Convex Optimization

Lecture 1 - Introduction

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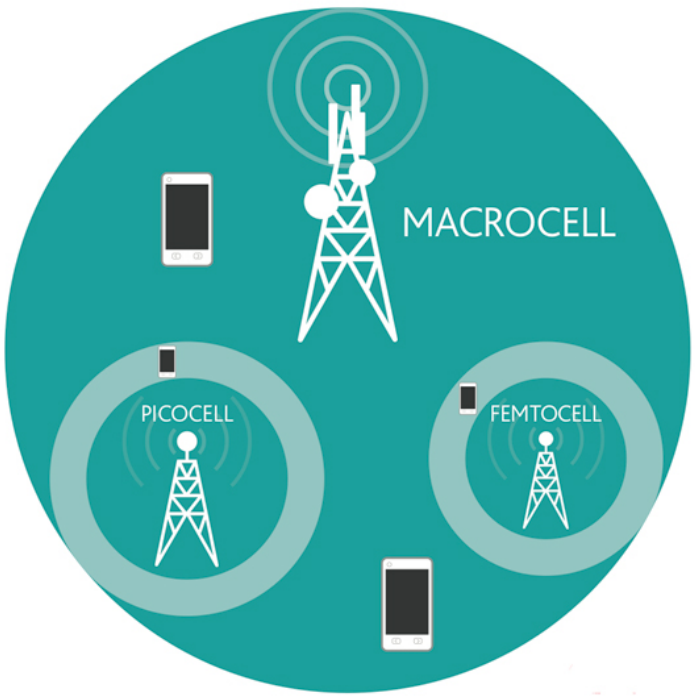
Today's Lecture

