Limited-feedback Modified Block Diagonalization for Multiuser MIMO Downlink With Time-Varying Channels

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***Abstract*—Block diagonalization (BD) is a low-complexity linear precoding scheme for multiuser MIMO (MU-MIMO) downlink, which can completely eliminate multi-user interference with perfect channel state information (CSI) at the transmitter. Under the assumption of block fading channels, BD with fixed amount of feedback is interference-limited. In this paper, we first introduce a low-complexity CSI feedback scheme, by exploiting the temporal correlation of practical channels, to improve the efficiency of feedback. Then, a modified BD algorithm is proposed to further utilize the channel correlation in time domain. Combined with the CSI feedback scheme, the modified BD algorithm handles the interference-limiting problem in a wide range of SNR with a fixed, small number of feedback bits.**

***Index Terms*—block diagonalization, precoding, limited feedback, multi-user MIMO, time-varying channels**

I. Introduction

The design of multiuser MIMO downlink systems has drawn considerable research interests, because of the potentially high capacity of these systems. To put the advantages of multiuser MIMO downlink into practice, various transmission schemes have been proposed. One of these schemes is the low-complexity linear precoding technique named block diagonalization [1] [2]. With perfect channel state information at the transmitter (CSIT), BD can achieve the full multiplexing gain, along with a sum rate close to the capacity of the multiuser downlink system [3].

In frequency-division duplexing (FDD) systems, the assumption of perfect CSIT is unrealistic because of the infinite amount of feedback required. As a result, the performance of block diagonalization with imperfect CSIT was studied in [4]. It was proved that, in block fading channels, limited-feedback BD with random quantization codebooks is interference-limited if the number of feedback bits is fixed, which means that the sum rate no longer increases with SNR beyond a certain threshold. To avoid this interference limitation, the number of feedback bits should scale linearly with SNR, which results in an increasingly large amount of feedback when SNR increases, and thus prohibitive complexity in choosing the optimal precoder from the codebook.

However, the assumption of block fading channels in [4] ignores the temporal correlation of channels that usually exists in practical systems. In fact, channel correlation in time domain have already been used for feedback reduction in single-user MIMO systems. In [5]–[8], the correlation between adjacent precoders in time domain is exploited to quantize the precoder efficiently at the receiver. But this approach is not feasible to multi-user MIMO systems, where the receiver can not calculate its corresponding precoder with only the knowledge of its own channel. In [9], the transmitter continuously updates the precoder according to the 1-bit feedback from the receiver. Nevertheless, the overhead is increased by the periodical broadcast of perturbation matrices.

Another benefit from exploiting temporal correlation of channels is the possibility of reducing the complexity in calculating the precoders. Given the precoders at the previous time slot, we are able to obtain the current precoders in a simple manner due to the correlation between them. In [10], the calculation of BD precoders in time-varying channels is viewed as a subspace tracking problem and reformulated as an optimization problem. Then various methods for subspace tracking problems, such as the steepest descent method with constant step sizes in [11] and the conjugate gradient method with optimal step sizes in [12], can be used to update the precoders. Nonetheless, conventional BD algorithms only require the precoder for a certain user to lie in the channel matrices of the other users'. Consequently, the resulting precoder is independent on the channel of the corresponding user. If the number of transmit antennas is larger than the sum of the number of receive antennas, which means that the dimension of the null space is larger than the number of columns in the precoding matrix, there is no inherent mechanism in conventional BD algorithms to select the best vectors from the null space.

In this paper, we propose a limited-feedback modified block diagonalization algorithm. First, we introduce a low-complexity CSI feedback scheme to efficiently quantize the temporally correlated channels, by using differential pulse

This work was supported by China's 863 Project (2006AA01Z282), National Basic Research Program of China(2007CB310608), Program for NCET (Shidong Zhou), and Tsinghua-Qualcomm Joint Research Program.

code modulation (DPCM in [13]). Then, we propose a modified BD algorithm to further reduce the complexity, by exploiting the correlation between adjacent precoders in time domain. The modified BD precoders are obtained by solving an optimization problem using steepest descent methods with optimal step sizes. The optimization problem is formulated properly, so that its solution inherently selects the vectors achieving relatively large sum rates from the null space when the number of transmit antennas exceeds that of all the receive antennas. Combined with the CSI feedback scheme, the modified BD algorithm handles the interference-limiting problem in a much larger range of SNR with a small, fixed amount of feedback, compared to random codebook based BD in [4].

