Towards a deeper understanding of deep learning

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    - (Please imagine some fancy pictures here)

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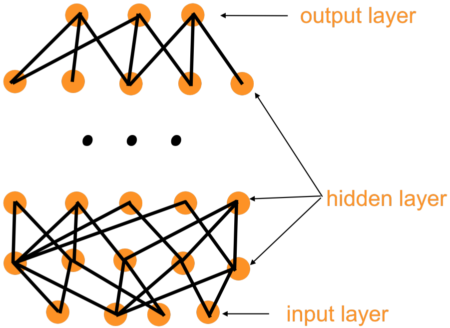

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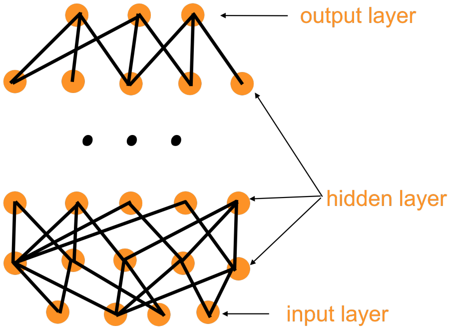

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- In this talk, I will particularly focus on neural networks with ReLU activations.

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    - Given any practical neural network, we can always come up with examples where we can't learn the network on them. (worst case examples)

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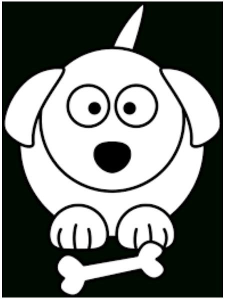
    - 

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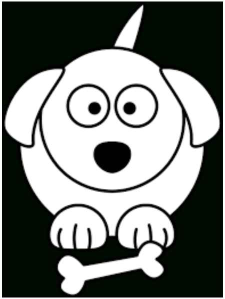
        - 
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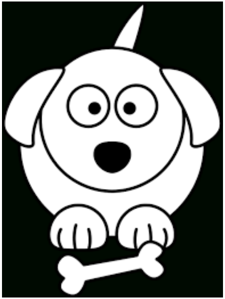
    - 
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- Make the analysis even harder.

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  - .... we will never have a chance to succeed.

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- But I am going to use a different approach.

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  - (In this talk): You should use a larger network!

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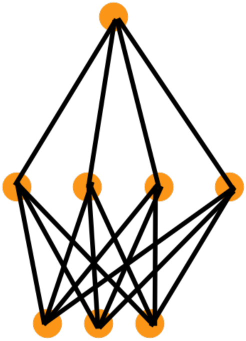
- Improves both the training and generalization.
- And it improves the theory.

Folklore example for training.

teacher
100 neurons



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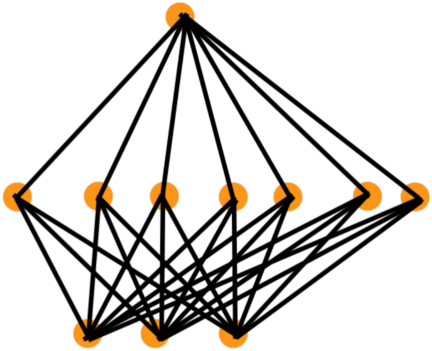

Example for generalization.

Widen factor	Number of parameters	Test error
1	0.6M	6.85
2	2.2M	5.33
4	8.9M	4.97
8	36.5M	4.66

Table: Depth 40 WideResNet on CIFAR-10 (0.05M training examples)

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- (2). And it generalizes to test data set.
- We begin with our theorem for (1), then we will see (2) as well.

Our theorem

Theorem (Sketched, (LL'18, ALS'18a,b))

Given N different training examples $x_1, \ldots, x_N$ with labels $y_1, \ldots, y_N$, then for every $\varepsilon > 0$, as long as the number of neurons (m) in the network satisfies

$$m \geq poly(N \log(1/\varepsilon))$$

then SGD starting from gaussian random initialization finds an ε-approximate optimal of the training objective in time poly(m/ε).

