# Unsupervised Channel Estimation with Dual Path Knowledge-Aware Auto-Encoder


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***Abstract*—In this paper, we propose a highly accurate unsupervised deep learning framework based on auto-encoder (AE) for channel estimation in massive MIMO systems. Our method builds on an improvement of the traditional AE by incorporating the knowledge of signal propagation models into the decoder. In our proposed knowledge-aware AE, instead of having learnable parameters, the decoder has fixed weights that implement the signal propagation model. Such modification forces the encoder to output meaningful physical parameters of interests (i.e., angle-of-arrivals, path gains and path angles), which cannot be achieved by standard AE. We rigorously analyze the multiplicity of global optima in unsupervised channel estimation problems. Our analysis informs the design of the encoder as a dual-path neural network, which uses the received signal and its correlation matrix to estimate the path gains and the angle-of-arrivals, respectively. In the training phase, we update the parameters in the dual paths alternatively to alleviate the issue of multiple global optima. To avoid the convergence to local optima, we propose an efficient method to compute good initial points for training. Numerical simulation results corroborate the analysis, and demonstrate the performance improvements of the proposed method over traditional channel estimation methods.**


## I. Introduction

With the popularity of fifth generation (5G) communication networks, massive multiple-input-multiple-output (MIMO) becomes one of the vital technologies to address the challenge of explosive data traffic growth and meet the requirements of high quality of service and energy efficiency. Theoretically, massive MIMO can enhance the capacity of a communication system with additional antennas [1]. In practice, accurate channel estimation is key to realize the potential gain of massive MIMO. However, there are challenges in accurate channel estimation, arising from sophisticated channel modeling [2], costly channel state information (CSI) [3], and high computational complexity due to the large amount of antennas.

We can divide existing channel estimation methods into two categories: (unsupervised) signal processing based methods [4]–[6] and (supervised) deep learning based methods [7]. Signal processing methods include classic algorithms such as ESPRIT [4] and MUSIC [5], and more recent compressed sensing algorithms [6]. These methods make certain assumptions on the channel model in order to exploit the resulting properties of the correlation matrix of the received signal (e.g., rotation invariance in ESPRIT, orthogonality between signal and noise subspaces in MUSIC, sparsity in compressive sensing). Subsequently, these methods apply signal processing techniques (e.g., eigenvalue decomposition in MUSIC) or optimization techniques (e.g., semidefinite programming in compressive sensing) to obtain channel estimation results. However, in massive MIMO, the correlation matrix has high dimensionality and results in prohibitive computational cost.

Recently, with the rapid development of deep learning, there is a growing literature of using *supervised* deep neural networks for channel estimation [8], [9]. These methods make minimum assumptions on the channel model and have low computational complexity in the deployment phase (i.e., after the model is trained). Given sufficient labeled training samples, these methods can train deep networks powerful enough to outperform traditional signal processing methods. In practice, however, it may be hard to obtain the training labels (e.g., angle-of-arrivals (AoAs), path gains, and path angles).

In this paper, we propose an *unsupervised* deep learning framework for channel estimation in massive MIMO. Our proposed method makes minimum assumptions on the channel model, has low computational complexity, achieves similar performance as the existing supervised deep learning based methods, and outperforms traditional signal processing methods. There are two major hurdles in developing unsupervised deep learning methods for channel estimation. A canonical unsupervised deep learning method is auto-encoder (AE), which consists of an encoder that learns low-dimensional embedding of the input (i.e., the received signal) and a decoder that reconstructs the input. The first hurdle is that the lower-dimensional embedding learned by the encoder usually has no physical meaning, and therefore will not be the channel parameters we want to estimate (i.e., AoAs, path gains, and path angles). This is because without the labels, the network can only take the difference between the input (i.e., the received signal) and the output (i.e., the reconstructed signal from the estimated features) as its loss function. Such a loss function also results in the second hurdle: there may be multiple sets of channel parameters that are local or global optima of the loss function. Without labels, it is hard to know which set of channel parameters is correct.

To the best of our knowledge, our work is the first to combine the unsupervised model-driven network based on auto-encoder [10] with channel estimation in massive MIMO. We propose a knowledge-aware auto-encoder (KA-AE) by modifying the decoder to implement the channel model. In


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this way, we break the first hurdle and enforce the output of encoder to be the channel parameters to estimate. To break the second hurdle of multiple local and global optima, we analyze the loss function in depth and discover key properties that guide our design of KA-AE. Specifically, we design the encoder as a dual path neural network, where one path estimates the AoAs from the correlation matrix of the received signals and the other path estimates the path gains and path angles from the received signals. We update the weights of both paths in an alternating fashion, which ensures the convergence of the training. We also propose a low-cost method to find good initial points prior to the training, so that KA-AE is more likely to converge to the global optima. In numerical simulations, we demonstrate the necessity of our modification of the decoder, our proposed alternating update manner for the dual path neural network, and our choices of initial points. We also evaluate the performance gains over traditional channel estimation methods.

## II. Channel Model and Problem Formulation

Consider a massive MIMO uplink system consisting of one base station with a uniform linear array of $N_t$ antennas and $K$ single-antenna users. The received signals at the base station during $M$ snapshots can be represented as follows:

$$\boldsymbol{Y} = \boldsymbol{A}\left(\boldsymbol{H} \odot \boldsymbol{S}\right) + \boldsymbol{N}, \tag{1}$$

where $\odot$ is the Hadamard product, $\boldsymbol{Y} \in \mathbb{C}^{N_t \times M}$ is the collection of received signals during $M$ snapshots, $\boldsymbol{A} \in \mathbb{C}^{N_t \times K}$ is the array response matrix, $\boldsymbol{H} \in \mathbb{C}^{K \times M}$ is the channel fading matrix, $\boldsymbol{S} \in \mathbb{C}^{K \times M}$ is the collection of $K$ transmit signals during $M$ snapshots, and $\boldsymbol{N} \in \mathbb{C}^{N_t \times M}$ is the Gaussian noise.

