** for all $n \in \mathcal{N}$

(C2) The stepsize ϵ is **small enough** to satisfy $\epsilon < \min\left\{1, \frac{2}{\|\mathbf{X}\|L+1}\right\}$ $L = \max_{n\in\mathcal{N}} L_n$, L_n: Lipschitz constant of ϕ_n

Then the closed-loop system admits a unique equilibrium which is *globally asymptotically stable*

Proof idea: **Contraction**

For any $\mathbf{v}(0), \mathbf{v}'(0)$, if (C1) and (C2) are satisfied, it has

$$\mathbf{v}(t+1) - \mathbf{v}'(t+1) = \underbrace{\left[\prod_{i=0}^{t}((1-\epsilon)\mathbf{I} - \epsilon\mathbf{X}\mathbf{M}(i))\right]}_{:=\mathbf{g}_t}(\mathbf{v}(0) - \mathbf{v}'(0))$$

$\mathbf{M}(i)$ is diagonal with each entry $\in [0, L]$

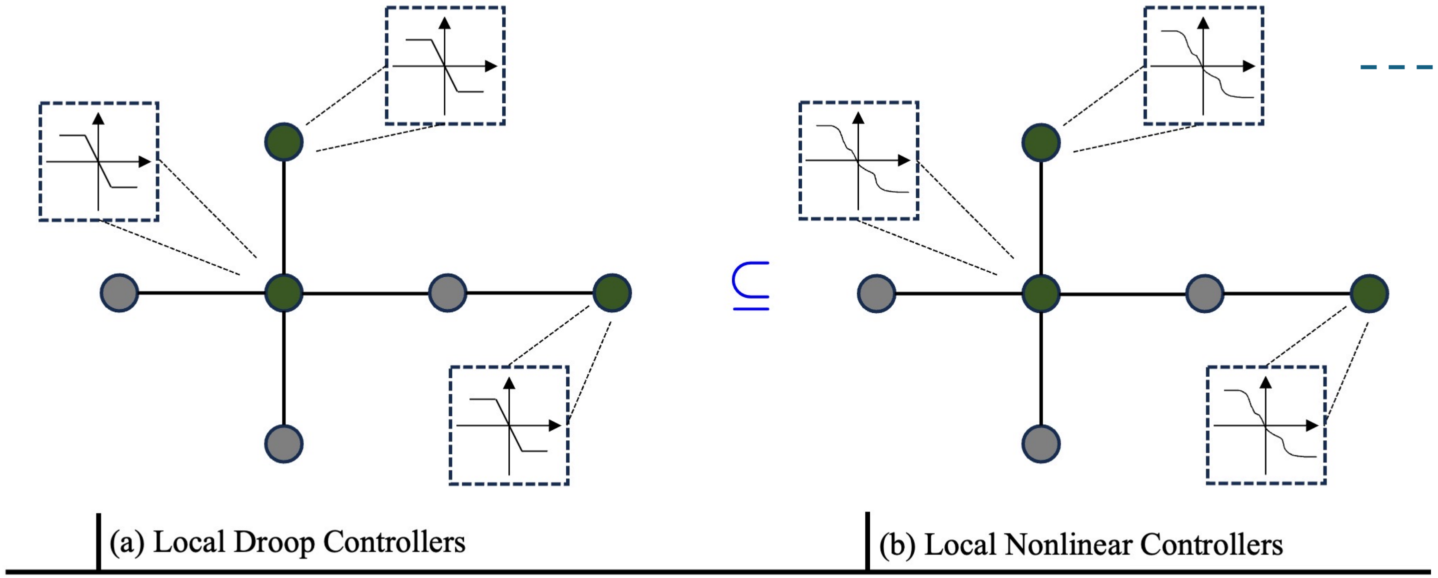
and $\mathbf{g}_t$ is a contraction when $t \to \infty$, i.e., $\|\mathbf{g}_t\| \to 0$

