Competition and Investment in On-Demand Networking Technology

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***Abstract*—Modern on-demand networking technologies (e.g., WiFi-Direct) improve users' networking experiences (e.g., enable them to connect to any WiFi enabled devices). Manufactures of wireless devices may choose to provide these technologies to make their products more appealing and charge a higher price. The perceived benefits from these technologies depend on how many other users are in the network, which we refer to as "positive network effect". In this paper, we study such a scenario and develop a price-competition based model for the market, where device manufacturers may choose to invest in on-demand technology with positive network effect and consumers choose whether to participate in the market and which manufacturer to buy from. While the positive network effect can give some firms a competitive advantage, leading to a monopolistic market outcome, we show that market equilibrium always exists and the social welfare is larger when the network effect is strong enough. This is because although only the "best" firms have positive market shares at equilibrium, competition from other firms limits the best firms' actions. For a case study of linear demand and network effects, we have analytical characterizations of the equilibrium and show that competition drives firms to invest more on technology and set lower prices at equilibrium, which improves equilibrium social welfare. Numerical studies suggest that these observations may hold in more general settings.**

I. Introduction

Device-to-device (D2D) communication technologies such as WiFi-Direct provide the ability for wireless users to communicate on demand without needing external infrastructure [1]. The potential advantages of such technologies include providing an alternative way of routing traffic when networks become congested [2], supporting collaborative streaming for high quality video [3], enabling communication when out of coverage or after a disaster [4], and providing a more efficient means for supporting various proximity services [5]. The value a user sees from such technologies and the services they enable depends in part on the number of other users that have access to such technologies. For example, a user will value having a device and software that easily supports D2D communication only if that user encounters other devices it can communicate with; the more such devices encountered, the greater the user will value this technology. This is an example of a *network effect* or a *positive externality*. Such effects are common in many communication services as well as other markets [6].

The focus of this paper is on network effects that arise with a D2D technology such as WiFi-Direct and in particular on the incentives of a device manufacturer (or a service provider) to enable such a technology in the devices it sells. Network effects can impact the competition among firms by enabling a firm to charge higher prices or further expand its customer base. Indeed there is a large economics literature that looks at the competition among firms which own separate (but similar) networks, with the results often being that a single monopolist emerges (e.g., see [7]). In this line of work the underlying network of each firm is "closed", i.e., it is for the exclusive use of a given firm's customers. An example is a social media site such as Facebook, where one has to be a user of that site to enjoy the positive externality of the network.

In this paper, motivated by WiFi-Direct, we study settings in which the underlying technology creates an "open" network, namely the positive externality depends on the number of customers of *all* the competing firms. For example, consider the case where firms are device manufactures. If a manufacture decides to provide WiFi-Direct enabled devices to its users, those users can communicate with not only other users of this manufacturer's devices, but users of any WiFi device made by any manufacturer. Indeed, using WiFi-Direct they can communicate directly with any other WiFi device even if that device does not support WiFi-Direct [8].[1] When a consumer is choosing which manufacturer to buy from, having WiFi-Direct is an appealing feature due to the improved connectivity and thus could give the firm an edge over the competition. Given such an open technology, we aim to understand how it affects the competition among firms and their investment decisions.

Our approach is based on the work in [9]–[12] for studying competition under negative externalities due to congestion effects. As in these works, we assume that there is a pool of infinitesimal users, whose valuation for a product depends on a *delivered price*, which is the sum of the price charged by the firm and a term that models the externality.[2] This externality term in our model has the opposite effect on the delivered price from these previous works, i.e., here, with more users a service becomes more valuable as opposed to less in the congestion case.

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[1] A manufacture could instead enable D2D communications in such a way to make the resulting network closed as in Apple's AirDrop protocol. We leave such considerations for future work.

[2] In other words we assume that each user's value for the product is separable into two terms, one that depends on the externality and one that does not.

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Using such a model we formulate a two-stage game. In the first stage, firms decide on the amount to invest in the given open D2D technology and the price they will charge for their devices. Here, the level of investment can model, for example, improvements in the underlying communication protocol but also improvements in the ease of use of the software supporting that protocol. Users of firms that invest more in the technology, experience a larger positive externality, which in turn can allow the firms to increase their prices. In the second stage, users select a single firm from which to purchase a device given the firms' investments and prices.

We characterize the equilibrium of such a market and show that in general only a subset of "best" firms will end up serving users in equilibrium, i.e., as with closed networks, open networks can still lead to more monopolistic markets. However, we also show that the firms out of the market still have a credible threat to the best firms. This causes the best firm to announce lower prices and improves welfare compared to a case without the open technology. In other words, the network effect generated by an open D2D technology can lead to a consolidation of market power, but due to the threat of entry, this is also beneficial for overall social welfare.

In addition to the aforementioned work on competition with congestion externalities, other related work includes an extensive literature on monopoly pricing with externalities such as [13], [14]. Another strand of related work is the literature on "club goods", which include both positive and negative externalities, e.g. [15]. In these works, the externalities only depend on the members of a given club, as opposed to the open networks we consider. Another work that studies open networks is [16], which considers a model for ad supported cloud services. Different from us, they focus on both positive and negative externalities and assume that firms do not charge for services as they are ad supported.

The rest of the paper is organized as follows. Section II describes the system model and defines the market equilibrium. Section III analyzes the equilibrium in the general setting, followed by a case study in Section IV. Section V presents numerical results. Finally, Section VI concludes the paper.

