

Downlink Linear Max-MSE Transceiver Design for Multiuser MIMO Systems Via Dual Decomposition

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Abstract—This paper addresses the problem of joint linear transceiver design in the downlink of multiuser MIMO systems. We define the performance criterion as minimizing the maximal mean square error among all the data streams (Max-MSE) under a total power constraint. The proposed algorithm is based on a dual decomposition technique and it iterates between close-formed precoder/decoder designs. The proposed algorithm outperforms most transceiver optimization algorithms in terms of BER performance. Compared to the algorithm which can achieve the same performance, it has a lower computational complexity.

I. INTRODUCTION

The design of MIMO systems has drawn considerable research interests, because of its high capacity and reliability. To put the advantages of MIMO systems into practice, we can use linear precoders and decoders at the transmitters and the receivers, and jointly optimize the transceivers.

For single-user MIMO systems, optimal transceiver designs have been developed in [1], where the transceiver optimization problem is reformulated and solved in the unified framework of convex optimization for most performance criteria.

For multiuser (MU) MIMO systems, most transceiver designs are proposed to minimize the sum MSE of all the data streams (Sum-MSE criterion). Several transceiver designs for the uplink of MU-MIMO systems have been proposed [2]-[4]. For the downlink of MU-MIMO systems, the Sum-MSE problem has been optimally solved out in [5]-[7]. Besides, a suboptimal algorithm has been proposed in [8]-[9], which iterates between close-formed precoder/decoder designs.

While the Sum-MSE criterion optimizes the overall performance, it may result in certain substreams with extremely low SINR or high MSE. Furthermore, since the average bit error rate (BER) is dominated by the performance of the worst substream, the BER performance obtained by optimal Sum-MSE transceivers is far from optimal. As a result, the criteria of maximizing the minimal SINR (Min-SINR) and minimizing the maximal MSE (Max-MSE) have been proposed in [6], [7] and [10]. The Min-SINR and Max-MSE criteria guarantee the performance of the worst substream and is superior to Sum-MSE criterion in terms of the average BER performance.

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For the systems where each receiver has one antenna (MU-MISO), the Min-SINR optimization problem was optimally solved out in [10]. For the MU-MIMO systems, several suboptimal transceiver designs have been proposed. In [7], a iterative Max-MSE transceiver design was proposed. In [6], an algorithm extended from [10] was proposed, which performs better than that in [7] but suffers from a relatively high complexity. Consequently, we desire to find a low-complexity algorithm with no worse performance than that in [6].

In this paper, we propose a Max-MSE transceiver design for the MU-MIMO downlink, using a dual decomposition method, similar to that used for the sum-capacity computation problem in [11]. The proposed algorithm achieves the same performance as the Min-SINR design in [6], with a lower computational complexity.

This paper is organized as follows. Section II introduces the signal model and formulates the problem. In section III, the proposed transceiver design method is described. Simulation results and complexity analysis are presented in section IV. Finally, section V concludes the paper.

II. SIGNAL MODEL AND PROBLEM FORMULATION

A. Signal Model

Consider a MU-MIMO downlink system consisting of one base station with M_T antennas and K users, where the k th user has M_{R_k} antennas. In the downlink, the received signal model for user k can be expressed as

$$\hat{\mathbf{s}}_k = \beta^{-1} \mathbf{R}_k^H \left(\mathbf{H}_k \sum_{i=1}^K \mathbf{T}_i \mathbf{s}_i + \mathbf{n}_k \right), \quad (1)$$

where $\mathbf{s}_k \in \mathbb{C}^{L_k \times 1}$ is the data vector transmitted to user k , and $\hat{\mathbf{s}}_k \in \mathbb{C}^{L_k \times 1}$ is the estimation of \mathbf{s}_k at user k . $\mathbf{T}_k \in \mathbb{C}^{M_T \times L_k}$ is the precoder for user k at the base station, while $\mathbf{R}_k \in \mathbb{C}^{M_{R_k} \times L_k}$ is the decoder at user k . β is a scaling factor common for all the data streams. The channel matrix and the noise vector are denoted as $\mathbf{H}_k \in \mathbb{C}^{M_{R_k} \times M_T}$ and $\mathbf{n}_k \in \mathbb{C}^{M_{R_k} \times 1}$. We assume that the data vector \mathbf{s}_k is zero-mean and unit-energy, with the correlation matrix denoted as $\mathbb{E}(\mathbf{s}_k \mathbf{s}_k^H) = \mathbf{I}_{L_k}$. We also assume that $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_{n_k}^2 \mathbf{I}_{M_{R_k}})$.