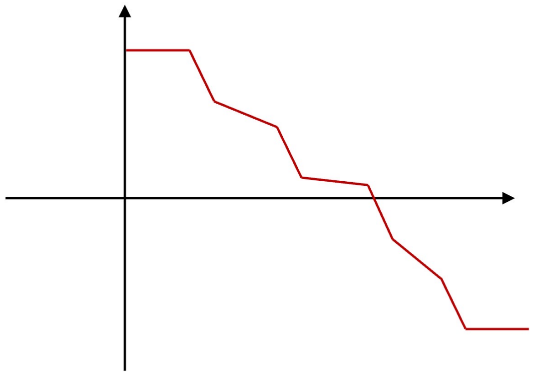
Invoking Banach's fixed point theorem to obtain the conclusions

If we make sure the equilibrium function $\{\phi_n\}_{n\in\mathcal{N}}$ and the stepsize ϵ satisfy (C1)-(C2), we guarantee v globally converges to a unique v^*

Learning Certificated (Nonlinear) Volt/Var Curves

The Volt/Var curves are generalized to be arbitrarily nonlinear:





Single-hidden-layer ReLU network

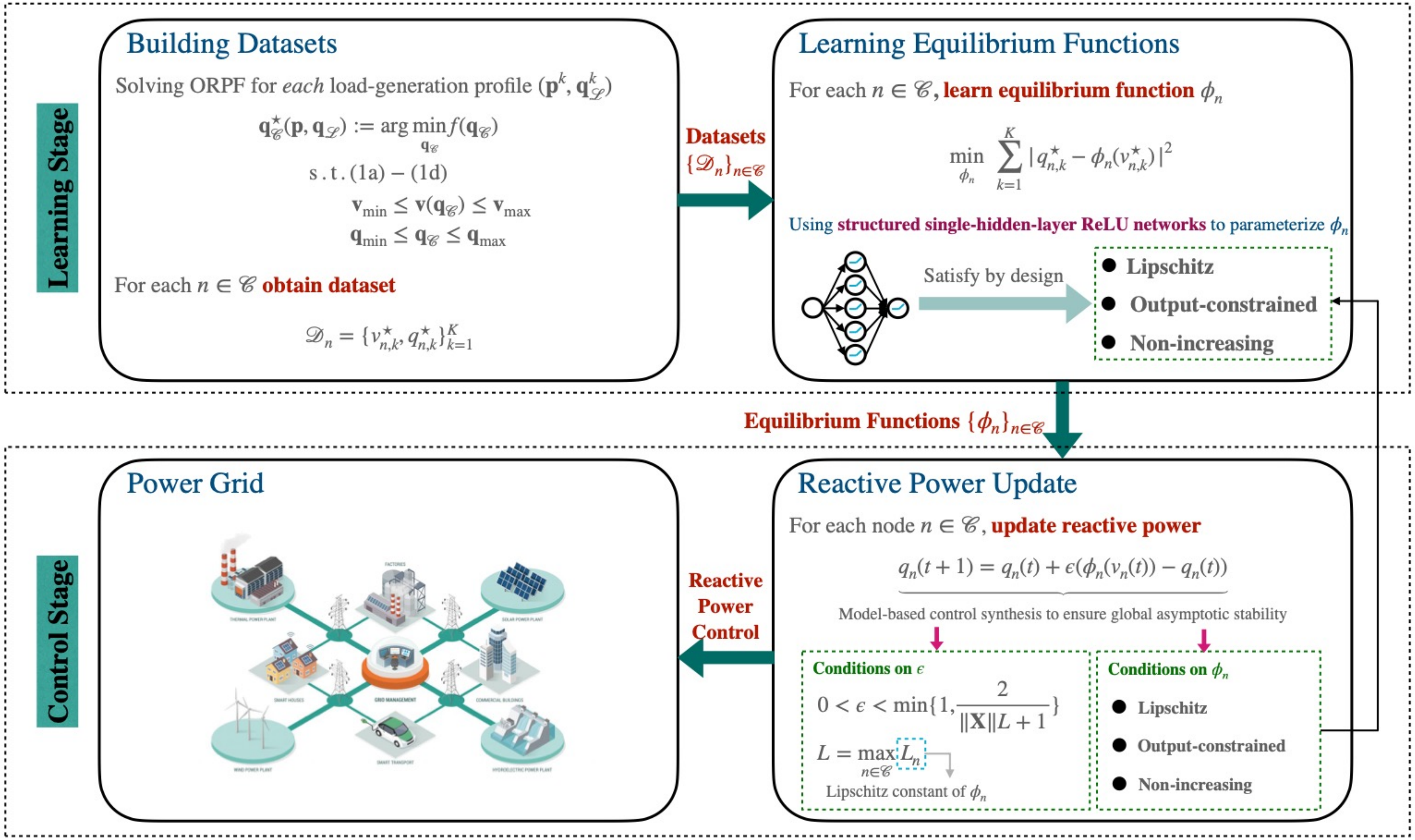
Universal approximation with sufficient segments