II. Model and Market Equilibrium

A. Basic Setup

We consider a market with a set $\mathcal{N} = \{1, \dots, N\}$ of N firms (e.g., device manufacturers). The firms provide products (e.g., cell phones) to a large population of infinitesimal users. Each firm i chooses an investment level[3] $I_i \in [0, 1]$ and a price $p_i \in \mathbb{R}_+$ for its product. Given investments $\boldsymbol{I} \triangleq (I_1, \dots, I_N)$ and prices $\boldsymbol{p} \triangleq (p_1, \dots, p_N)$, each firm i gets a population x_i of customers. Firm i's *market share* is the population x_i of users choosing its product, and the *total market size* is the total population $X \triangleq \sum_{i=1}^{N} x_i$ of users in the market.

The investment has two conflicting effects for the firm. One effect is the investment cost given by $c_i(I_i) \cdot x_i$. The investment cost is the cost per product $c_i(I_i)$ multiplied by the market share x_i. In other words, the investment cost is incurred for each product sold. This is reasonable, for example if the investment models the hardware put into a product (e.g., a better WiFi chipset) and not on the innovation of technology (e.g., the research leading to a better chip), which is one-time and does not scale with the market share.

The other effect of investment is the benefit from positive network externality: a user of firm i's product enjoys a benefit from the positive network externality of $b_i(I_i, X)$. The benefit depends on the total market size (not just on firm i's market share) and the level of investment by firm i. This captures the "openness" of the D2D technology such as WiFi-Direct as we have discussed.[4]

Given the investment level I_i, price p_i, and market share x_i, firm i's profit $\pi_i(I_i, p_i, x_i)$ is simply the revenue minus the cost, calculated as:

$$\pi_i (I_i, p_i, x_i) = p_i \cdot x_i - c_i(I_i) \cdot x_i = [p_i - c_i(I_i)] \cdot x_i. \quad (1)$$

A final ingredient of our model is the users' demand, which is characterized by the inverse demand $P(X)$. $P(X)$ indicates the price that a market size of X users are willing to pay for the product. We next introduce the some notations and assumptions, and illustrate this setup via an example .

B. Useful Notions and Assumptions

Now we define three useful notions. The first notion is firm i's *delivered price*, $p_i - b_i(I_i, X)$, defined as the price p_i of the product minus the benefit $b_i(I_i, X)$ from the positive network externality. Firm i's delivered price is the effective price perceived by a user, taking the positive network externality into account.

The second notion is the *effective inverse demand* for firm i's product, defined as

$$P(X) + b_i(I_i, X).$$

The effective inverse demand is the price the Xth user is willing to pay for firm i's product, again taking the positive network externality into account.

The third notion is firm i's *monopolistic market share*, $Y_i(I_i, p_i)$, which is the *largest* nonnegative solution x_i to the equation

$$p_i - b_i (I_i, x_i) = P(x_i). \quad (2)$$

The monopolistic market share $Y_i(I_i, p_i)$ is the largest number of users choosing firm i's product if firm i was the only firm in the market and chose the investment I_i and the price p_i.

We also define firm i's *maximum monopolistic market share* $\hat{Y}_i$ as the maximum of $Y_i(I_i, p_i)$ over all the decisions (I_i, p_i) that result in nonnegative profit:

$$\hat{Y}_i = \max_{I_i, p_i} \; Y_i(I_i, p_i) \quad \text{s.t.} \quad p_i \geq c_i(I_i). \quad (3)$$

[3]The fact that the maximum investment level is 1 is just a normalization.

[4]In some settings, one could also argue that even users of a different firm j's technology might benefit from firm i's technology, as firm i's users might be able to establish better connections with them. Here, we ignore these secondary effects and focus on the case where a customer only sees benefits from the investment of the firm it chooses.

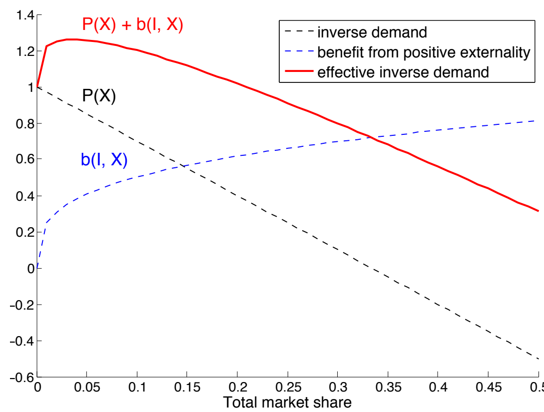

Fig. 1. The inverse demand, the benefit from positive externality, and the effective inverse demand for the model in Example 1 with $\theta = 3$, $\beta_i = 1$, and $\alpha_i = 0.3$.

Note that each firm i's maximum monopolistic market share $\hat{Y}_i$ exists and is unique, and hence is well defined (see Lemma 1 in our online appendix [17]). Given $\{\hat{Y}_i\}_{i=1}^N$, we can define

$$\mathcal{M} = \arg\max_{i \in \mathcal{N}} \hat{Y}_i$$

as the set of firms with the highest maximum monopolistic market shares.

Assumption 1: We make the following assumptions throughout this paper:

- the inverse demand $P(X)$ is continuous and strictly decreasing in X,
- the benefit from positive externality $b_i(I_i, X)$ is continuous and increasing in I_i and X,
- the effective inverse demand $P(X) + b_i(I_i, X)$ is quasi-concave in X, and is strictly decreasing for large enough X under any $I_i \in [0, 1]$,
- the investment cost per product $c_i(I_i)$ is continuous and increasing in I_i,
- $P(0) > 0$, $b_i(I_i, X) \geq 0$ for all i, I_i, X, and $c_i(0) = 0$ for all i.

The conditions in Assumption 1 are mild. In fact, we only need the effective inverse demand to be decreasing in the number of users when there are many users and to be quasi-concave, which is less stringent than the usual assumptions of monotonicity in any number of users and concavity.

