Towards deeper understandings of deep learning

Yuanzhi Li

Stanford University

date: Today

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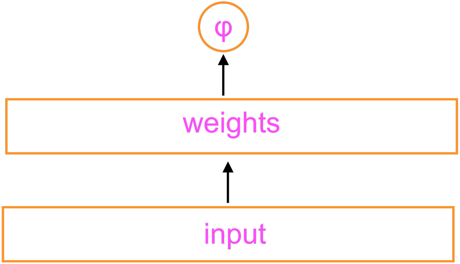

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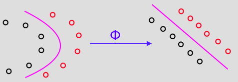

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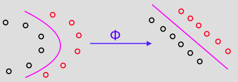

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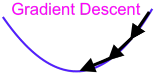

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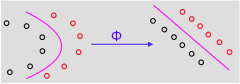

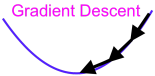

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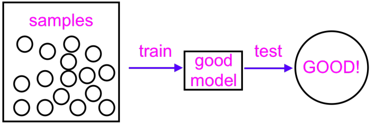

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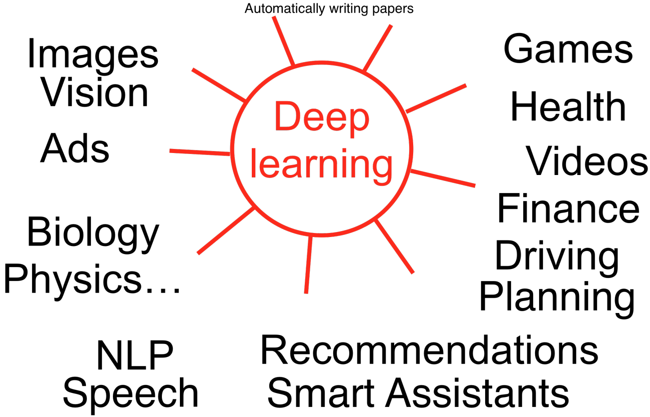

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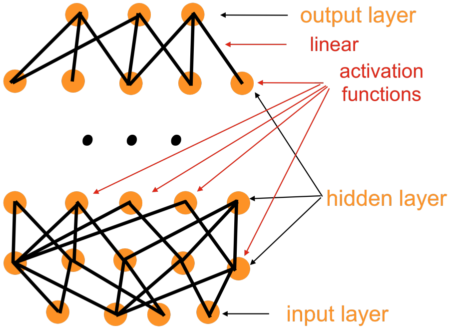

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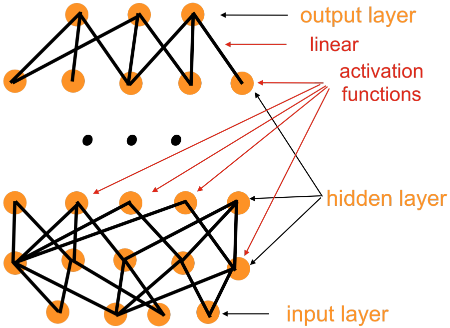

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- In this talk, I will particularly focus on neural networks with ReLU activations.

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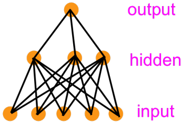
  - 

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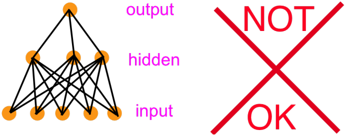
  - 

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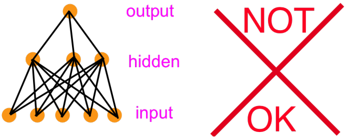
    - 

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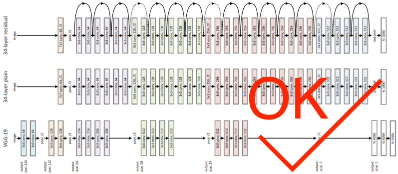
    - 

