Supply Function Equilibrium in Power Markets: Mesh Networks

Yuanzhang Xiao, *Member, IEEE,* Chaithanya Bandi, and Ermin Wei, *Member, IEEE,*

***Abstract*—We study deregulated power markets with strategic power suppliers. In deregulated markets, each supplier submits its supply function (i.e., the amount of electricity it is willing to produce at various prices) to the independent system operator (ISO), who takes the submitted supply functions as the true marginal cost functions, and dispatches the suppliers to clear the market. If all suppliers reported their true marginal cost functions, the market outcome would be efficient (i.e., the total generation cost is minimized). However, when suppliers are strategic and aim to maximize their own profits, the reported supply functions are not necessarily the marginal cost functions, and the resulting market outcome may be inefficient. The efficiency loss depends crucially on the topology of the underlying transmission network. This paper provides an analytical upper bound of the efficiency loss due to strategic suppliers, and proves that the bound is tight under a large class of transmission networks (i.e., weakly cyclic networks). Our upper bound sheds light on how the efficiency loss depends on the transmission network topology (e.g., the degrees of nodes, the admittances and flow limits of transmission lines).**

***Index Terms*—Electricity markets, supply function bidding, price of anarchy**

I. Introduction

A special feature of the power system is that supply and demand must be balanced at any time, because imbalance may cause serious consequences such as blackout [1]. Therefore, due to the lack of large-scale energy storage, electricity markets become the major instrument in balancing supply and demand and maintaining the stability of power systems.

Nowadays, most of the major electricity markets in United States and Europe are deregulated. In deregulated markets, the electricity suppliers submit bids to the independent system operator (ISO). A bid, also called a supply function, specifies the amounts of electricity a supplier is willing to produce at different prices. After receiving bids from the suppliers, the ISO considers the bids as their marginal cost functions, calculates the cost functions (by integration), and dispatches the suppliers such that the demand is met and the total generation cost is minimized. This procedure is called economic dispatch.

If the suppliers submitted their true marginal cost functions as their bids, the economic dispatch would result in the socially optimal outcome that minimizes the true total cost. However, the suppliers aim to maximize their own profits, and for this purpose, may choose bids that are different from their true marginal cost functions. In this case, the outcome of the economic dispatch is inefficient. The goal of this paper is to analytically quantify this inefficiency due to strategic behavior of generators.

To analyze the efficiency loss in deregulated electricity markets, we first characterize the supply function equilibrium (SFE) of the market. We show that the supply profile (i.e., amounts of electricity produced by each supplier) at the equilibrium is unique. Following the literature, we define the efficiency loss as price of anarchy (PoA), namely the ratio of the total generation cost at the SFE to the total cost at social optimum. By definition, the efficiency loss is larger when the PoA is larger. Then we provide an analytical upper bound of the PoA. Our bound has the following desirable properties:

- Our upper bound depends on, among other factors, the topology of the underlying transmission networks (e.g., the numbers of lines connected to each generator, the admittances and flow limits of transmission lines).
- Our bound generalizes existing bounds on the PoA at SFE, and reduces to these bounds under special cases.
- Our bound is proved to be "tight"[1] under a large class of transmission networks, i.e., weakly cyclic networks (the networks in which one line belongs to at most one cycle).

It is important to note that our upper bound depends on the topology of the transmission networks. Therefore, our results provide insights on how to optimize the transmission network in order to reduce the efficiency loss caused by strategic generators. Simulations on realistic power system test cases demonstrate that our bound is tighter than existing ones.

The rest of this paper is organized as follows. We discuss related works in Section II. In Section III, we will describe our model of electricity markets and define the supply function equilibrium. We analyze the equilibrium in Section IV. We provide numerical results in Section V. Finally, Section VI concludes the paper.

II. Related Works

Two models have been used in most of the works that study strategic behavior in deregulated electricity markets. The first model is the Cournot competition model, where each generator submits the amount of electricity to produce (i.e., a quantity) [2][3]. These works suggest that the network topology plays an important role in the efficiency loss. However, in the Cournot competition model, the generators act quite differently from the way they bid in reality. Hence, we want to analyze the efficiency loss under a model with a more realistic bidding format.

Y. Xiao and E. Wei are with Department of Electrical Engineering and Computer Science, Northwestern University, Evanston, IL 60208 USA (e-mail: {yuanzhang.xiao, ermin.wei}@northwestern.edu).

C. Bandi is with Kellogg School of Management, Northwestern University, Evanston, IL 60208 USA (e-mail: c-bandi@kellogg.northwestern.edu).

[1]It will be clear what do we mean by "tight" in Theorem 2.

The second model commonly used in the literature is the supply function equilibrium model, where each generator submits the amounts of electricity to produce at different prices (i.e., a curve of price versus quantity) [4][5][6][7]. The SFE model is closer to the real bidding formats in electricity markets. However, most existing works using the SFE model do not study the impact of the transmission network topology on the efficiency loss [4][5][6]. Their upper bounds of the PoA depend on the number of generators only [4], or on the number and the capacity limits of generators [5][6].

The work that is most closed to this paper is our prior work [7], where we analyze the efficiency loss using the SFE model, and quantify the impact of *certain aspects* of the transmission network topology. Specifically, in the analysis in [7], we treated the transmission network as a *radial* network by ignoring the cycles. Consequently, the upper bound of the PoA in [7] does not depend on the admittances of transmission lines, and may be loose when there are cycles in the transmission network (which is usually the case).

III. System Model

We model a power system as a graph $(\mathcal{N}, \mathcal{E})$, where each node in $\mathcal{N}$ is a bus[2] with a generator or a load or both, and each edge in $\mathcal{E}$ is a transmission line connecting two buses. We assume that each generator is owned by a different supplier. Therefore, we can use "generator" and "supplier" interchangeably, and will use "generator" to emphasize the physical aspects and "supplier" to emphasize the strategic aspects. A representative power system, namely IEEE 14-bus system, is shown in Fig. 1 [8]. Denote the set of buses that have a generator by $\mathcal{N}_g \subseteq \mathcal{N}$ (For the IEEE 14-bus system, we have $\mathcal{N}_g = \{1, 2, 3, 6, 8\}$). We assume that there are more than two generators (i.e., $|\mathcal{N}_g| > 2$). Since the majority of the load in the electricity market is inelastic [9], we assume that the load is inelastic, and denote the inelastic load profile by $\boldsymbol{d} = (d_1, \ldots, d_{|\mathcal{N}|})$. The total demand is then $D \triangleq \sum_{j \in \mathcal{N}} d_j$.

