Joint Power and Channel Resource Allocation for F/TDMA Decode and Forward Relay Networks

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***Abstract*—In this paper, we study the joint power and channel resource allocation problem for a multiuser F/TDMA decode-and-forward (DF) relay network under per-node power constraints and a total channel resource constraint. Our goal is to maximize the total throughput achieved by the systems. To that end, we formulate a joint power and channel resource allocation problem. We develop an iterative optimization algorithm to solve this problem, whose convergence and optimality are guaranteed. Due to the per-node power constraints, more than one relay node may be needed for a single data stream. Our solution also provides a way of finding the optimal relays among the assisting relay nodes.**

I. Introduction

Cooperative relaying is a promising technique for providing cost effective enhancements of network coverage and throughput [1]. The relay nodes exploit the broadcast feature of wireless channels. They can "hear" the transmitted signals of the source nodes and assist forwarding the information [2].

In wireless access networks, the transmission power of the nodes and the channel resources (time and frequancy) are limited. Hence, appropriate power and channel resource allocation is needed to fully utilize the available radio resource. It has been shown that power and channel resource allocation can result in significant performance gains for *single user relay networks* [3]-[5].

The study of *multiuser relay networks* is more crucial for wireless access networks. When multiple relay nodes are involved in the network, the number of access links increases greatly. How to select proper access links and allocate power and channel resource for them is very important for the system performance of wireless relay networks.

In [6], the authors considered relaying strategy selection and power allocation at the relay nodes for F/TDMA relay networks, where the power allocation at the source node and the relay node selection are not jointly considered. The power and channel resource allocation for orthogonal multiple access relay networks was addressed in [7], where the data rate of one user is maximized subject to target rate requirements for the other nodes.

In this paper, we focus on the joint power and channel resource allocation problem of a multi-user F/TDMA decode-and-forward (DF) relay network. We adopt the assumption that each node is subject to separate power constraints [4]. Further, we suppose that the total channel resource of the network is limited[1]. We show that the joint power and channel resource allocation problem is a convex optimization problem. Therefore, we can develop a fast iterative algorithm for this problem based on the duality theory. The dual method is beneficial since the dual problem not only has fewer variables and simpler constraints but also is easily decomposable.

However, this problem is hard because the objective function of the problem is neither differentiable nor strictly concave even if only the power allocation subproblem is considered. In this paper, the non-differentiability of the objective function is solved by using auxiliary variables. This approach is equivalent to the max-min method [4]. The *proximal optimization method* is used to handle the non-strict concavity of the primal objective function [8], [9]. The channel resource allocation problem is given by the root of the Karush-Kuhn-Tucker (KKT) condition [10, pp. 243].

By performing power allocation and channel resource allocation iteratively, the joint optimal power and channel resource allocation solution is derived. The convergence and global optimality of this iterative optimization algorithm are guaranteed by using similar argument as in [4]. Due to the per-node power constraints, more than one relay nodes may be needed for a single data stream. The optimal relay nodes selection is derived simultaneously in our algorithm.

The outline of this paper is given as follows:

In Section II, the system model is introduced. In Section III, we show that the joint power and channel resource allocation problem is a convex optimization problem. In Section IV, we present the iterative optimization algorithm for this problem. The numerical results are shown in Section V. And finally, in Section VI, we give the conclusion.

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[1]For distributed controlled network, the channel resource is pre-assigned to the source nodes. The source nodes can allocate the channel resource among different relay links. The distributed implentation of this problem is out of the scope of this paper.

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II. System Model

We consider a multiuser F/TDMA DF relay network, which consists of R relay nodes and N user nodes. Here, the term user nodes encompasses all possible source and destination nodes. Let $\mathcal{R}$ be the set of relay nodes and $\mathcal{N}$ be the set of user nodes, i.e. $\mathcal{R} = \{1, 2, \ldots, R\}$ and $\mathcal{N} = \{1, 2, \ldots, N\}$. In each time frame, certain data streams, each of which is between a source-destination pair, are scheduled. Let $m = (s, d)$ $(s, d \in \mathcal{N})$ be a source-destination pair, and let $\mathcal{M}$ be the set of data streams, which satisfies $\mathcal{M} \subseteq \{(i, k) | i, k \in \mathcal{N}, i \neq k\}$. Each data stream $m = (i, k) \in \mathcal{M}$ can be assisted by some nearby relay nodes, which is the practical situation. Clearly, these relay nodes only compose a sub-set of $\mathcal{R}$. However, we assume all R relay nodes are potential relay nodes for each data stream, and let the joint optimization algorithm to select the best relay nodes for each data stream.

Suppose the network operates in a slow fading environment. Each node performs channel estimation and the channel strength information is fed back to a central node (e.g., the base station of the relay-assisted cellular network). The central node performs the power and channel resource allocation, and then broadcasts the result to the other nodes.