Each user $k$'s location is specified by the distance to the base station and the AoA, defined as the angle of user $k$'s impinging signal relative to the broadside of the antenna array (i.e., the line perpendicular to the antenna array). We assume that the users are stationary during the $M$ snapshots. Therefore, user $k$'s AoA $\theta_k \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ stays the same during the $M$ snapshots. Given the AoAs, the array response matrix can be written as $\boldsymbol{A} = \left[\boldsymbol{a}\left(\theta_1\right), \boldsymbol{a}\left(\theta_2\right), \ldots, \boldsymbol{a}\left(\theta_K\right)\right]$ with

$$\boldsymbol{a}(\theta_k) = \left[1, e^{-j\frac{2\pi d}{\lambda}\sin\theta_k}, \ldots, e^{-j\frac{2\pi d}{\lambda}(N_t-1)\sin\theta_k}\right]^T,$$

where $j = \sqrt{-1}$, $\lambda$ is the wavelength and $d \geq \frac{\lambda}{2}$ is the distance between adjacent antenna elements. For notational simplicity, we define

$$\boldsymbol{\Psi}(\theta_k) \triangleq \frac{2\pi d}{\lambda}\sin\theta_k \cdot [0, 1, \ldots, N_t - 1]^T,$$

so that we have $\boldsymbol{a}(\theta_k) = e^{-j\boldsymbol{\Psi}(\theta_k)}$.

The distances from users to the base station, along with shadowing and small-scale fading, determine the channel fading matrix $\boldsymbol{H}$. The channel gain from user $k$ to the base station in snapshot $m$ is $\left[\boldsymbol{H}\right]_{k,m} = \alpha_{k,m}e^{j\phi_{k,m}}$, where $\alpha_{k,m}$ and $\phi_{k,m}$ are the magnitude and the phase of the complex channel gain. Unlike the AoAs, the path gains $\alpha_{k,m}$ and path angles $\phi_{k,m}$ are time varying during the $M$ snapshots due to random fading effects.

In this paper, we aim to estimate the AoAs $\theta_1, \ldots, \theta_K$, the path gains $\alpha_{k,m}$ and the path angles $\phi_{k,m}$, $k = 1, \ldots, K, m = 1, \ldots, M$ from the received signals $\boldsymbol{Y}$ during the $M$ snapshots. We collect all $K$ users' AoAs into a vector $\boldsymbol{\theta} \in \mathbb{R}^K$ and all $K$ users' path gains and path angles during $M$ snapshots into matrices $\boldsymbol{\alpha} \in \mathbb{R}^{K \times M}$ and $\boldsymbol{\phi} \in \mathbb{R}^{K \times M}$.

We make an innocuous assumption that different users have different AoAs.

*Assumption 1:* There is no two users with the same AoA, i.e. $\theta_i \neq \theta_k$ for all $i \neq k$.

## III. Proposed Solution

In unsupervised learning, we aim to obtain estimates of AoAs $\widehat{\boldsymbol{\theta}}$, path gains $\widehat{\boldsymbol{\alpha}}$, and path angles $\widehat{\boldsymbol{\phi}}$, so that the reconstructed signals $\widehat{\boldsymbol{Y}}$ (using (1)) is close to the received signals $\boldsymbol{Y}$. Therefore, the loss function is the mean squared error between the reconstructed and received signals as follows:

$$loss^{(train)} = \mathbb{E}\left[\left\|\boldsymbol{Y} - \widehat{\boldsymbol{Y}}\right\|_F^2\right], \tag{2}$$

where $\|\cdot\|_F$ is the Frobenius norm, and $\mathbb{E}(\cdot)$ is the expectation operator. Here the expectation is taken over the parameters to estimate and the noise.

We illustrate our proposed knowledge-aware auto-encoder in Fig. 1. Next, we summarize the key ingredients of our solution, and will elucidate our design choices with theoretical analysis in the next section.

First, to enforce the output of the encoder to be the channel parameters to estimate, we let the decoder reconstruct the signal using (1), instead of having learnable parameters. The fixed decoder implementing the channel model (1) is the key difference from the standard auto-encoder.

Second, the encoder of the proposed KA-AE is a dual path neural network. One path is the channel estimation network with the input as the received signal matrix $\boldsymbol{Y}$ and the output as the estimated path gains $|\widehat{\alpha}|$ and path angles $\widehat{\phi}$. The other path is the AoA estimation network with the input as the correlation matrix $\boldsymbol{R}_{yy} = \frac{1}{M}\boldsymbol{Y}\boldsymbol{Y}^H$ and the output as the estimated AoAs $\widehat{\theta}$. By having a dual path neural network, we can update the weights of each network in an alternating fashion. This updating schedule is shown to be the key to the convergence of KA-AE (see Proposition 1 and Proposition 2).

Finally, due to the existence of multiple local and global optima (see Proposition 2 and Proposition 3), we need a good initial point prior to the training phase. Therefore, we use an initialization phase to obtain pseudo-labels of AoAs $\widetilde{\theta}$ and train the networks under the supervision of the pseudo-labels we generated. In the initialization phase, the loss function is:

$$loss^{(init)} = \mathbb{E}\left[\left\|\boldsymbol{Y} - \widehat{\boldsymbol{Y}}\right\|_F^2 + \left(\widetilde{\boldsymbol{\theta}} - \widehat{\boldsymbol{\theta}}\right)^2\right] \tag{3}$$

## IV. Performance Analysis

### A. Analysis of The Loss Function and Global Optima

One may expect that there are multiple sets of channel parameters that result in the same received signal as the true channel parameters. All these channel parameters minimize