Example 1: A model that satisfies Assumption 1 is

$$P(X) = 1 - \theta X, \ b_i(I_i, X) = \beta_i I_i X^\alpha, \ \text{and} \ c_i(I_i) = \gamma_i I_i,$$

where $\theta > 0$, $\beta_i \in [0, \theta)$, $\alpha \in [0, 1]$, and $\gamma_i > 0$. Note that when $\alpha \in (0, 1)$, the effective inverse demand $1 - \theta X + \beta_i I_i X^\alpha$ is increasing in the total market size X when X is small, and then is decreasing in X. This externality model is motivated by [16]. See Fig. 1 for an illustration.

C. Market Equilibrium

Next, we describe the market equilibrium. We model the market as the following two-stage game: In stage one, the firms choose their investment levels and the prices for their products. In stage two, the users choose whether to participate in the market and if so, which firm to buy from.

In stage two, the firms' decisions $(\boldsymbol{I}, \boldsymbol{p})$ and the market shares $\boldsymbol{x} \triangleq (x_1, \ldots, x_N)$ must satisfy:

$$\begin{aligned} p_i - b_i(I_i, X) &\geq P(X), \forall i \in \mathcal{N}; \\ p_i - b_i(I_i, X) &= P(X), \forall i \text{ such that } x_i > 0. \end{aligned} \quad (4)$$

The conditions in (4) require that:

i.) the inverse demand cannot be higher than any delivered price (otherwise, more users would be willing to enter the market and buy products), and

ii.) any firm with positive market share must have a delivered price equal to the inverse demand (a strictly higher delivered price would prevent users from buying its product).

Note that given the decisions $(\boldsymbol{I}, \boldsymbol{p})$, the market shares $\boldsymbol{x}$ that satisfy the conditions in (4) may not be unique. This is because the conditions in (4) impose constraints on the total market size X, which may be shared among the firms in different ways. Hence, given a total market size X that satisfies (4), we need to specify a rule of dividing the total market size. One class of market division rules is as follows:

Definition 1 (Market Division Based on Maximum Monopolistic Market Share ($\hat{Y}_i$)): The market is shared only among the firm(s) with the highest maximum monopolistic market share (i.e., the firms in $\mathcal{M}$).

The market division rules in Definition 1 allocate no market share to firms outside the set $\mathcal{M}$, even to a firm that has a delivered price equal to the inverse demand. Note that the division of the market among the firms in $\mathcal{M}$ can be arbitrary. It turns out that this class of market division rules are the only "meaningful" rules, in the sense that there is no equilibrium at which a firm outside $\mathcal{M}$ has a positive market share (see Proposition 1 in Section III).

For an arbitrary market division rule Γ, let $\boldsymbol{x}^\Gamma(\boldsymbol{I}, \boldsymbol{p})$ be a profile of market shares that satisfy the conditions in (4) and this division rule Γ. We write the market shares as a function of investment and prices $(\boldsymbol{I}, \boldsymbol{p})$ to make the dependence explicit.

Now we can formally define the market equilibrium.

Definition 2 (Market Equilibrium): Fix a market division rule Γ. The market equilibrium is a triple $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$ of investment levels, prices, and market shares, such that

- the investment $\boldsymbol{I}^*$, the prices $\boldsymbol{p}^*$, and the total market size $X^* = \sum_{i=1}^N x_i^*$ satisfy (4), and the total market size X^* is divided into $\boldsymbol{x}^*$ according to the division rule Γ;
- each firm i's profit is maximized, namely

$$\pi_i(I_i^*, p_i^*, x_i^*) \geq \pi_i\left[I_i, p_i, x_i^\Gamma\left((I_i, \boldsymbol{I}_{-i}^*), (p_i, \boldsymbol{p}_{-i}^*)\right)\right] \quad (5)$$

for any investment $I_i \in [0, 1]$, any price $p_i \geq 0$, and any market share $x_i^\Gamma\left((I_i, \boldsymbol{I}_{-i}^*), (p_i, \boldsymbol{p}_{-i}^*)\right)$, where $\boldsymbol{I}_{-i}^*$ and $\boldsymbol{p}_{-i}^*$ are the investment and prices of all firms other than i.

Definition 2 ensures that at the market equilibrium, the users in the market will not leave, new users will not enter the market, and the firms maximize their own profits.

D. Performance Benchmark

We are interested in the social welfare at market equilibrium. The social welfare is the sum payoff of the users and the firms. Given the investment and prices $(\boldsymbol{I}, \boldsymbol{p})$ and the market shares $\boldsymbol{x}$, the social welfare can be calculated as follows:

$$
\begin{aligned}
SW(\boldsymbol{I}, \boldsymbol{p}, \boldsymbol{x}) \;\; = \;\; & \underbrace{\int_0^{\sum_{i=1}^N x_i} P(y)dy}_{\text{valuation of service}} - \underbrace{\sum_{i=1}^N c_i(I_i) \cdot x_i}_{\text{investment cost}} \\
+ \;\; & \underbrace{\sum_{i=1}^N b_i\left(I_i, X\right) \cdot x_i}_{\text{benefit from positive externality}}
\end{aligned}
$$

The market equilibrium $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$ depends on the functions $\{b_i(\cdot)\}_{i=1}^N$ which characterize the impact of investment on the positive externality, among other things. Therefore, one goal in this work is to understand how the social welfare depends on the positive externality.

III. Equilibrium Analysis

In this section, we first prove that any market equilibrium exhibits a monopolistic structure, where one or only a few firms may have positive market shares, while the other firms are out of the market. Next, we compare with the scenario where there is no positive externality. We show that when there is no externality, all the firms share the market at the market equilibrium. Finally, we show that despite the fact that the market equilibrium under positive externality exhibits a monopolistic structure, the resulting social welfare is higher than that under no externality.