For practical consideration [3], all nodes are assumed to operate in half-duplex mode. In order to prevent inter-stream interference and facilitate simpler transmitter design, we require that all source and relay nodes transmit in orthogonal sub-channels [6] and [11]-[12]. When some data stream $m = (i, k)$ is assisted by a relay node j, the source node i transmits in one sub-channel with channel resource proportion $\theta_{mj}/2$ and the relay node j transmits in another orthogonal sub-channel with channel resource proportion $\theta_{mj}/2$. The received signal of the destination k in the first sub-channel is

$$y_{m1}^{d} = \sqrt{2P_{mj}^{s}/\theta_{mj}}\beta_m x_{m1}^{s} + n_{m1}^{d}, \tag{1}$$

where x_{m1}^{s} is the transmitted signal of the source node i, P_{mj}^{s} is total transmitted energy of the source node i, β_m denotes the normalized channel gain of the source-destination pair m with n_{m1}^{d} as the zero mean AWGN with unit variance. Similarly, the received signal at relay node j is given by

$$y_{mj}^{r} = \sqrt{2P_{mj}^{s}/\theta_{mj}}\alpha_{mj} x_{m1}^{s} + n_{mj}^{r}, \tag{2}$$

where α_{mj} denotes the normalized channel gain between the source node i and the relay node j, and n_{mj}^{r} is the zero mean AWGN with unit variance at the relay node j. In the second sub-channel, the received signal at the destination k is

$$y_{m2}^{d} = \sqrt{2P_{mj}^{r}/\theta_{mj}}\gamma_{mj} x_{mj}^{r} + n_{m2}^{d}, \tag{3}$$

where x_{mj}^{r} is the transmitted signal of the relay node j, P_{mj}^{r} is the total transmitted energy of relay node j, γ_{mj} denotes the normalized channel gain between relay node j and destination k with n_{m2}^{d} as the zero mean AWGN with unit variance.

We also allow the user to transmit to the destination directly. Suppose the source node i transmits in one sub-channel with channel resource proportion θ_m, the received signal of the destination k is

$$y_{m}^{d} = \sqrt{P_{m}^{s}/\theta_m}\beta_m x_{m}^{s} + n_{m}^{d}, \tag{4}$$

where x_m^s and P_m^s are the transmitted signal and total transmitted energy of the source node i and n_m^d is the zero mean AWGN with unit variance.

III. Joint Power and Channel Resource Allocation Problem

The achievable data rate of decode-and-forward (DF) relaying strategy given in [13] is

$$\begin{aligned} C_{mj} = \theta_{mj}/2 \min\big\{ & C\left[2(P_{mj}^{s}\beta_m^2 + P_{mj}^{r}\gamma_{mj}^2)/\theta_{mj}\right], \\ & C\left[2P_{mj}^{s}\alpha_{mj}^2/\theta_{mj}\right]\big\}, \end{aligned} \tag{5}$$

where $C(x) = \log_2(1 + x)$. The data rate of direction transmission (DT) is simply the capacity of adaptive white Gaussian noise channel

$$C_m = \theta_m C\left(P_m^s \beta_m^2/\theta_m\right). \tag{6}$$

If $\alpha_{mj} \leq \beta_m$, using the property that the function $\theta \mapsto \theta C(a/\theta)$ is increasing, we can show that C_{mj} is no larger than the C_m when $\theta_m = \theta_{mj}$. Therefore, we only adopt the DF relaying strategy when $\alpha_{mj} > \beta_m$ as in [6]. If $\alpha_{mj} \leq \beta_m$, we simply let $P_{mj}^s = P_{mj}^r = 0$.

One can show that both C_m and C_{mj} are strictly concave with respect to the power allocation variables. Using the property of *perspective function* given in [10, pp. 89], it is easy to prove that C_m and C_{mj} are also concave with respect to the power and channel resource variables. Our objective is to maximize the achievable sum rate of all the data streams. The rate of a data stream is the sum rate of one DT link and R DF relay links. We note that DF and DT use orthogonal channels. Their power and channel resource allocation variables are independent variables, even for one source-destination pair. Suppose that each node is subject to separate average power constraints (or total transmission energy in a scheduling frame). The total channel resource of the F/TDMA relay network is limited to 1. Therefore, the joint power and channel resource allocation problem is described by the following convex optimization problem

$$\max_{P_m^s, \theta_m, P_{mj}^s, P_{mj}^r, \theta_{mj}} \sum_{m \in \mathcal{M}} (C_m + \sum_{j \in \mathcal{R}} C_{mj}) \tag{7}$$

$$s.t. \sum_{m \in S_l} (P_m^s + \sum_{j \in \mathcal{R}} P_{mj}^s) \leq P_{l,max}^s, \forall\, l \in \mathcal{N} \tag{8}$$

$$\sum_{m \in \mathcal{M}} P_{mj}^r \leq P_{j,max}^r, \forall\, j \in \mathcal{R} \tag{9}$$

$$\sum_{m \in \mathcal{M}} (\theta_m + \sum_{j \in \mathcal{R}} \theta_{mj}) \leq 1 \tag{10}$$

$$P_m^s, P_{mj}^s, P_{mj}^r, \theta_m, \theta_{mj} \geq 0, \forall\, m, j \tag{11}$$

where C_{mj} and C_m are given by (5) and (6), $S_l \triangleq \{(i, k) \in \mathcal{M} : i = l\}$ is defined as the set of data streams with the same source node l, $P_{l,max}^s$ and $P_{l,max}^r$ are the average transmitted power constraints of source node and relay node.

