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Design and Analysis of Intervention Mechanisms in Power Control Games

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Abstract*—We study the power control problem in wireless ad hoc networks with *selfish* users. Without incentive mechanisms, selfish users transmit at their maximum power levels at the Nash equilibrium (NE), causing significant interference to each other. In order to induce users to transmit at desired power levels, existing works have proposed pricing and auctions as incentive mechanisms. With pricing or auctions, it is explicitly stated or implicitly assumed that the users are *obedient*, in that they adopt the utility functions designed by the system and accept the prices as control signals. In this paper, we use the intervention mechanism to incentivize selfish users to achieve efficient outcomes as the (unique) NE. In the intervention mechanism, a system designer prescribes a intervention rule and uses a intervention device to execute it. Depending on the monitoring technology and intervention capability of the intervention device, we propose two types of intervention rules with different performance and complexity tradeoffs. We study the performance achievable by the proposed intervention rules, as well as the design principles for different intervention rules. We prove that all the Pareto boundary can be achieved as the NE or even the unique NE of the game with intervention. Simulation results demonstrate the performance improvement achieved when using different intervention rules and illustrate performance analysis on different intervention rules.

I. INTRODUCTION

Power control is the essential resource allocation scheme to control the signal-to-interference-and-noise ratio (SINR) for efficient transmission in wireless networks. Extensive studies have been done on power control; see [1] and the reference therein for an overview of this topic. The early works on power control assign a fixed SINR requirement to each user, where each user minimizes its transmit power subject to the fixed minimum SINR requirement [1, Ch. 2] [2] [3]. This formulation is suitable for fixed-rate communications with voice applications. However, with the growth of data and multimedia applications, most recent works formulate the problem in a network utility maximization framework. In this framework, a central controller can calculate the optimal transmit power levels, when the utility functions are such that the network utility maximization problem is convex, and then assigns the optimal power levels to the users. Assuming that the users cooperate with the central controller, the problem can also be solved in a distributed fashion [1, Ch. 4] [4]- [7].

Besides the network utility maximization framework, many works use noncooperative games to model the distributed power control problem, in which each user maximizes its own utility, instead of maximizing the sum utility. In the noncooperative game model, each user tends to transmit at its maximum power level to get higher throughput, causing significant interference to each other. This outcome may be far from the global optimality of the social welfare [1] [5] [8], especially when the interferences are strong [9]. To improve the non-cooperative outcome, several incentive schemes, such as pricing and auctions, have been proposed [10]- [14]. However, we argue that in the works with pricing or auction mechanisms, noncooperative games are just used to model the distributed power control problem, whereas the users are obedient but not selfish.

Pricing mechanisms [10]- [13] and auction mechanisms [14] impose cost in the users' utility by charging for the transmit power. In either mechanism, the prices are used as control signals rather than real monetary exchanges. In this regard, the users's utility functions are designed by the system, so that the outcome of the game can be efficient. Therefore, the users are *obedient*, in that they maximize the utility functions given by the system. On the contrary, if the users are *selfish*, they should maximize their own innate utility functions such as the throughput, ignoring such control signals as pricing if by doing so they get better off.

If the pricing or auction mechanism does involve real monetary charges to the selfish users, in order to achieve an efficient outcome, the system designer needs to know how each user values their Quality of Service (QoS) in money. In other words, the designer needs to know the user's utility function, which is the private information that selfish users have no incentive to expose.

Repeated game has been proposed in [15] [16] as another incentive mechanism to improve the non-cooperative outcomes. However, in repeated games, the users have long-run frequent interactions before the equilibrium is achieved, which usually requires an infinite horizon and sufficiently patient users [21].

In this paper, we apply intervention mechanisms [17]- [20] in power control games to enforce selfish users to transmit at desired power levels. In intervention mechanisms, there is an *intervention device* operating according to the *intervention rule* designed by the system: it estimates the individual transmit power of each user or the aggregate receive power at the intervention device, and then transmits at a certain power level, determined as a function of its estimation. If the users are transmitting at the desired power levels, the intervention device will transmit minimum, probably zero, power. Once the users deviate, the intervention device will transmit at a positive power level, which causes interference to the users and directly reduces the users' SINR. In this way, the intervention mechanism presents a credible threat to the selfish users without knowing their utility functions. It is worthwhile noting that ideally, at the equilibrium, the intervention device will not transmit due to the good behavior of the users. Hence, there is no performance loss due to possible interference from the intervention device. This can be achieved when the intervention device has perfect monitoring and sufficient intervention capability.