The rest of this paper is organized as follows. Section II introduces the system model. In section III, the CSI feedback scheme is described. Section IV presents the modified BD algorithm, followed by simulation results in Section V. Finally, section VI concludes the paper.

II. System Model

Consider a MU-MIMO downlink system consisting of one base station with M_T antennas and K users, where the kth user has M_{R_k} antennas. At time t, the received signal at user k can be expressed as

$$\mathbf{y}_k[t] = \mathbf{H}_k[t] \cdot \textstyle\sum_{i=1}^{K} \mathbf{T}_i[t]\mathbf{s}_i[t] + \mathbf{n}_k[t] \ \ , \tag{1}$$

where $\mathbf{s}_k[t] \in \mathbb{C}^{L_k \times 1}$ and $\mathbf{T}_k[t] \in \mathbb{C}^{M_T \times L_k}$ are the data vector and the precoder for user k at the base station, respectively. $\mathbf{H}_k[t] \in \mathbb{C}^{M_{R_k} \times M_T}$ is the channel matrix from the base station to user k, and $\mathbf{n}_k[t] \in \mathbb{C}^{M_{R_k} \times 1}$ is the noise vector distributed as $\mathcal{CN}(\mathbf{0}, \sigma_{n_k}^2 \mathbf{I}_{M_{R_k}})$.

We impose a total transmit power constraint denoted as

$$\mathbb{E}\left\{\| \textstyle\sum_{k=1}^{K} \mathbf{T}_k[t]\mathbf{s}_k[t]\|_2^2\right\} \leq P_t \ \ . \tag{2}$$

Equal power allocation among all the data streams is assumed, which is asymptotically optimal at high SNR in terms of ergodic capacity. The data vector $\mathbf{s}_k[t]$ has zero mean, with the correlation matrix expressed as

$$\mathbb{E}\left\{\mathbf{s}_k[t]\mathbf{s}_k^H[t]\right\} = \frac{P_t}{L}\mathbf{I}_{L_k}, \tag{3}$$

where $L = \sum_{k=1}^{K} L_k$ is the total number of data streams. To satisfy the power constraint, we assume that $\mathbf{T}_k^H \mathbf{T}_k = \mathbf{I}_{L_k}, k = 1, \ldots, K$.

We also assume that $\sum_{k=1}^{K} M_{R_k} \leq M_t$, so that the multi-user interference can be completely eliminated.

III. CSI Feedback Scheme

In this section, we propose a CSI feedback scheme for time-varying channels, which uses DPCM in [13] for channel quantization and feedback reduction. First, the transmitter and the receivers estimate the current channel according to previous channel estimates. Then, the estimation errors are calculated and quantized at the receivers and sent back to the transmitter through zero-delay error-free feedback channels. Finally, the estimated CSI is adjusted based on the quantized estimation error. The details of this scheme are given below.

Let $h[t]$ be an arbitrary channel element in $\mathbf{H}_k[t]$ at time t, then its estimate $\hat{h}[t]$ is calculated at the transmitter and the kth receiver as

$$\hat{h}[t] = \textstyle\sum_{j=1}^{p} a_j^*[t] \cdot \hat{h}[t-j] + \mathcal{Q}(e[t]) \ \ , \tag{4}$$

where the coefficients $a_1[t], \ldots, a_p[t]$ of the p-order autoregressive (AR) model can be solved by Yule-Walker equation

$$\begin{bmatrix} r_t(0) & r_t(1) & \cdots & r_t(p-1) \\ r_t^*(1) & r_t(0) & \cdots & r_t(p-2) \\ \vdots & \vdots & & \vdots \\ r_t^*(p-1) & r_t^*(p-2) & \cdots & r_t(0) \end{bmatrix} \begin{bmatrix} a_1[t] \\ a_2[t] \\ \vdots \\ a_p[t] \end{bmatrix} = \begin{bmatrix} r_t(1) & r_t(2) & \ldots & r_t(p) \end{bmatrix}^H, \tag{5}$$

where $r_t(i) = \mathbb{E}\{\hat{h}[t]\hat{h}^*[t-i]\}$ is the autocorrelation function.