Each generator $n \in \mathcal{N}_g$ has a cost of $c_n(s_n)$ in providing s_n unit of electricity. We make the following standard assumption about cost functions.

Assumption 1: Each generator n's cost function $c_n(s_n)$ is convex and strictly increasing in supply $s_n \in [0, +\infty)$.

Due to physical constraints, each generator n's supply s_n must be in a range $[\underline{s}_n, \bar{s}_n]$, namely

$$\underline{s}_n \leq s_n \leq \bar{s}_n. \tag{1}$$

In addition, the supply profile $\boldsymbol{s} = (s_n)_{n \in \mathcal{N}_g}$ must satisfy physical constraints of the electrical network. First, in a power system, it is crucial to balance the supply and the demand at all time for the stability of the system [1]. Hence, we need

$$\textstyle\sum_{n \in \mathcal{N}_g} s_n = D. \tag{2}$$

Second, the flow on each transmission line, which depends on the supply profile, cannot exceed the flow limit of the line. In economic dispatch, the ISO uses the linearized power flow model, where the flow on each line is the linear combination of injections from each node [1][3]. Hence, the line flow constraints can be written as follows:

$$-\boldsymbol{f} \leq \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \leq \boldsymbol{f}, \tag{3}$$

where $\boldsymbol{f} \in \mathbb{R}^{|\mathcal{E}|}$ is the vector of flow limits, $\mathbf{A}_g \in \mathbb{R}^{|\mathcal{E}| \times |\mathcal{N}_g|}$ and $\mathbf{A}_\ell \in \mathbb{R}^{|\mathcal{E}| \times |\mathcal{N}|}$ are shift-factor matrices. The shift-factor matrices $\mathbf{A}_g$ and $\mathbf{A}_\ell$ depend on the underlying transmission network topology (e.g., the degrees of nodes and the admittance of transmission lines).

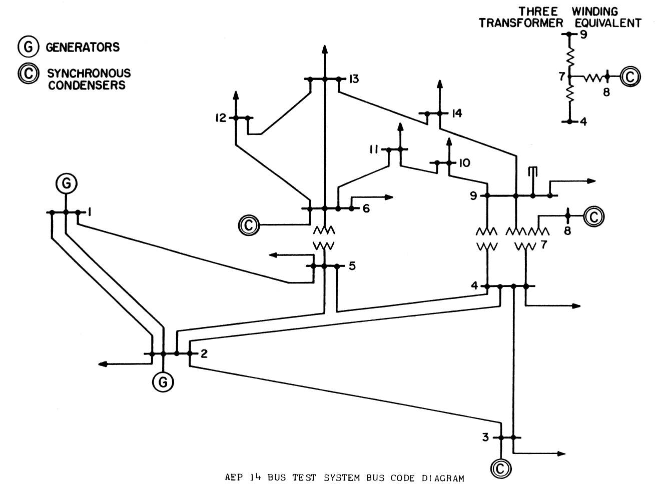

Fig. 1. Illustration of the IEEE 14-bus system, which will serve as a running example and be used in the simulation.

A. Benchmark - Social Optimum

If the ISO knew the true cost functions of each generator, it would solve for the optimal supply profile $\boldsymbol{s}^*$ to minimize the total generation cost subject to the constraints (1)–(3). We summarize this optimization problem as follows:

$$\begin{aligned} \max_{\boldsymbol{s}} \quad & \textstyle\sum_{n \in \mathcal{N}_g} c_n(s_n) \\ s.t. \quad & \textstyle\sum_{n \in \mathcal{N}_g} s_n = D, \\ & \underline{s}_n \leq s_n \leq \bar{s}_n, \quad \forall n \in \mathcal{N}_g, \\ & -\boldsymbol{f} \leq \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \leq \boldsymbol{f}. \end{aligned} \tag{4}$$

To avoid triviality, we assume that the feasible set of supply profiles is non-empty and is not a singleton.

Assumption 2: There exists a strictly feasible supply profile $\boldsymbol{s}$ (i.e., a profile $\boldsymbol{s}$ that satisfies constraints (1)–(3) strictly).

We write a solution to the optimization problem (4) as $\boldsymbol{s}^* = (s_n^*)_{n \in \mathcal{N}_g}$, and call it the socially optimal supply profile.

B. Deregulated Markets and Supply Function Bidding

In practice, each supplier submits a supply function (i.e., a bid) to the ISO. A supply function is a mapping from the unit selling price of electricity to the amount of electricity produced by a supplier. In practice, the supply function is usually a step function. For analytical tractability, we assume that each supplier n submits a parametrized supply function of the following form: [4][5][7]

$$S_n(p, w_n) = D - \frac{w_n}{p}, \tag{5}$$

[2]In this paper, we will use "node" and "bus" interchangeably.

where $w_n \in \mathbb{R}_+$ is supplier n's strategic action, and $p \in \mathbb{R}_+$ is the unit price of electricity. To clear the market, (i.e., to find the price p that satisfies the condition $\sum_{n\in\mathcal{N}_g} S_n(p, w_n) = D$), the ISO sets the price p as follows:

$$p(\boldsymbol{w}) = \frac{\sum_{n\in\mathcal{N}_g} w_n}{(|\mathcal{N}_g| - 1)\, D}. \tag{6}$$

where $\boldsymbol{w} = (w_n)_{n\in\mathcal{N}_g} \in \mathbb{R}_+^{|\mathcal{N}_g|}$ is the action profile.