1. Motivation – Why Study Convex Optimization?
2. Basics and Demos
3. Course Organization

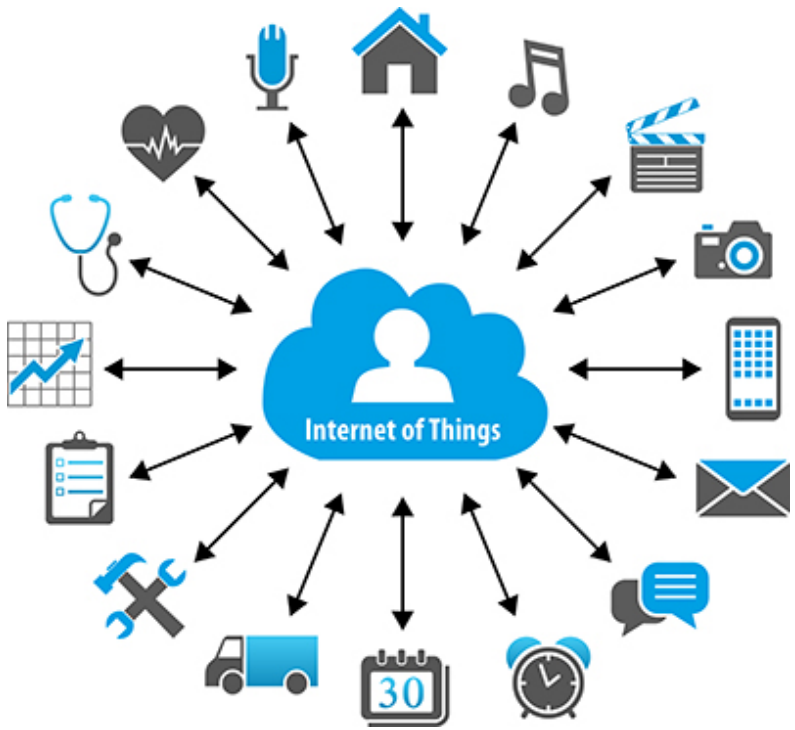
Outline

Fundamental Tool in Electrical Engineering

Communications and Networks



Small cell networks



Internet of Things

DSP is ubiquitous in electrical engineering

Image processing

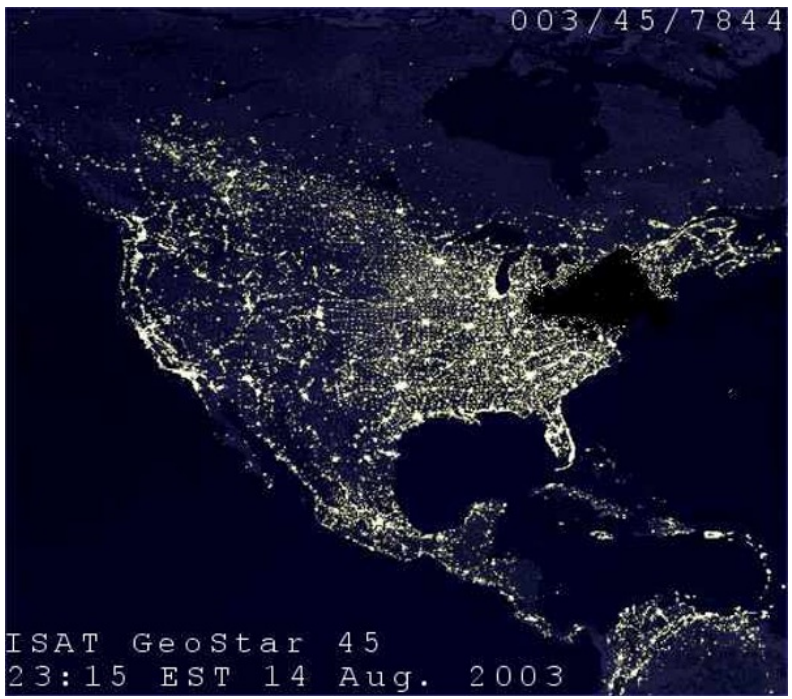
Image enhancement

Special effects

Image compression

Fundamental Tool in Electrical Engineering

Electric Power Systems



Transmission networks

Distribution networks