Designing neural networks to parameterize the equilibrium functions $\{\phi_n\}_{n\in\mathcal{N}}$ which satisfy monotonicity conditions

Relevant ML algorithms using structured NNs which satisfy monotonicity by design

Supervised Learning Realization

Offline learning + Online implementation:



Simulation Results: Improved Optimality

IEEE 37-bus feeder (5 DERs), 1440 minute-based data (one day)

Consider OPF problem minimizing $f(\mathbf{q}) = \alpha\|\mathbf{v}(\mathbf{q}) - \mathbf{1}\| + (1-\alpha)(\mathbf{p}^\top \mathbf{R}\mathbf{p} + \mathbf{q}^\top \mathbf{R}\mathbf{q})$

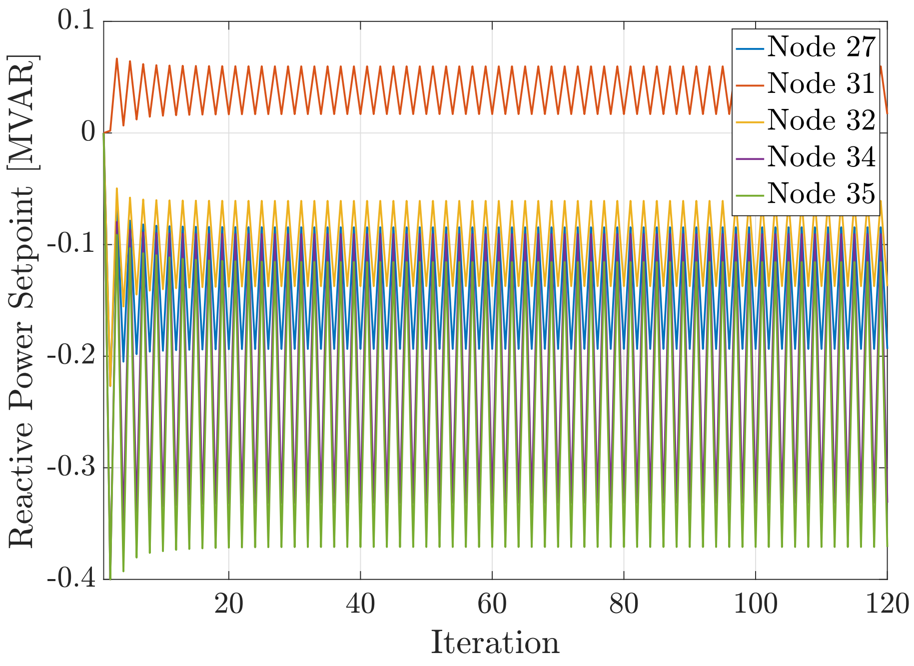
Tested using training data with random perturbations

α	0	1/3	1/2	2/3	1
Learning-based	$\mathbf{0.1185}$	$\mathbf{0.0985}$	$\mathbf{0.0784}$	$\mathbf{0.1115}$	$\mathbf{0.1645}$
Optimal Droop	0.2786	0.2474	0.3728	0.4410	0.4854
Standard Droop	0.2886	0.3311	0.4076	0.4699	0.5047
No Control	0.3160	0.5081	0.6842	0.8029	0.8358