Assuming perfect CSI at the receiver, the estimation error $e[t]$ can be obtained by

$$e[t] = h[t] - \textstyle\sum_{j=1}^{p} a_j^*[t] \cdot \hat{h}[t-j] \ \ . \tag{6}$$

The receiver quantizes $e[t]$ and sends back the quantized estimation error $\mathcal{Q}(e[t])$ to the transmitter. To reduce the feedback, we basically use the following 2-bit quantization

$$\begin{aligned} \mathcal{Q}(e[t]) \ &= \ \begin{cases} +\delta, & \text{real}(e[t]) \geq 0 \\ -\delta, & \text{real}(e[t]) < 0 \end{cases} \\ &+ \ \ i \cdot \begin{cases} +\delta, & \text{imag}(e[t]) \geq 0 \\ -\delta, & \text{imag}(e[t]) < 0 \end{cases}, \end{aligned} \tag{7}$$

where the optimal value of δ is determined by simulation.

To further reduce the feedback overhead, we generate feedback information in a longer period than every time slot. Between two consecutive feedbacks, we use the same quantized estimation error to adjust the channel estimates. As a result, the average feedback bits of user k at each sample is given by

$$2 \cdot f_b T_s \cdot M_T M_{R_k}, \tag{8}$$

where f_b is the feedback frequency and T_s is the symbol interval.

IV. Modified Block Diagonalization

In this section, we first reformulate the calculation of precoders as an optimization problem. Then, we describe the modified BD precoding algorithm for time-varying channels. Finally, we analyze the complexity of the proposed algorithm.

A. Problem Formulation

We would like to maximize the sum rate of the MU-MIMO downlink system, under the constraint that no user experiences interference from other users. The constraint of zero multi-user interference results in a suboptimal solution, but it significantly reduces the computational complexity, compared to the optimal iterative precoding algorithms. For the purpose of further complexity reduction, we adopt a simple expression, instead of the sum rate, as the objective function, which makes it easy to compute the gradient. To sum up, we formulate the calculation of $\mathbf{T}_k$ for user k as the follwing optimization problem

$$\begin{aligned} \max_{\mathbf{T}_k} \quad & \operatorname{tr}\left(\mathbf{T}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{T}_k\right) \\ s.t. \quad & \mathbf{H}_j \mathbf{T}_k = \mathbf{0}, j = 1, \ldots, K, j \neq k \\ & \mathbf{T}_k^H \mathbf{T}_k = \mathbf{I}, \end{aligned} \tag{9}$$

where we drop the time index t of the channel matrices and precoders for brevity.

The optimal solution to this optimization problem is equivalent to the conventional BD precoders in [1] and [2], when the number of transmit antennas equals the total number of receive antennas. When the number of transmit antennas is larger than that of receive antennas, the solution to (9) achieves a higher sum rate than the conventional BD precoder does, with negligible increase in complexity.

B. Algorithm Description

To speed up the convergence, we decompose the original problem (9) into L_k subproblems and solve them in sequence. The solution $\mathbf{t}_{k,i}$ to the ith subproblem is the ith column of $\mathbf{T}_k$, which is also the precoder for the ith data stream of user k. The ith subproblem is as follows

$$\begin{aligned} \min_{\mathbf{t}_{k,i}} \quad & -\mathbf{t}_{k,i}^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{t}_{k,i} \\ s.t. \quad & \mathbf{H}_j \mathbf{t}_{k,i} = \mathbf{0}, j = 1, \ldots, K, j \neq k \\ & \mathbf{t}_{k,m}^H \mathbf{t}_{k,i} = 0, m = 1, \ldots, i-1 \\ & \mathbf{t}_{k,i}^H \mathbf{t}_{k,i} = 1, \end{aligned} \tag{10}$$

where $\mathbf{t}_{k,1}, \ldots, \mathbf{t}_{k,i-1}$ are the solutions to the first $i-1$ subproblems.