The rest of the paper is organized as follows. In section II, we will describe the system model and formulate the problem of designing the intervention mechanisms. In section III, we study the proposed intervention rules and provide guidelines for the design of the proposed intervention rules. Simulation results are shown in section IV. Finally, section V concludes the paper.

II. System Model and Problem Formulation

A. System Model

We consider a wireless ad hoc network with N users (see Fig. 1 for an example two-user network). The set of the users is denoted by $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$. Each user has a transmitter and a receiver. Each user i chooses its transmit power p_i in the set $\mathcal{P}_i \triangleq [0, P_i]$, where $P_i > 0$ for all $i \in \mathcal{N}$. The power profile of the users is denoted by $\mathbf{p} = (p_1, \ldots, p_N) \in \mathcal{P} \triangleq \prod_{i=1}^{N} \mathcal{P}_i$, and the power profile of all the users other than user i is denoted by $\mathbf{p}_{-i}$.

In the network, there is an *intervention device* that consists of a transmitter and a receiver. The receiver of the intervention device can monitor the power profile of the users, while the transmitter can create interference to the users by transmitting power. After observing the users' power profile, the intervention device chooses its own transmit power p_0 in the set $\mathcal{P}_0 \triangleq [0, P_0]$, where $P_0 > 0$ is the *intervention capability* of the intervention device.

We assume that all the users and the intervention device transmit in the same frequency. Hence, no user can avoid the interference (punishment) of the intervention device by hopping to another frequency channel. If there were multiple frequency channels available, the designer could place one intervention device in each frequency channel. In this way, the system with multiple channels can be decomposed into independent single-channel systems studied in this paper. We also assume that the

We index the intervention device by 0. For $i, j \in \mathcal{N} \cup \{0\}$, let $h_{ij} > 0$ be the channel gain from user j's transmitter to user i's receiver, and let $n_i > 0$ be the noise power at user i's receiver. When the intervention device chooses a power p_0 and the users choose a power profile $\mathbf{p}$, the SINR of user $i \in \mathcal{N}$ is given by

$$\gamma_i(p_0, \mathbf{p}) = \frac{h_{ii} p_i}{h_{i0} p_0 + \sum_{j \neq i} h_{ij} p_j + n_i}. \tag{1}$$

We assume that each user $i \in \mathcal{N}$ has monotonic preferences on its own SINR in the sense that it weakly prefers γ_i to γ_i' if and only if $\gamma_i \geq \gamma_i'$. Our analysis does not require any other properties of preferences (for example, preferences do not need to be represented by a concave utility function).

In our setting, the intervention device has a receiver to measure the aggregate receive power from all the users. Furthermore, if the receiver moves and takes measurement at different locations, it can estimate the individual transmit power of each user as well. Thus, in this paper we will focus on two types of *monitoring technology* with which the intervention device can estimate individual transmit powers $\mathbf{p}$ or an aggregate receive power $\sum_{i=1}^{N} h_{0i} p_i$. Here we assume that estimation is perfect; the scenarios with imperfect estimation will be studied in future work.

The strategy of the intervention device can be represented by a mapping $f : \mathcal{P} \to \mathcal{P}_0$, which is called an intervention rule. The SINR of user i when the intervention device uses an intervention rule f and the users choose a power profile $\mathbf{p}$ is given by $\gamma_i(f(\mathbf{p}), \mathbf{p})$. With an abuse of notation, we will use $\gamma_i(f, \mathbf{p})$ to mean $\gamma_i(f(\mathbf{p}), \mathbf{p})$. Given an intervention rule f, the interaction among the users that choose their own power levels selfishly can be modeled as a non-cooperative game, whose strategic form is given by

$$\Gamma_f = \langle \mathcal{N}, (\mathcal{P}_i)_{i \in \mathcal{N}}, (\gamma_i(f, \cdot))_{i \in \mathcal{N}} \rangle. \tag{2}$$

We can predict the power profile chosen by the users given an intervention rule using the concept of Nash equilibrium.