We define the k th user's MSE matrix \mathbf{E}_k as

$$\mathbf{E}_k \triangleq \mathbb{E}((\hat{\mathbf{s}}_k - \mathbf{s}_k)(\hat{\mathbf{s}}_k - \mathbf{s}_k)^H). \quad (2)$$

Then the quality of service (QoS) of the i th data stream of user k is measured by its MSE, which can be denoted as

$$\text{MSE}_{k,i} \triangleq \mathbb{E}(|\hat{s}_{k,i} - s_{k,i}|^2) = [\mathbf{E}_k]_{ii}, \quad (3)$$

where $[\mathbf{E}_k]_{ii}$ is \mathbf{E}_k 's i th diagonal element.

B. Problem Formulation

As mentioned before, we consider an optimization problem of minimizing the maximal MSE of all the data streams (Max-MSE) under a total power constraint, which can be noted as

$$\sum_{k=1}^K \mathbb{E}(\|\mathbf{T}_k \mathbf{s}_k\|^2) = \sum_{k=1}^K \text{tr}(\mathbf{T}_k \mathbf{T}_k^H) \leq P_t, \quad (4)$$

where $\text{tr}(\cdot)$ denotes the trace of a matrix. Thus, the problem can be formulated as

$$\begin{aligned} \min_{\{\mathbf{T}_k\}, \{\mathbf{R}_k\}, \beta} \quad & \max_{k,i} \{\text{MSE}_{k,i}\} \\ \text{s.t.} \quad & \sum_{k=1}^K \text{tr}(\mathbf{T}_k \mathbf{T}_k^H) \leq P_t. \end{aligned} \quad (5)$$

III. TRANSCEIVER DESIGN USING DUAL DECOMPOSITION

A. Problem Simplification

Before solving the original optimization problem (5), we can first simplify it by exploiting the feature of the optimal solution to the Max-MSE problem. It is stated in [3] that the optimal solution is characterized by equal MSE's among all the data streams. Thus, the optimal solution satisfies

$$\text{MSE}_{k,i} = \text{MSE}_k / L_k, i = 1, \dots, L_k, k = 1, \dots, K, \quad (6)$$

where $\text{MSE}_k = \text{tr}(\mathbf{E}_k)$, the sum MSE of user k .

Furthermore, we can set equal the MSE's of all the data streams within one user without affecting the other users. As a result, we can first solve the following optimization problem:

$$\begin{aligned} \min_{\{\mathbf{T}_k\}, \{\mathbf{R}_k\}, \beta} \quad & \max_k \{\text{MSE}_k / L_k\} \\ \text{s.t.} \quad & \sum_{k=1}^K \text{tr}(\mathbf{T}_k \mathbf{T}_k^H) \leq P_t. \end{aligned} \quad (7)$$

After that, we set equal the MSE's of the data streams within each user. In this way, we can transform the original per-stream MSE balancing problem (5), which balances the MSE's of all the $\sum_{k=1}^K L_k$ data streams, into a simplified per-user sum MSE balancing problem (7), which balances the sum MSE's of all the K users.

Now we will show the algorithm that sets equal the MSE's within one user. The k th user's sum MSE is

$$\begin{aligned} \text{MSE}_k &= \text{tr}(\mathbf{E}_k) \\ &= \beta^{-2} \cdot \text{tr}(\mathbf{R}_k^H (\mathbf{H}_k \sum_{i=1}^K \mathbf{T}_i \mathbf{T}_i^H \cdot \mathbf{H}_k^H + \sigma_{n_k}^2 \mathbf{I}) \mathbf{R}_k) \\ &- \beta^{-1} \cdot \text{tr}(\mathbf{R}_k^H \mathbf{H}_k \mathbf{T}_k) - \beta^{-1} \cdot \text{tr}(\mathbf{T}_k^H \mathbf{H}_k^H \mathbf{R}_k) + L_k. \end{aligned} \quad (8)$$