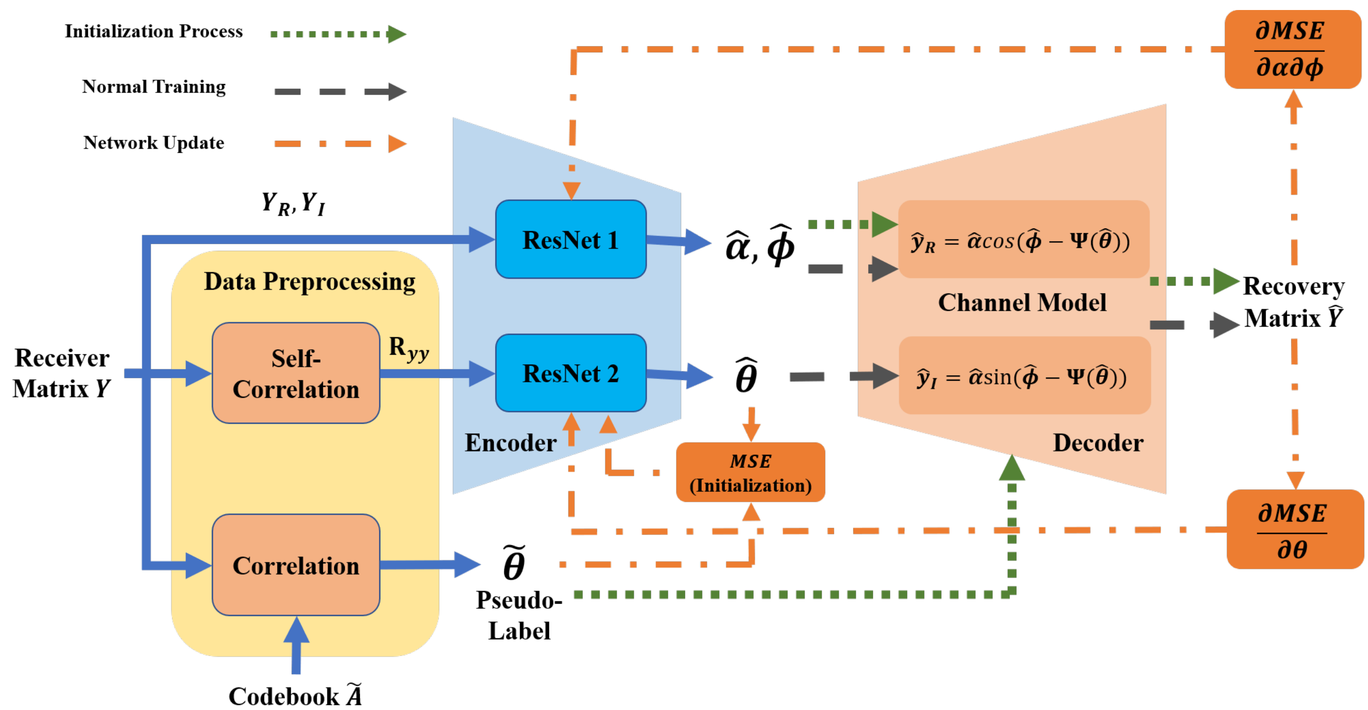



Fig. 1: The KA-AE network is composed of data preprocessing, the encoder and the decoder. The encoder consists of channel estimation ResNet and AoA estimation ResNet. The decoder implements the channel model in (1). The training process is divided into initialization and training phases. The pseudo-labels are only used in the initialization phase.

the loss function (2). Therefore, it seems impossible to get accurate channel estimation without supervision of true labels. However, we discover a key property that gives us guidelines of how to alleviate the problem of multiple global optima.

*Proposition 1:* When the AoA estimation is accurate (i.e., $\widehat{\theta} = \theta$), the estimates of path gains and path angles minimize the loss function in (2) if and only if $\widehat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}$ and $\widehat{\phi}_{k,m} = \phi_{k,m} + 2\pi l_{k,m}$ with $l_{k,m} \in \mathbb{Z}$ for all $k, m$.

*Proof:* See Appendix A. ■

Proposition 1 indicates that once we obtain the accurate AoA estimates, the unique minimize of the loss function is the true path gains and path angles. This motivates us to design the encoder as a dual path neural network, where the AoA estimation is separated from the estimation of path gains and path angles. It allows us to update the weights of the two networks in an alternating fashion. In this way, if the AoA estimation network converges to the optimal solution, the other network has a unique global optimum.

Now that we know the path gains and path angles can be estimated accurately once the AoA estimation is accurate, we turn to the analysis of AoA estimation.

*Proposition 2:* Suppose that the channel estimation is accurate for all users except $k$, namely $\widehat{\theta}_i = \theta_i$, $\widehat{\alpha}_{i,m} = \alpha_{i,m}$, and $\widehat{\phi}_{i,m} = \phi_{i,m}$, for all $i \neq k$ and for all $m$, and that user $k$'s path gain estimation is accurate, namely $\widehat{\alpha}_{k,m} = \alpha_{k,m}$. User $k$'s AoA estimation minimizes the loss function if and only if for all $m = 1, \dots, M$ and $n = 0, \dots, N_t - 1$, there exists an integer $l_{m,n} \in \mathbb{Z}$ such that

$$\left[\left(\phi_{k,m} - \widehat{\phi}_{k,m}\right) - \frac{2\pi(n-1)d}{\lambda}\left(\sin\theta_k - \sin\widehat{\theta}_k\right)\right] = 2\pi l_{m,n}. \quad (4)$$

*Proof:* See Appendix B. ■

Proposition 2 indicates that even if the estimation of path gains is accurate, there are multiple sets of AoAs and path angles that minimize the loss function. In fact, the multiplicity of global optima in AoA estimation is more severe than as indicated by Proposition 2. Proposition 3 shows that even if both path gains and path angles are accurately estimated, there may be multiple AoAs that minimize the loss function.