Fundamental Tool in Electrical Engineering

Socio-Technological Networks

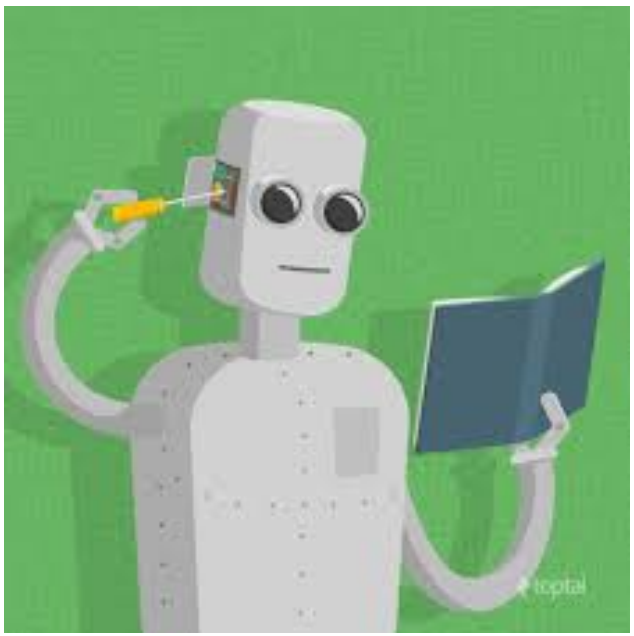
Sharing economy

Crowdsourcing

Fundamental Tool in Electrical Engineering

Machine Learning

Big data analytics



Reinforcement learning

Other disciplines – Finance

Financial engineering

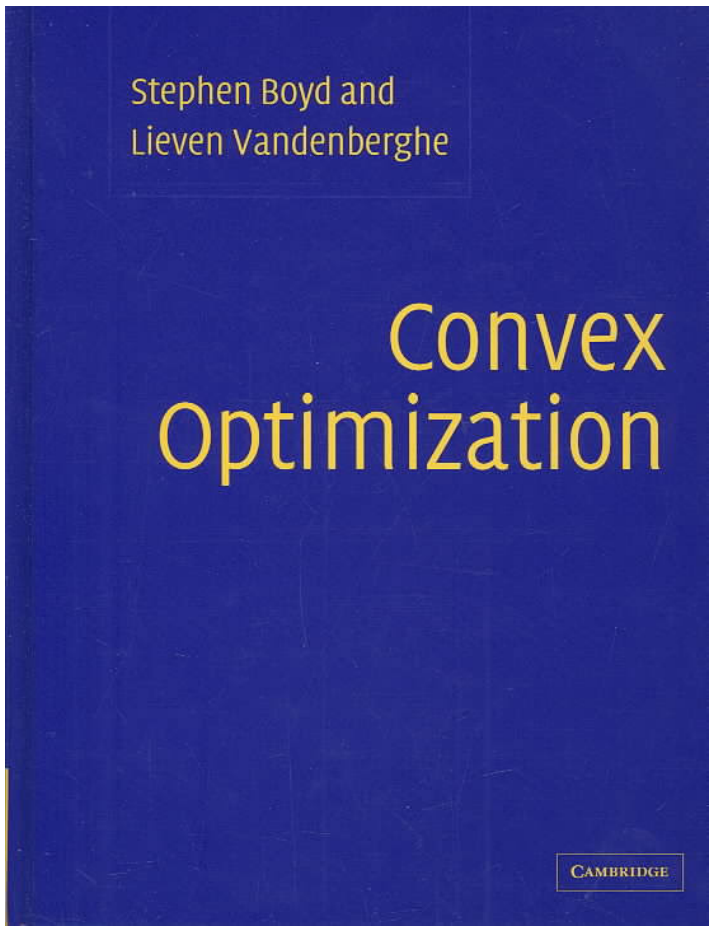
Savings

Stock prices

Widely Used in Research

Google scholar "convex optimization" – 1,950,000 results

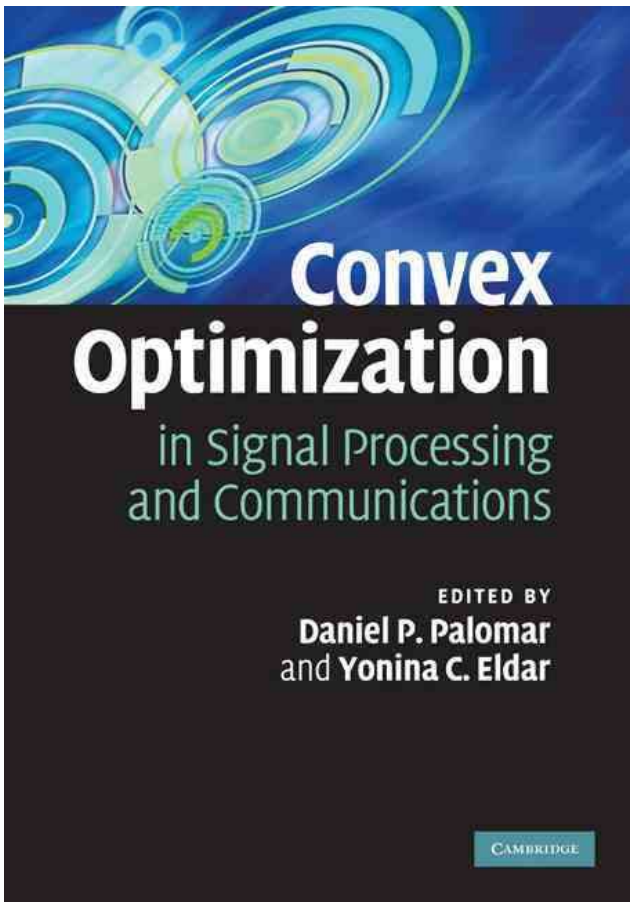
"The book" - Over 36,000 citations and counting...