Each supplier n aims to maximizes its profit $u_n(w_n, \boldsymbol{w}_{-n})$, where $\boldsymbol{w}_{-n}$ is the action profile of all the suppliers other than n. Supplier n's profit is the revenue minus cost as follows:

$$\begin{aligned} u_n(w_n, \boldsymbol{w}_{-n}) &= \underbrace{p(w_n, \boldsymbol{w}_{-n}) \cdot S_n\left[p(w_n, \boldsymbol{w}_{-n}), w_n\right]}_{\text{revenue}} \\ &- \underbrace{c_n\left(S_n\left[p(w_n, \boldsymbol{w}_{-n}), w_n\right]\right)}_{\text{cost}}. \end{aligned} \tag{7}$$

Now we formally define the supply function equilibrium.

Definition 1: An action profile $\boldsymbol{w}^{**}$ is a supply function equilibrium, if each supplier n's action w_n^{**} is a solution to the following profit maximizing problem:[3]

$$\begin{aligned} \max_{w_n} \quad & u_n\left(w_n, \boldsymbol{w}_{-n}^{**}\right) \\ s.t. \quad & \underline{s}_n \le S_n\left[p\left(w_n, \boldsymbol{w}_{-n}^{**}\right), w_n\right] \le \bar{s}_n, \forall n \in \mathcal{N}_g \\ & -\boldsymbol{f} \le \\ & \qquad \mathbf{A}_\ell \cdot \boldsymbol{d} + [\mathbf{A}_g]_{*n} \cdot S_n\left[p(w_n, \boldsymbol{w}_{-n}^{**}), w_n\right] \\ & \qquad + \sum_{\substack{m\in\mathcal{N}_g \\ m\neq n}} [\mathbf{A}_g]_{*m} \cdot S_m\left[p(w_n, \boldsymbol{w}_{-n}^{**}), w_m^{**}\right] \\ & \quad \le \boldsymbol{f}. \end{aligned} \tag{8}$$

In a SFE, each supplier's action maximizes its own profit given the others' actions. Note that the set of feasible actions of each supplier depends on the others' actions. Therefore, the SFE is a generalized Nash equilibrium [10].

IV. Efficiency Loss at SFE

A. Uniqueness of Equilibrium Supply Profile

Now we will show that the SFE exists and that there is a unique equilibrium supply profile at any SFE.

Proposition 1: The SFE exists. At any SFE, the resulting equilibrium supply profile is the unique solution to the modified social welfare (MSW) maximization problem

$$\begin{aligned} & \texttt{(MSW)}: \\ & \min_{\boldsymbol{s}} \quad \textstyle\sum_{n\in\mathcal{N}_g} \hat{c}_n(s_n) \\ & s.t. \quad \textstyle\sum_{n\in\mathcal{N}_g} s_n = D, \\ & \qquad \underline{s}_n \le s_n \le \bar{s}_n, \forall n \in \mathcal{N}_g \\ & \qquad -\boldsymbol{f} \le \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \le \boldsymbol{f}, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \hat{c}_n(s_n) &= \left(1 + \frac{s_n}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D}\right) \cdot c_n(s_n) \\ &- \frac{1}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D} \cdot \int_0^{s_n} c_n(x)dx. \end{aligned} \tag{10}$$

[3]We denote the nth column of a matrix $\mathbf{A}$ by $[\mathbf{A}]_{*n}$.

Proof: See Appendix VII-A. ■

Proposition 1 ensures the existence of SFE. In general, the generalized Nash equilibrium is not unique [10], which is the case for SFE. However, Proposition 1 indicates that although there may be multiple SFE, the resulting equilibrium supply profile is always unique. This is important because the system efficiency is measured by the total general cost, which depends on the supply profile, not on the action profile. As a result, the total general cost at the equilibrium is unique.

B. Analysis of Efficiency Loss

We quantify the efficiency loss by PoA defined below:

Definition 2: PoA is the ratio of the total cost at SFE to the total cost at social optimum:

$$\frac{\sum_{n\in\mathcal{N}_g} c_n(s_n^{**})}{\sum_{n\in\mathcal{N}_g} c_n(s_n^{*})}.$$

By definition, the PoA is never smaller than 1. A larger PoA indicates a larger efficiency loss at the equilibrium. Note that the PoA is well defined because the equilibrium supply profile $\boldsymbol{s}^{**}$ is unique.

1) Preliminaries on Graph Theory: We derive an upper bound of the PoA that depends crucially on the topology of the transmission networks. Therefore, to better describe our bound, we need to introduce several useful concepts from graph theory. We will use the IEEE 14-bus system in Fig. 1 as a running example to illustrate these concepts.

Definition 3 (Cycle): A cycle C of a graph is a sequence of nodes $n_1, n_2, \ldots n_k, n_1$ ($k \ge 3$) that satisfies: 1) there is an edge between every consecutive nodes (i.e., n_i and n_{i+1} for $i = 1, \ldots, k-1$) and between n_k and n_1, and 2) the nodes $n_1, \ldots, n_k$ are distinct.

In short, a cycle is a group of distinct nodes that form a loop.

Example 1 (Cycles): Take the IEEE 14-bus system in Fig. 1 for example. The sequence $1, 2, 5, 1$ forms a cycle. The sequence $2, 1, 5, 2, 4, 3, 2$ does not form a cycle (although all consecutive nodes are connected), because node 2 is visited twice in the loop.

Denote the set of node n's neighbors by $\mathcal{N}(n)$. We then define a partition of $\mathcal{N}(n)$, denoted by $\mathcal{P}(n) \subset 2^{\mathcal{N}(n)}$.

Definition 4 (Partition of neighbors): A partition $\mathcal{P}(n)$ is a set of singletons and duples of nodes that satisfy:

1) the sets in $\mathcal{P}(n)$ are mutually exclusive, and the union of all sets in $\mathcal{P}(n)$ is $\mathcal{N}(n)$;
2) any duple of nodes $\{i, j\} \in \mathcal{P}(n)$ are in a cycle with node n, namely i, n, j or j, n, i are in a cycle;
3) any two singletones $\{i\}, \{j\} \in \mathcal{P}(n)$ are not in the same cycle.