Using the Lagrange method, we put the linear equality constraints onto the objective function as penalty. Then the new objective function can be written as

$$\begin{aligned} f_{k,i}(\mathbf{t}_{k,i}) \quad = \quad & -\mathbf{t}_{k,i}^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{t}_{k,i} + c_1 \cdot \sum_{j \neq k} \|\mathbf{H}_j \mathbf{t}_{k,i}\|_2^2 \\ & + c_2 \cdot \sum_{m=1}^{i-1} \|\mathbf{t}_{k,m}^H \mathbf{t}_{k,i}\|_2^2, \end{aligned} \tag{11}$$

where c_1 and c_2 are positive penalty parameters. Now the problem (10) can be rewritten as

$$\begin{aligned} \min_{\mathbf{t}_{k,i}} \quad & f_{k,i}(\mathbf{t}_{k,i}) \\ s.t. \quad & \mathbf{t}_{k,i}^H \mathbf{t}_{k,i} = 1, \end{aligned} \tag{12}$$

The above problem can be solved by a gradient projection method of the following form [14]

$$\mathbf{t}_{k,i}^{n+1} = [\mathbf{t}_{k,i}^n + \alpha_{k,i}^n \cdot \mathbf{d}_{k,i}^n]_{\mathcal{X}}^+, \tag{13}$$

where the superscript n indicates the iteration number, $\mathbf{d}_{k,i}^n$ is the negative gradient direction of the objective function calculated as

$$\begin{aligned} \mathbf{d}_{k,i}^n \quad = \quad & -(-\mathbf{H}_k^H \mathbf{H}_k + c_1 \cdot \sum_{j \neq k} \mathbf{H}_j^H \mathbf{H}_j + c_2 \cdot \sum_{m=1}^{i-1} \mathbf{t}_{k,m}^n \mathbf{t}_{k,m}^{nH}) \\ & \cdot \mathbf{t}_{k,i}^n, \end{aligned} \tag{14}$$

TABLE I
ALGORITHM 1 - LIMITED-FEEDBACK MODIFIED BD ALGORITHM FOR USER k

1	**Initialize**: $t = 0$	
	a) Set $\mathbf{T}_k[0]$ as the first L_k columns of the identity matrix $\mathbf{I}_{M_t}$.	
2	**Receive the CSI feedback**: $t \leftarrow t + 1$	
	a) The base station receives the quantized estimation errors, which are generated by the CSI feedback scheme in Section III.	
	b) Updates the channel estimates $\widehat{\mathbf{H}}_1[t], \ldots, \widehat{\mathbf{H}}_K[t]$.	
3	**Update the precoder for user** k:	
	for $i = 1, 2, \ldots, L_k$, solve (10) as follows	
	a) Calculate the descent direction	$O(M_T \sum_i M_{R_i})$
	$\mathbf{d}_{k,i}[t] = -(-\widehat{\mathbf{H}}_k^H[t]\widehat{\mathbf{H}}_k[t] + c_1 \cdot \sum_{j \neq k} \widehat{\mathbf{H}}_j^H[t]\widehat{\mathbf{H}}_j[t]$ $+\ c_2 \cdot \sum_{m=1}^{i-1} \mathbf{t}_{k,m}[t]\mathbf{t}_{k,m}^H[t]) \cdot \mathbf{t}_{k,i}[t-1]$.	
	b) Calculate the step size $\alpha_{k,i}[t]$ according to (16).	$O(M_T^2)$
	c) Update the precoder	$O(M_T)$
	$\mathbf{t}_{k,i}[t] = [\mathbf{t}_{k,i}[t-1] + \alpha_{k,i}[t] \cdot \mathbf{d}_{k,i}[t]]_{\mathcal{X}}^+$.	
4	Go to Step 2.	

$[\cdot]_{\mathcal{X}}^+$ denotes the projection on the feasible set $\mathcal{X}$ denoted as

$$\mathcal{X} = \{\mathbf{x} | \mathbf{x}^H \mathbf{x} = 1\}, \tag{15}$$

and $\alpha_{k,i}^n$ is the step size determined by the minimization rule

$$\alpha_{k,i}^n = \arg\min_{\alpha_{k,i}^n} f_{k,i}(\mathbf{t}_{k,i}^{n+1}). \tag{16}$$

The projection of an arbitrary vector $\mathbf{x}_0$ on the feasible set $\mathcal{X}$ is simply given as

$$[\mathbf{x}_0]_{\mathcal{X}}^+ = \mathbf{x}_0 / (\mathbf{x}_0^H \mathbf{x}_0)^{\frac{1}{2}}. \tag{17}$$

The step size $\alpha_{k,i}^n$ can be calculated analytically. The detailed derivation and expression for $\alpha_{k,i}^n$ can be found in [12, Appendix].