Definition 1: A power profile $\mathbf{p}^* \in \mathcal{P}$ is a Nash equilibrium (NE) of the game Γ_f if

$$\gamma_i(f, \mathbf{p}^*) \geq \gamma_i(f, p_i, \mathbf{p}_{-i}^*) \tag{3}$$

for all $p_i \in \mathcal{P}_i$, for all $i \in \mathcal{N}$.

When a power profile $\mathbf{p}^*$ is a NE of Γ_f, no user has an incentive to deviate from $\mathbf{p}^*$ unilaterally provided that the intervention device uses intervention rule f. Moreover, if $\mathbf{p}^*$ is a unique NE of Γ_f, intervention has added robustness in that we do not need to worry about coordination failure (i.e., the possibility that the users get stuck in a "wrong" equilibrium).

B. Problem Formulation

In this paper, we assume that the designer desires to achieve a target power profile, denoted by $\mathbf{p}^\star$, as in [13], with minimum possible intervention. Thus, the design problem is to find an intervention rule f such that $\mathbf{p}^\star$ is the (unique) NE of Γ_f and $f(\mathbf{p}^\star) = 0$. For better exposition, we define the concept of *(strongly) sustainment* as follows.

Definition 2: An intervention rule f *(strongly) sustains* a power profile $\mathbf{p}^\star$ if $\mathbf{p}^\star$ is a (unique) NE of the game Γ_f.

We use $\mathcal{E}(f)$ to denote the set of all power profiles sustained by f. Then the design problem can be stated mathematically as finding an intervention rule f such that $\mathbf{p}^\star \in \mathcal{E}(f)$ (or $\{\mathbf{p}^\star\} = \mathcal{E}(f)$) and $f(\mathbf{p}^\star) = 0$.

Given a target power profile $\mathbf{p}^\star$, there are potentially many intervention rules f that satisfy the design criteria $\mathbf{p}^\star \in \mathcal{E}(f)$ and $f(\mathbf{p}^\star) = 0$. Thus, below we propose three performance metrics with which we can evaluate different intervention rules satisfying the design criteria.

- Monitoring requirement: The information about the power profile (individual transmit powers or aggregate receive power) required for the intervention device to execute a given intervention rule.
- Intervention capability requirement: The minimum intervention capability needed for the intervention device to execute a given intervention rule, i.e., $\sup_{\mathbf{p}\in\mathcal{P}} f(\mathbf{p})$. (Even though there is no intervention at an equilibrium, the intervention device should have an intervention capability $P_0 \geq \sup_{\mathbf{p}\in\mathcal{P}} f(\mathbf{p})$ in order to make the intervention rule f credible to the users.)
- Strong sustainment: Whether a given intervention rule strongly sustains the target power profile $\mathbf{p}^\star$.

Without loss of generality, we can express an intervention rule f satisfying $f(\mathbf{p}^\star) = 0$ as $f(\mathbf{p}) = [g(\mathbf{p})]_0^{P_0}$, where $[x]_a^b = \min\{\max\{x, a\}, b\}$, for some function $g : \mathcal{P} \to \mathbb{R}$ such that $g(\mathbf{p}^\star) = 0$. Also, since the designer desires to achieve $\mathbf{p}^\star$, it is natural to consider functions g that increase as the users deviate from $\mathbf{p}^\star$. Hence, we consider the following two simple class of intervention rules,

$$\mathcal{F}_I(\mathbf{p}^\star) = \left\{ f_I : f_I(\mathbf{p}) = \left[\sum_{i=1}^N \alpha_i |p_i - p_i^\star|\right]_0^{P_0}, \alpha_i \geq 0 \right\} \quad (4)$$

and

$$\mathcal{F}_A(\mathbf{p}^\star) = \left\{ f_A : f_A(\mathbf{p}) = \left[\alpha_0 \left| \left(\sum_{i=1}^N h_{0i} p_i\right) - p_A^\star \right|\right]_0^{P_0} \right\} \quad (5)$$

with $\alpha_0 \geq 0$ and $p_A^\star = \sum_{i=1}^N h_{0i} p_i^\star$.