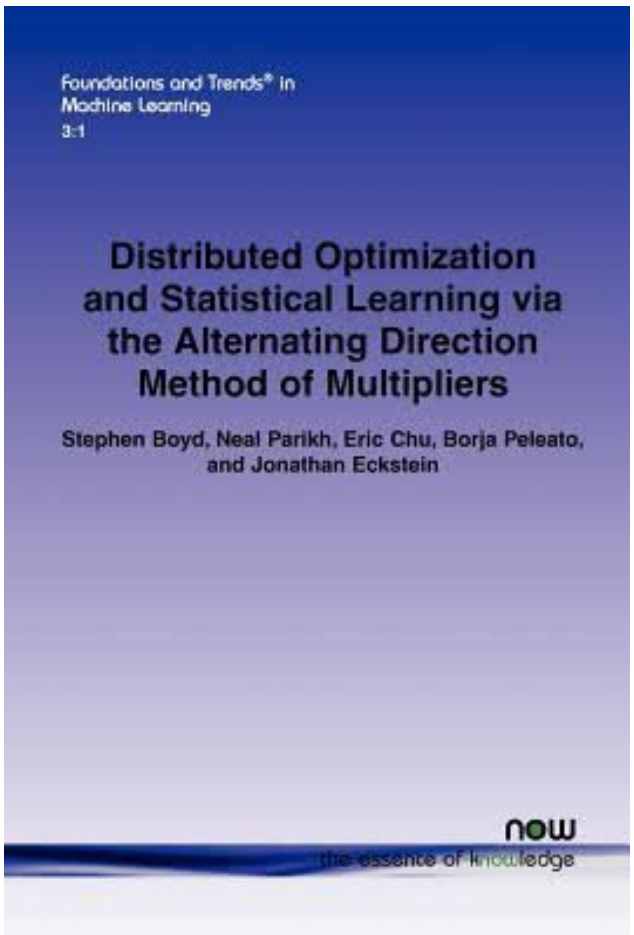


[BOOK] **Convex optimization**
S Boyd, L Vandenberghe - 2004 - books.google.com
Convex optimization problems arise frequently in many different fields. This book provides a comprehensive introduction to the subject, and shows in detail how such problems can be solved numerically with great efficiency. The book begins with the basic elements of convex
Cited by 36348 Related articles All 41 versions Cite Save More

Widely Used in Research

Applied in numerous scholarly works





Why is convex optimization so useful?

Mathematical optimization

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \leq b_i, \ \ i = 1, \ldots, m \end{array}$$

- $x = (x_1, \ldots, x_n)$: optimization variables
- $f_0 : \mathbb{R}^n \to \mathbb{R}$: objective function
- $f_i : \mathbb{R}^n \to \mathbb{R}, \ \ i = 1, \ldots, m$: constraint functions

Very general formulation - pervasive in engineering

Convex optimization - can be solved very efficiently and reliably

Hopefully you are motivated...

Outline

Convex Optimization Problem

Convex optimization

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \leq b_i, \ i = 1, \ldots, m \end{array}$$

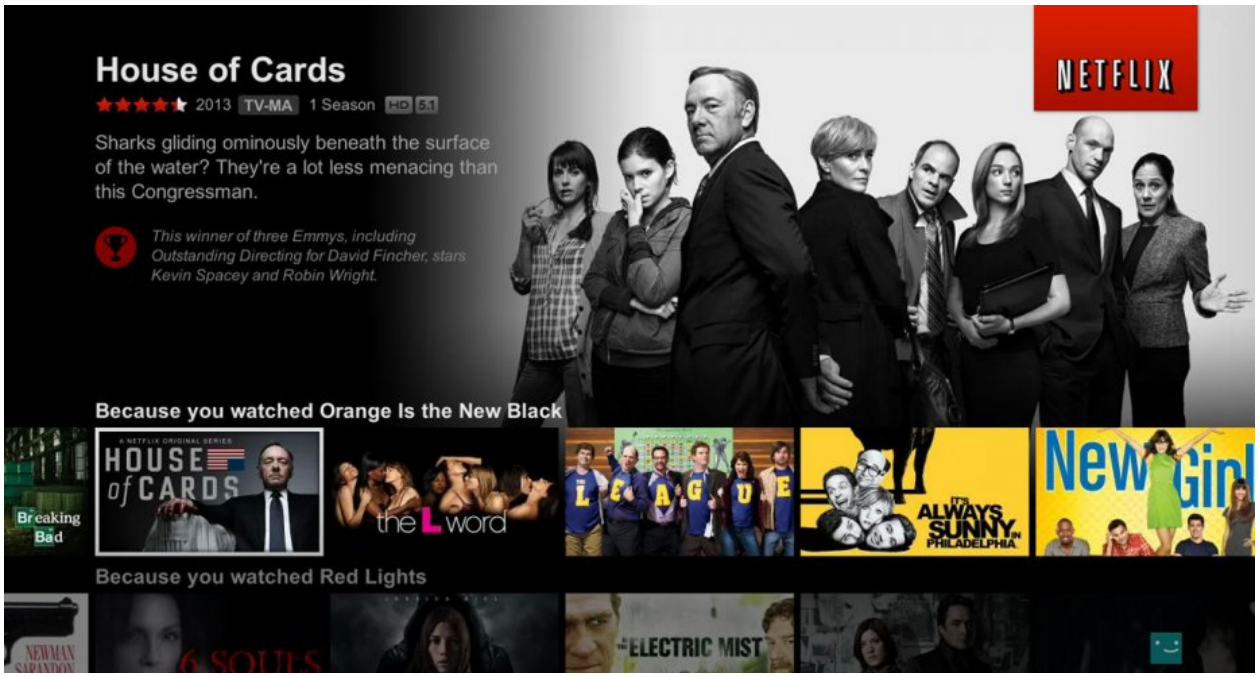
- f_0 and f_i, ı $= 1, \ldots, m$ are convex functions
- linear functions - linear program
- quadratic functions (e.g., $f(x) = x^2$) – quadratic program