In practice, we do not solve the optimization problem (9) completely at each time slot. Instead, using the precoders at the previous time slot as initial conditions, we only perform one iteration for the optimization problem at the current time slot. The modified BD precoding algorithm with proposed CSI feedback scheme is summarized in Table I. The computational complexity is also listed for the corresponding operations.

C. Complexity Issue

In this subsection, we compare the computational complexity of the proposed BD algorithm with that of conventional BD and codebook-based BD. We can see from Table I that the proposed algorithm has a complexity of $O(M_T^2 L_k)$ for user k at each time slot. So the overall complexity is $O(M_T^2 \sum_{k=1}^K L_k)$ at each time slot. For the conventional BD algorithm, K SVD operations have to be performed at each sample, leading to an overall complexity of $O(K M_T^3)$. Since $\sum_{k=1}^K L_k \leq M_T$, the complexity of conventional BD is at least K times that of the proposed algorithm. For the codebook-based BD algorithm, the dominant complexity comes from the selection of precoders. Given a codebook of size 2^B and the selection rule of minimum Chordal distance [4], $O(2^B M_T \sum_{k=1}^K M_{R_k}^2)$

operations are performed in the selection. Thus, codebook-based BD usually has the highest complexity due to the large values of B in practice.

V. Simulation Results

A. System Configuration

In the simulation, we consider a downlink system consisting of one base station with $M_T = 6$ transmit antennas and $K = 2$ or $K = 3$ users, each of which is equipped with $M_{R_k} = 2$ receive antennas.

The channels from the base station to different users are independent and Rayleigh flat fading with temporal correlation given by Jakes model. For the proposed algorithm, we use the order-1 AR model to estimate the channel elements $h[t]$, where the autocorrelation functions in the Yuler-Walker equation (5) are approximated by the autocorrelation functions of the actual channels given by

$$r_t(i) \approx \mathbb{E}\{h[t]h^*[t-i]\} = J_0(2\pi f_d T_s i), \tag{18}$$

where $J_0(\cdot)$ is the zero-th order Bessel function of the first kind and f_d is the maximum Doppler shift. Since the channel correlation $r_t(i)$ remains unchanged, the coefficients $a_i[t]$ in the estimation (4) is fixed during the simualtion. The optimal δ in (7) are obtained through simulation. The initial estimate of an arbitrary channel element, $\hat{h}[1]$, is obtained by a 2-bit quantization on the real and imaginary parts of $h[1]$ with the set of reconstruction values as $\{-1, -0.5, 0, 0.5\}$. Consequently, user k should generate a feedback of $4M_T M_{R_k}$ bits for the first channel estimate $\widehat{\mathbf{H}}_k[1]$. After that, 2 bits are assigned for each channel element at each feedback.

B. Performance Comparison

In this subsection, we compare the proposed limited-feedback block diagonalization algorithm with conventional BD (BD in [1] [2]) and the random quantization codebook-based BD (BD-RQC in [4]), in terms of their throughput. The throughput of user k is calculated as

$$R_k = \mathbb{E}\left\{\log_2 \det\left(\mathbf{I}_{L_k} + \frac{P_t}{L \cdot \sigma_{n_k}^2}\mathbf{T}_k^H \mathbf{H}_k^H \mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{T}_k\right)\right\}, \tag{19}$$

where $\mathbf{R}_k$ is the interference-plus-noise covariance matrix

$$\mathbf{R}_k = \sum_{j\neq k} \frac{P_t}{L \cdot \sigma_{n_k}^2}\mathbf{H}_k\mathbf{T}_j\mathbf{T}_j^H\mathbf{H}_k^H + \mathbf{I}_{M_T}. \tag{20}$$

Fig. 1 compares the throughput of the three algorithms under different Doppler shifts in a $K = 3$ system. For the proposed algorithm, we choose $f_b T_s = 1/3$, which indicates a feedback interval of 3 samples. Neglecting the amount of feedback for the initial estimates, we can calculate the average per-user feedback rate as 8 bits per sample. For the conventional BD, we assume perfect CSIT at all the samples. For BD-RQC, we use an 8-bit random codebook to quantize the channel at each sample.