IV. Iterative Optimization Algorithm

In this section, we develop an iterative algorithm to solve the joint power and channel resource allocation problem. We have mentioned that the data rate of DF relaying (5) is a concave function, but it is neither differentiable nor strictly concave. Because the objective function is not strictly concave, the primal optimal solution is not unique and the dual function is non-differentiable [8]. Using the standard dual solution will cause the primal variables to oscillate during the dual iterations. This is explained in detail in [9].

To alleviate this difficulty, we use the proximal optimization method to solve the power allocation subproblem. The basic idea is to make the primal objective function strictly concave by subtracting a quadratic term from it. This ensures that the dual optimzation algorithm is stablized and converges quite fast, and the converged point is one of the optimal solutions of the original problem [8, Section 3.4.3]. The non-differentiable property of (5) is handled by using auxiliary variables. Such a method is equivalent to the max-min method given in [4].

The channel resource allocation solution is given by the root of the KKT condition. The famous rapidly convergent Newton's method [14, Section 5.5.3] is used to solve the KKT condition. By performing power allocation and channel resource allocation iteratively, we can prove that the joint optimal power and channel resource allocation solution is derived.

A. Proximal optimization method for power allocation

In this subsection, we utilize the dual decomposition based proximal optimization method [8], [9] to solve the power allocation subproblem for fixed channel resource variables.

The proximal optimization method considers the following modified problem

$$\max_{P_m^s, P_{mj}^s, P_{mj}^r, Q_{mj}^s, Q_{mj}^r} \sum_{m\in\mathcal{M}} [C_m + \sum_{j\in\mathcal{R}} C_{mj} - \frac{c_{mj}}{2}(P_{mj}^s - Q_{mj}^s)^2 - \frac{c_{mj}}{2}(P_{mj}^r - Q_{mj}^r)^2] \quad (12)$$

$$s.t.\ P_m^s, P_{mj}^s, P_{mj}^r \in W,\ Q_{mj}^s, Q_{mj}^r \geq 0, \forall\ m, j, \quad (13)$$

where c_{mj} is a positive number chosen for each m, j, and

$$W = \{P_m^s, P_{mj}^s, P_{mj}^r | \sum_{m\in\mathcal{M}} P_{mj}^r \leq P_{j,max}^r, \forall\ j \in \mathcal{R}, \sum_{m\in S_l}(P_m^s + \sum_{j\in\mathcal{R}} P_{mj}^s) \leq P_{l,max}^s, \forall\ l \in \mathcal{N}, P_m^s, P_{mj}^s, P_{mj}^r \geq 0, \forall\ m, j\} \quad (14)$$

One can show that the optimal power allocation variables of the original problem (7)-(11) coincide with the optimal solution of (12)-(13) [8, Section 3.4.3].

The *proximal optimization algorithm* [8], [9] as applied in our problem is given by:

Algorithm $\mathcal{A}$: At the t-th iteration,

(A1) Fix $Q_{mj}^s(t), Q_{mj}^r(t)$ and maximize the objective function (12) with respect to $P_m^s, P_{mj}^s, P_{mj}^r$. More precisely, this step solves the problem

$$\max_{P_m^s, P_{mj}^s, P_{mj}^r} \sum_{m\in\mathcal{M}} \{C_m + \sum_{j\in\mathcal{R}} C_{mj} - \frac{c_{mj}}{2}[P_{mj}^s - Q_{mj}^s(t)]^2 - \frac{c_{mj}}{2}[P_{mj}^r - Q_{mj}^r(t)]^2\} \quad (15)$$

$$s.t.\ P_m^s, P_{mj}^s, P_{mj}^r \in W \quad (16)$$

Since the objective function (15) is strictly concave, the solution to the problem exists and is unique.

(A2) Suppose the solution of (A1) is $P_m^s(t), P_{mj}^s(t), P_{mj}^r(t)$. Let $Q_{mj}^s(t+1) = P_{mj}^s(t),\ Q_{mj}^r(t+1) = P_{mj}^r(t)$.