A. Characteristics of Market Equilibrium

We first establish the monopolistic structure of the market equilibrium.

Proposition 1 (Monopolistic Structure): At any market equilibrium, the market is shared among the firms with the highest monopolistic market share, i.e., $x_j^* = 0$ for any $j \notin \mathcal{M}$.

Proof: See Appendix A. ■

Proposition 1 ensures that at any equilibrium, the firms outside the set $\mathcal{M}$ have no market share. In this sense, the equilibrium has a monopolistic structure, where the market is shared only among a subset $\mathcal{M}$ of firms (unless all firms the same maximum monopolistic market share).

Proposition 1 also implies that there exists no equilibrium under any market division rule other than that in Definition 1. Therefore, in the rest of this paper, we assume that the division rule in Definition 1 is used.

Next, we give a full characterization of the market equilibrium. Before that, we write $(\hat{I}_i, \hat{p}_i)$ as firm i's decision that results in its maximum monopolistic market share, namely

$$Y_i(\hat{I}_i, \hat{p}_i) = \hat{Y}_i. \tag{6}$$

Moreover, since the effective inverse demand $P(X)+b_i(I_i, X)$ is strictly decreasing in the market size X, we must have

$$\hat{p}_i = c_i(\hat{I}_i). \tag{7}$$

Proposition 2 (Full Characterization of Equilibrium): The market equilibrium always exists and satisfies:

- Suppose that $|\mathcal{M}| > 1$. At any equilibrium, at least two firms $i, i' \in \mathcal{M}$ choose
$$(I_i^*, p_i^*) = (\hat{I}_i, \hat{p}_i), \text{ and } (I_{i'}^*, p_{i'}^*) = (\hat{I}_{i'}, \hat{p}_{i'}),$$
and firm $j \neq i, i'$ chooses (I_j^*, p_j^*), arbitrarily. All the firms have zero profit at the equilibrium. The total market size $X^* = \hat{Y}_i$ is arbitrarily shared among the firms in $\mathcal{M}$ that choose $(I_i^*, p_i^*) = (\hat{I}_i, \hat{p}_i)$.
- Suppose that $|\mathcal{M}| = 1$. At any equilibrium, firm $i \in \mathcal{M}$ chooses (I_i^*, p_i^*) as a solution to the following problem
$$
\begin{array}{ll}
\max_{I_i, p_i} & [p_i - c_i(I_i)] \cdot Y_i(I_i, p_i) \\
s.t. & Y_i(I_i, p_i) \geq \hat{Y}_{i'}, \ \forall i' \neq i,
\end{array}
$$
at least one firm $j \in \arg\max_{n \notin \mathcal{M}} \hat{Y}_n$ with the second highest maximum monopolistic market share chooses
$$(I_j^*, p_j^*) = (\hat{I}_j, \hat{p}_j),$$
and firm $k \neq i, j$ chooses (I_k^*, p_k^*) arbitrarily. The total market size $X^* = Y_i(I_i^*, p_i^*)$ belongs to firm i only.

Proof: See Appendix B. ■

Proposition 2 proves the existence of the market equilibrium, and fully characterizes the decisions and market shares at any equilibrium. In general, there exist multiple equilibria. The multiplicity occurs because the firms outside the set $\mathcal{M}$, namely the firms with no market share, can make arbitrary decisions. The decisions of the firms with positive market shares are unique. In this sense, the market equilibrium is essentially unique. Note also that when $|\mathcal{M}| > 1$, the market is arbitrarily shared among the firms $i \in \mathcal{M}$ that choose $(I_i^*, p_i^*) = (\hat{I}_i, \hat{p}_i)$. Although the equilibrium market shares of these firms are arbitrary, the social welfare at any equilibrium is the same (see Lemma 2 in our online appendix [17]).

B. Positive Exteranlity Improves Equilibrium Social Welfare

A major goal of this paper is to show that a positive network externality, as long as it is not too "weak", improves the social welfare at the equilibrium. Before describing our result, we need to make it precise what we mean by "no network externality" and "strong positive network externality".

Definition 3 (No Network Externality): We say that there is no network externality if for any firm i, its users receive zero benefit $b_i(I_i, X) = 0$ for all I_i and X.

Under no network externality, there can not be a monopoly in equilibrium. The reason is as follows. With no network externality, each firm i's monopolistic market share $Y_i(I_i, p_i)$ is the solution X to $p_i = P(X)$. Since $P(X)$ is strictly decreasing in X, to reach the maximum monopolistic market share, firm i should choose a price p_i as low as possible, subject to the constraint that $p_i \geq c_i(I_i)$. Therefore, firm i should choose $I_i = 0$ and $p_i = c_i(0)$ to reach its maximum monopolistic market share. We denote firm i's maximum monopolistic market share by $\hat{Y}_i^0$ (the superscript 0 indicates no network externality). Then we have

$$P(\hat{Y}_i^0) = c_i(0) = 0, \ \forall i. \tag{8}$$

Therefore, all the firms have the same maximum monopolistic market share $\hat{Y}_i^0 = P^{-1}(0)$, namely $\mathcal{M} = \mathcal{N}$, meaning that the market is shared among all the firms in the equilibrium.

Definition 4 (Strong Positive Network Externality): We say that there is strong positive network externality, if the second highest maximum monopolistic market share is larger than the maximum monopolistic market share under no network externality, namely

$$\max_{n \notin \mathcal{M}} \hat{Y}_n > P^{-1}(0). \tag{9}$$

The requirement of strong positive network externality is not very restrictive. In fact, we allow most firms to have the same maximum monopolistic market share as that under no network externality (i.e., $\hat{Y}_i = \hat{Y}_i^0$).