The optimal decoder \mathbf{R}_k can be easily derived as

$$\mathbf{R}_k = \beta \cdot \left(\mathbf{H}_k \sum_{i=1}^K \mathbf{T}_i \mathbf{T}_i^H \cdot \mathbf{H}_k^H + \sigma_{n_k}^2 \mathbf{I} \right)^{-1} \cdot \mathbf{H}_k \mathbf{T}_k. \quad (9)$$

Then, we can write the k th MSE matrix \mathbf{E}_k as

$$\mathbf{E}_k = \mathbf{I}_{L_k} - \mathbf{T}_k^H \mathbf{H}_k^H \mathbf{W}_k \mathbf{H}_k \mathbf{T}_k \quad (10)$$

where \mathbf{W}_k is defined as

$$\mathbf{W}_k = \left(\mathbf{H}_k \sum_{i=1}^K \mathbf{T}_i \mathbf{T}_i^H \cdot \mathbf{H}_k^H + \sigma_{n_k}^2 \mathbf{I} \right)^{-1} \quad (11)$$

TABLE I
ALGORITHM 1 - MSE BALANCING FOR USER k

1	Eigenvalue decomposition of \mathbf{E}_k : $\mathbf{E}_k = \mathbf{V}_k \mathbf{\Lambda}_k \mathbf{V}_k^H$.
2	Obtain the rotation matrix \mathbf{Q}_k such that $\mathbf{Q}_k^H \mathbf{\Lambda}_k \mathbf{Q}_k = \text{tr}(\mathbf{\Lambda}_k) / L_k \cdot \mathbf{I}_{L_k}$. We can use the algorithm described in [12, Sec. IV-A].
3	$\mathbf{T}_k^* = \mathbf{T}_k \mathbf{V}_k \mathbf{Q}_k$

Thus, we can use Algorithm 1 in Table I to balance $\text{MSE}_{k,1}, \dots, \text{MSE}_{k,L_k}$ of user k . Note that multiplying \mathbf{T}_k with a unitary matrix does not change the other users' MSE.

B. Dual Decomposition

In order to make the notations succinct, we assume that each user has the same number of data streams. We can easily extend the algorithm to the case where the users have different numbers of data streams. As a result, we further simplify problem (7) into the following problem:

$$\begin{aligned} \min_{\{\mathbf{T}_k\}, \{\mathbf{R}_k\}, \beta} \quad & \max_k \{\text{MSE}_k\} \\ \text{s.t.} \quad & \sum_{k=1}^K \text{tr}(\mathbf{T}_k \mathbf{T}_k^H) \leq P_t. \end{aligned} \quad (12)$$

Since the current objective function is difficult to optimize, we rewrite the problem in the following form

$$\begin{aligned} \min_{\{\mathbf{T}_k\}, \{\mathbf{R}_k\}, \beta, t} \quad & t \\ \text{s.t.} \quad & \text{MSE}_k \leq t, \quad k = 1, 2, \dots, K \\ & \sum_{k=1}^K \text{tr}(\mathbf{T}_k \mathbf{T}_k^H) \leq P_t. \end{aligned} \quad (13)$$

Now we derive the Lagrangian of the optimization problem with respect to the K constraints on the MSE's as

$$\begin{aligned} L(\mathbf{T}_1, \dots, \mathbf{T}_K, \mathbf{R}_1, \dots, \mathbf{R}_K, \beta, t, \lambda_1, \dots, \lambda_K) \\ = t + \sum_{k=1}^K \lambda_k (\text{MSE}_k - t). \end{aligned} \quad (14)$$