We call an intervention rule $f \in \mathcal{F}_I(\mathbf{p}^\star)$ a first-order intervention rule based on individual transmit power, and an intervention rule $f \in \mathcal{F}_A(\mathbf{p}^\star)$ a first-order intervention rule based on aggregate receive power. As we can see from their definitions, the distinction between these two classes of intervention rules comes from the monitoring technology of the intervention device. We could also define intervention rules with higher orders by using $|p_i - p_i^\star|^k$, which contains more intervention rules, but at the same time complexity increases. Simple intervention rules are desirable for the designer, the users, and the intervention device. Thus, our analysis mainly focuses on first-order intervention rules.

Let $\tilde{\mathcal{F}}_I(\mathbf{p}^\star)$ ($\tilde{\mathcal{F}}_I^s(\mathbf{p}^\star)$) be the set of first-order intervention rules based on individual transmit powers that (strongly) sustains $\mathbf{p}^\star$, i.e., $\tilde{\mathcal{F}}_I(\mathbf{p}^\star) = \{f \in \mathcal{F}_I(\mathbf{p}^\star) : \mathbf{p}^\star \in \mathcal{E}(f)\}$ and $\tilde{\mathcal{F}}_I^s(\mathbf{p}^\star) = \{f \in \mathcal{F}_I(\mathbf{p}^\star) : \{\mathbf{p}^\star\} = \mathcal{E}(f)\}$. [1] We define the *minimum power budget* for a first-order intervention rule based on individual transmit powers to (strongly) sustain $\mathbf{p}^\star$ by

$$PB_I(\mathbf{p}^\star) = \inf_{f\in\tilde{\mathcal{F}}_I(\mathbf{p}^\star)} \sup_{\mathbf{p}\in\mathcal{P}} f(\mathbf{p}) \quad (6)$$

and

$$PB_I^s(\mathbf{p}^\star) = \inf_{f\in\tilde{\mathcal{F}}_I^s(\mathbf{p}^\star)} \sup_{\mathbf{p}\in\mathcal{P}} f(\mathbf{p}). \quad (7)$$

Thus, with an intervention capability $P_0 > PB_I(\mathbf{p}^\star)$ ($P_0 > PB_I^s(\mathbf{p}^\star)$), there exists a first-order intervention rule based on individual transmit powers that (strongly) sustains $\mathbf{p}^\star$. We set $PB_I(\mathbf{p}^\star) = +\infty$ ($PB_I^s(\mathbf{p}^\star) = +\infty$) if there is no such intervention rule that (strongly) sustains $\mathbf{p}^\star$. The difference $PB_I^s(\mathbf{p}^\star) - PB_I(\mathbf{p}^\star)$ can be interpreted as the price of strong sustainment in terms of the minimum power budget.

III. Design and Analysis of First-Order Intervention

A. Intervention Rules Based on Individual Transmit Powers

We consider first-order intervention rules of the form

$$f_I(\mathbf{p}) = \left[\sum_{i=1}^N \alpha_i |p_i - p_i^\star|\right]_0^{P_0}. \quad (8)$$

Under the above intervention rule, the intervention device increases its transmit power linearly with the deviation of each user from the target power, $|p_i - p_i^\star|$, in the range of its intervention capability. We call α_i the *intervention rate* for user i, which measures how sensitive intervention reacts to a deviation of user i. Let $\tilde{\mathcal{N}}(\mathbf{p}^\star) = \{i \in \mathcal{N} : p_i^\star < P_i\}$. Without loss of generality, we label the users in such a way that $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$ if and only if $i \leq N'$, where $N' = |\tilde{\mathcal{N}}(\mathbf{p}^\star)|$. Since the users have natural incentives to choose their maximum powers in the absence of intervention, we need to provide incentives only for the users in $\tilde{\mathcal{N}}(\mathbf{p}^\star)$. The following theorem shows that when the intervention capability is sufficiently large, the designer can always find intervention rates to have a given target power profile $\mathbf{p}^\star$ sustained by a first-order intervention rule.