Now we state our main result.

Theorem 1: Under Assumption 1, the equilibrium social welfare under strong positive network externality is strictly larger than the equilibrium social welfare under no network externality.

Proof: See Appendix C. ■

Theorem 1 says that a positive network externality, as long as it is not too weak, improves the social welfare at the market equilibrium. This is somewhat surprising when considering the monopolistic structure of the market equilibrium under a positive network externality, and the lack of monopoly at the equilibrium under no network externality. In particular, Theorem 1 holds even when the positive externality is so asymmetric among the firms that one firm emerges as the monopolist at the equilibrium. The intuition behind this result is that although a firm is the monopolist at the equilibrium, it is the "best" firm and its decision is limited due to the competition of the other firms (i.e., $Y_i(I_i^*, p_i^*) \geq \hat{Y}_j, \forall j \neq i$) and the externality enlarges the total market size at equilibrium.

IV. Case Study – The Linear Model

In this section, we focus on the following linear model:

$$\begin{aligned} P(X) &= 1 - \theta X, \\ b_i(I_i, X) &= \beta_i I_i X, \\ c_i(I_i) &= \gamma_i I_i, \end{aligned} \tag{10}$$

where $\theta > 0$, $\beta_i \in [0, \theta)$, and $\gamma_i > 0$.

Under the linear model, we can obtain analytical expressions of the equilibrium decisions, market shares, and social welfare. As a result, we can provide more insights on the market equilibrium, which were not available from the results on the general model in Section III. In particular, we would like to understand how competition affects the market equilibrium.

To understand the impact of competition, we focus on the case of a single firm i with the highest maximum monopolistic market share (i.e., $\mathcal{M} = \{i\}$). We write $\hat{Y}_{-i} = \max_{n \neq i} \hat{Y}_n$ as the second highest maximum monopolistic market share. We would like to see how the market equilibrium changes as $\hat{Y}_{-i}$ approaches $\hat{Y}_i$, namely as competition becomes more intense.

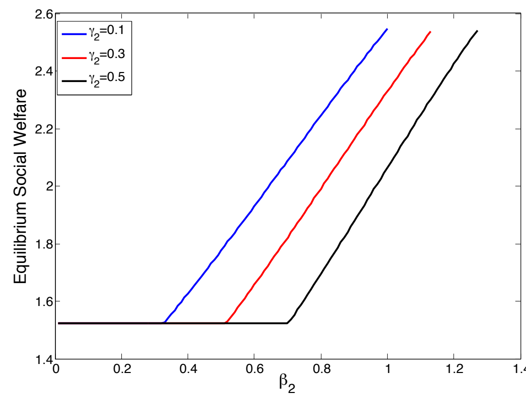

Fig. 2. Illustration of equilibrium social welfare as competition increases (i.e., as β_2 increases). We fix $\theta = 1.1$, $\alpha = 0.5$, $\beta_1 = 1$, and $\gamma_1 = 0.1$.

Due to space limitations, we provide the detailed expressions of equilibrium decisions, equilibrium market shares, and equilibrium social welfare in our online appendix [17]. In this paper, we summarize the insights from the analytical characterization of the equilibrium under the linear model.

Proposition 3: Suppose that there is a single firm i with the highest monopolistic market share (i.e., $\mathcal{M} = \{i\}$). Then we have the following:

- Firm i's equilibrium investment level is always 1;
- The equilibrium price is non-increasing in $\hat{Y}_{-i}$;
- The equilibrium market share and social welfare are non-decreasing in $\hat{Y}_{-i}$.

Proof: See our online appendix [17] for the analytical expressions of equilibrium decisions, equilibrium market shares, and equilibrium social welfare. The above observations follow directly from the expressions. ■

This proposition gives us some insights about the equilibrium under the linear model. Specifically, competition drives firm i to fully invest in the externality and to decrease the price of its product, which increases the number of users in the market and improves the social welfare.

V. Simulation

In this section, we present some numerical results to illustrate the properties of the market equiibrium. In the simulation, we consider a two-firm market and the model in Example 1.

We first look at how competition affects the equilibrium social welfare. To this end, we let firm 1 to be the one in the market (i.e., $\hat{Y}_1 > \hat{Y}_2$), and increase firm 2's $\hat{Y}_2$ by increasing β_2. Fig. 2 shows the equilibrium social welfare as we increase β_2, namely as competition becomes more intense. We can see that under different parameters γ_2, the equilibrium social welfare exhibits the same trend: it increases as competition becomes more intense. We conjecture that social welfare increasing with competition may hold under more general models than the linear one.

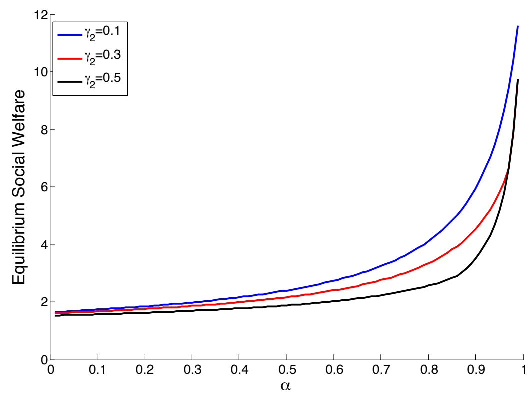

Fig. 3. Illustration of equilibrium social welfare as α increases. We fix $\theta = 1.1$, $b_1 = 1$, $b_2 = 0.9$, and $\gamma_1 = 0.1$.