The partition $\mathcal{P}(n)$ divides node n's neighbors into several subsets. Roughly speaking, we divide the neighbors by their affiliation to the cycles. Since node n appears only once in a cycle, it has exactly two neighbors in the cycle. Therefore, each subset is either a duple of two nodes (in the case these two nodes are in the same cycle with node n), or a singleton (in the case this node is not in a cycle with either node n or node n's remaining neighbors).

Note that the partition is not unique, but any partition can be chosen for the purpose of deriving our upper bound on PoA. Once a partition $\mathcal{P}(n)$ has been chosen for each node $n \in \mathcal{N}_g$, we define a mapping $\mathcal{C}_n : \{i,j\} \mapsto C$ for each duple $\{i,j\} \in \mathcal{P}(n)$. The mapping indicates which cycle C nodes i,j,n belong to. In the case that nodes i,j,n belong to multiple cycles, this mapping arbitrarily selects one of them. Again, this mapping is not unique, but any mapping can be chosen for our purpose. Hence, we will fix one mapping $\mathcal{C}_n$ for each node $n \in \mathcal{N}_g$ in the following.

Example 2 (Partition and Mapping to Cycles): Take the IEEE 14-bus in Fig. 1 for example again. Node 1 has two neighbors: nodes 2 and 5, which are in cycle $1,2,5,1$ with node 1. Hence, the partition of node 1's neighbors is unique, namely $\mathcal{P}(1) = \{\{2,5\}\}$. Since nodes 1, 2 and 5 belong to multiple cycles, the mapping $\mathcal{C}_1$ is not unique. In this case, we can choose $\mathcal{C}_1(\{2,5\})$ to be either $1,2,5,1$ or $1,2,4,5,1$. Node 2 has four neighbors: nodes $1,3,4,5$. For node 2, the partition of its neighbors is not unique. We could choose either $\mathcal{P}(2) = \{\{1,5\},\{3,4\}\}$ or $\mathcal{P}'(2) = \{\{1\},\{3\},\{4,5\}\}$. If we choose $\mathcal{P}(2)$, we can set the mapping $\mathcal{C}_2$ as $\mathcal{C}_2(\{1,5\}) = 1,2,5,1$ and $\mathcal{C}_2(\{3,4\}) = 2,3,4,2$.

All the above definitions of cycles, partitions, and mappings to cycles are preparations for the next crucial definition of *effective flow limit*. For each supplier $n \in \mathcal{N}_g$, we define an effective flow limit $\hat{f}_{nm}$ for each outgoing link m from supplier n.

Definition 5 (Effective Flow Limit): For each supplier n, fix a partition $\mathcal{P}(n)$ of its neighbors and the mapping $\mathcal{C}_n$. Then the effective flow limit from n to its neighbor $m \in \mathcal{N}(n)$ is defined as follows:

- If m is a singleton in the partition (i.e., $\{m\} \in \mathcal{P}(n)$), the effective flow limit is the same as the conventional flow limit, namely $\hat{f}_{nm} = f_{nm}$;
- If m is in a duple in the partition (i.e., $\{m,i\} \in \mathcal{P}(n)$), the effective flow limit is as follows:

$$\hat{f}_{nm} = \min\left\{ f_{nm}, \left(\sum_{\substack{j,k \in \mathcal{C}_n(\{m,i\}) \\ \{j,k\} \neq \{n,m\}}} \frac{f_{jk}}{B_{jk}} \right) \cdot B_{nm} \right\}, \quad (11)$$

  where B_{nm} is the admittance of the transmission line between bus n and bus m.

The effective flow limit is the minimum between the conventional flow limit and another term that depends on the flow limits and admittances of the other lines in the cycle. The latter term is the flow from n to m when the other lines in the cycle reach flow limits. The flow from n to m cannot exceed this term, even if the flow limit f_{nm} allows.

Note that the effective flow limit depends on the direction of the flow, namely $\hat{f}_{nm} \neq \hat{f}_{mn}$ in general. This is because the partition of the neighbors and the associated cycles are different for different nodes n and m. For example, for the IEEE 14-bus system in Fig. 1, we have

$$\hat{f}_{12} = \min\left\{ f_{12}, \left(\frac{f_{25}}{B_{25}} + \frac{f_{51}}{B_{51}} \right) \cdot B_{12} \right\}$$

under the partition $\mathcal{P}(1) = \{\{2,5\}\}$, and

$$\hat{f}_{21} = f_{21} = f_{12}$$

under the partition $\mathcal{P}'(2) = \{\{1\},\{3\},\{4,5\}\}$.

2) Analytical Upper Bound: With the effective flow limit defined, we are ready to provide an analytical upper bound of the PoA.

Theorem 1: The PoA is upper bounded by

$$1 + \max_{n \in \mathcal{N}_g} \frac{\min\left\{ \bar{s}_n, D - \sum_{\substack{m \in \mathcal{N}_g \\ m \neq n}} \underline{s}_m, d_n + \sum_{m \in \mathcal{N}(n)} \hat{f}_{nm} \right\}}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D}.$$

Proof: See Appendix VII-B. ■

The upper bound in Theorem 1 gives us insights on the key factors that influence the efficiency loss. First, the upper bound is higher if one supplier has a significantly higher capacity limit than the others. In this case, this supplier may have larger market power, especially when the total capacity from the other suppliers are barely enough to fulfill the demand. Second, the upper bound is higher if one generator has higher local demand and higher effective flow limits of its outgoing links. In this case, this generator has advantage over the other generators in fulfilling its local demand (because incoming flow limits constrain the import of electricity from the other generators), and can more easily export its electricity generation to other nodes due to higher outgoing effective flow limits. Therefore, this generator has more influence on the market outcome.

Our upper bound in Theorem 1 recovers the bounds in prior work as special cases. When all the generators have the same capacity limits of $\underline{s}_n = 0$ and $\bar{s}_n = D$ for all $n \in \mathcal{N}_g$, and when we ignore the network topology (i.e., ignore the term $d_n + \sum_{m \in \mathcal{N}(n)} \hat{f}_{nm}$), the bound reduces to the one in [4]. If we allow generators to have different capacity limits $\bar{s}_n$, the bound reduces to the one in [5]. If we ignore the cycles in the transmission network, we have $\hat{f}_{nm} = f_{nm}$ for all n and m, and hence recover the bound in [7].