Then, the dual function is [13]

$$\begin{aligned} g(\lambda_1, \dots, \lambda_K) \\ = \min_{\{\mathbf{T}_k\}, \{\mathbf{R}_k\}, \beta, t} L(\cdot) \\ = \min_{\{\mathbf{T}_k\}, \{\mathbf{R}_k\}, \beta, t} \left(\sum_{k=1}^K \lambda_k \text{MSE}_k + (1 - \sum_{i=1}^K \lambda_i) \cdot t \right), \end{aligned} \quad (15)$$

which can be determined as

$$\begin{aligned} g(\lambda_1, \dots, \lambda_K) \\ = \begin{cases} \min_{\{\mathbf{T}_k\}, \{\mathbf{R}_k\}, \beta} \sum_{k=1}^K \lambda_k \text{MSE}_k & \sum_{i=1}^K \lambda_i = 1 \\ -\infty & \text{otherwise.} \end{cases} \end{aligned} \quad (16)$$

Thus, the Lagrange dual problem associated with the problem (13) is

$$\begin{aligned} \max_{\lambda} \quad & g(\lambda) \\ \text{s.t.} \quad & \|\lambda\|_1 = 1, \lambda \succeq 0, \end{aligned} \quad (17)$$

where $\lambda = [\lambda_1, \dots, \lambda_K]^T$ and ' \succeq ' is the component-wise inequality for vectors.

Using the dual decomposition technique [14], the dual problem (17) can be decomposed into a subproblem and a

master problem. The subproblem, which calculates the dual function $g(\lambda)$ for the given λ , can be written as

$$\min_{\{\mathbf{T}_k\}, \{\mathbf{R}_k\}, \beta} \sum_{k=1}^K \lambda_k \cdot \text{MSE}_k \quad (18)$$

$$s.t. \quad \sum_{k=1}^K \text{tr}(\mathbf{T}_k \mathbf{T}_k^H) \leq P_t.$$

The master problem, which adjusts the dual variables $\lambda_1, \dots, \lambda_K$ according to the optimal MSE values $\{\text{MSE}_k^*\}$ obtained from the subproblem, is denoted as

$$\max_{\lambda} \sum_{k=1}^K \lambda_k \cdot \text{MSE}_k^* \quad (19)$$

$$s.t. \quad \|\lambda\|_1 = 1, \lambda \succeq 0.$$

C. Solution to the Subproblem

The subproblem (18) can be seen as a weighted sum MSE minimization problem. We can use an iterative algorithm similar to that used for the Sum-MSE problem in [8]-[9]. This algorithm iteratively optimizes the transmit precoding matrices $\{\mathbf{T}_k\}$ and the receive decoding matrices $\{\mathbf{R}_k\}$. By fixing the other variables, the optimal precoder and decoder for user k can be derived as

$$\begin{cases} \mathbf{T}_k = \beta \left(\sum_{i=1}^K \lambda_i \mathbf{H}_i^H \mathbf{R}_i \mathbf{R}_i^H \mathbf{H}_i + \alpha \mathbf{I} \right)^{-1} \cdot \lambda_k \mathbf{H}_k^H \mathbf{R}_k \\ \mathbf{R}_k = \beta \left(\mathbf{H}_k \sum_{i=1}^K \mathbf{T}_i \mathbf{T}_i^H \cdot \mathbf{H}_k^H + \sigma_{n_k}^2 \mathbf{I} \right)^{-1} \cdot \mathbf{H}_k \mathbf{T}_k \end{cases}, \quad (20)$$

where

$$\begin{cases} \mathbf{A} = \sum_{i=1}^K \lambda_i \mathbf{H}_i^H \mathbf{R}_i \mathbf{R}_i^H \mathbf{H}_i \\ \alpha = \text{tr} \left(\sum_{k=1}^K \lambda_k \sigma_{n_k}^2 \mathbf{R}_k^H \mathbf{R}_k \right) / P_t \\ \beta = \sqrt{\frac{P_t}{\text{tr}((\mathbf{A} + \alpha \mathbf{I})^{-1} \mathbf{A} (\mathbf{A} + \alpha \mathbf{I})^{-1})}} \end{cases}. \quad (21)$$