Theorem 1: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$, $\mathbf{p}^\star \in \mathcal{E}(f_I)$ if and only if for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$,

$$\alpha_i \geq \left(\sum_{j\neq i} h_{ij} p_j^\star + n_i\right) / (p_i^\star h_{i0}) \quad (9)$$

and

$$P_0 \geq (P_i - p_i^\star)\left(\sum_{j\neq i} h_{ij} p_j^\star + n_i\right) / (p_i^\star h_{i0}). \quad (10)$$

Proof: See Appendix B of [23]. ■

We can explain the minimum intervention rate for user i, expressed in the right-hand side of (9), as follows. As h_{i0} is larger, intervention causes more interference to user i with the same transmit power, and thus the intervention rate for user i can be chosen smaller to yield the same interference. When $\sum_{j\neq i} h_{ij} p_j^\star + n_i$ is large, interference to user i from other users and its noise power are already strong, and thus the intervention rate for user i should be large in order for intervention to be effective. Hence, $h_{i0} / (\sum_{j\neq i} h_{ij} p_j^\star + n_i)$ can be interpreted as the effectiveness of intervention to user

[1] For the intervention rules based on aggregate receive power, we use similar definitions on $\tilde{\mathcal{F}}_A(\mathbf{p}^\star)$ ($\tilde{\mathcal{F}}_A^s(\mathbf{p}^\star)$) and $PB_A(\mathbf{p}^\star)$ ($PB_A^s(\mathbf{p}^\star)$). We omit them due to the space limit.

i. Without intervention, the users have natural incentives to increase their transmit powers. Thus, as the target power for user i, $p_i^\star$, is smaller, the incentive for user i to deviate is stronger, and thus a larger intervention rate is needed to prevent deviation. In summary, α_i should be chosen larger as intervention is less effective to user i and user i has a stronger incentive to deviate. Note that $(P_i - p_i^\star)$ is the maximum possible deviation by user i (in the direction where it has a natural incentive to deviate). The minimum intervention capability, expressed in the right-hand side of (10), is increasing with the maximum possible deviation and the strength of the incentive to deviate while decreasing with the effectiveness of intervention. Note that the minimum intervention capability is independent of the choice of intervention rates.

A first-order intervention rule f_I satisfying the conditions in Theorem 1 may have a NE other than the target power profile $\mathbf{p}^\star$. For example, if $P_0 \leq \sum_{j\neq i} \alpha_j (P_j - p_j^\star)$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$, $\mathbf{P}$ is also sustained by f_I. The presence of this extra NE is undesirable since it brings a possibility that the users still choose $\mathbf{P}$ while the intervention device causes interference to the users by transmitting its maximum power P_0. Obviously, this outcome $(P_0, \mathbf{P})$ is worse for every user than the outcome at the unique NE without intervention $(0, \mathbf{P})$. In order to eliminate this possibility, the designer may want to choose an intervention rule that strongly sustains the target power profile. The following theorem provides a sufficient condition for a first-order intervention rule to strongly sustain a given target power profile.

Theorem 2: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$, $\{\mathbf{p}^\star\} = \mathcal{E}(f_I)$ if

$$\alpha_i > \frac{1}{p_i^\star} \sum_{j>i} \alpha_j (P_j - p_j^\star) + \frac{\sum_{j<i} h_{ij} p_j^\star + \sum_{j>i} h_{ij} P_j + n_i}{p_i^\star h_{i0}} \tag{11}$$

and

$$P_0 > \frac{P_i}{p_i^\star} \sum_{j>i} \alpha_j (P_j - p_j^\star) + \frac{(P_i - p_i^\star)(\sum_{j<i} h_{ij} p_j^\star + \sum_{j>i} h_{ij} P_j + n_i)}{p_i^\star h_{i0}} \tag{12}$$

for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$.[2]

Proof: See Appendix C of [23]. ■

By comparing Theorems 1 and 2, we can see that the requirements for the intervention rates and the intervention capability is higher when we impose strongly sustainment. For any given power profile, the intervention rates can be chosen sequentially to satisfy the condition (11) starting from user N' down to user 1. We can set $\alpha_i = 0$ for all $i \notin \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Unlike Theorem 1, the choice of the intervention rates affects the minimum required intervention capability. For strong sustainment, the intervention capability is required to be larger as the designer chooses larger intervention rates.

[2] We define $\sum_{j\in J} x_j = 0$ and $\prod_{j\in J} x_j = 1$ if J is empty.