Now, we use standard duality techniques to solve (15) in Step (A1). Let $\mu_l\ (l \in \mathcal{N})$ and $\nu_j\ (j \in \mathcal{R})$ be the Lagrange dual variables for constraints (8) and (9), respectively. The Lagrangian of (15) can be given in a dual decomposition form

$$\begin{aligned} &L(P_m^s, P_{mj}^s, P_{mj}^r, \mu_l, \nu_j) \\ =& \sum_{l\in\mathcal{N}}\sum_{m\in S_l}(C_m - \mu_l P_m^s) \\ &+ \sum_{l\in\mathcal{N}}\sum_{m\in S_l}\sum_{j\in\mathcal{R}}\Big\{C_{mj} - \frac{c_{mj}}{2}[P_{mj}^s - Q_{mj}^s(t)]^2 \\ &- \frac{c_{mj}}{2}[P_{mj}^r - Q_{mj}^r(t)]^2 - \mu_l P_{mj}^s - \nu_j P_{mj}^r\Big\} \\ &+ \sum_{l\in\mathcal{N}} \mu_l P_{l,max}^s + \sum_{j\in\mathcal{R}} \nu_j P_{j,max}^r. \end{aligned} \quad (17)$$

Therefore, the objective function of the dual problem is

$$\begin{aligned} D(\mu_l, \nu_j) =& \max_{P_m^s, P_{mj}^s, P_{mj}^r \geq 0} L(P_m^s, P_{mj}^s, P_{mj}^r, \mu_l, \nu_j) \\ =& \sum_{l\in\mathcal{N}}\sum_{m\in S_l} H_m(\mu_l) + \sum_{l\in\mathcal{N}}\sum_{m\in S_l}\sum_{j\in\mathcal{R}} I_{mj}(\mu_l, \nu_j) \\ &+ \sum_{l\in\mathcal{N}} \mu_l P_{l,max}^s + \sum_{j\in\mathcal{R}} \nu_j P_{j,max}^r, \end{aligned} \quad (18)$$

where

$$H_m(\mu_l) = \max_{P_m^s \geq 0}(C_m - \mu_l P_m^s), \quad (19)$$

$$\begin{aligned} I_{mj}(\mu_l, \nu_j) =& \max_{P_{mj}^s, P_{mj}^r \geq 0}\Big\{C_{mj} - \frac{c_{mj}}{2}[P_{mj}^s - Q_{mj}^s(t)]^2 \\ &- \frac{c_{mj}}{2}[P_{mj}^r - Q_{mj}^r(t)]^2 - \mu_l P_{mj}^s - \nu_j P_{mj}^r\Big\}. \end{aligned} \quad (20)$$

The dual problem of (15) is given by

$$\min_{\mu_l, \nu_j \geq 0} D(\mu_l, \nu_j). \quad (21)$$

Since the objective function of (15) is strictly concave, the dual function is differentiable on the whole region [8]. The gradient of the dual function D is

$$\frac{\partial D}{\partial \mu_l} = P_{l,max}^s - \sum_{m\in S_l}[P_m^{s\star}(u) + \sum_{j\in\mathcal{R}} P_{mj}^{s\star}(u)], \quad (22)$$

$$\frac{\partial D}{\partial \nu_j} = P_{j,max}^r - \sum_{m\in\mathcal{M}} P_{mj}^{r\star}(u), \quad (23)$$

where $P_m^{s\star}(u), P_{mj}^{s\star}(u), P_{mj}^{r\star}(u)$ solve (19) and (20) for $\mu_l = \mu_l(u)$ and $\nu_j = \nu_j(u)$. Therefore, the dual problem (21) can be solve by the gradient project algorithm [8, Section 3.3.2]

$$\mu_l(u+1) = \left\{\mu_l(u) + \rho_l[\sum_{m\in S_l}(P_m^{s\star} + \sum_{j\in\mathcal{R}} P_{mj}^{s\star}) - P_{l,max}^s]\right\}^\dagger$$

$$\nu_j(u+1) = \left[\nu_j(u) + \sigma_j(\sum_{m\in\mathcal{M}} P_{mj}^{r\star} - P_{j,max}^r)\right]^\dagger \quad (24)$$

where $(\cdot)^\dagger = \max\{\cdot, 0\}$. It can be shown that the dual iterations (24) converge to the optimal solution, if the step size $\rho_l,\ \sigma_j$ are small enough [8, Section 3.3.2].