D. Solution to the Master Problem

Now we will deal with the master problem (19). The master problem, which properly adjusts the dual variable λ , can be solved by a simple subgradient algorithm. The subgradient method produces a sequence of feasible points $\{\lambda^n\}$ (where n indicates the iteration number) as [14, Sec. 6.3.1]

$$\lambda^{n+1} = [\lambda^n - \alpha^n \mathbf{s}^n]_{\mathcal{X}}^+, \quad (22)$$

where α^n is the positive scalar stepsize, \mathbf{s}^n is the subgradient of the objective function of the master problem $f(\lambda) = \sum_{k=1}^K \lambda_k \text{MSE}_k^*$ at λ^n , and $[\cdot]_{\mathcal{X}}^+$ denotes the projection onto the constraint set of the master problem

$$\mathcal{X} = \{\lambda \mid \|\lambda\|_1 = 1, \lambda \succeq 0\}, \quad (23)$$

which is a simplex in \mathbb{R}^K .

The master problem (19) can be solved out efficiently. First, the subgradient \mathbf{s} of $f(\lambda)$ at λ can be expressed as

$$\mathbf{s} = \mathbf{MSE}^* = [\text{MSE}_1^*, \dots, \text{MSE}_K^*]^T, \quad (24)$$

where $\{\text{MSE}_k^*\}$ are the optimal MSE's of the subproblem (18) with the weighting vector λ .

Second, for the diminishing step size rule

$$\alpha^n = \alpha^0 \cdot \frac{1+m}{n+m}, \quad (25)$$

where $\alpha^0 \in (0, 1]$ is the initial stepsize and m is a fixed positive integer, the subgradient algorithm is guaranteed to converge according to [14].

TABLE II
ALGORITHM 2 - MAX-MSE TRANSCEIVER OPTIMIZATION ALGORITHM

1	Initialization: $\lambda^0 = \frac{1}{K} [1, \dots, 1]^T$, $\mathbf{R}_k = [\mathbf{1}, \dots, \mathbf{1}]$, $k = 1, \dots, K$
2	Subproblem: Calculate the dual function $g(\lambda) = \sum_{k=1}^K \lambda_k \text{MSE}_k$ a). Transmit precoding matrices (for $k = 1, \dots, K$) $\mathbf{T}_k = \beta \left(\sum_{i=1}^K \lambda_i \mathbf{H}_i^H \mathbf{R}_i \mathbf{R}_i^H \mathbf{H}_i + \alpha \mathbf{I} \right)^{-1} \cdot \lambda_k \mathbf{H}_k^H \mathbf{R}_k$ where α and β can be calculated according to (21). b). Receive decoding matrices (for $k = 1, \dots, K$) $\mathbf{R}_k = \beta \left(\mathbf{H}_k \sum_{i=1}^K \mathbf{T}_i \mathbf{T}_i^H \cdot \mathbf{H}_k^H + \sigma_{n_k}^2 \mathbf{I} \right)^{-1} \cdot \mathbf{H}_k \mathbf{T}_k$ c). If converges to ϵ_{sub} -optimal, go on to step 3. Otherwise, go back to a).
3	Master problem: Update the dual variable λ $\lambda^{n+1} = [\lambda^n - \alpha^n \mathbf{s}^n]_{\mathcal{X}}^+$, with \mathbf{s}^n and α^n defined in (24) and (25).
4	If $\max_k \{\text{MSE}_k\} - \sum_k \lambda_k \text{MSE}_k \leq \epsilon$, stop. Otherwise, go to step 2.
5	MSE balancing using Algorithm 1 in Table I.

Finally, the projection on a simplex is very easy. We can use the projection algorithm described in [15, Algorithm 1].

Now we can summarize the downlink Max-MSE transceiver optimization algorithm in Table II.

E. Convergence Issue

The convexity of the dual problem (17) ensures that its global optimum can be achieved by the subgradient algorithm. However, we should notice that the primal problem (13) is not a convex optimization problem. Therefore, there may exist a positive duality gap between the the optimal solutions to the primal problem and the dual problem. In other words, the algorithm is not guaranteed to converge to the global optimum of the original Max-MSE problem (12).