From Theorem 1, we obtain

$$PB_I(\mathbf{p}^\star) = \max_i \frac{(P_i - p_i^\star)(\sum_{j\neq i} h_{ij} p_j^\star + n_i)}{p_i^\star h_{i0}}. \tag{13}$$

Since Theorem 2 gives a sufficient condition for strong sustainment, we obtain an upper bound on $PB_I^s(\mathbf{p}^\star)$,

$$\overline{PB}_I^s(\mathbf{p}^\star) = \sum_{i=1}^{N} \left[\left(\prod_{j=1}^{i-1} \frac{P_j}{p_j^\star} \right) \cdot \frac{(P_i - p_i^\star)(\sum_{j<i} h_{ij} p_j^\star + \sum_{j>i} h_{ij} P_j + n_i)}{p_i^\star h_{i0}} \right]. \tag{14}$$

Note that $PB_I(\mathbf{p}^\star) \leq \overline{PB}_I^s(\mathbf{p}^\star)$ with equality if and only if $N' \leq 1$. Combining these results, we can bound $PB_I^s(\mathbf{p}^\star)$ by

$$PB_I(\mathbf{p}^\star) \leq PB_I^s(\mathbf{p}^\star) \leq \overline{PB}_I^s(\mathbf{p}^\star). \tag{15}$$

By Theorems 1 and 2, we know that all the feasible power profiles can be (strongly) sustained. This suggests that we gain nothing by using higher-order intervention rules, in terms of what power profile they can sustain.

B. Intervention Rules Based on Aggregate Receive Power

The results in this section so far relies on the ability of the intervention device to estimate individual transmit powers. However, estimating individual transmit powers requires larger monitoring overhead for the intervention device than estimating aggregate receive power. In order to study intervention rules that can be executed with the monitoring of aggregate receive power, we consider the intervention rules based on aggregate receive power

$$f_A(\mathbf{p}) = \left[\alpha_0 \left| \left(\textstyle\sum_{i=1}^N h_{0i} p_i \right) - p_A^\star \right| \right]_0^{P_0} \tag{16}$$

with $\alpha_0 \geq 0$. We call α_0 the aggregate intervention rate, and call $p_A^\star$ the target aggregate power, which is set as the aggregate receive power at the target power profile, i.e., $p_A^\star = \sum_{i=1}^N h_{0i} p_i^\star$. We first give a necessary and sufficient condition for an intervention rule based on aggregate power to sustain a target power profile.

Theorem 3: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$, $\mathbf{p}^\star \in \mathcal{E}(f_A)$ if and only if for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$,

$$\alpha_0 \geq (\textstyle\sum_{j\neq i} h_{ij} p_j^\star + n_i)/(h_{0i} p_i^\star h_{i0}) \tag{17}$$

and

$$P_0 \geq (P_i - p_i^\star)(\textstyle\sum_{j\neq i} h_{ij} p_j^\star + n_i)/(p_i^\star h_{i0}). \tag{18}$$

Proof: See Appendix H of [23]. ■

The minimum intervention capability required to sustain a target profile is not affected by using aggregate receive power instead of individual transmit powers. However, the aggregate intervention rate should be chosen high enough to prevent a deviation of any user, whereas with the monitoring of individual transmit powers the intervention rates can be chosen individually for each user. This suggests that strong sustainment is more difficult with intervention rules based on aggregate power. For example, $\mathbf{P}$ is also sustained by

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f_A if $P_0 \leq \alpha_0 \sum_{j \neq i}(h_{0j}P_j - p_j^\star)$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$, which is weaker than the corresponding condition in the case of intervention rules based on individual powers, $P_0 \leq \sum_{j \neq i} \alpha_j (P_j - p_j^\star)$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$. With the monitoring of individual powers, a deviation of each user can be detected and punished. This leads to the property that the best response of user i is almost always either $p_i^\star$ or P_i under first-order intervention rules based on individual powers. This implies that a power profile sustained by a first-order intervention rule based on individual powers almost always belongs to the set $\prod_i \{p_i^\star, P_i\}$. In contrast, with the monitoring of aggregate power, only an aggregate deviation can be detected. This yields a possibility that an intervention rule based on aggregate power sustains a power profile that is different from the target but yields the same aggregate power. This possibility makes the problem of coordination failure more worrisome because if the users are given only the target aggregate power $p_A^\star$ they may not know which power profile to select among those that yield the aggregate power $p_A^\star$.[3] The problem arising from the increased degree of non-uniqueness can be considered as the cost of reduced monitoring overhead. To state the result formally, let $\alpha_0^i = (\sum_{j \neq i} h_{ij} p_j^\star + n_i)/(h_{0i} p_i^\star h_{i0})$ and $P_0^i = (P_i - p_i^\star)(\sum_{j \neq i} h_{ij} p_j^\star + n_i)/(p_i^\star h_{i0})$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Also, let $\bar{\alpha}_0 = \max_{i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)} \alpha_0^i$ and $\bar{P}_0 = \max_{i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)} P_0^i$.