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B. Recovery of the power allocation variables

In this subsection, we consider the power allocation variables, which optimize the problems (19) and (20). The solution of (19) is just the common "water-filling" result [10, pp. 245]

$$P_m^s = \theta \left(\frac{1}{\mu_l \ln 2} - \frac{1}{\beta_m^2} \right)^{\dagger}, \text{ if } m \in S_l. \tag{25}$$

For the problem of (20), we first define

$$R_1 = \frac{\theta}{2} C \left[\frac{2}{\theta} (P^s \beta^2 + P^r \gamma^2) \right], \ R_2 = \frac{\theta}{2} C \left(\frac{2}{\theta} P^s \alpha^2 \right). \tag{26}$$

We omit the subscripts of the variables in this subsection for brevity. substituting the data rate of DF relaying with auxiliary variable t in (20), we can change the problem (20) to a differentiable convex optimization problem with two new inequality constraints (like [10, pp. 150-151]). Then, by calculating the Lagrangian of the two new constraints and the derivative on the auxiliary variable t, it is easy to verify that (20) is equivalent to the following problem

$$\begin{aligned} \max_{P^s, P^r, \tau} \ & \tau R_1 + (1-\tau) R_2 - \mu_l P^s - \nu_j P^r \\ & - \frac{c}{2}[P^s - Q^s(t)]^2 - \frac{c}{2}[P^r - Q^r(t)]^2 \\ s.t. \ \ & 0 \le \tau \le 1, P^s, P^r \ge 0, \end{aligned} \tag{27}$$

with the relationship of R_1 and R_2 determined by $\tau^\star$

$$\begin{cases} \text{if} \quad \tau^\star = 0, \quad R_1 \ge R_2; \\ \text{if} \quad \tau^\star = 1, \quad R_1 \le R_2; \\ \text{if } 0 < \tau^\star < 1, R_1 = R_2; \end{cases} \tag{28}$$

where $\tau^\star$ is the optimal value of τ in (27). We note that this method is equivalent with the technique presented in [4, Sec. III.A], which has a geometric interpretation. When $c_{mj} = 0$, the problem (27) reduces to standard Lagrange problem, and then the solution is expected to be similar to the water-filling solution, but have some difficulty for convergence.

Using the equivalent result of (20), which given in (27) and (28), we can deduce the solution of (20) in a case by case basis. First, when $\alpha^2 \le \beta^2$, we just let $P^s = P^r = 0$. If $\alpha^2 > \beta^2$, the solution of (20) is divided into 3 cases:

Case 1: if $\tau^\star = 0$, we have $R_1 \ge R_2$, the KKT conditions of (27) are given by

$$\frac{\alpha^2}{\ln 2(1 + 2\alpha^2 P^s/\theta)} \le \mu_l + c[P^s - Q^s(t)], \quad \text{with equality if } P^s > 0, \tag{29}$$

$$0 \le \nu_j + c[P^r - Q^r(t)], \text{with equality if } P^r > 0. \tag{30}$$

1) If $-\nu_j/c + Q^r(t) > 0$, (30) yields $P^r > 0$, thus $P^r = -\nu_j/c + Q^r(t)$. Solving (29), P^s is given by

$$P^s = f(\theta, c, \mu_l, Q^s(t), \alpha^2, 1), \tag{31}$$

where

$$f(\theta, c, \mu, Q, h, v) \triangleq \frac{1}{2} \left(\frac{\theta}{\mu \ln 2} - \frac{\theta}{h} + \sqrt{x^2 + y} - x \right)^{\dagger} \tag{32}$$

$$x = \frac{\mu}{cv} - \frac{Q}{v} + \frac{\theta}{\mu \ln 2} - \frac{\theta}{2h}, \tag{33}$$

$$y = \frac{2Q\theta}{\mu v \ln 2} + \frac{\theta^2}{h\mu \ln 2} - \frac{\theta^2}{\mu^2 \ln 2^2}. \tag{34}$$

It is interesting to look at the case $c \to 0$ in (31). One can show that $\sqrt{x^2 + y} - x \to 0$ as $c \to 0$, and (31) returns to the simple water-filling solution.

We note that this case happens when

$$\gamma^2 P^r \ge (\alpha^2 - \beta^2) P^s. \tag{35}$$

2) If $-\nu_j/c + Q^r(t) < 0$, (30) cannot be equal. Thus, $P^r = 0$. Since $P^s \ge 0$, relation (35) can be satisfied only when $P^s = 0$. This subcase is summarized in case 3.

3) If $-\nu_j/c + Q^r(t) = 0$, and suppose $P^r > 0$, (30) should be equal. But it cannot happen for (30). Hence, P^r has to be zero. This subcase is also summarized in case 3.

Case 2: if $\tau^\star = 1$, we have $R_1 \le R_2$, the KKT conditions of (27) are given by

$$\frac{\beta^2}{\ln 2(1 + 2\beta^2 P^s/\theta + 2\gamma^2 P^r/\theta)} \le \mu_l + c[P^s - Q^s(t)], \quad \text{with equality if } P^s > 0, \tag{36}$$

$$\frac{\gamma^2}{\ln 2(1 + 2\beta^2 P^s/\theta + 2\gamma^2 P^r/\theta)} \le \nu_j + c[P^r - Q^r(t)], \quad \text{with equality if } P^r > 0. \tag{37}$$

1) If $\{\mu_l + c[P^s - Q^s(t)]\}/\beta^2 < \{\nu_j + c[P^r - Q^r(t)]\}/\gamma^2$, (37) cannot be equal, thus $P^r = 0$, and P^s is given by

$$P^s = f(\theta, c, \mu_l, Q^s(t), \beta^2, 1). \tag{38}$$

Equation (38) also reduces to the water-filling solution as $c \to 0$. Note that such a solution always satisfies $R_1 \le R_2$.