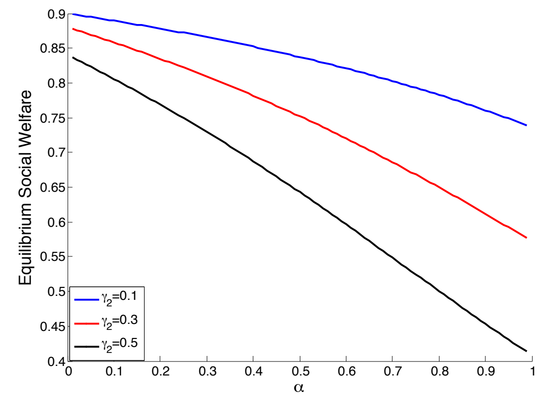

Fig. 5. Illustration of equilibrium social welfare as α increases. We fix $\theta = 2$, $b_1 = 1$, $b_2 = 0.9$, and $\gamma_1 = 0.1$.

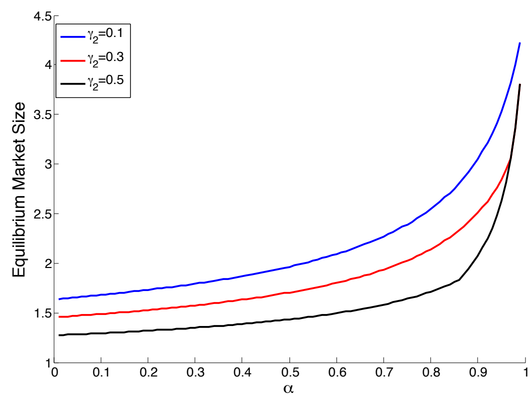

Fig. 4. Illustration of equilibrium market share as α increases. We fix $\theta = 1.1$, $b_1 = 1$, $b_2 = 0.9$, and $\gamma_1 = 0.1$.

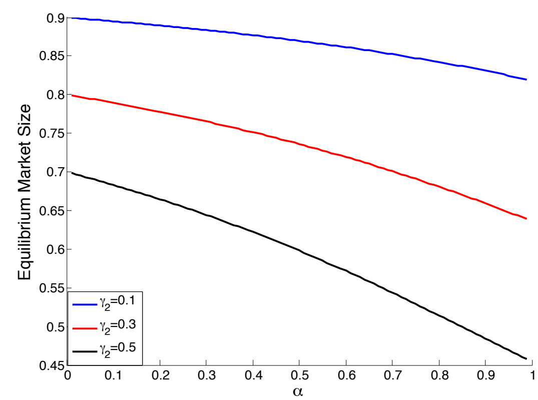

Fig. 6. Illustration of equilibrium market share as α increases. We fix $\theta = 2$, $b_1 = 1$, $b_2 = 0.9$, and $\gamma_1 = 0.1$.

Next, we look at different forms of positive externality. Again, we let firm 1 to be the one in the market (i.e., $\hat{Y}_1 > \hat{Y}_2$), and fix all the parameters other than the exponent α in the benefit function $b_i(I_i, X) = \beta_i I_i X^{\alpha}$. When $\alpha = 0$, the benefit does not depend on the market share X. When $\alpha = 1$, the benefit scales linearly with the market share X. Note that as we increase α, the benefit b_i may increase or decrease, depending on the size of X. If $X > 1$, a higher α results in a higher benefit. In contrast, if $X < 1$, a higher α results in a lower benefit.

Figures 3 and 4 show the equilibrium social welfare and market size as α increases, respectively. Fig. 4 shows that $X^* > 1$, which means the benefit increases with α. Fig. 3 shows that the equilibrium social welfare increases with α.

Under a different set of parameters, we show the equilibrium social welfare and market share as α increases in Fig. 5 and Fig. 6, respectively. Note from Fig. 6 that $X^* < 1$, which means the benefit decreases as α increases. Fig. 5 shows that the equilibrium social welfare decreases with α.

In summary, Fig. 3–6 indicate that the equilibrium social welfare increases with the benefit from positive network externality. However, the impact of the form of the benefit functions (i.e., α) depends on the size of the equiibrium market share.

VI. Conclusion

This paper introduced a general model of investment and competition in on-demand networking technology. Our model captured a key feature of on-demand networking: as firms invest in advanced on-demand networking technology, the users benefit from the resulting positive network externality. We show that under no network externality, the market is shared among all the firms at the equilibrium, namely there is no monopoly. In contrast, under positive network externality, the market is shared among a subset of firms at the equilibrium, and exhibits a monopolistic structure. Our main finding is that although a positive network externality induces such a monopolistic structure, it improves the equilibrium social welfare compared to the case with no network externality. Our result motivates the development of more advanced on-demand networking technology. Finally, we investigate the market equilibrium in more detail under a linear model, and show that competition drives firms to invest more in technology and set lower prices at equilibrium, which improves equilibrium social welfare. Simulations results suggest that these observations may hold in more general settings. Future directions include comparing consumer welfare and firms' profit with and without externality and extending the framework to dynamic network, where demand is time varying.