To reach an accuracy of ϵ , the stopping criterion is set as

$$\max_k \{\text{MSE}_k\} - \sum_{k=1}^K \lambda_k \text{MSE}_k \leq \epsilon. \quad (26)$$

Then we have

$$\begin{aligned} \max_k \{\text{MSE}_k\} - p^* &\leq \max_k \{\text{MSE}_k\} - d^* \\ &\leq \max_k \{\text{MSE}_k\} - \sum_{k=1}^K \lambda_k \text{MSE}_k \\ &\leq \epsilon, \end{aligned}$$

where p^* and d^* are the optimal values of the original and dual problem, respectively. Consequently, the proposed algorithm has a reliable stopping criterion.

To better illustrate the convergence behavior of the proposed algorithm, we provide a geometric interpretation of this algorithm in Fig. 1. For the given channel matrices $\{\mathbf{H}_k\}$, noise powers $\{\sigma_k^2\}$, and the total power constraint P_t , we define the feasible MSE region as

$$\mathcal{O} = \{ (\text{MSE}_1, \dots, \text{MSE}_K) \mid \sum_{k=1}^K \text{tr}(\mathbf{T}_k \mathbf{T}_k^H) \leq P_t \}. \quad (27)$$

We also define the hyperplane with normal vector $\lambda = [\lambda_1, \dots, \lambda_K]^T$ and a point $\mathbf{MSE}^0 = (\text{MSE}_1^0, \dots, \text{MSE}_K^0)$ in the hyperplane as [13, Sec. 2.2.1]

$$\mathcal{P} = \{ \mathbf{MSE} \mid \lambda^T \mathbf{MSE} = \lambda^T \mathbf{MSE}^0 \}. \quad (28)$$

Suppose the hyperplane \mathcal{P} with normal vector λ is tangent to the MSE region \mathcal{O} at \mathbf{MSE}^0 , then \mathbf{MSE}^0 is the optimal

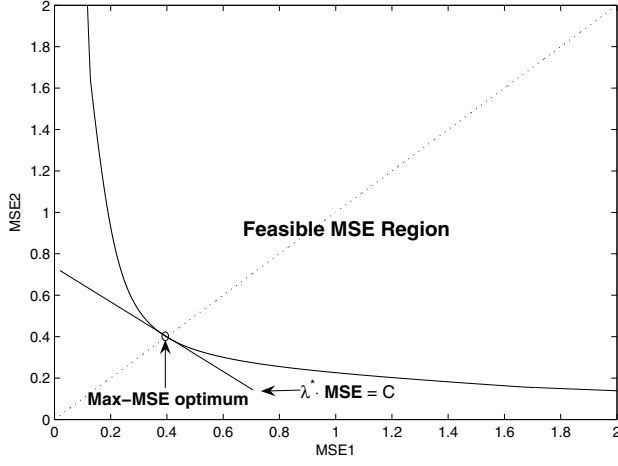


Fig. 1. Geometric Interpretation of the proposed algorithm for a system with 2 users, each of which has 2 substreams.

solution to the subproblem (18) with the dual variable λ . We assume that the hyperplane with normal vector λ^* is tangent to \mathcal{O} at MSE^* , the optimal solution to the Max-MSE problem. Then the goal of the master problem (19) is to obtain this optimal λ^* by gradually adjusting λ . We can see that as long as the feasible MSE region is convex, the optimal dual variable λ^* can be found and the global optimum of the Max-MSE problem can be achieved.

IV. SIMULATION RESULTS

In this section, we provide simulation results to illustrate the performance of the proposed algorithm.

A. BER Performance

We compare the BER performance of the proposed algorithm with the Sum-MSE design, the Max-MSE design in [7, Algorithm 5], and the Min-SINR design.

For the Sum-MSE design, we first exploit the uplink-downlink duality in [5]-[6] to convert the downlink problem into its uplink dual, then we use the semidefinite programming (SDP) algorithm in [4], with its individual power constraints replaced by a total power constraint. This algorithm achieves the global optimum of the sum MSE minimization problem.

For the Min-SINR design, we slightly modify the algorithm for power minimization problem in [6, Table I]. Since the power minimization problem and the Min-SINR problem are essentially the same, we only change the uplink and downlink power allocation in [6] for the Min-SINR algorithm used in the simulation.

In the simulation, the channel is assumed to be flat Rayleigh fading and the noise is zero mean white Gaussian. We assume that perfect channel state information (CSI) is known at the transmitter and receivers. The transmit SNR is the total transmit power to noise ratio, namely $10 \cdot \log(P_t/\sigma_n^2)$.