2) If $\{\mu_l + c[P^s - Q^s(t)]\}/\beta^2 > \{\nu_j + c[P^r - Q^r(t)]\}/\gamma^2$, (36) cannot be equal, thus $P^s = 0$. Since $P^r \ge 0$, $R_1 \le R_2$ can be satisfied only when $P^r = 0$. This subcase is summarized in case 3.

3) If $\{\mu_l + c[P^s - Q^s(t)]\}/\beta^2 = \{\nu_j + c[P^r - Q^r(t)]\}/\gamma^2$, there are two possibilities:

For the first case, both (36) and (37) achieve equality. After some manipulations, we obtain that

$$\begin{aligned} \beta^2 P^s + \gamma^2 P^r = f(&\theta(\beta^4 + \gamma^4), c, \beta^2 \mu_l + \gamma^2 \nu_j, \\ &\beta^2 Q^s(t) + \gamma^2 Q^r(t), \beta^4 + \gamma^4, 1). \end{aligned} \tag{39}$$

We note that as c approaches 0, one can obtain

$$\beta^2 P^s + \gamma^2 P^r \to \frac{\theta}{2} \left(\frac{\beta^4 + \gamma^4}{\ln 2(\beta^2 \mu_l + \gamma^2 \nu_j)} - 1 \right)^{\dagger}.$$

Since the condition of this subcase limits to $\mu_l/\beta^2 = \nu_j/\gamma^2$, we have

$$P^s + \gamma^2 P^r/\beta^2 \to \frac{\theta}{2} \left(\frac{1}{\ln 2\mu_l} - \frac{1}{\beta^2} \right)^{\dagger}$$

just like [4, Eq. 49]. Therefore, when $c = 0$, the values of P^s and P^r are not unique and are hard to recover. Hence, some difficulties arise in the dual iterations (24). Let $a = \beta^2 P^s + \gamma^2 P^r$ and substituting it into the condition $\{\mu_l + c[P^s - Q^s(t)]\}/\beta^2 = \{\nu_j + c[P^r - Q^r(t)]\}/\gamma^2$, we obtain

$$P^s = \frac{-\gamma^4 \mu_l + \beta^2 \gamma^2 \nu_j}{(\beta^4 + \gamma^4)c} + \frac{\gamma^4 Q^s(t) - \beta^2 \gamma^2 Q^r(t) + \beta^2 a}{\beta^4 + \gamma^4}$$

$$P^r = \frac{-\beta^4 \nu_j + \beta^2 \gamma^2 \mu_l}{(\beta^4 + \gamma^4)c} + \frac{\beta^4 Q^r(t) - \beta^2 \gamma^2 Q^s(t) + \gamma^2 a}{\beta^4 + \gamma^4}$$

This case happens when

$$0 \leq \gamma^2 P^r \leq (\alpha^2 - \beta^2)P^s. \quad (40)$$

If none of (36) and (37) achieves equality, we have $P^s = P^r = 0$, which is summarized in case 3.

Case 3: if $0 < \tau^\star < 1$, we have $R_1 = R_2$, the KKT conditions of (27) are given by

$$\frac{\tau\beta^2}{\ln 2(1 + 2\beta^2 P^s/\theta + 2\gamma^2 P^r/\theta)} + \frac{(1-\tau)\alpha^2}{\ln 2(1 + 2\alpha^2 P^s/\theta)} \leq \mu_l + c[P^s - Q^s(t)], \quad \text{with equality if } P^s > 0, \quad (41)$$

$$\frac{\tau\gamma^2}{\ln 2(1 + 2\beta^2 P^s/\theta + 2\gamma^2 P^r/\theta)} \leq \nu_j + c[P^r - Q^r(t)], \quad \text{with equality if } P^r > 0. \quad (42)$$

Since $R_1 = R_2$, we have

$$\gamma^2 P^r = (\alpha^2 - \beta^2)P^s. \quad (43)$$

Two subcases need to be considered:

1) If $P^r > 0$ and $P^s > 0$, (41) and (42) achieve equality. From (41)-(43), we derive

$$P^s = f\left(\theta, c, \mu_l + \nu_j(\alpha^2 - \beta^2)/\gamma^2, Q^s(t) + Q^r(t)(\alpha^2 - \beta^2)/\gamma^2, \alpha^2, 1 + (\alpha^2 - \beta^2)^2/\gamma^4\right)$$

$$P^r = P^s(\alpha^2 - \beta^2)]/\gamma^2,$$

$$\tau = \frac{\alpha^2[\nu_j + c(P^r - Q^r(t))]}{\gamma^2[\mu_l + c(P^s - Q^s(t))] + (\alpha^2 - \beta^2)[\nu_j + c(P^r - Q^r(t))]}$$

This case happens when $0 < \tau < 1$. The power allocation variable P^s also reduces to the water-filling solution in this case as $c \to 0$.