The theorem holds for multi-layer DNN, CNN, ResNet and Recurrent Neural Networks (all with ReLU activation functions), the training loss can be given by ℓ_2 loss, cross entropy, hinge loss etc.

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  - Not a trivial theorem, but much simpler than the next question.

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  - So why does SGD it still generalize?

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- The inductive bias of SGD: SGD biases towards generalizable solutions instead of the solutions that simply memorize the training data.
  - We are going to prove it for certain neural networks.

Labels are not random?

- Assume that the labels are realizable by a simple neural network:

$$F^* = (f_1^*, \cdots, f_k^*)$$
$$\text{each } f_r^*(x) = \sum_{i=1}^{p} a_{r,i}^* \phi_i(\langle w_i^*, x \rangle)$$

k is the output dimension, ϕ_i are smooth activation functions (such as $\sin, \cos, \exp$ and low degree polynomials), w_i^* are unit vectors, $|a_{r,i}^*| \leq 1$ and $\|x\|_2 \leq 1$.

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- Such that the average loss of network F^* on the training data set is $\leq \varepsilon$.

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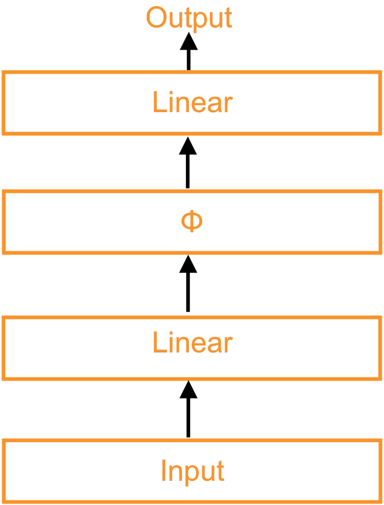
$$\text{each } f_r^*(x) = \sum_{i=1}^{p} a_{r,i}^* \phi_i(\langle w_i^*, x \rangle)$$

  k is the output dimension, ϕ_i are smooth activation functions (such as $\sin, \cos, \exp$ and low degree polynomials), w_i^* are unit vectors, $|a_{r,i}^*| \le 1$ and $\|x\|_2 \le 1$.
- Such that the average loss of network F^* on the training data set is $\le \varepsilon$.
- Two-layer network with p hidden neurons.

Picture of the Network.



Our theorem

Theorem (Sketched (LL'18, ALL'18))

Suppose the labels of the data can be realized as in the previous slides, then as long as the number of training examples N satisfies:

$$N \geq poly(kp/\varepsilon) \times poly(log(m))$$

Then SGD on a two layer (one hidden layer) fully-connected neural networks (with m neurons and ReLU activation functions) finds a solution with generalization gap $\leq \varepsilon$ in time $poly(m/\varepsilon)$.

Generalization gap = average error on test data - average error on training data.

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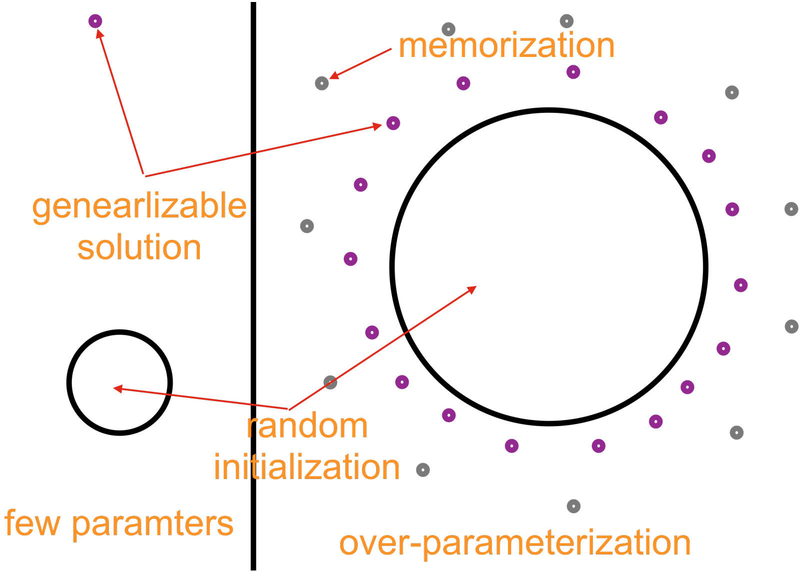
- The theorem also applies to convolution nets.