3) Tightness: We show that our upper bound in Theorem 1 is tight under a large class of networks, namely weakly cyclic networks defined as follows.

Definition 6 (Weakly-Cyclic Networks): A weakly-cyclic network is a graph in which no edge belongs to more than one cycle.

The tightness result is described in the following theorem.

Theorem 2: For any weakly-cyclic network, our upper bound is tight in the following sense: Given any small number $\varepsilon > 0$, there are instances of demand profiles $\boldsymbol{d}$, sets of generators $\mathcal{N}_g$, cost functions $\{c_n\}_{n \in \mathcal{N}_g}$, generator capacity limits $\{\underline{s}_n, \bar{s}_n\}_{n \in \mathcal{N}_g}$, flow limits $\{f_{nm}\}_{nm \in \mathcal{E}}$, and admittances $\{B_{nm}\}_{nm \in \mathcal{E}}$, under which

- the upper bound of PoA is equal to

$$1 + \max_{n \in \mathcal{N}_g} \frac{d_n + \sum_{m \in \mathcal{N}(n)} \hat{f}_{nm}}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D}.$$

- the difference between the PoA and the upper bound is within ε.

Proof: See Appendix VII-E ■

V. Simulation

VI. Conclusion

We analyzed the efficiency loss in decentralized electricity markets. The distinct feature of our work is our consideration of the topology of the mesh transmission network. We show that there exists a unique equilibrium supply profile, and gave an analytical upper bound of the efficiency loss at the equilibrium. Our upper bound suggests that to reduce the efficiency loss, we should evenly distribute the generation capacity and outgoing effective flow limits among the generators.

VII. Appendix

A. Proof of Proposition 1

Step 1: At any SFE, there exist at least two generators with strictly positive bids.

Suppose that all the generators bid zero, namely $w_n = 0$ for all n. Then each generator n supply D, resulting in a total supply of $|\mathcal{N}_g|\, D$. This supply profile violates the constraint that the supply equals the demand. Hence, at least one generator submits strictly positive bids.

Suppose that generator n is the only generator with a strictly positive bid. Then generator n needs to supply

$$D - \frac{w_n}{w_n / \left[(|\mathcal{N}_g - 1|) D \right]} = -\left(|\mathcal{N}_g - 2| \right) D < 0,$$

which violates the constraint that $s_n \geq 0$. Hence, at least two generators submit strictly positive bids.

Step 2: The payoff function $u_n(w_n, \boldsymbol{w}_{-n})$ of each generator n is strictly convex in w_n at any equilibrium.

We write down the expression of the payoff function as

$$\begin{aligned} & u_n(w_n, \boldsymbol{w}_{-n}) \qquad (12) \\ = \; & p(w_n, \boldsymbol{w}_{-n}) \cdot S_n \left[p(w_n, \boldsymbol{w}_{-n}), w_n \right] \\ & - c_n \left(S_n \left[p(w_n, \boldsymbol{w}_{-n}), w_n \right] \right) \\ = \; & \frac{\sum_{m \in \mathcal{N}_g} w_m}{|\mathcal{N}_g| - 1} - w_n - c_n \left(S_n \left[p(w_n, \boldsymbol{w}_{-n}), w_n \right] \right), \end{aligned}$$

where

$$S_n \left(p(w_n, \boldsymbol{w}_{-n}), w_n \right) = D - \frac{w_n}{\sum_{m \in \mathcal{N}_g} w_m} \cdot \left(|\mathcal{N}_g| - 1 \right) D, \quad (13)$$

Since $c_n(s_n)$ is strictly convex and increasing in s_n, and $S_n \left(p(w_n, \boldsymbol{w}_{-n}), w_n \right)$ is strictly convex in w_n when $\sum_{m \neq n} w_m > 0$, the payoff function $u_n(w_n, \boldsymbol{w}_{-n})$ is strictly convex in w_n when $\sum_{m \neq n} w_m > 0$. In Step 1, we have proved that $\sum_{m \neq n} w_m > 0$ at an equilibrium. Hence, the payoff function $u_n(w_n, \boldsymbol{w}_{-n})$ is strictly convex in w_n at any equilibrium.

Step 3: Since the payoff function $u_n(w_n, \boldsymbol{w}_{-n})$ is strictly convex in w_n, each generator n's optimization problem is a convex optimization problem. Therefore, $\boldsymbol{w}$ is a SFE if and only if it satisfies the KKT conditions for all n.

We write the Lagrangian multipliers associated with the local constraints (**??**) and (**??**) as $\underline{\mu}_n^{\texttt{SFE}} \in \mathbb{R}_+$ and $\bar{\mu}_n^{\texttt{SFE}} \in \mathbb{R}_+$, respectively. In addition, we write the Lagrangian multipliers associated with the common constraints (**??**) and (**??**) as $\boldsymbol{\lambda}_1^{\texttt{SFE}} \in \mathbb{R}_+^{|\mathcal{E}|}$ and $\boldsymbol{\lambda}_2^{\texttt{SFE}} \in \mathbb{R}_+^{|\mathcal{E}|}$, respectively. Then the action profile $\boldsymbol{w}$ is a SFE if and only if there exists $\underline{\mu}_n^{\texttt{SFE}}$, $\bar{\mu}_n^{\texttt{SFE}}$, $\boldsymbol{\lambda}_1^{\texttt{SFE}}$, and $\boldsymbol{\lambda}_2^{\texttt{SFE}}$ for all n such that the following KKT conditions are satisfied for all n:

$$\frac{\partial u_n(w_n, \boldsymbol{w}_{-n})}{\partial w_n} + \left[\bar{s}_n + \left(|\mathcal{N}_g| - 2 \right) D \right] \underline{\mu}_n^{\texttt{SFE}} - \left(|\mathcal{N}_g| - 2 \right) \bar{\mu}_n^{\texttt{SFE}} + \left(\boldsymbol{\lambda}_1^{\texttt{SFE}} \right)^T$$