APPENDIX

A. Proof of Proposition 1

We prove the proposition by contradiction. Specifically, suppose that at an equilibrium $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$, we have $x_j^* > 0$ for a firm $j \notin \mathcal{M}$. Based on conditions in (4), we have

$$p_j^* - b_j(I_j^*, X^*) = P(X^*). \tag{11}$$

Since firm j can always choose a large enough p_j to exit the market and get zero profit, its equilibrium profit must be nonnegative, namely $p_j^* \geq c_j(I_j^*)$. Therefore, the decision (I_j^*, p_j^*) is a feasible solution to (3), which leads to

$$X^* = Y_j(I_j^*, p_j^*) \leq \hat{Y}_j. \tag{12}$$

Consider a firm $i \in \mathcal{M}$, namely a firm with the highest maximum monopolistic market share. First, we show that firm i can secure a positive profit when the other firms choose $(\boldsymbol{I}_{-i}^*, \boldsymbol{p}_{-i}^*)$. Suppose that firm i's maximum monopolistic market share $\hat{Y}_i$ is achieved at $(\hat{I}_i, \hat{p}_i)$, namely

$$Y_i(\hat{I}_i, \hat{p}_i) = \hat{Y}_i > \hat{Y}_j \geq X^*. \tag{13}$$

Then firm i can increase its price by a small amount $\varepsilon > 0$, such that we still have $Y_i(\hat{I}_i, \hat{p}_i + \varepsilon) > X^*$. Then for any firm $k \neq i$, since the effective inverse demand is strictly decreasing in the market share, we have

$$\begin{aligned} p_k^* &\geq b_k\left(I_k^*, X^*\right) + P\left(X^*\right) \qquad (14) \\ &> b_k\left[I_k^*, Y_i\left(\hat{I}_i, \hat{p}_i + \varepsilon\right)\right] + P\left[Y_i\left(\hat{I}_i, \hat{p}_i + \varepsilon\right)\right]. \end{aligned}$$

In other words, by choosing $(\hat{I}_i, \hat{p}_i + \varepsilon)$, firm i grabs the entire market share of $Y_i\left(\hat{I}_i, \hat{p}_i + \varepsilon\right)$. Since $\hat{p}_i \geq c_i(\hat{I}_i)$, we have $\hat{p}_i + \varepsilon > c_i(\hat{I}_i)$. In this way, firm i secures a positive profit.

Since firm i can secure a positive profit when the other firms choose $(\boldsymbol{I}_{-i}^*, \boldsymbol{p}_{-i}^*)$, it must have a positive profit at the equilibrium $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$. Therefore, we must have $p_i^* > c_i(x_i^*)$ and $x_i^* > 0$. Since $x_i^* > 0$, we have

$$p_i^* - b_i(I_i^*, X^*) = P(X^*). \tag{15}$$

Now firm i can decrease its price by a small $\delta > 0$ such that $p_i^* - \delta > c_i(x_i^*)$. At the new price $p_i^* - \delta$, we also have

$$Y_i(I_i^*, p_i^* - \delta) > X^*, \tag{16}$$

and hence

$$\begin{aligned} p_k^* &\geq b_k(I_k^*, X^*) + P(X^*) \qquad (17) \\ &> b_k\left[I_k^*, Y_i(I_i^*, p_i^* - \delta)\right] + P\left[Y_i(I_i^*, p_i^* - \delta)\right], \ \forall k \neq i. \end{aligned}$$

In other words, if firm i chose $(I_i^*, p_i^* - \delta)$, it would grab the entire market share of $Y_i(I_i^*, p_i^* - \delta) > X^* \geq x_i^* + x_j^*$, and get a new profit of

$$\left[p_i^* - \delta - c_i(I_i^*)\right] \cdot Y_i(I_i^*, p_i^* - \delta). \tag{18}$$

Compared to the equilibrium profit $[p_i^* - c_i(I_i^*)] \cdot x_i^*$, the increase in the new profit is

$$\begin{aligned} &\left[p_i^* - c_i(I_i^*)\right] \cdot \left[Y_i(I_i^*, p_i^* - \delta) - x_i^*\right] - \delta \cdot Y_i(I_i^*, p_i^* - \delta) \\ > \; &\left[p_i^* - c_i(I_i^*)\right] \cdot x_j^* - \delta \cdot \hat{Y}_i, \qquad (19) \end{aligned}$$

which can be made positive by setting δ to be small enough. However, this is contradictory to the fact that $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$ is an equilibrium, which concludes the proof.

B. Proof of Proposition 2

1) The Case of $|\mathcal{M}| > 1$: We first show that any triple $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$ specified in the proposition is a market equilibrium, which also implies the existence of the market equilibrium. Consider any firm k. There exists another firm $i \neq k, i \in \mathcal{M}$ that chooses $(I_i^*, p_i^*) = (\hat{I}_i, \hat{p}_i)$. Therefore, we have $X^* = \hat{Y}_i \geq \hat{Y}_k$. If firm $k \notin \mathcal{M}$, since $Y_k(I_k, p_k) \leq \hat{Y}_k < \hat{Y}_i$ for any I_k, p_k such that $p_k \geq c_k(I_k)$, firm k gets no market share. If firm $k \in \mathcal{M}$, it can get a positive market share only if it chooses $(\hat{I}_k, \hat{p}_k)$, at which it receives zero profit. In summary, firm k always gets zero profit, and cannot improve the zero profit it receives at equilibrium. Hence, the triple $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$ specified in the proposition is an equilibrium.

Next, we show that any equilibrium must satisfy the conditions in the proposition.

- Suppose that only one firm $i \in \mathcal{M}$ chooses $(I_i^*, p_i^*) = (\hat{I}_i, \hat{p}_i)$ in $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$. Then for any firm $j \neq i$, we have

  $$Y_j(I_j^*, p_j^*) < \hat{Y}_j = \hat{Y}_i = Y_i(I_i^*, p_i^*), \text{ if } j \in \mathcal{M}, \tag{20}$$

  and

  $$Y_j(I_j^*, p_j^*) \leq \hat{Y}_j < \hat{Y}_i = Y_i(I_i^*, p_i^*), \text{ if } j \notin \mathcal{M}. \tag{21}$$