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```
optimizer = optim.SGD(para, lr)
optimizer.zero_grad()
loss.backward()
optimizer.step()
```

Yes!

Face recognition

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- Makes the theoretical work even harder (understand the universality of SGD).

My work

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Tensor decomposition
Mixture of regressions
k-means
ICA NMF
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Bellman equation
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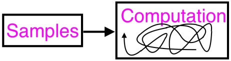

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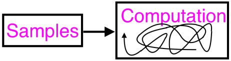

  - Noisy computation.
- And how can they help in learning neural networks, provably.

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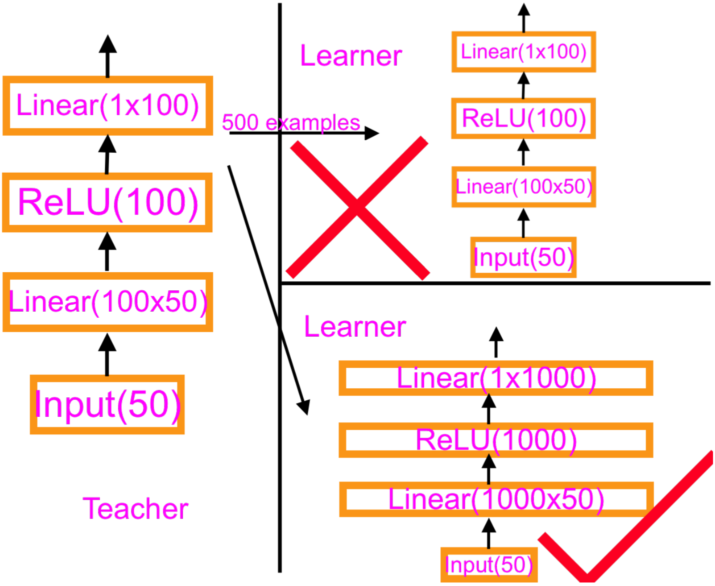
- And it improves the theory.

Folklore example for training.



Example for generalization

Widen factor	Number of parameters	Test error
1	0.6M	6.85
2	2.2M	5.33
4	8.9M	4.97
8	36.5M	4.66

Table: Depth 40 WideResNet on CIFAR-10 (0.05M training examples), training errors are all 0.

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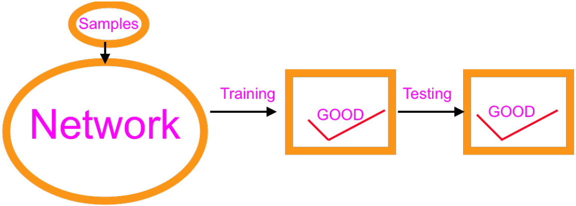
- 

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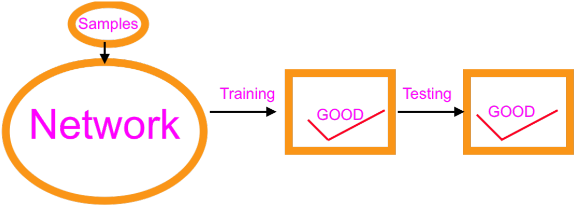
- 