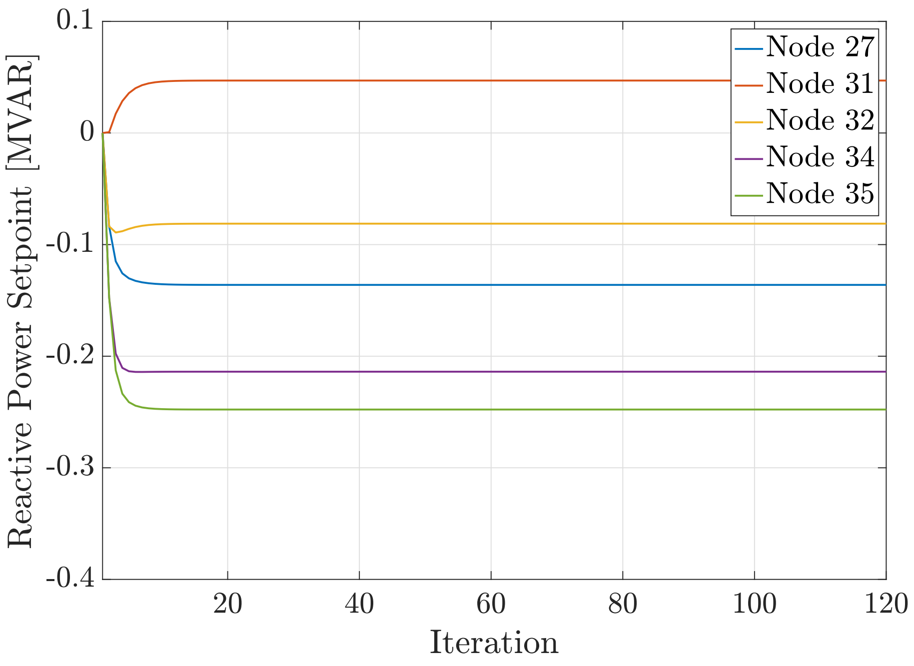
Table 1: Average distances between actual reactive power setpoints and OPF solutions, i.e., $\|\mathbf{q} - \mathbf{q}^*\|$

Significantly improved optimality compared to (linear) droop methods in all situations