$$0 \le \underline{\mu}_n^{\texttt{SFE}} \perp \left[\bar{s}_n + \left(|\mathcal{N}_g| - 2 \right) D \right.$$

$$0 \le \bar{\mu}_n^{\texttt{SFE}} \perp - ($$

By plugging in the expression of the payoff function $u_n(w_n, \boldsymbol{w}_{-n})$, we can rewrite (14) as

$$\begin{aligned} 0 \;=\; & \frac{dc_n \left(S_n \left[p(w_n, \boldsymbol{w}_{-n}), w_n \right] \right)}{ds_n} \cdot \left(1 + \frac{S_n \left[p(w_n, \boldsymbol{w}_{-n}), w_n \right]}{\left(|\mathcal{N}_g| - 2 \right) D} \right) - p(\boldsymbol{w} \\ + \; & \frac{\sum_{m \in \mathcal{N}_g} \bar{s}_m - D}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D} \cdot p(\boldsymbol{w}) \cdot \underline{\mu}_n^{\texttt{SFE}} - \left(\sum_{m \in \mathcal{N}_g} \bar{s}_m - D \right) \cdot p(\boldsymbol{w}) \\ + \; & \frac{\sum_{m \in \mathcal{N}_g} \bar{s}_m - D}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D} \cdot p(\boldsymbol{w}) \cdot \left(\boldsymbol{\lambda}_1^{\texttt{SFE}} \right)^T \cdot \left(\boldsymbol{b}_n + \boldsymbol{c}_1 \right) - \frac{\sum_{m \in \mathcal{N}_g}}{\sum_{m \in \mathcal{N}_g, m} } \end{aligned}$$

Step 4: Since the optimization problem (`MSW`) is a convex optimization problem, $\boldsymbol{s}$ is a solution if and only if it satisfies the KKT conditions.

We write the Lagrangian multipliers associated with the constraints $\mathbf{0} \le \boldsymbol{s}$ and $\boldsymbol{s} \le \bar{\boldsymbol{s}}$ in (10) as $\underline{\boldsymbol{\mu}}^{\texttt{MWS}} = (\underline{\mu}_n^{\texttt{MWS}})$ and $\bar{\boldsymbol{\mu}}^{\texttt{MWS}} = (\bar{\mu}_n^{\texttt{MWS}})$, respectively. Similarly, we write the Lagrangian multiplier associated with the constraint $\sum_{n \in \mathcal{N}_g} s_n = D$ in (10) as $p^{\texttt{MSW}}$. Finally, we write the Lagrangian multipliers associated with the constraints $-\boldsymbol{c}_1 \le \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d}$ and $\mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \le \boldsymbol{c}_2$ in (10) as $\boldsymbol{\lambda}_1^{\texttt{MWS}}$ and $\boldsymbol{\lambda}_2^{\texttt{MWS}}$, respectively.

The KKT conditions are

$$\frac{dc_n(s_n)}{ds_n} \cdot \left(1 + \frac{s_n}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D} \right) - p^{\texttt{MSW}} - \underline{\mu}_n^{\texttt{MSW}} + \bar{\mu}_n^{\texttt{MSW}} - \left(\boldsymbol{\lambda}_1^{\texttt{MSW}} \right)$$

$$\mathbf{0} \le$$

$$\mathbf{0} \le$$

Step 5: Given any SFE $\boldsymbol{w}$, the allocation $\boldsymbol{s}$, where $s_n = \bar{s}_n - \frac{w_n}{p(\boldsymbol{w})}, \forall n$, is a solution to (`MSW`).

For any SFE $\boldsymbol{w}$, there exists $\underline{\mu}_n^{\texttt{SFE}}$, $\bar{\mu}_n^{\texttt{SFE}}$, $\boldsymbol{\lambda}_1^{\texttt{SFE}}$, and $\boldsymbol{\lambda}_2^{\texttt{SFE}}$ such that the KKT conditions are satisfied.

Define $s_n = \bar{s}_n - \frac{w_n}{p(\boldsymbol{w})}, \forall n$ and the Lagrangian multipliers of the problem MSW as follows

$$\begin{aligned}
p^{\texttt{MSW}} &= p(\boldsymbol{w}) \\
&\cdot \left\{1 - \frac{\sum_{m\in\mathcal{N}_g} \bar{s}_m - D}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D} \cdot \left[\left(\boldsymbol{\lambda}_1^{\texttt{SFE}}\right)^T \cdot (\mathbf{A}_g \cdot \bar{\boldsymbol{s}}_m + \mathbf{A}_\ell \cdot \boldsymbol{d} + \boldsymbol{f}) + \left(\boldsymbol{\lambda}_2^{\texttt{SFE}}\right)^T \cdot (\mathbf{A}_g \cdot \bar{\boldsymbol{s}}_m + \mathbf{A}_\ell \cdot \boldsymbol{d} - \boldsymbol{f})\right]\right\} \\
\underline{\mu}_n^{\texttt{MSW}} &= \left(\sum_{m\in\mathcal{N}_g} \bar{s}_m - D\right) \cdot p(\boldsymbol{w}) \cdot \bar{\mu}_n^{\texttt{SFE}} \\
\bar{\mu}_n^{\texttt{MSW}} &= \frac{\sum_{m\in\mathcal{N}_g} \bar{s}_m - D}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D} \cdot p(\boldsymbol{w}) \cdot \underline{\mu}_n^{\texttt{SFE}} \\
\boldsymbol{\lambda}_1^{\texttt{MSW}} &= \frac{\left(\sum_{m\in\mathcal{N}_g} \bar{s}_m - D\right)^2}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D} \cdot p(\boldsymbol{w}) \cdot \boldsymbol{\lambda}_1^{\texttt{SFE}} \\
\boldsymbol{\lambda}_2^{\texttt{MSW}} &= \frac{\left(\sum_{m\in\mathcal{N}_g} \bar{s}_m - D\right)^2}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D} \cdot p(\boldsymbol{w}) \cdot \boldsymbol{\lambda}_2^{\texttt{SFE}}
\end{aligned}$$

It is not difficult to check that the KKT conditions of MSW are satisfied by $\boldsymbol{s}$, $p^{\texttt{MSW}}$, $\underline{\mu}_n^{\texttt{MSW}}$, $\bar{\mu}_n^{\texttt{MSW}}$, $\boldsymbol{\lambda}_1^{\texttt{MSW}}$, and $\boldsymbol{\lambda}_2^{\texttt{MSW}}$.