- We begin with our theorem for (1), then we will see (2) as well.

Our Theorem

Theorem (Sketched, (LL'18, ALS'18a,b))

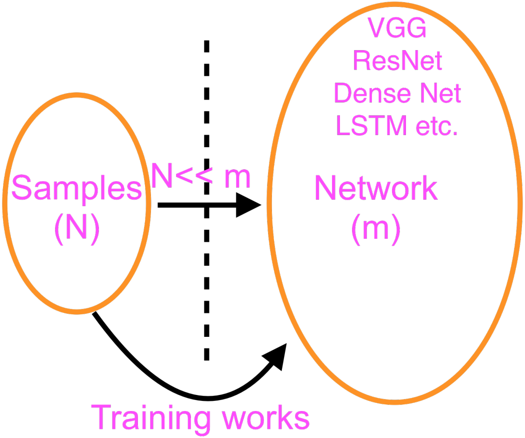
Given N different training examples $x_1, \ldots, x_N$ with labels $y_1, \ldots, y_N$, then for every $\varepsilon > 0$, as long as the number of neurons (m) in the network satisfies

$$m \geq poly(N \log(1/\varepsilon))$$

then SGD starting from gaussian random initialization finds an ε-approximate global optimal of the training objective in time poly(m/ε).

The theorem holds for multi-layer DNNs, CNNs, ResNet and Recurrent Neural Networks (all with ReLU activation functions), the training loss can be given by ℓ_2 loss, cross entropy etc.

Our theorem by picture



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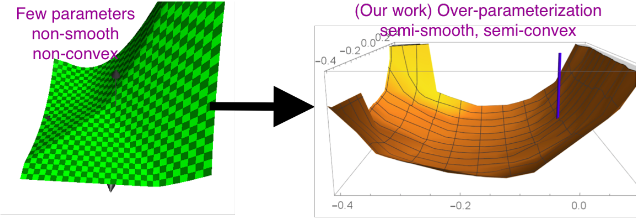

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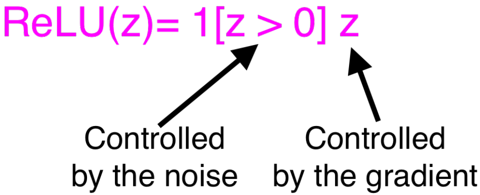
- 

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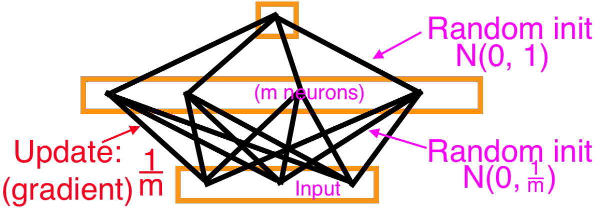
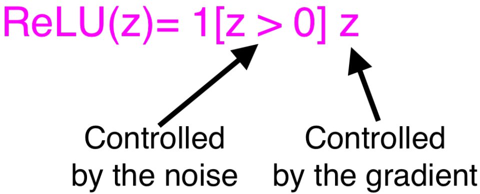
- 

- 

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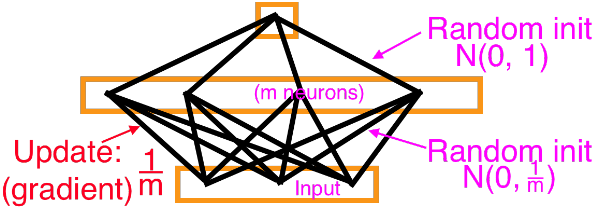
- 

- Sign controlled by the noise in SGD (principle of noisy computation).

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  - Over-parameterized networks (AlexNet) can fit CIFAR-10 data with random labels.