Simulation Results: Necessity of Stability Conditions



$0 < \epsilon < \min\left\{1, \frac{2}{\|\mathbf{X}\|L+1}\right\}$ not satisfied

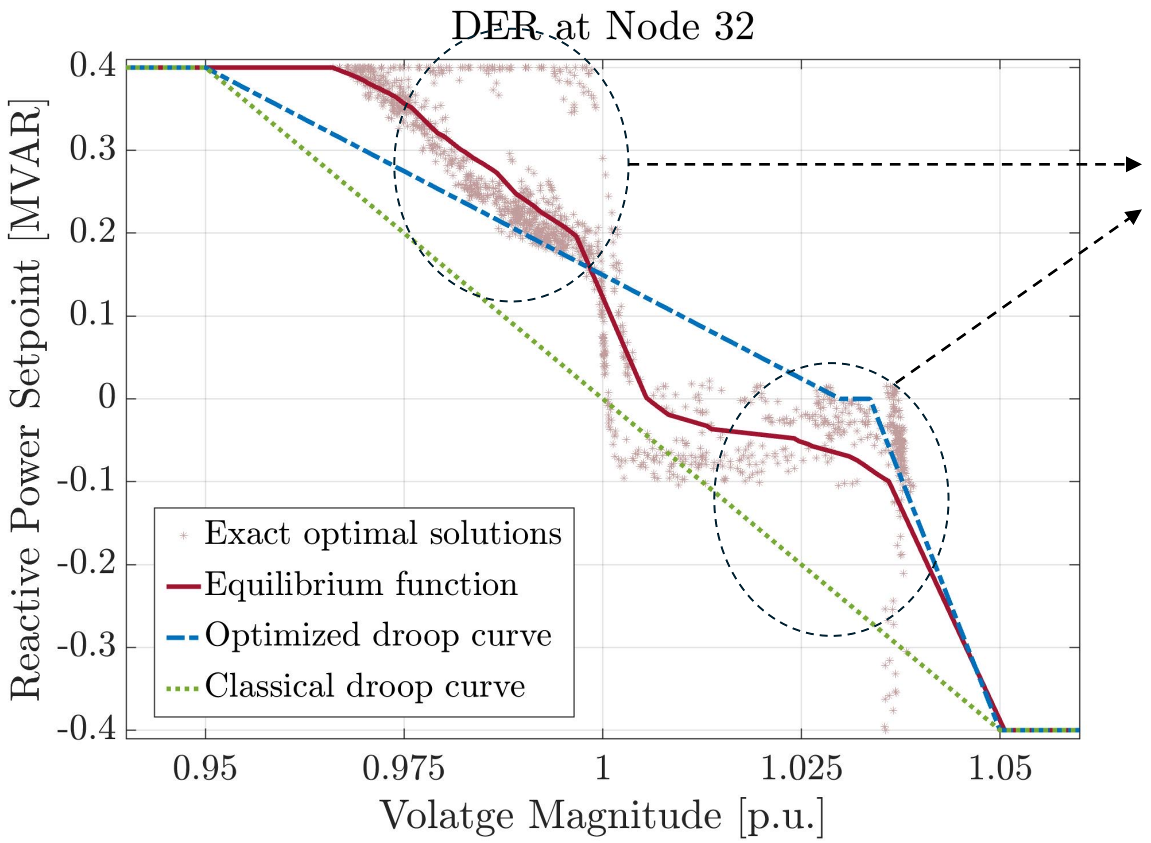


$0 < \epsilon < \min\left\{1, \frac{2}{\|\mathbf{X}\|L+1}\right\}$ satisfied

Necessity of the monotonicity condition?

Consider the case that the control action is increasing w.r.t. voltage magnitude around a fixed point!

Simulation Results: Learned Curves ($\alpha = 1/3$)



Data incosistency:

- The same local voltage magnitude might correspond to multiple OPF solutions

Break the labeled data pair $(\mathbf{q}^*, \mathbf{v}^*)$ into $\{(q_n^*, v_n^*)\}_{n \in \mathcal{N}}$

- The optimal solutions do not form an obvious non-increasing shape

Cost function mainly about power losses

Intrinsic limitations of local design

- Guaranteed to be non-increasing (ensure stability)
- Better approximation of optimal setpoints (OPF)

Go Beyond Decentralized Design

Data incosistency:

- The same local voltage magnitude might correspond to multiple OPF solutions

Make these points more distinguishable

Break the labeled data pair $(\mathbf{q}^*, \mathbf{v}^*)$ into $\{(q_n^*, v_n^*)\}_{n\in\mathcal{N}}$

- The optimal solutions do not form an obvious non-increasing shape

Potentially relax the monotonicity constraint

Cost function mainly about power losses

What if we have some (limited) communication capabilities?

Why not use distributed algorithms to find the exact OPF solutions?

The practical communication infrastructure mostly can not meet the requirements!

Rethink the Stability Conditions

If the following conditions hold

ϕ is **monotonically decreasing** w.r.t. $\mathbf{v}$, i.e., $\left(\phi(\mathbf{v}) - \phi(\mathbf{v}')\right)^\top (\mathbf{v} - \mathbf{v}') \leq 0$

(C1) The function ~~ϕ_n is Lipschitz and **monotonically decreasing** for all $n \in \mathcal{N}$~~

(C2) The stepsize ϵ is **small enough** to satisfy $\cancel{\epsilon < \min\left\{1, \frac{2}{\|\mathbf{X}\| L + 1}\right\}}$ $\epsilon < \min\left\{1, \frac{2}{\|\mathbf{X}\|^2 L^2 + 1}\right\}$

Then the closed-loop system admits a unique equilibrium which is *globally asymptotically stable*