Step 6: Given a solution $\boldsymbol{s}$ to (MSW), we can construct a SFE $\boldsymbol{w}$.

This can be proved similarly as in Step 5, using the mapping between the Lagrangian multipliers.

B. Proof of Theorem 1

In the proof, we will first prove an upper bound of the PoA, based on the properties of the modified cost functions and the equilibrium supply profile. Then we will do further relaxation to obtain two looser, but analytical, upper bounds. One upper bound contains information about the generators' capacity limits, while the other one contains information about the network topology. The minimum of these two upper bounds yields the upper bound of the PoA stated in Theorem 1.

C. Preparation

First, we show that for any $s_n \in [\underline{s}_n, \bar{s}_n]$, we have

$$c_n(s_n) \leq \hat{c}_n(s_n) \leq \left(1 + \frac{s_n}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D}\right) \cdot c_n(s_n). \quad (31)$$

For convenience, we rewrite $\hat{c}_n(s_n)$ here:

$$\begin{aligned}
\hat{c}_n(s_n) &= \left(1 + \frac{s_n}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D}\right) \cdot c_n(s_n) \\
&- \frac{1}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D} \cdot \int_0^{s_n} c_n(x) dx.
\end{aligned}$$

It is clear that $\hat{c}_n(s_n) \leq \left(1 + \frac{s_n}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D}\right) \cdot c_n(s_n)$ because the integral in the expression of $\hat{c}_n(s_n)$ is nonnegative.

To see $c_n(s_n) \leq \hat{c}_n(s_n)$, we look at

$$\hat{c}_n(s_n) - c_n(s_n) = \frac{s_n}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D} \cdot c_n(s_n) - \frac{1}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D} \int_0^{s_n} c_n(x) dx.$$

Note that $\hat{c}_n(0) - c_n(0) = 0$, and that the derivative of $\hat{c}_n(s_n) - c_n(s_n)$ with respect to s_n is

$$\frac{s_n}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D} \cdot \frac{dc_n(s_n)}{ds_n} \geq 0. \quad (26)$$

Therefore, we have $\hat{c}_n(s_n) - c_n(s_n) \geq 0$ for all s_n. (27)

Using (31) and the property of the equilibrium supply profile, we have the following inequality

$$\begin{aligned}
\sum_{n\in\mathcal{N}_g} c_n(s_n^{**}) &\leq \sum_{n\in\mathcal{N}_g} \hat{c}_n(s_n^{**}) && (28) \\
&\leq \sum_{n\in\mathcal{N}_g} \hat{c}_n(s_n^{*}) && (29) \\
&\leq \sum_{n\in\mathcal{N}_g} \left(1 + \frac{s_n^*}{\sum_{m\in\mathcal{N}_g, m\neq n} \bar{s}_m - D}\right) \cdot c_n(s_n^*) && (30) \\
&\leq \max\left\{1 + \frac{s_k^*}{\sum_{m\in\mathcal{N}_g, m\neq k} \bar{s}_m - D}\right\} \cdot \sum_{n\in\mathcal{N}_g} c_n(s_n^*),
\end{aligned}$$

where the first and third inequalities come from the inequality (31), and the second inequality follows from the fact that $(s_n^{**})_{n\in\mathcal{N}_g}$ is the optimal solution to the modified cost minimization problem.

Based on the above inequality, we can see that an upper bound of the PoA is

$$1 + \max_{k\in\mathcal{N}_g} \left\{\frac{s_k^*}{\sum_{m\in\mathcal{N}_g, m\neq k} \bar{s}_m - D}\right\}. \quad (32)$$

Our next step is to compute this upper bound analytically.

D. Upper Bounds of PoA

To calculate the upper bound of PoA in (32) analytically, we need to calculate the socially optimal supply profile analytically, which is a difficult task. Therefore, we aim to further relax the upper bound in (32). Instead of using the socially optimal supply profile $\boldsymbol{s}^*$ in (32), we replace $\boldsymbol{s}^*$ with an arbitrary feasible supply profile $\boldsymbol{s}$, and find the maximum of the resulting term over all feasible supply profiles $\boldsymbol{s}$, namely:

$$\begin{aligned}
\max_{\boldsymbol{s}} \quad & 1 + \max_{k\in\mathcal{N}_g} \left\{\frac{s_k}{\sum_{m\in\mathcal{N}_g, m\neq k} \bar{s}_m - D}\right\} && (33) \\
s.t. \quad & \textstyle\sum_{n\in\mathcal{N}_g} s_n = D, \\
& \underline{s}_n \leq s_n \leq \bar{s}_n, \quad \forall n \in \mathcal{N}_g, \\
& -\boldsymbol{f} \leq \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \leq \boldsymbol{f}.
\end{aligned}$$

The optimal value of the above optimization problem must be an upper bound of (32), because the socially optimal supply profile $\boldsymbol{s}^*$ is a feasible solution to the above optimization problem.