2) $P^r = P^s = 0$. It happens when all of previous cases are not satisfied.

C. Channel resource allocation

We now fix the power allocation variables and perform channel resource allocation. By using the dual decomposition method as in (17), and auxiliary variables as in (27), the KKT conditions of channel resource allocation are given by

$$C(\beta_m^2 P_m^s/\theta_m) - \frac{\beta_m^2 P_m^s/\theta_m}{\ln 2(1 + \beta_m^2 P_m^s/\theta_m)} \leq \varepsilon \quad \text{with equality if } \theta_m > 0, \quad (44)$$

$$(1-\tau)C(2\alpha_{mj}^2 P_{mj}^s/\theta_{mj}) - \frac{(1-\tau)2\alpha_{mj}^2 P_{mj}^s/\theta_{mj}}{\ln 2(1 + 2\alpha_{mj}^2 P_{mj}^s/\theta_{mj})} + \tau C(2(\beta_{mj}^2 P_{mj}^s + \gamma_{mj}^2 P_{mj}^r)/\theta_{mj}) - \frac{\tau 2(\beta_{mj}^2 P_{mj}^s + \gamma_{mj}^2 P_{mj}^r)/\theta_{mj}}{\ln 2[1 + 2(\beta_{mj}^2 P_{mj}^s + \gamma_{mj}^2 P_{mj}^r)/\theta_{mj}]} \leq 2\varepsilon \quad \text{with equality if } \theta_{mj} > 0. \quad (45)$$

where $\varepsilon \geq 0$ is the dual variable for the channel resource constraint. Considering the different cases of $\tau^\star$ given in (28), one can show that (45) is equivalent to

$$C(2\min\{\alpha_{mj}^2 P_{mj}^s, \beta_{mj}^2 P_{mj}^s + \gamma_{mj}^2 P_{mj}^r\}/\theta_{mj}) - \frac{2\min\{\alpha_{mj}^2 P_{mj}^s, \beta_{mj}^2 P_{mj}^s + \gamma_{mj}^2 P_{mj}^r\}/\theta_{mj}}{\ln 2[1 + 2\min\{\alpha_{mj}^2 P_{mj}^s, \beta_{mj}^2 P_{mj}^s + \gamma_{mj}^2 P_{mj}^r\}/\theta_{mj}]} \leq 2\varepsilon, \quad (46)$$

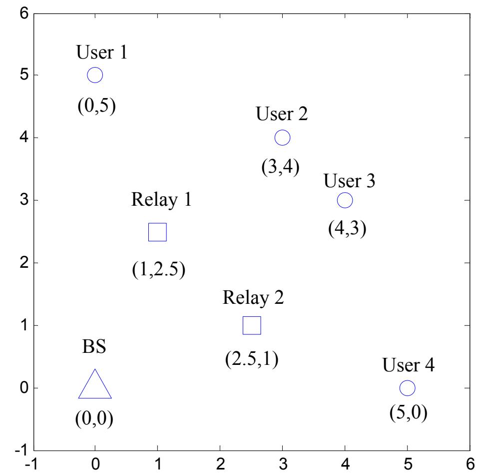

Fig. 1. The topology of an uplink F/TDMA cellular relay network.

the proof of which is omitted for space limitation. The left side of (44) is zero only when $P_m^s = 0$ or $\theta_m = \infty$, the left side of (46) is zero only when $P_{mj}^s = P_{mj}^r = 0$ or $\theta_{mj} = \infty$. Therefore, ε should be positive since there is at least one link with positive power allocation.

When $P_m^s > 0$, the left side of (44) grows to infinite when $\theta_m \to 0$. Thus, $\theta_m = 0$ only if $P_m^s = 0$. For (46), we have that $\theta_{mj} = 0$ happens only if $P_{mj}^s = P_{mj}^r = 0$. One can set a minimal value for θ_{mj} and θ_m to avoid the problem that the objective function has no definition for $\theta_{mj} = 0$ or $\theta_m = 0$. Let $\theta^0(a, b)$ be the root of equation

$$C(a/\theta) - \frac{a/\theta}{\ln 2(1 + a/\theta)} = b \quad (47)$$

For the case $\theta_m, \theta_{mj} > 0$, the optimal value of θ_m is given by $\theta^0(\beta_m^2 P_m^s, \varepsilon)$ and the optimal value of θ_{mj} is given by $\theta^0(2\min\{\alpha_{mj}^2 P_{mj}^s, \beta_{mj}^2 P_{mj}^s + \gamma_{mj}^2 P_{mj}^r\}, 2\varepsilon)$. The famous rapidly convergent Newton's method [14, Section 5.5.3], which requires only several iterations, is used to solve (47) and the dual variable ε is obtained by bisection method.