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  - So why does the solution found by SGD still generalize?

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- The labels are not random (necessary).

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  - We are going to prove these claims for certain neural networks.
  - We start with two layer networks, and then multi-layer ones.

Labels are not random? (Two layer networks)

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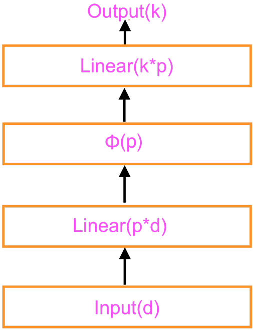

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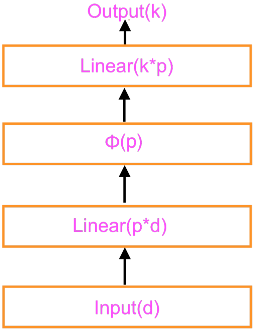

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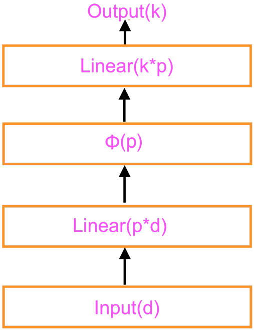

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- Two-layer network with p hidden neurons, where p is something small (e.g. 100). Φ is a smooth activation function (such as $\sin, \cos, \exp, \tanh, sigmoid$, and low degree polynomials).
- Such that the average loss of this network on the training data set is $\leq \varepsilon$.

Our Theorem

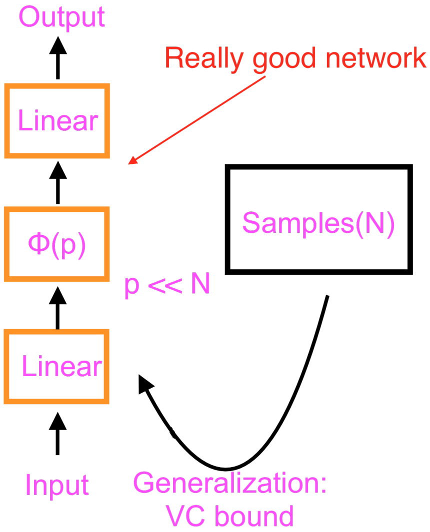
Theorem (Sketched (LL'18, ALL'18))

Suppose the labels of the data can be realized as in the previous slides, then as long as the number of training examples N satisfies:

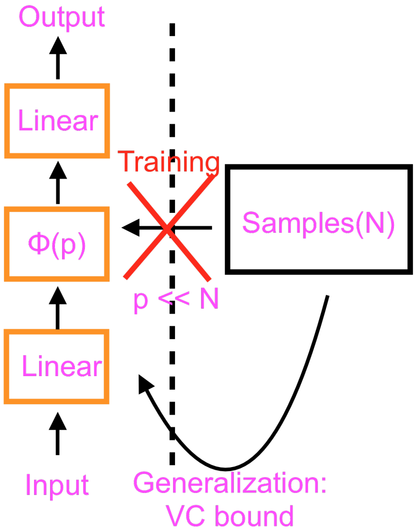