Proof idea: Lyapunov

Uniqueness: $(\mathbf{v}^* - \mathbf{v}^\sharp)^\top (\mathbf{q}^* - \mathbf{q}^\sharp) = (\mathbf{v}^* - \mathbf{v}^\sharp)^\top \left(\phi(\mathbf{v}^*) - \phi(\mathbf{v}^\sharp)\right) \leq 0$

$(\mathbf{v}^* - \mathbf{v}^\sharp)^\top (\mathbf{q}^* - \mathbf{q}^\sharp) = (\mathbf{v}^* - \mathbf{v}^\sharp)^\top \mathbf{X}^{-1} (\mathbf{v}^* - \mathbf{v}^\sharp) > 0$

GAS: $D(t) = (\mathbf{v}(t) - \mathbf{v}^\star)^\top \mathbf{X}^{-1} (\mathbf{v}(t) - \mathbf{v}^\star)$ and $D(t+1) - D(t) < 0, \ \forall t \geq 0, \ \mathbf{v} \neq \mathbf{v}^\star$

Less restrictive condition on equilibrium functions ϕ and more restrictive condition on stepsize ϵ

Implications from the New Stability Conditions

$$(\boldsymbol{\phi}(\mathbf{v}) - \boldsymbol{\phi}(\mathbf{v}'))^\top (\mathbf{v} - \mathbf{v}') \leq 0 \Leftrightarrow (\phi_i(v_i) - \phi_i(v_i'))(v_i - v_i') \leq b_i, \ \sum_{i \in \mathcal{N}} b_i = 0$$

Rather than asking each ϕ_n to be non-increasing, it only asks all ϕ_n's to be *collectively* non-increasing

$\sum_{i \in \mathcal{N}} b_i = 0$ can be enforced via coordination among buses with non-zero b_i's

Potentially relax the monotonicity constraint per DER in decentralized case

Neighboring information can also be included to determine the output of ϕ_n

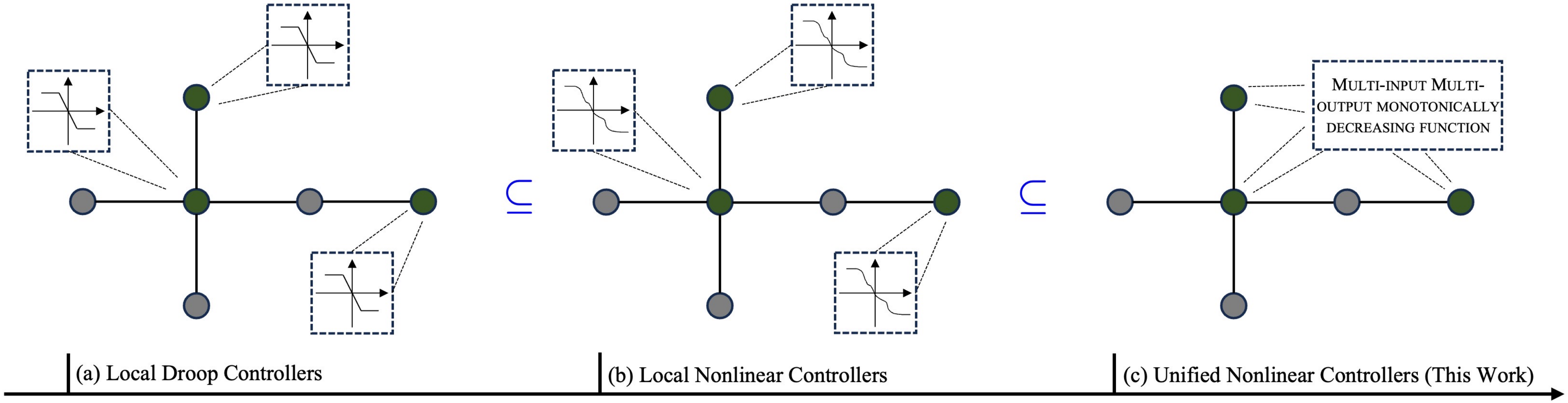
$$\left(\phi_i(v_i, \{v_j\}_{j \in \mathcal{N}_i}) - \phi_i(v_i', \{v_j'\}_{j \in \mathcal{N}_i})\right)(v_i - v_i') \leq b_i, \ \sum_{i \in \mathcal{N}} b_i = 0$$