The optimization problem (33) can be decomposed into $|\mathcal{N}_g|$ problems, with the ith problem looking for the maximum of $\frac{s_i}{\sum_{m\in\mathcal{N}_g, m\neq i} \bar{s}_m - D}$

over all feasible supply profiles, which is equivalent to looking for the maximum feasible supply s_i from generator i. Therefore, we will focus on finding generator i's maximum feasible supply, which is formulated as follows:

$$\begin{aligned} \max_{\boldsymbol{s}} \quad & s_i \\ s.t. \quad & \textstyle\sum_{m\in\mathcal{N}_g} s_m = D, \\ & \underline{s}_m \le s_m \le \bar{s}_m, \quad \forall m \in \mathcal{N}_g, \\ & -\boldsymbol{f} \le \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \le \boldsymbol{f}. \end{aligned} \tag{34}$$

The first two constraints in the above problem, namely $\sum_{m\in\mathcal{N}_g} s_m = D$ and $\underline{s}_m \le s_m \le \bar{s}_m, \forall m \in \mathcal{N}_g$, imply that

$$\underline{s}_i \le s_i \le \min\left\{\bar{s}_i, D - \textstyle\sum_{j\neq i} \underline{s}_j\right\}.$$

We replace the first two constraints in (34) with the above inequality, which is a relaxation of the two original constraints. Then the optimal value of (34) is upper bounded by the optimal value of the resulting optimization problem:

$$\begin{aligned} \max_{\boldsymbol{s}} \quad & s_i \\ s.t. \quad & \underline{s}_i \le s_i \le \min\left\{\bar{s}_i, D - \textstyle\sum_{j\neq i} \underline{s}_j\right\}, \\ & -\boldsymbol{f} \le \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \le \boldsymbol{f}. \end{aligned} \tag{35}$$

Since the shift matrix $\mathbf{A}$ is complicated and does not reflect the network topology, it is hard to solve (35) directly. Hence, we work on an equivalent formulation where we introduce additional variables of the flows on each line $\boldsymbol{p} = (p_{mn})_{(m,n)\in\mathcal{E}}$.

$$\begin{aligned} \max_{\boldsymbol{s},\boldsymbol{p}} \quad & s_i \\ s.t. \quad & \underline{s}_i \le s_i \le \min\left\{\bar{s}_i, D - \textstyle\sum_{j\neq i} \underline{s}_j\right\}, \\ & -f_{mn} \le p_{mn} \le f_{mn}, \ \forall (m,n) \in \mathcal{E}, \\ & \sum_{(m,n)\in L} \frac{p_{mn}}{B_{mn}} = 0, \ \text{for any loop } L, \\ & s_m - d_m = \sum_{n\in\mathcal{N}_m} p_{mn}, \forall m \in \mathcal{N}_g. \end{aligned} \tag{36}$$

Note that the second constraint in (35) is replaced by three new constraints, namely flow limit constraints (the second constraint), voltage angle constraints (the third one), and energy conservation at each node (the fourth one).

In order to write down the optimal value of (36) analytically, we further relax the optimization problem by removing some constraints. Specifically, we remove the voltage angle constraints for loops that do not include generator i, and the energy conservation constraints for generators other than i. The relaxed optimization problem is as follows

$$\begin{aligned} \max_{\boldsymbol{s},\boldsymbol{p}} \quad & s_i \\ s.t. \quad & \underline{s}_i \le s_i \le \min\left\{\bar{s}_i, D - \textstyle\sum_{j\neq i} \underline{s}_j\right\}, \\ & -f_{mn} \le p_{mn} \le f_{mn}, \ \forall (m,n) \in \mathcal{E}, \\ & \sum_{(m,n)\in L} \frac{p_{mn}}{B_{mn}} = 0, \ \text{for any loop } L \in \mathcal{L}_i, \\ & s_i - d_i = \sum_{j\in\mathcal{N}_i} p_{ij}. \end{aligned} \tag{37}$$

We observe that the first constraint in (37) is independent of the other three constraints. Therefore, the optimal value of (37) is the minimum of the optimal values of the following two optimization problems:

$$\begin{aligned} \max_{\boldsymbol{s},\boldsymbol{p}} \quad & s_i \\ s.t. \quad & \underline{s}_i \le s_i \le \min\left\{\bar{s}_i, D - \textstyle\sum_{j\neq i} \underline{s}_j\right\}. \end{aligned} \tag{38}$$

and

$$\begin{aligned} \max_{\boldsymbol{s},\boldsymbol{p}} \quad & s_i \\ s.t. \quad & -f_{mn} \le p_{mn} \le f_{mn}, \ \forall (m,n) \in \mathcal{E}, \\ & \sum_{(m,n)\in L} \frac{p_{mn}}{B_{mn}} = 0, \ \text{for any loop } L \in \mathcal{L}_i, \\ & s_i - d_i = \sum_{j\in\mathcal{N}_i} p_{ij}. \end{aligned} \tag{39}$$

The optimal value of the first optimization problem is clearly $\min\left\{\bar{s}_i, D - \sum_{j\neq i} \underline{s}_j\right\}$. Hence, the optimal value of (37) is

$$\min\left\{\bar{s}_i, D - \textstyle\sum_{j\neq i} \underline{s}_j, \hat{s}_i\right\},$$

where $\hat{s}_i$ is defined as the optimal value of (39). Now it remains to calculate $\hat{s}_i$.

Note that the optimization problem (39) is equivalent to the one below

$$\begin{aligned} \max_{\boldsymbol{p}} \quad & d_i + \sum_{j\in\mathcal{N}_i} p_{ij} \\ s.t. \quad & -f_{mn} \le p_{mn} \le f_{mn}, \ \forall (m,n) \in \mathcal{E}, \\ & \sum_{(m,n)\in L} \frac{p_{mn}}{B_{mn}} = 0, \ \text{for any loop } L \in \mathcal{L}_i. \end{aligned} \tag{40}$$

In summary, we have proved that the optimal value of (34) is upper bounded by

$$\begin{aligned} \Delta \ &\triangleq\ \min\left\{\max_{m\in\mathcal{N}_g} \bar{s}_m, \max_{m\in\mathcal{N}_g}\left\{d_m + \sum_{(m,n)\in\mathcal{E}} f_{mn}\right\}\right\} \\ &= \max_{m\in\mathcal{N}_g} \min\left\{\bar{s}_m, d_m + \sum_{(m,n)\in\mathcal{E}} f_{mn}\right\}, \end{aligned}$$

which yields the upper bound of the PoA $1 + \frac{\Delta}{D-2\Delta}$ in Theorem 1.

E. Proof of Theorem 2

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