By performing power allocation and channel resource allocation iteratively, the joint optimal power and channel resource allocation solution is derived. The proof for this is quite similar with the one given in [4, pp. 3440]. It is omitted here for space limitation.

V. Numerical Results

In this section, we present numerical results to demonstrate the performance of the proposed joint optimal power and channel resource allocation for F/TDMA DF relay networks. We compare our method with two other schemes:

Scheme 1 is F/TDMA cellular network without the assistance of relay nodes. The optimal power and channel resource allocation is utilized. Scheme 2 is F/TDMA relay network with optimal power allocation and equal channel resource allocation among the relay links. Our joint power and channel resource allocation scheme for F/TDMA DF relay networks is denoted as Scheme 3.

We consider an uplink F/TDMA cellular relay network with 4 users, 2 relay nodes, and 1 base station, whose topology is

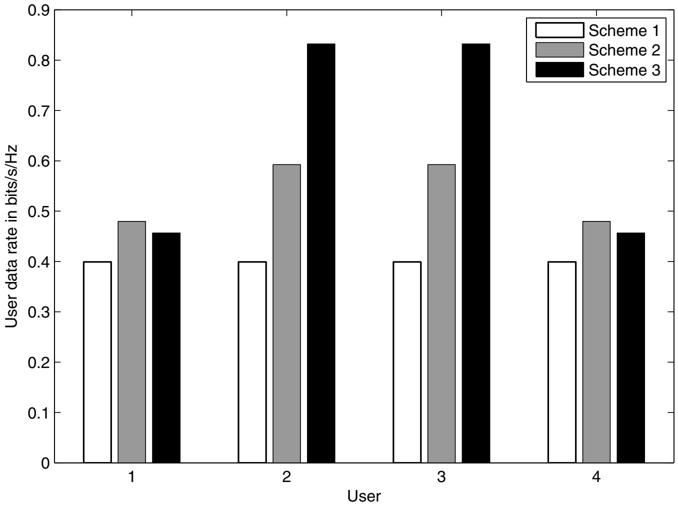

Fig. 2. The data rates of each user in three schemes.

TABLE I
RELAY SELECTION RESULTS OF THE USERS

	User 1	User 2	User 3	User 4
Scheme 2	1,2	1,2	1,2	1,2
Scheme 3	1	1,2	1,2	2

shown in Fig. 1. The total channel resource in a scheduling frame is normalized to 1. Each node is subject to a separate average power (or sum transmission energy) constraint during the scheduling frame. The channel gain between two nodes is given by a large-scale path loss component with path loss factor of 4. We assume that each user or relay node has the same maximal average power, and the received SNR at unit distance from a transmitting node is 25dB if this node occupies all the unit channel resources. Since the distance between the S-D pair is large, this assumption corresponds to a low-SNR environment for the direct transmission.

Figure 2 shows the data rates of each user in these three schemes. We can see that by jointly optimizing the power and channel resources, our proposed scheme outperforms the first two schemes in terms of the sum rates by 61.5% and 20.2%, respectively. Our numerical results suggest that such an increase is more evident for low-SNR environment. This observation is also consistent with observations in previous works that cooperative relaying is more beneficial for the users with poor channel conditions, e.g. [15]-[16]. We note that the system sum data rate does not reflect fairness among the users. Thus, the achievable data rates of the users in Scheme 3 are very different. The data rates of User 1 and User 4 sacrifice a little in order to get a higher sum data rate.

Finally, we consider the relay selection result given in Table I. In Scheme 2, both relays are used to assist each user's transmission. On the other hand, by optimizing the channel resources, Scheme 3 can select the optimal subset of relay nodes to assist each user. For example, User 1 only utilizes the help of a nearby relay 1 to forward its message. The optimal Scheme 3 does not waste channel resource on Relay 2 which is far from User 1.

Finally, we note that when a source node has a nearby relay node, it may still require the help of some other relay nodes if the nearby relay node's power is not large enough to assist the source node's transmission.

VI. CONCLUSION

In this paper, we have solved the joint power and channel resource allocation problem for a multiuser F/TDMA DF relay network under the per-node power constraints and a sum channel resource constraint. The difficulties that the objective function is neither strict concave and nor differentiable have been carefully handled in our iterative optimzation algorithm. The optimal relay selection result can be derived simultaneously in our algorithm. It has been shown that more than one relay node may be needed for a single data stream due to the per-node power constraints. A distributed cross-layer solution which could guarantee the fairness among the users will be considered in our future work.

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