Picture of intuition



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- We are training a network with $1M$ parameters to match the performance of a network with $1K$ parameters, and that is easy!

Networks with more hidden layers?

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Networks with more hidden layers?

- Our theorem also works for three layer networks (two hidden layers).
- Theoretical reasoning on three layer networks is much harder due to the extremely non-convex interactions between the two hidden layers.

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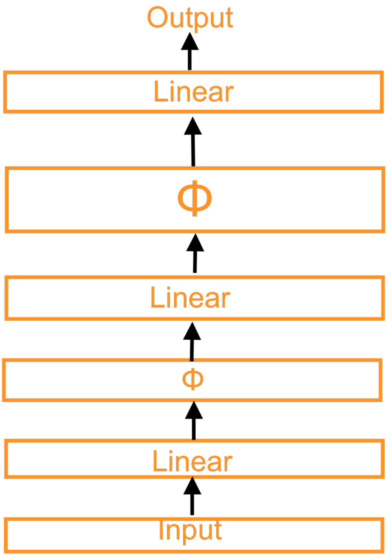

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- Despite the fact that the paper is 67 pages long (all proofs), the proof is conceptually very simple. Believe me :)

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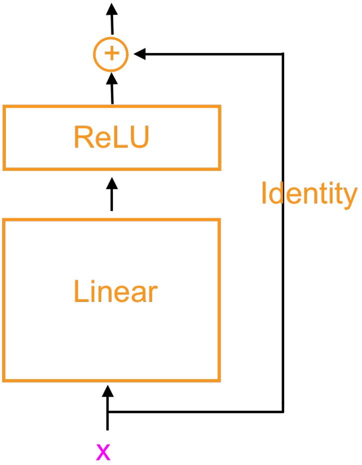

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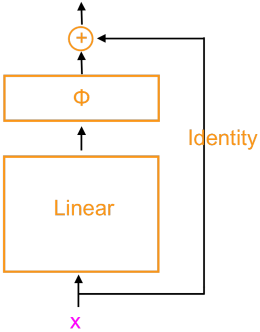

-

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Theorem (Sketched (corollary of ALL'18))

Suppose the labels of the data can be realized as in the previous slides, then as long as the number of training examples N satisfies:

$$N \geq poly(kL/\varepsilon) \times poly(log(mL))$$

Then SGD on a L-layer single-skip ResNet with ReLU activation functions (each layer is fully connected with m neurons) finds a solution with generalization gap $\leq \varepsilon$ in time $poly(mL/\varepsilon)$.

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- Also applies to ResNet with convolution layers. But not double skip or triple skip yet.

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- Question: Why always SGD?
  - Can we modify SGD so it biases towards solutions with even better generalization?

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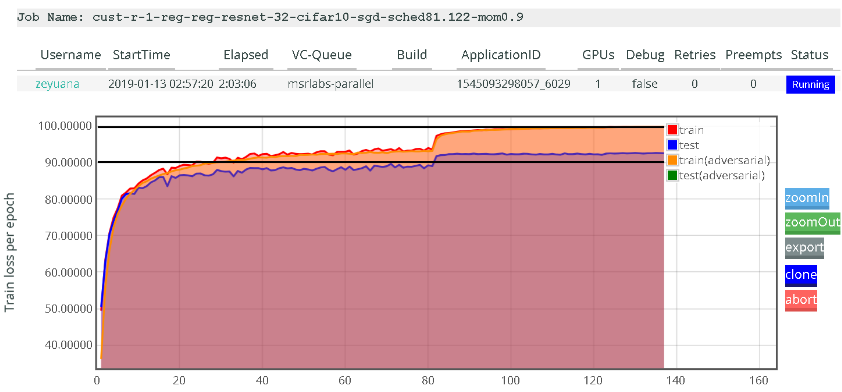
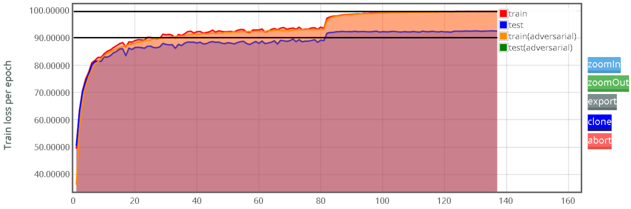
- 