Leveraging neighboring information to make more accurate predictions

In Summary: The role of information in improving the control performance

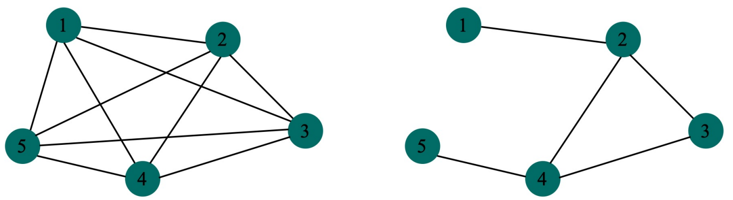
Further Improvement on Provably Stable Control Design

The Volt/Var curves are further generalized to be only *collectively* non-increasing:



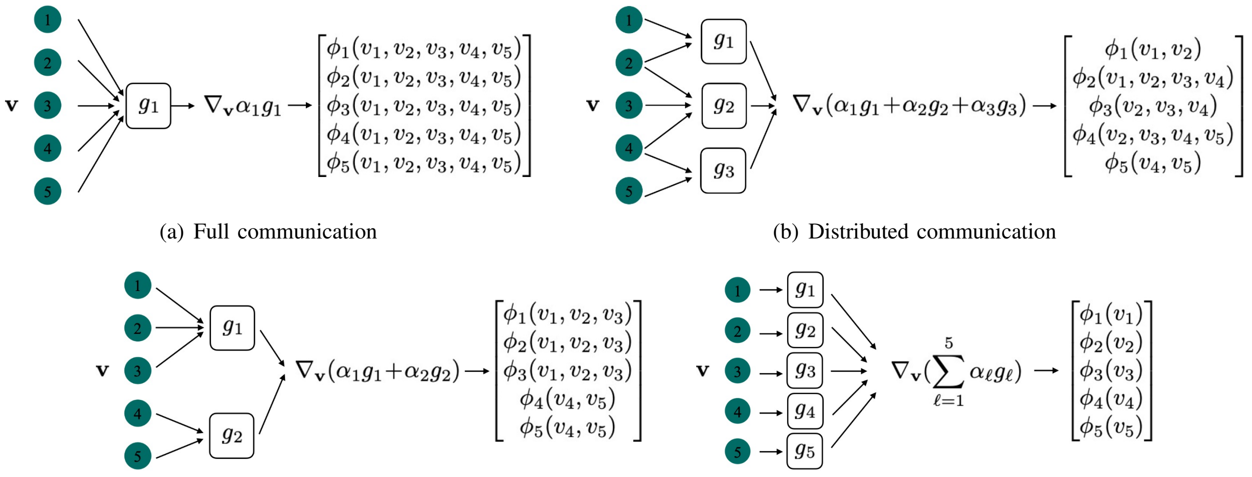
How to design NNs (for each DER) to *collectively* satisfy the MIMO monotonicity constraint under different coommunication networks?

Monotone MIMO NN Design



(a) Full communication (b) Distributed communication

(c) Hybrid communication (d) No communication



(a) Full communication (b) Distributed communication

(c) Hybrid communication (d) No communication

Step 1: Partition the communication graph into subgraphs which are all-to-all connected. Note that one bus may appear in multiple subgraphs

Step 2: Suppose there are in total S subgraphs indexed by $\ell \in \mathcal{S} \triangleq \{1, ..., S\}$ and each contains m_ℓ buses. Select functions $g_\ell : \mathbb{R}^{m_\ell} \to \mathbb{R}$ for all $\ell \in \mathcal{S}$, where the inputs are voltage magnitude measurements available in the corresponding subgraph

Input Convex NN (ICNN)!