Fig. 2 shows the BER performance of the Sum-MSE design, the Max-MSE design in [7], the Min-SINR design, and the proposed algorithm in a 4-user MIMO system (QPSK). The average channel powers of all the users are the same. We can

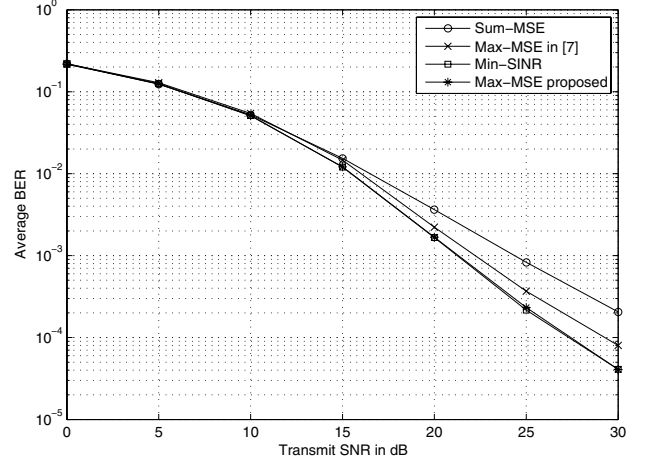


Fig. 2. Comparison of BER performance. $M_T = 8$, $K = 4$, $M_{R_k} = 2$, and $L_k = 2$, $\forall k$. The average channel powers of all the users are the same.

see that, in the high-SNR scenario, the proposed algorithm is 3-5 dB better than the Sum-MSE design and is 1 dB better than the Max-MSE design in [7]. The performance advantage over the algorithm in [7] might result from the different methods of updating the Lagrangian multipliers in (14). The proposed algorithm can guarantee that, after each update, the solution to the subproblem is closer to the optimum, while this is not always achieved by the algorithm in [7].

We can also notice that the proposed algorithm is equivalent to the Min-SINR design with respect to BER performance. This is because of the one-to-one relationship between the optimal $\text{MSE}_{k,i}$ and $\text{SINR}_{k,i}$, namely $\text{MSE}_{k,i} = 1/(1 + \text{SINR}_{k,i})$ [5][12].

Fig. 3 shows the BER performance of the four algorithms in a 3-user MIMO system (16QAM) with near-far effect. The average channel powers of user 2 and user 3 are four times that of user 1. In this case, the proposed algorithm is superior to the Sum-MSE design and the Max-MSE design in [7], while it achieves the same performance as the Min-SINR design.

B. Complexity Analysis

We have shown above that the proposed algorithm has better BER performance than the Sum-MSE design and Max-MSE design in [7]. Now we demonstrate that the proposed algorithm has lower computational complexity than the Min-SINR design.

First, we calculate the per-iteration complexity of these two algorithms. For the proposed algorithm, we can see from Table II that the major computational complexity comes from the matrix-inversion operations when computing $\{\mathbf{T}_k\}_{k=1}^K$ and $\{\mathbf{R}_k\}_{k=1}^K$. To calculate $\{\mathbf{T}_k\}$, we have to do a $M_T \times M_T$ matrix inversion, which requires $O(M_T^3)$ operations. To calculate $\{\mathbf{R}_k\}$, we should do K matrix inversions, which has a total number of $\sum_{k=1}^K O(M_{R_k}^3)$ operations. So the overall computational complexity in one iteration of the proposed algorithm is