  In summary, we have $Y_j(I_j^*, p_j^*) < Y_i(I_i^*, p_i^*)$ for any $j \neq i$. In this case, firm i can increase its price by a small amount ϵ, such that we still have $Y_i(I_i^*, p_i^* + \epsilon) > Y_j(I_j^*, p_j^*)$ for all $j \neq i$. Then firm i would grab the entire market share of $Y_i(I_i^*, p_i^* + \epsilon)$ and get a positive profit, which improves its zero profit at $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$. Hence, this $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$ cannot be an equilibrium.
- Suppose that no firm $i \in \mathcal{M}$ chooses $(I_i^*, p_i^*) = (\hat{I}_i, \hat{p}_i)$ in $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$. Then we must have $X^* < \hat{Y}_i$ for $i \in \mathcal{M}$. In addition, since $|\mathcal{M}| > 1$, there must be one firm $i' \in \mathcal{M}$ that does not get the entire market X^*. Following the same logic of the proof in Appendix A, firm i' can secure a positive profit against $(\boldsymbol{I}_{-i}^*, \boldsymbol{p}_{-i}^*)$ by choosing $(\hat{I}_i, \hat{p}_i + \varepsilon)$, and can improve its profit by choosing $(I_i^*, p_i^* - \delta)$. Hence, this $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$ cannot be an equilibrium.

2) The Case of $|\mathcal{M}| = 1$: We first show that any triple $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$ specified in the proposition is a market equilibrium, which also implies the existence of equilibrium.

- Consider any firm $j \neq i$. Since firm i chooses (I_i^*, p_i^*) such that $Y_i(I_i^*, p_i^*) \geq \hat{Y}_j$, firm j cannot get a positive market share by any decision (I_j, p_j). Therefore, firm j always gets zero profit and is indifference in any decision.

- Consider firm i. Since firm j chooses $(I_j^*, p_j^*) = (\hat{I}_j, \hat{p}_j)$, firm i has to make sure that $Y_i(I_i^*, p_i^*) \geq \hat{Y}_j$. Otherwise, the entire market will go to firm j. Therefore, firm i maximizes its profit subject to the constraint that $Y_i(I_i^*, p_i^*) \geq \hat{Y}_j$ for all $j \neq i$.

Next, we show that any equilibrium must satisfy the conditions in the proposition.

- Suppose that firm i chooses (I_i^*, p_i^*) such that $Y_i(I_i^*, p_i^*) < \hat{Y}_j$. Then if the market belongs to firm i, firm j can grab the market by choosing $(\hat{I}_j, \hat{p}_j + \varepsilon)$ and gets a positive profit, which improves its profit at $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$. If the market belongs to firm j, firm i can grab the market by choosing $(\hat{I}_i, \hat{p}_i+\varepsilon)$ and gets a positive profit, which improves its profit at $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$. Hence, this $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$ cannot be an equilibrium.
- Suppose that no firm $j \in \arg\max_{n \notin \mathcal{M}} \hat{Y}_n$ chooses $(I_j^*, p_j^*) = (\hat{I}_j, \hat{p}_j)$ in $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$. Then firm i can relax the constraint in its optimization problem to improve its profit. Hence, this $(\boldsymbol{I}^*, \boldsymbol{p}^*, \boldsymbol{x}^*)$ cannot be an equilibrium.

C. Proof of Theorem 1

In our online appendix [17], we provide a graphical proof, which is easy to understand. Here we give a rigorous proof.

We first calculate the equilibrium social welfare under no network externality. In Section III, we have argued that all the firms have the same maximum monopolistic market share $\hat{Y}_i^0 = P^{-1}(0)$. Hence, at any equilibrium, the total market size is $X^{*0} = P^{-1}(0)$, and the equilibrium social welfare, denoted by SW^{*0}, can be calculated as

$$SW^{*0} = \int_0^{X^{*0}} P(y)dy. \tag{22}$$

We next consider the equilibrium social welfare under strong positive network externality. From Proposition 2, we know that the equilibrium market share X^* is no smaller than the second highest maximum monopolistic market share. According to our definition of strong positive network externality, the total market size at the equilibrium satisfies

$$X^* \geq \max_{n \notin \mathcal{M}} \hat{Y}_n > P^{-1}(0) = X^{*0}. \tag{23}$$

Since the equilibrium social welfare is independent of how the market is shared among the firm(s) in $\mathcal{M}$ (see Lemma 2 in our online appendix [17]), we assume that firm i grabs the entire market share of X^*. Then the equilibrium social welfare is

$$SW^* = \int_0^{X^*} P(y)dy - c_i(I_i^*) \cdot X^* + b_i\left(I_i^*, X^*\right) \cdot X^*. \tag{24}$$

The difference in the equilibrium social welfare is

$$SW^* - SW^{*0} = \int_{X^{*0}}^{X^*} P(y)dy - c_i(I_i^*)X^* + b_i\left(I_i^*, X^*\right)X^*.$$

Since $X^* > X^{*0}$, $P(X^{*0}) = 0$, and $P(X)$ is strictly decreasing in X, we have $P(X^*) < P(X) < 0$, for all $X \in (X^{*0}, X^*)$, and thus have

$$\int_{X^{*0}}^{X^*} P(y)dy > P(X^*) \cdot (X^* - X^{*0}) \geq P(X^*) \cdot X^*. \tag{25}$$

Therefore, the difference in the equilibrium social welfare

$$\begin{aligned} SW^* - SW^{*0} &> P(X^*)X^* - c_i(I_i^*)X^* + b_i\left(I_i^*, X^*\right)X^* \\ &= [p_i^* - c_i(I_i^*)] \cdot X^* \geq 0, \end{aligned} \tag{26}$$

where the equality follows from conditions in (4) (i.e., $p_i^* - b_i(I_i^*, X^*) = P(X^*)$), and the last inequality follows from the fact that firm i has nonnegative profit at the equilibrium.

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