Step 3: Define $\phi = \nabla_{\mathbf{v}} \sum_{\ell\in\mathcal{S}} \alpha_\ell g_\ell(\mathbf{v})$, where $\{\alpha_\ell\}_{\ell\in\mathcal{S}}$ are weighting parameters

$$(\boldsymbol{\phi}(\mathbf{v}) - \boldsymbol{\phi}(\mathbf{v}'))^\top (\mathbf{v}-\mathbf{v}') = \left(\sum_{\ell\in\mathcal{S}} \alpha_\ell \nabla_{\mathbf{v}} g_\ell(\mathbf{v}) - \sum_{\ell\in\mathcal{S}} \alpha_\ell \nabla_{\mathbf{v}'} g_\ell(\mathbf{v}')\right)^\top (\mathbf{v}-\mathbf{v}')$$

$$= \sum_{\ell\in\mathcal{S}} \alpha_\ell \left(\nabla_{\mathbf{v}} g_\ell(\mathbf{v}) - \nabla_{\mathbf{v}'} g_\ell(\mathbf{v}')\right)^\top (\mathbf{v}-\mathbf{v}') \le 0$$

Given $\{g_\ell\}_{\ell\in\mathcal{S}}$ are all convex and $\{\alpha_\ell\}_{\ell\in\mathcal{S}} \le 0$

Simulation Results: Different Communication Levels

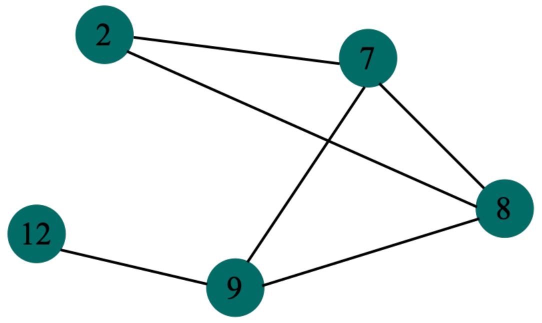
IEEE 13-bus feeder (5 DERs), 1440 minute-based data

Consider OPF problem penalizing voltage deviation and control effort, use RL for training

Table 1: Average performance of decentralized, distributed, centralized controllers on 100 voltage violation scenarios for IEEE 13-bus system.

Method	Volt. Cost	Act. Cost	Total Cost	Improvement
Decentralized	1777.02	358.53	**2135.55**	-
Distributed	1417.73	478.05	**1895.77**	**11.23%**
Centralized	1262.56	407.95	**1670.52**	**21.78%**

Distributed communication: $\mathcal{S}_1 = \{2, 7, 8\}$, $\mathcal{S}_2 = \{7, 8, 9\}$, and $\mathcal{S}_3 = \{9, 12\}$



Connected network

Validate the role of information in improving the control performance!

Summary and Outlook

Summary:

- Data-driven learning framework to design efficient and reliable Volt/Var controllers in DGs
- Unified NN design to take advantage of arbitrary communication infrastructure to enhance performance

Outlook:

$$P_{mn} - \sum_{(n,k)\in\mathcal{E}} P_{nk} = -\left(p_n + \Re(d_n)\right) + r_{mn}\frac{P_{mn}^2 + Q_{mn}^2}{v_m^2},$$

$$Q_{mn} - \sum_{(n,k)\in\mathcal{E}} Q_{nk} = -\left(q_n + \Im(d_n)\right) + x_{mn}\frac{P_{mn}^2 + Q_{mn}^2}{v_m^2},$$

$$v_m^2 - v_n^2 = 2\left(r_{mn}P_{mn} + x_{mn}Q_{mn}\right) - \left(r_{mn}^2 + x_{mn}^2\right)\frac{P_{mn}^2 + Q_{mn}^2}{v_m^2}.$$

$$\mathbf{q}(t+1) = (1-\epsilon)\mathbf{q}(t) + \epsilon\boldsymbol{\phi}(\mathbf{v}(t))$$
$$\mathbf{v}(t+1) = \mathbf{X}\mathbf{q}(t+1) + \hat{\mathbf{v}}.$$

Depends on $\mathbf{v}$ and $\boldsymbol{\theta}$

Linearized PF: GAS $\Rightarrow$ Nonlinear PF: LAS?

Region of attraction?

Q&A

Thank you!

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