*Proposition 3:* Suppose that the channel estimation is accurate for all users except $k$, namely $\widehat{\theta}_i = \theta_i$, $\widehat{\alpha}_{i,m} = \alpha_{i,m}$, and $\widehat{\phi}_{i,m} = \phi_{i,m}$, for all $i \neq k$ and for all $m$, and that user $k$'s path gain and path angle estimation is accurate, namely $\widehat{\alpha}_{k,m} = \alpha_{k,m}$ and $\widehat{\phi}_{k,m} = \phi_{k,m}$. There exist at least $N_{loc} = 2 \cdot \lfloor 2\frac{d}{\lambda} \rfloor + 1$ AoAs of user $k$ that minimize the loss function. More specifically, these $N_{loc}$ AoAs are as follows:

$$\theta^l = \arcsin\left(\sin\theta - \left\lceil -2\frac{d}{\lambda}\right\rceil + l\right), \;\forall l = 0, 1, ..., N_{loc} - 1. \quad (5)$$

*Proof:* See Appendix B. ■

Proposition 2 and Proposition 3 stress the importance of having good initial points for AoA estimation.

From the characterization of possible AoAs in Proposition 3, the number of global optima does not depend on the number of antenna elements, and increases with the carrier frequency.

Therefore, increasing the number of antenna elements may not solve the problem, and the problem is more severe in millimeter wave systems of higher frequencies.

For the reason that there is no inter-users interference after initialization process, i.e. $\widehat{\theta} \rightarrow \theta$ holds, the muti-user estimation reduces to the stacking of multiple single-user estimation problem. The number of multiple global optima is $K \times N_{loc}$. To overcome this problem, this paper imposes the spatial filter in [11]. Thus, the sectorization method [12] is introduced into the proposed KA-AE network where the whole AoA estimation range is devided into $Q$ sectors and the $q$th sector's range is $[-\pi/2 + (q-1)\pi/Q, -\pi/2 + q\pi/Q]$.

To further improve the initial points, we propose a low-cost method to find the initial points, which is described in the next subsection.

### B. Initialization Based on Correlation Codebook

In order to avoid the randomness of the network at the beginning of the training which will affect the convergence of the loss function, this paper divides the network training process into two phase, initialization and normal training. The initialization process is to make $\widehat{\boldsymbol{\theta}} \rightarrow \boldsymbol{\theta}$ before normal training.

The initialization process builds a codebook to provide the pseudo-label for the proposed KA-AE network. Specifically, we define the correlation codebook as $\overline{\boldsymbol{A}} = \left[e^{-j\boldsymbol{\Psi}(\theta_{min})}, e^{-j\boldsymbol{\Psi}(\theta_{min}+\Delta\theta)}, \ldots, e^{-j\boldsymbol{\Psi}(\theta_{max})}\right]$, where $\theta_{min}$ is the minimum AoA, $\theta_{max}$ is the maximum AoA, and $\Delta\theta$ is the precision of the initialization. Then we calculate the correlation between the received signal matrix and the codebook $\overline{\boldsymbol{c}}_{\boldsymbol{YA}} = \boldsymbol{1}^T \boldsymbol{Y} \overline{\boldsymbol{A}}^H$, The initial $\widetilde{\boldsymbol{\theta}}$ are the $K$ angles that result in the $K$ largest values in $\overline{\boldsymbol{c}}_{\boldsymbol{YA}}$.

The proposed KA-AE network uses the initial $\widetilde{\boldsymbol{\theta}}$ in the loss function (3) during the initialization process. Thus, we have $\widehat{\boldsymbol{\theta}} \rightarrow \boldsymbol{\theta}$ before the normal training.

## V. Numerical Simulation

This section verifies the theoretical analysis on the multiplicity of global optima, demonstrates the effectiveness of our proposed method in dealing with multiple global optima, and evaluate the performance against existing methods. Unless specified otherwise, we set $d/\lambda = 0.5$, the number of snapshots $M = 40$, the number of antennas $N_t = 32$, the number of sectors $Q = 3$, and the number of users $K = 3$. We use 40000 samples for training and 10000 samples for testing.

### A. Efficacy in Dealing with Multiplicity of Global Optima

Proposition 3 characterized the multiplicity of global optima and their positions in the AoA domain. Fig. 3 show the loss surface under different numbers of antennas, wavelengths, and antenna spacing. We can see that there are 3 and 9 distinct AoAs at global optima under $d/\lambda = 0.5$ and $d/\lambda = 2$, respectively, which is consistent with Proposition 3. We can also see that the multiplicity of global optima does not disappear as we increase the number of antennas, which validates our analysis.

Next, Fig. 2 shows that our proposed method effectively resolves the issue of global optima. In Fig. 2, "sector free" is the method without sectorization and has the highest estimation errors. Then we compare different algorithms under the sectorized scenario. The "Initial free" algorithm chooses the initial point randomly and achieves only negligible improvement over the "sector free" scenario. The "Pseudo-Label" method indicates the performance after the initialization phase, which further improves the performance. Our proposed method continues to minimize the training loss by alternating between improving the estimation of AoAs and path gains, and achieves significant performance improvement after the initialization.

### B. Performance Improvement over Existing Works

Finally, this paper compares the channel estimation accuracy of the KA-AE network with the following benchmarks:

- the MUSIC algorithm [5];
- a supervised learning algorithm, for which all labels of true AoAs, path angles, and path gains (called "All-Label Network (All-LN)") are used and the loss function is the mean squared error between the estimates and the labels.
- "AoA-Label Network", an variation of standard AE with labels of true AoAs, whose loss function is:

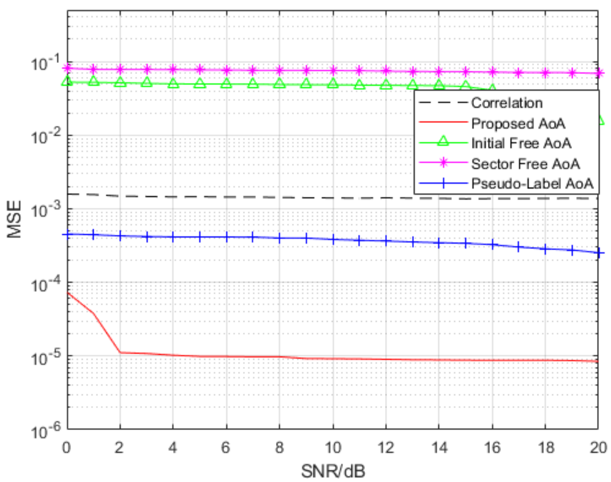
$$\mathbb{E}\Big[(\boldsymbol{Y}_R - \hat{\boldsymbol{Y}}_R)^2 + (\boldsymbol{Y}_I - \hat{\boldsymbol{Y}}_I)^2\Big] + \mathbb{E}\left[(\boldsymbol{\theta} - \widehat{\boldsymbol{\theta}})^2\right]. \quad (6)$$

The AoA-Label Network (AoA-LN) is used to demonstrate that the proposed KA-AE is able to enforce meaningful output of the encoder by the novel design of the decoder.