From the figure, we can see that the proposed BD with perfect CSIT has approximately the same throughput as that of conventional BD, except for the small rate loss in high SNR regime. This rate loss may come from the residual interference, which is caused by the fact that the proposed algorithm can not keep track on the optimal solution perfectly. With limited feedback and small Doppler shifts, the proposed algorithm performs closely to the case with perfect CSIT. But its performance degrades when the Doppler shift becomes larger. We can observe that the sum rate of BD-RQC stops growing at the SNR of about 20 dB and remains nearly constant at a relatively small value. On the contrary, the throughput of the proposed algorithm increases linearly in a larger range of SNR and achieves a higher value at high SNR.

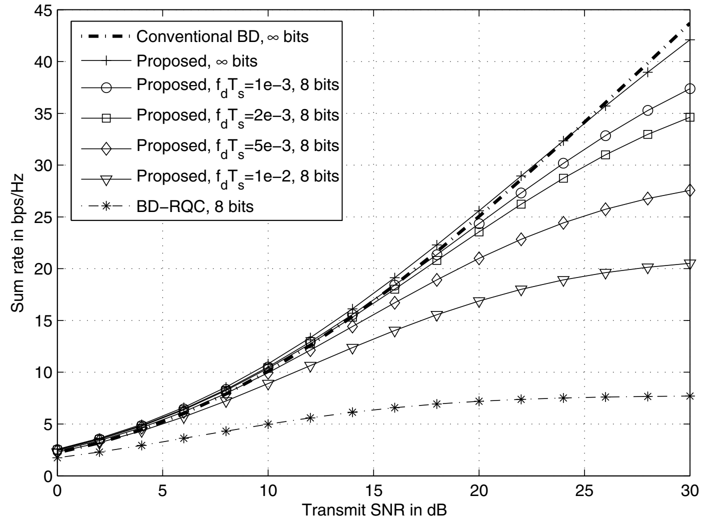

Fig. 1. Comparison of throughput under different Doppler shifts. $M_T = 6$, $K = 3$, $M_{R_k} = 2$, and $L_k = 2$, $\forall k$.

Fig. 2 shows the sum rates of the three algorithms in the $K = 2$ system with all the other parameters identical to those in Fig. 1. In this case, we can see that the proposed algorithm, even with limited feedback when the Doppler shift is relatively small, outperforms the conventional BD. This is because the proposed algorithm selects $L_k = 2$ vectors, which possibly achieves higher throughput, from the 4-dimensional null space. Similar to the $K = 3$ case, the proposed algorithm outperforms BD-RQC in 2-user systems.

Fig. 3 and Fig. 4 compare the sum rates of the proposed algorithm and BD-RQC with different amount of feedback in the coherence time of 1000 samples and 500 samples, respectively. The transmit SNR is fixed at 30 dB. Given a total feedback constraint, we adjust the codebook size and the feed-back interval of BD-RQC, and find the optimal combination of these two parameters. Note that we only consider the codebook with a size smaller than 20 bits, because a very large codebook is not feasible in practice due to the extremely high complexity in searching the optimal codeword. From the figure, we can observe that the proposed algorithm is advantageous over BD-RQC only when the total amount of feedback is relatively large. We can also notice that, when the amount of feedback is small, the throughput of 2-user systems are larger than that

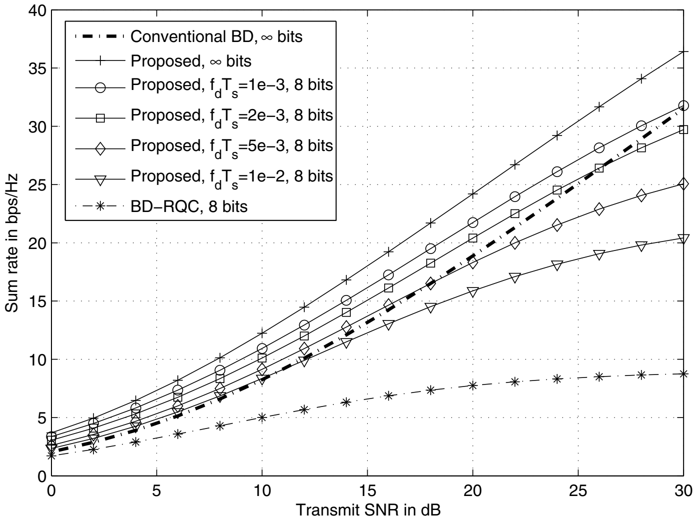

Fig. 2. Comparison of throughput under different Doppler shifts. $M_T = 6$, $K = 2$, $M_{R_k} = 2$, and $L_k = 2$, $\forall k$.