Theorem 4: Suppose that, for $\mathbf{p}^\star \in \prod_i (0, P_i]$, there exist $i, j \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$ such that (i) $\bar{\alpha}_0 = \alpha_0^i > \alpha_0^j$ or $\alpha_0^i, \alpha_0^j < \bar{\alpha}_0$, and (ii) $\bar{P}_0 = P_0^i > P_0^j$ or $P_0^i, P_0^j < \bar{P}_0$. Then for any f_A such that $\mathbf{p}^\star \in \mathcal{E}(f_A)$ and for any $\epsilon > 0$, there exists $\tilde{\mathbf{p}} \neq \mathbf{p}^\star$ such that $\tilde{\mathbf{p}} \in \mathcal{E}(f_A)$, $\sum_{i=1}^N h_{0i} p_i^\star = \sum_{i=1}^N h_{0i} \tilde{p}_i$, and $|\tilde{\mathbf{p}} - \mathbf{p}^\star| < \epsilon$.

Proof: See Appendix I of [23]. ■

Theorem 4 provides a sufficient condition under which the strong sustainment of a given target power profile is impossible with intervention rules based on aggregate power. We argue that the sufficient condition is mild. First, note that, for almost all $\mathbf{p}^\star \in \prod_i (0, P_i]$, α_0^i's and P_0^i's can be ordered strictly. With strict ordering of α_0^i's and P_0^i's, we can always find a pair of users $i, j \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$ satisfying the condition in Theorem 4 if there are at least three users in $\tilde{\mathcal{N}}(\mathbf{p}^\star)$. That is, strong sustainment is generically impossible with intervention rules based on aggregate power when $|\tilde{\mathcal{N}}(\mathbf{p}^\star)| \geq 3$.

IV. Simulation Results

We consider a two-user network shown in Fig. 1. The transmitter and the receiver of the intervention device are co-located. User 2's transmitter is near to user 1's receiver, causing significant interference to user 1. The distance from user 1's transmitter to its receiver is normalized to 1. Originally, the distance from user 2's transmitter to its receiver is 0.5. The distance between the two users' receivers is 0.5. Without specific notice, we assume that the positions of the transmitters and receivers of both users remain the same. In the simulation for Fig. 2, we let user 2's transmitter moves away from its receiver (as shown by the dash left arrow), resulting in less interference to user 1. We assume that the path loss exponent is 3. The power budgets of both users are 10.

[3] A way to overcome this problem is to broadcast the target power profile $\mathbf{p}^\star$ to the users in order to make $\mathbf{p}^\star$ as a focal point [22].

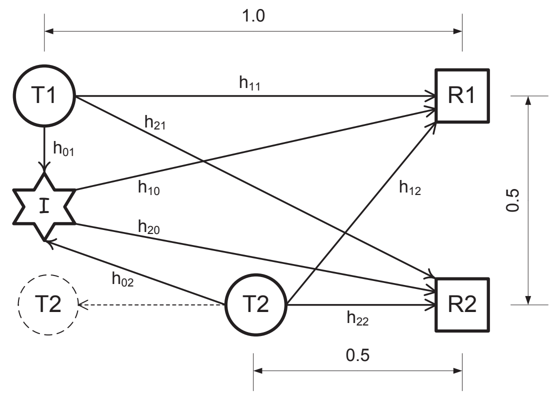

Fig. 1. An example wireless ad-hoc network with two users.

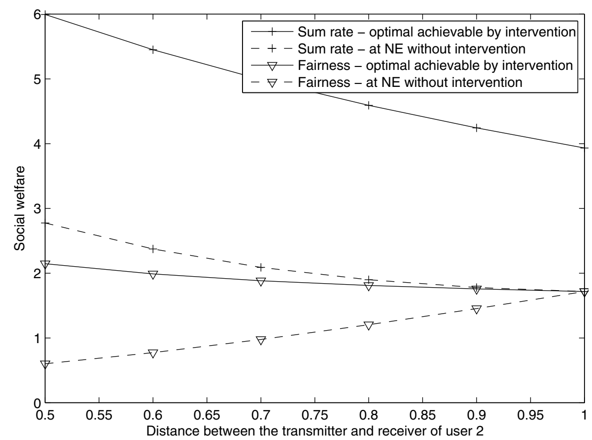

Fig. 2. The optimal social welfare achievable by intervention and the social welfare at NE without intervention, when user 2's transmitter moves away from its receiver.