Solving Convex Optimization Problems

- May not have analytical solution
- Efficient and reliable algorithms
- Many problems can be converted to convex problems
- Reliable (and free) software (demo1)
- Easy to program (demo2)

Demo1 - Machine Learning

(Overly-simplied) Netflix recommendation systems:



Demo1 - Machine Learning

Regularized least square problems:

$$\begin{aligned} \text{minimize} \quad & \|Ax - b\|_2^2 + \beta \cdot \|x\|_1 \\ \text{subject to} \quad & x \geq 0. \end{aligned}$$

- Each row of A: features (movie genre, gender, zip code, income, history, etc.)
- b: results (whether watched, or how long watched)
- x: weights

Demo2 - Optimal Power Flow

$$\begin{aligned}
\min_{\mathbf{s}} \quad & \sum_{n\in\mathcal{N}_g} c_n(s_n) && \text{(minimize total generation cost)}\\
s.t. \quad & \textstyle\sum_{n\in\mathcal{N}_g} s_n = D, && \text{(supply equals demand)}\\
& \underline{\mathbf{s}} \le \mathbf{s} \le \bar{\mathbf{s}}, && \text{(generator capacity limits)}\\
& -\mathbf{f} \le \mathbf{A}_g \cdot \mathbf{s} + \mathbf{A}_\ell \cdot \mathbf{d} \le \mathbf{f}. && \text{(flow limit constraints)}
\end{aligned}$$

- $c_n(\cdot)$ are not in standard forms supported in CVX

Outline

What to Learn

Formulate and recognize convex optimization problems

Convert non-convex optimization problems to convex optimization

Solve convex optimization problems (software or your own codes)

Topics and Tentative Schedule

Theory:	Week 1. Motivation and basics
	Week 2. Convex sets and convex functions
	Week 3. Convex optimization problems
	Week 4. Duality
Applications:	Week 5. Applications in machine learning
	Week 6. Applications in signal processing
	Week 7. Applications in communications
	Week 8. Applications in smart grids
Mid-Term:	Week 9. Mid-tern exam
Computation:	Week 10. Project proposals
	Week 11. Unconstrained minimization
	Week 12. Equality constrained minimization
	Week 13. Interior-point methods
	Week 14. CVX, CVXPY
	Week 15. Project presentations

Course organization

Course website:
`http://yuanzhangxiao.com/convex-optimization-fall2017`

Contact: `xyz.xiao@gmail.com`

Prerequisites: *very simple* linear algebra

Office hour: Tuesday and Thursday 3:00pm-4:15pm (or by appointment), POST 201G

Textbook:
Convex Optimization by Stephen Boyd and Lieven Vandenburghe
https://web.stanford.edu/ boyd/cvxbook/ (free)

Software:
CVX in Matlab, or
CVXPY in Python (free)

Grades

- Homework – 50%
  - Homework 1-2 on theory
  - Homework 3 on applications
  - Homework 4-5 on computation
- Mid-term exam – 20%
  - On theory and applications
- Project – 30%
  - A research problem in your research, or
  - A research problem in a paper (I can provide a list)

You are free