SGD with small learning rate

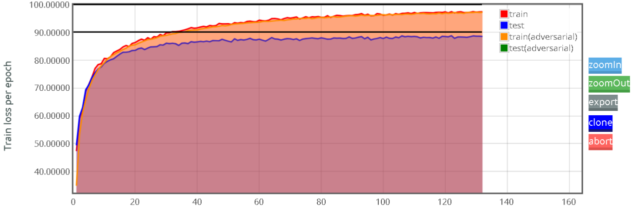
Job Name: cust-r-1-reg-reg-resnet-32-cifar10-sgd-sched81.122-mom0.9

Username	StartTime	Elapsed	VC-Queue	Build	ApplicationID	GPUs	Debug	Retries	Preempts	Status
zeyuana	2019-01-13 02:57:20	2:03:06	msrlabs-parallel		1545093298057_6029	1	false	0	0	Running



Job Name: cust-r-1-reg-reg-resnet-32-cifar10-sgd-lr0.01-sched300.301-mom0.9

Username	StartTime	Elapsed	VC-Queue	Build	ApplicationID	GPUs	Debug	Retries	Preempts	Status
zeyuana	2019-01-13 02:58:56	2:02:01	msrlabs-parallel		1545093298057_6032	1	false	1	0	Running



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Our current work

- A concrete example of a data set such that:

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- After weight decay it then learns the symbols.

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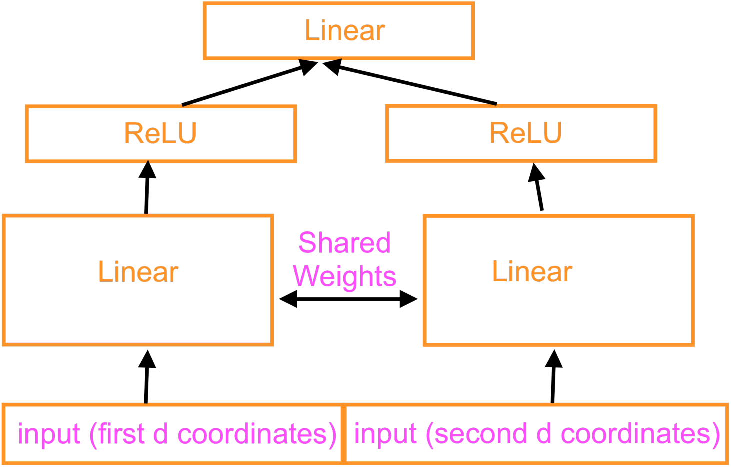
- With probability $1 - p$,

$$x_2 = \begin{cases} z \pm \delta & \text{if } y(x) = 0; \\ z & \text{if } y(x) = 1. \end{cases}$$

Where z, δ are two vectors in $\mathbb{R}^d$ with $\|z\|_2 = 1$ and $\|\delta\|_2 = \frac{1}{d^{\Theta(1)}}$.

The model

We use the following model:



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- For $p = o(1)$, large learning rate provably generalize better.
- But in practice, probably $p = 0.9$ (so we ignore 10 percent of the data) would make a huge difference.

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    - Matrix/Operator scaling, ℓ_p regressions, parallel(distributed) optimization.