A. Improvement on Social Welfare by Intervention

Now we examine the performance improvement by using intervention mechanisms. We let user 2's transmitter moves away from its receiver. In Fig. 2, we show the performance achieved by intervention and that at the NE without intervention, under two criteria for social welfare. The sum rate is define by $\log(1 + \gamma_1) + \log(1 + \gamma_2)$, and the fairness is defined by $\log(1 + \min\{\gamma_1, \gamma_2\})$.

As we can see from Fig. 2, the sum rate achievable by intervention doubles that at the NE without intervention in all the cases. The fairness achievable by intervention is much larger than that at the NE without intervention in most cases. When the distance from user 2's transmitter to its receiver is 1.0, the network is symmetric. Only at this point is the NE without intervention optimal.

B. Minimum Power Budget

Now we show the power budget requirement for different intervention rules. In Fig. 3-4, we show the contour of the

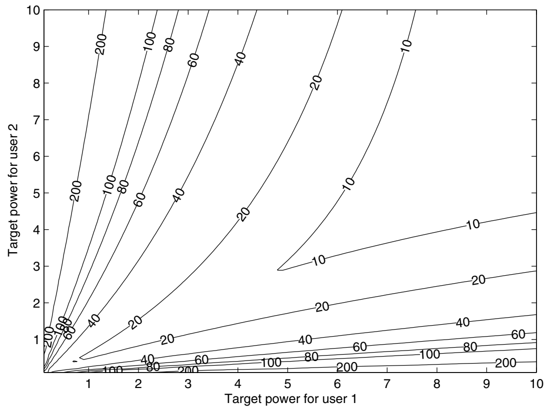

Fig. 3. Contour of the minimum power budget of first-order intervention that sustains a target power profile, when user 2's transmitter is near to user 1's receiver. Obtained by Theorem 1 and Theorem 3.

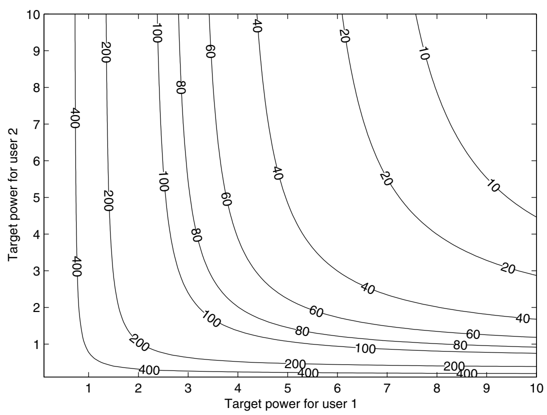

Fig. 4. Contour of the minimum power budget of first-order intervention based on individual transmit powers that strongly sustains a target power profile, when user 2's transmitter is near to user 1's receiver. Obtained by Theorem 2.

minimum power budget for different intervention rules under different target power profiles, when user 2's transmitter is at its original location. As we expect, a larger power budget is required to strongly sustain a target power profile.

V. Conclusion

In this paper, we studied the power control problem in wireless ad hoc networks with selfish users. Without incentive mechanisms, selfish users transmit at their maximum power levels at the Nash equilibrium (NE), resulting in inefficient operating points. To achieve efficient outcomes, we proposed intervention mechanism to induce selfish users to transmit at desired power levels. Different from other incentive mechanisms such as pricing and auctions, intervention mechanism punishes the users by directly decreasing their SINR's, thus provides a more credible threat to regulate the users' behaviors. We proposed two types of intervention rules with different monitoring technologies. We analyzed the performance of the proposed intervention rules in terms of the intervention capability requirement and the strong sustainment, and provided the design principles. All the Pareto boundary can be (strongly) sustained with intervention. Simulation results demonstrated the performance improvement achieved by using intervention and validated the performance analysis on different intervention rules.

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