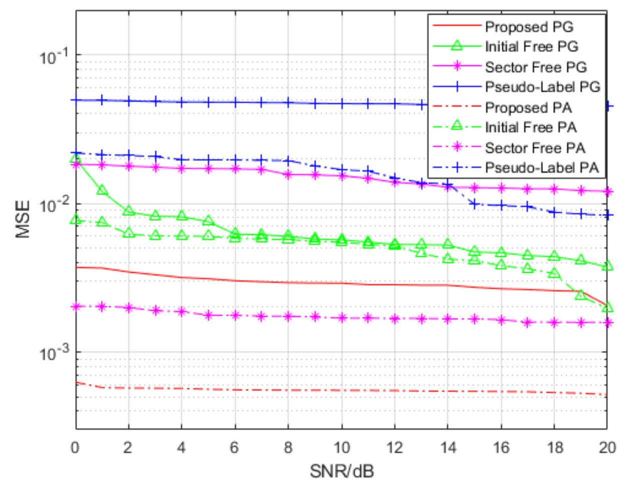
Fig. 4 summarizes the performance evaluation in terms of the signal reconstruction error and the estimation errors of AoAs, path gains, and path angles. Note that for the fully supervised All-LN, the signal reconstruction error is not shown because there is no decoder to reconstruct the signal. For the MUSIC algorithm, the reconstruction error and the estimation errors of path gains and path angles are not shown because the algorithm does not produce these estimates.

The key observation is that the proposed KA-AE achieves almost identical performance as the fully supervised All-LN. This demonstrates the advantage of the proposed method: the removal of labels in our method comes at almost no cost. This achievement is not trivial, because the classic unsupervised MUSIC algorithm has much higher AoA estimation errors.

The performance comparison with the AoA-LN illustrates that the proposed decoder enforces meaningful output of the encoder by incorporating the knowledge of the signal propagation model. The AoA-LN is the standard AE with a modified loss function that includes the MSE of the AoA estimation. Note that in the AoA-LN, the decoder is the same as the one in the standard AE, which has learnable parameters. For standard AEs, the physical meanings of the embedding vector (i.e., the output of the encoder) cannot be controlled. For the AoA-LN, part of the embedding vector is enforced to be the AoA through the addition of the MSE of AoA estimation, but have no control over the rest of the embedding vector. As expected and as demonstrated in Fig. 4, the AoA-LN has comparable performance as the proposed KA-AE in AoA estimation, but much worse performance in signal reconstruction and estimation of path gains and path angles.

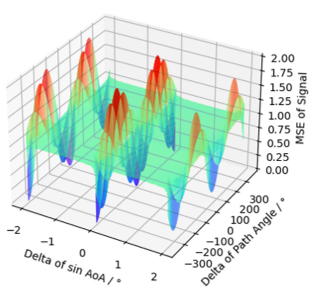



(a) Estimation of AoA

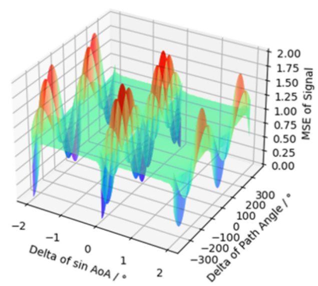



(b) Estimation of path gain and path angle

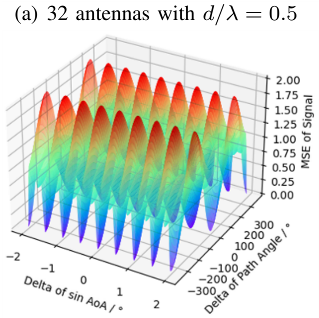
Fig. 2: Proposed is the performance of KA-AE and Initial Free means the network training without initialization process while Sector Free without sectorization. The Pseudo-Label uses (3) as loss function in the whole training process. Correlation means the AoA estimated by the correlation codebook in data preprocessing.



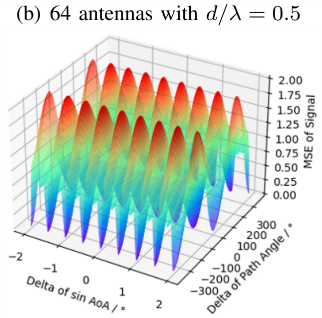

(a) 32 antennas with $d/\lambda = 0.5$




(b) 64 antennas with $d/\lambda = 0.5$




(c) 32 antennas with $d/\lambda = 2$




(d) 64 antennas with $d/\lambda = 2$

Fig. 3: Multiplicity of Global Optima.

## VI. Conclusions

To the best of our knowledge, our work is the first unsupervised deep learning method for channel estimation in massive MIMO systems. The key design idea is to replace the decoder of the standard AE with the known signal propagation model. Such knowledge is instilled in the proposed decoder by hardwiring it with the signal propagation model. Furthermore, a correlation codebook is imposed to generate the pseudo-label and the training process is divided into initialization and normal training to eliminate the inter-users inference. Thereafter, the issue of multiple global optima is analyzed in the unsupervised estimation problem by separating the encoder into channel estimation network and AoAs estimation network as dual path network, and adopted the sectorization method to alleviate the issue. Numerical simulations were carried out to validate the theoretical analysis. Comparisons with carefully designed benchmark algorithms demonstrated that our proposed method achieves almost identical performance as the fully supervised method, and that the achievement is enabled by the well-designed decoder.