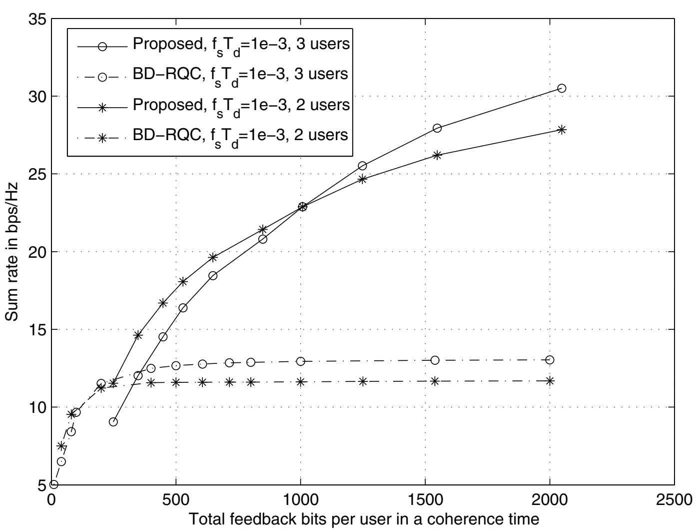

Fig. 3. Throughput comparison for the proposed algorithm and BD-RQC under different total feedback bits in a coherence time (1000 symbols). $M_T = 6$, $M_{R_k} = 2$, and $L_k = 2$, $\forall k$. The transmit SNR is fixed at 30dB.

of 3-user systems for both the proposed BD and BD-RQC, because of less multi-user interference in 2-user systems.

VI. Conclusion

In this paper, we have proposed a limited-feedback modified block diagonalization algorithm. First, we introduced a DPCM-based CSI feedback scheme to efficiently quantize the temporally correlated channels. Then, we proposed a modified BD to further reduce the complexity of calculating the precoders, which are obtained by solving a properly-designed optimization problem. Simulation results have shown that the optimization problem achieves higher throughput than that of conventional BD when the number of transmit antennas exceeds that of receive antennas. It has also been shown that the modified BD algorithm handles the interference-limiting

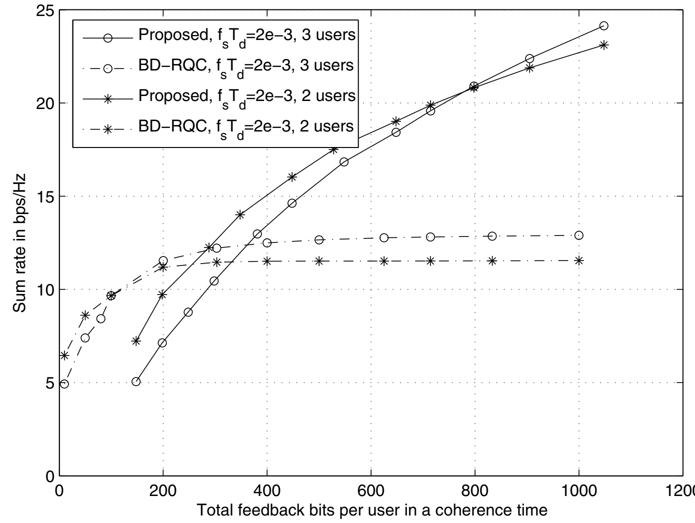

Fig. 4. Throughput comparison for the proposed algorithm and BD-RQC under different total feedback bits in a coherence time (500 symbols). $M_T = 6$, $M_{R_k} = 2$, and $L_k = 2$, $\forall k$. The transmit SNR is fixed at 30dB.

problem in a much larger range of SNR with a small, fixed amount of feedback, compared to random codebook based BD.

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