$$O(M_T^3) + \sum_{k=1}^K O(M_{R_k}^3) \quad (29)$$

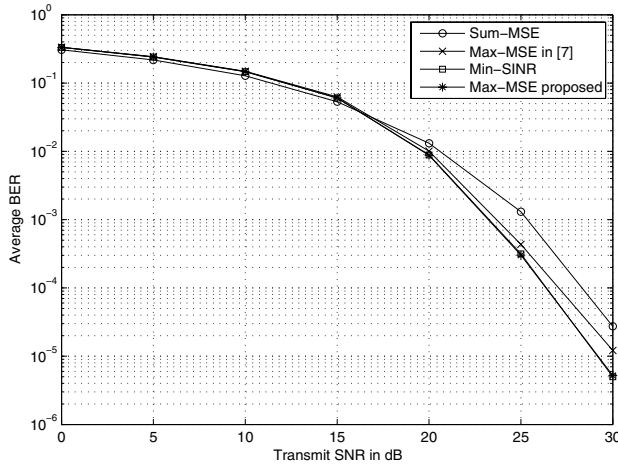


Fig. 3. Comparison of BER performance. $M_T = 8$, $K = 3$, $M_{R_k} = 2$, and $L_k = 2$, $\forall k$. The average channel powers of user 2 and user 3 are four times that of user 1.

For the Min-SINR design, the per-iteration complexity is dominated by the matrix inversions when calculating $\{\mathbf{T}_k\}$ and $\{\mathbf{R}_k\}$. The precoder \mathbf{t}_k and the decoder $\mathbf{r}_{k,i}$ for the i th substream of the k th user are calculated by [6]

$$\mathbf{t}_{k,i} = \left(\sum_{l=1}^K \mathbf{H}_l^H \mathbf{R}_l \mathbf{R}_l^H \mathbf{H}_l + \sigma_n^2 \mathbf{I} - \mathbf{H}_k^H \mathbf{r}_{k,i} \mathbf{r}_{k,i}^H \mathbf{H}_k \right)^{-1} \mathbf{H}_k^H \mathbf{r}_{k,i}$$

and

$$\mathbf{r}_{k,i} = \left(\mathbf{H}_k \sum_{l=1}^K \mathbf{T}_l \mathbf{T}_l^H \cdot \mathbf{H}_k^H + \sigma_n^2 \mathbf{I} - \mathbf{H}_k \mathbf{t}_{k,i} \mathbf{t}_{k,i}^H \mathbf{H}_k^H \right)^{-1} \mathbf{H}_k \mathbf{t}_{k,i}.$$

So the matrix inversion for the calculation of precoders $\{\mathbf{T}_k\}$ and decoders $\{\mathbf{R}_k\}$ requires

$$\sum_{k=1}^K L_k \cdot O(M_T^3) + \sum_{k=1}^K L_k \cdot O(M_{R_k}^3) \quad (30)$$

operations. Obviously, our Max-MSE algorithm has a lower per-iteration complexity than the Min-SINR design. This reduction in complexity is significant, especially when the number of users K is so large that the total number of data streams $\sum_{k=1}^K L_k$ approaches the number of transmit antennas M_T . In this case, the per-iteration complexity of the Min-SINR design will be dominated by $O(M_T^4)$, while the complexity of the proposed algorithm remains to be $O(M_T^3)$.

Now we compare the convergence rates of this two algorithms in different channel conditions and SNR's. The Average iteration numbers required to reach within 10^{-4} accuracy of the optimal values are listed in Table III. We can see that the convergence speed of the proposed algorithm is faster than that of the Min-SINR design. Combined with the fact that the proposed algorithm requires less computation per iteration, we can conclude that it has a lower complexity than the Min-SINR design.

V. CONCLUSION

In this paper, we have proposed a linear transceiver design algorithm for the downlink of multiuser MIMO systems, which

TABLE III
AVERAGE ITERATION NUMBERS REQUIRED TO REACH WITHIN 10^{-4} ACCURACY OF THE OPTIMAL VALUES

Channel	Channel in Fig. 2			Channel in Fig. 3		
SNR	15dB	20dB	25dB	15dB	20dB	25dB
Min-SINR	38.2	58.6	73.9	67.1	111.8	147.9
Proposed	33.8	45.0	67.8	27.5	45.8	84.6

minimizes the maximal MSE (Max-MSE) under a total power constraint. We use a dual decomposition method to decompose the original problem into a subproblem (which iterates between close-formed precoder/decoder designs) and a master problem (which can be solved out by a simple subgradient method). The proposed algorithm outperforms the Sum-MSE design and the Max-MSE optimization algorithm in [7] in terms of BER performance. It achieves the same performance as the Min-SINR design, but has a lower computational complexity.

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