## Appendix A
## Proof of Proposition 1

*Assumption 2:* The noise $\boldsymbol{n}_m$ in each snap shot $m$ follows the Gaussian distribution of $\boldsymbol{n}_m \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2 \mathbf{I}_{N_t}\right)$.

*Assumption 3:* The number of users is no larger than the number of antennas at the base station, namely $K \leq N_t$.

Based on the channel model (1), we can write the received signal $\boldsymbol{y}_m$ in snapshot $m$ (i.e., the $m$-th column of $\boldsymbol{Y}$) as

$$\boldsymbol{y}_m = \boldsymbol{A}\boldsymbol{h}_m + \boldsymbol{n}_m,$$

where $\boldsymbol{h}_m$ is the $m$-th column of the channel matrix $\boldsymbol{H}$. Based on Assumption 2, the received signal $\boldsymbol{y}_m$ is Gaussian with the following distribution

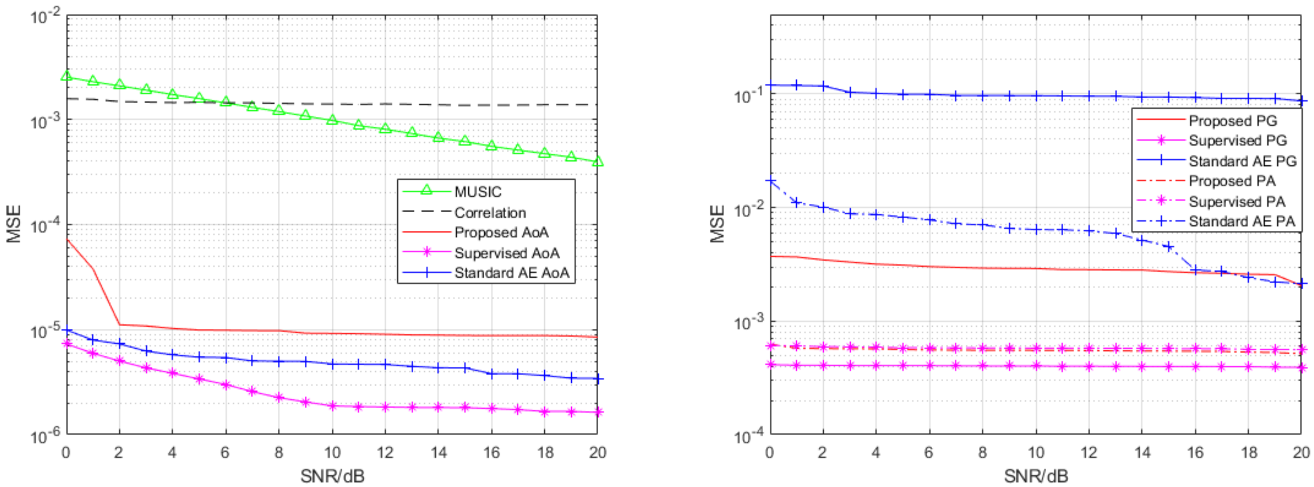
$$\boldsymbol{y}_m \sim \mathcal{CN}\left(\boldsymbol{A}\boldsymbol{h}_m, \sigma^2 \mathbf{I}_{N_t}\right).$$

Given the estimates $\widehat{\boldsymbol{\theta}} = [\widehat{\theta}_1, \ldots, \widehat{\theta}_K]^T$, $\widehat{\boldsymbol{\alpha}}_m = [\widehat{\alpha}_{1,m}, \ldots, \widehat{\alpha}_{K,m}]^T$, and $\widehat{\boldsymbol{\phi}}_m = [\widehat{\phi}_{1,m}, \ldots, \widehat{\phi}_{K,m}]^T$, the reconstructed signal is

$$\widehat{\boldsymbol{y}}_m = \widehat{\boldsymbol{A}}\widehat{\boldsymbol{h}}_m,$$

where

$$\widehat{\boldsymbol{A}} = \left[e^{-j\Psi(\widehat{\theta}_1)}, \ldots, e^{-j\Psi(\widehat{\theta}_K)}\right]$$




(a) Estimation of AoA

(b) Estimation of path gain and path angle

Fig. 4: Performance improvement of proposed KA-AE over benchmarks.

and

$$\widehat{\boldsymbol{h}}_m = \left[\widehat{\alpha}_{1,m} e^{-j\widehat{\phi}_{1,m}}, \ldots, \widehat{\alpha}_{K,m} e^{-j\widehat{\phi}_{K,m}}\right]^T.$$

Therefore, the reconstruction error is a Gaussian random vector, namely

$$\boldsymbol{y}_m - \widehat{\boldsymbol{y}}_m \sim \mathcal{CN}\left(\boldsymbol{A}\boldsymbol{h}_m - \widehat{\boldsymbol{A}}\widehat{\boldsymbol{h}}_m, \sigma^2 \mathbf{I}_{N_t}\right).$$

We have

$$\mathbb{E}\left(\|\boldsymbol{y}_m - \widehat{\boldsymbol{y}}_m\|_2^2\right) = \left\|\boldsymbol{A}\boldsymbol{h}_m - \widehat{\boldsymbol{A}}\widehat{\boldsymbol{h}}_m\right\|_2^2 + N_t\sigma^2. \tag{7}$$

The loss function (2) can be written as

$$\begin{aligned}\mathbb{E}\left(\left\|\boldsymbol{Y} - \widehat{\boldsymbol{Y}}\right\|_F^2\right) &= \sum_{m=1}^{M} \mathbb{E}\left(\|\boldsymbol{y}_m - \widehat{\boldsymbol{y}}_m\|_2^2\right) \\ &= \sum_{m=1}^{M} \left\|\boldsymbol{A}\boldsymbol{h}_m - \widehat{\boldsymbol{A}}\widehat{\boldsymbol{h}}_m\right\|_2^2 + MN_t\sigma^2.\end{aligned} \tag{8}$$

When the AoA estimation is accurate (i.e., $\widehat{\boldsymbol{\theta}} = \boldsymbol{\theta}$, we have $\widehat{\boldsymbol{A}} = \boldsymbol{A}$. In this case, the loss function reduces to

$$\sum_{m=1}^{M} \left\|\boldsymbol{A}\left(\boldsymbol{h}_m - \widehat{\boldsymbol{h}}_m\right)\right\|_2^2 + MN_t\sigma^2. \tag{9}$$

We analyze the term

$$\left\|\boldsymbol{A}\left(\boldsymbol{h}_m - \widehat{\boldsymbol{h}}_m\right)\right\|_2^2 = \left(\boldsymbol{h}_m - \widehat{\boldsymbol{h}}_m\right)^H \boldsymbol{A}^H\boldsymbol{A}\left(\boldsymbol{h}_m - \widehat{\boldsymbol{h}}_m\right). \tag{10}$$

Since $\boldsymbol{A}^H\boldsymbol{A}$ is positive semidefinite, the above term is non-negative, and reaches zero if and only if $\boldsymbol{h}_m - \widehat{\boldsymbol{h}}_m = \boldsymbol{0}$ or $\boldsymbol{A}\left(\boldsymbol{h}_m - \widehat{\boldsymbol{h}}_m\right) = \boldsymbol{0}$. Due to Assumption 3, the matrix $\boldsymbol{A}$ has full rank of $K$. So $\boldsymbol{A}\left(\boldsymbol{h}_m - \widehat{\boldsymbol{h}}_m\right) = \boldsymbol{0}$ implies $\boldsymbol{h}_m - \widehat{\boldsymbol{h}}_m = \boldsymbol{0}$, too. In other words, the loss function reaches its minimum if and only if $\widehat{\alpha}_{k,m} e^{-j\widehat{\phi}_{k,m}} = \alpha_{k,m} e^{-j\phi_{k,m}}$ for all $k$ and $m$. This concludes our proof.

## Appendix B
## Proof of Proposition 2 and Proposition 3

Similar with the derivation of the convergence of the channel estimation network, the loss function of the AoAs estimation network is:

$$\begin{aligned} loss^{(\theta)} = &\mathbb{E}\left[\sum_{m=0}^{M-1}\sum_{k=0}^{K-1}\left(\alpha_{m,k}^2 + \overline{\alpha}_{m,k}^2 - 2\alpha_{m,k}\overline{\alpha}_{m,k}\right.\right., \\ &\left.\left.\cos\left(\left(\phi_{m,k} - \overline{\phi}_{m,k}\right) - \left(\boldsymbol{\Psi}\left(\theta_k\right) - \boldsymbol{\Psi}\left(\widehat{\theta}_k\right)\right)\right)\right)\right].\end{aligned} \tag{11}$$

Let $loss^{(\theta)} \to 0$, the following equations still hold.

$$\left[\left(\phi - \overline{\phi}\right) - \frac{2\pi(n-1)d}{\lambda}\left(\sin\theta - \sin\widehat{\theta}\right)\right] \to 2\pi l, l \in Z, \tag{12}$$

where $n = 1, 2, ..., N$. In this problem, the difference between target AoA and the estimated AoA and the difference target path angle and estimated path angle should be placed most attention. Let $\Delta\overline{\phi} = \phi - \overline{\phi}$ and $\Delta\widehat{\theta} = \sin\theta - \sin\widehat{\theta}$. Then $f(\widehat{\phi}, \widehat{\theta}) = f(\Delta\overline{\phi}, \Delta\widehat{\theta}) = \Delta\overline{\phi} - \frac{2\pi dk}{\lambda}\Delta\widehat{\theta}$, and from what has been discussed above, let $f(\Delta\overline{\phi}, \Delta\widehat{\theta}) = 2l_i\pi$, for $i = 0, 1, ..., N_t - 1$ and $l_i \in Z$. Turn it into matrix format $\boldsymbol{H}\boldsymbol{x} = \boldsymbol{b}$:

$$\begin{bmatrix} 1 & 0 \\ 1 & \frac{2\pi d}{\lambda} \\ \vdots & \vdots \\ 1 & \frac{2\pi d}{\lambda}(N_t - 1)\end{bmatrix}\begin{bmatrix}\Delta\overline{\phi} \\ -\Delta\widehat{\theta}\end{bmatrix} = \begin{bmatrix} 2l_0\pi \\ 2l_1\pi \\ \vdots \\ 2l_{N_t-1}\pi\end{bmatrix}. \tag{13}$$

Perform row transformation on the augmented matrix $[\boldsymbol{H}|\boldsymbol{b}]$ to get $[\boldsymbol{H}'|\boldsymbol{b}']$ which can be shown as (14).

As a result, $\boldsymbol{H}'\boldsymbol{x} = \boldsymbol{b}'$ can be expressed as (15).Then it can be find out that the multiplicity of global optima is suffered from the difference between target path angle and estimated path angle $\Delta\overline{\phi}$ and the difference between target AoA and estimated AoA $\Delta\widehat{\theta}$, while has nothing to do with the number of antennas of the ULA. Furthermore, $\Delta\overline{\phi} = \phi - \overline{\phi} = 2l_0\pi$, where $\phi, \overline{\phi} \in [-\pi, \pi], l_0 \in Z$, means $\Delta\overline{\phi} \in [-2\pi, 2\pi]$ and $l_0 = 0$ or $\pm 1$. Because of $\Delta\widehat{\theta} = \sin\theta - \sin\widehat{\theta} \in [-2, 2]$ and $l_1 \in Z$, there will be a convergence point for the network if $\Delta\widehat{\theta} = \frac{\lambda}{d}(l_0 - l_1)$ where $l_0 = 0$ or $\pm 1$. Furthermore, $l_i$ can be derived by iteration, where $i = 2, 3, ..., N_t - 1$.

If $\Delta\overline{\phi} = 0$ and $\Delta\widehat{\theta} = 0$, there will be a correction estimation on the channel feature. However, if $\Delta\overline{\phi} = \pm 2\pi$ and $\Delta\widehat{\theta} = 0$, there will also be a correction estimation, for $\Delta\overline{\phi} = \pm 2\pi$ means that $\phi = \pm\pi$ and $\overline{\phi} = \mp\pi$, respectively, which means $\overline{\phi} = \phi$ is still hold, i.g.$\Delta\overline{\phi} = 0$ is equivalent to $\Delta\overline{\phi} = \pm 2\pi$, which is the same as the conclusion discussed in *Appendix* **??**. Thus, (15) can be simplified as:

$$\begin{cases} \Delta\overline{\phi} = 2l_0\pi \\ \Delta\widehat{\theta} = \dfrac{\lambda}{d}(l_0 - l_1) \\ l_2 = 2l_1 - l_0 \\ \vdots \\ l_{N_t-1} = 2l_{N_t-2} - l_{N_t-3} \end{cases} \Rightarrow \begin{cases} \Delta\overline{\phi} = 0 \\ \Delta\widehat{\theta} = -\dfrac{\lambda}{d}l_1 \\ l_2 = 2l_1 \\ \vdots \\ l_{N_t-1} = (N_t - 1)l_1 \end{cases}. \quad (16)$$

Where $l_1 = -\frac{d}{\lambda}\Delta\widehat{\theta} \in [-2\frac{d}{\lambda}, 2\frac{d}{\lambda}], l_1 \in Z$. For the reason that $d/\lambda \geq 1/2$, which means $[-1, 1] \subset [-2\frac{d}{\lambda}, 2\frac{d}{\lambda}]$, then $l_1 = 0$ and $\pm 1$ must be the solution of (16). So, there must be muti-convergence point in the unsupervised KA-AE network. With the increase of the $d/\lambda$, the number of the solution for (16) can be expressed as $N_{loc} = 2 \times \left\lfloor 2\frac{d}{\lambda} \right\rfloor + 1$, where $\lfloor x \rfloor$ denotes the maximum integer less than or equal to $x$.

For $l_1 = -\frac{d}{\lambda}\Delta\widehat{\theta} \in [-2\frac{d}{\lambda}, 2\frac{d}{\lambda}], l_1 \in Z$ and there are $N_{loc}$ solution for (16), then let $l_1^0 = l_1^{min} = \left\lceil -2\frac{d}{\lambda} \right\rceil$ and $l_1^{N_{loc}-1} = l_1^{max} = \left\lfloor 2\frac{d}{\lambda} \right\rfloor$, where $\lceil x \rceil$ denotes the minimum integer greater than or equal to $x$. Furthermore, for $l_1 \in Z$, then the set of all $l_1$ satisfying (16) is as follows:

$$\begin{aligned} \boldsymbol{l_1} &= [l_1^0, l_1^1, ..., l_1^{N_{loc}-1}] \\ &= [\underbrace{\left\lceil -2\frac{d}{\lambda} \right\rceil, \left\lceil -2\frac{d}{\lambda} \right\rceil + 1, ..., -1}_{(N_{loc}-1)/2}, 0, \underbrace{1, ..., \left\lfloor 2\frac{d}{\lambda} \right\rfloor - 1, \left\lfloor 2\frac{d}{\lambda} \right\rfloor}_{(N_{loc}-1)/2}]. \end{aligned} \quad (17)$$

Consider that when $\theta$, $\widehat{\theta} \in [-\pi/2, \pi/2]$ and the target AoA $\theta$ is set, $\Delta\widehat{\theta} = \sin(\theta) - \sin(\widehat{\theta}) = C - \sin(\widehat{\theta})$, where $C$ is a constant which means that $\Delta\widehat{\theta}$ is a monotonic function of $\widehat{\theta}$. According to (17), when the target AoA is $\theta$, the estimation AoA $\widehat{\theta}_i$ can be:

$$\widehat{\theta}^l == \arcsin(\sin(\theta) - \left\lceil -2\frac{d}{\lambda} \right\rceil + l). \quad (18)$$

Where $l = 0, 1, ..., N_{loc} - 1$. And only when $\widehat{\theta}_k \in [-\pi/2, \pi/2]$, it can be retained as a solution.

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$$[\boldsymbol{H}|\boldsymbol{b}] = \left[\begin{array}{cc|c} 1 & 0 & 2l_0\pi \\ 1 & \frac{2\pi d}{\lambda} & 2l_1\pi \\ \vdots & \vdots & \vdots \\ 1 & \frac{2\pi d}{\lambda}(N_t - 1) & 2l_{N_t-1}\pi \end{array}\right] \xrightarrow{transform} \left[\begin{array}{cc|c} 1 & 0 & 2l_0\pi \\ 0 & \frac{2\pi d}{\lambda} & 2(l_1 - l_0)\pi \\ \vdots & \vdots & \vdots \\ 0 & 0 & 2(l_{N_t-1} + l_{N_t-3} - 2l_{N_t-2})\pi \end{array}\right] = [\boldsymbol{H}'|\boldsymbol{b}']. \quad (14)$$

$$\begin{cases} \Delta\overline{\phi} = 2l_0\pi \\ -\dfrac{2\pi d}{\lambda}\Delta\widehat{\theta} = 2(l_1 - l_0)\pi \\ 0 = 2(l_2 + l_0 - 2l_1)\pi \\ \vdots \\ 0 = 2(l_{N_t-1} + l_{N_t-3} - 2l_{N_t-2})\pi \end{cases} \Rightarrow \begin{cases} \Delta\overline{\phi} = 2l_0\pi \\ \Delta\widehat{\theta} = \dfrac{\lambda}{d}(l_0 - l_1) \\ l_2 = 2l_1 - l_0 \\ \vdots \\ l_{N_t-1} = 2l_{N_t-2} - l_{N_t-3} \end{cases}. \quad (15)$$

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