$$N \geq poly(kp/\varepsilon) \times poly(\log(m))$$

Then SGD on a two layer neural network (with m neurons and ReLU activation functions) finds a solution with generalization gap $\leq \varepsilon$ in time $poly(m/\varepsilon)$.

Generalization gap = average error on test data - average error on training data.


Output
Really good network
Linear
Samples(N)
Φ(p)
p << N
Linear
Input
Generalization:
VC bound

Picture of the Theorem



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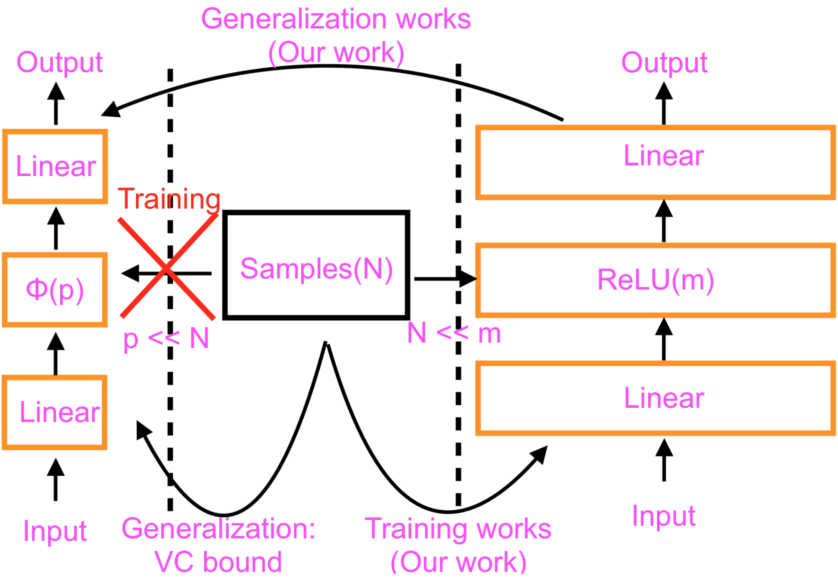

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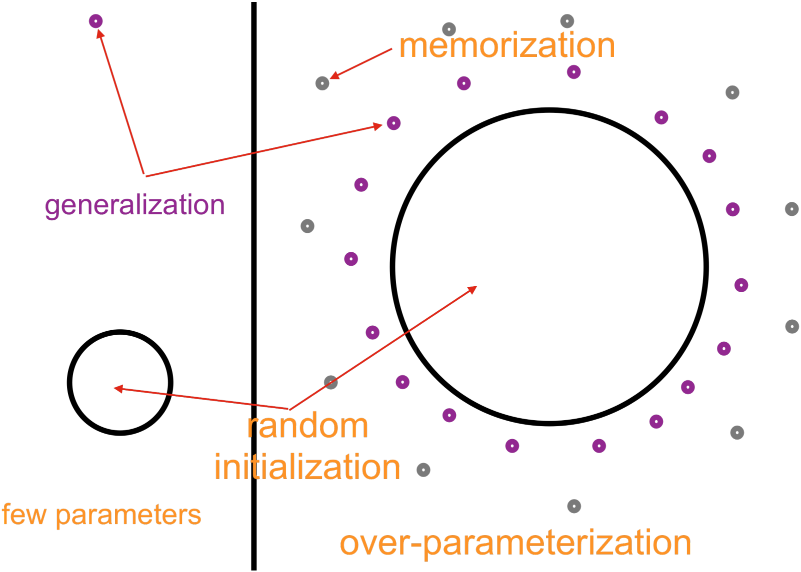
- We are training a network with $1M$ parameters to match the performance of a network with $1K$ parameters, and that is easy!
- This is what behind the principle of over-parameterization.

Intuition



memorization
generalization
random
initialization
few parameters
over-parameterization

Intuition

Generalization

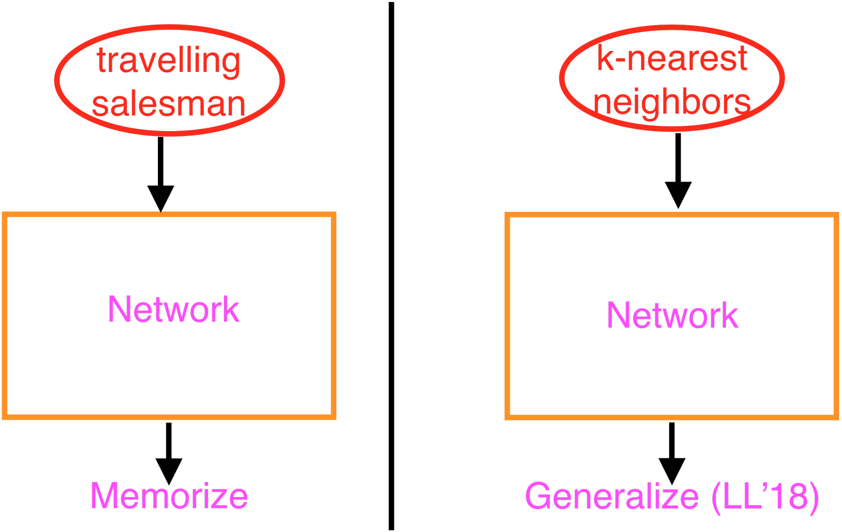
$y = \cos(x)$

Simple!

Memorization

if x = 1, y = 0.54
if x = 2, y = -0.42
if x = 3, y = -0.99
if x = 4, y = -0.65
if x = 5, y = 0.28
if x = 6, y = 0.96
if x = 7, y = 0.75
if x = 8, y = -0.15
if x= 9, y = -0.91
if x = 10, y = -0.84
I gave up typing
those things…
too tired…

Intuition



Existence of small networks is somewhat necessary

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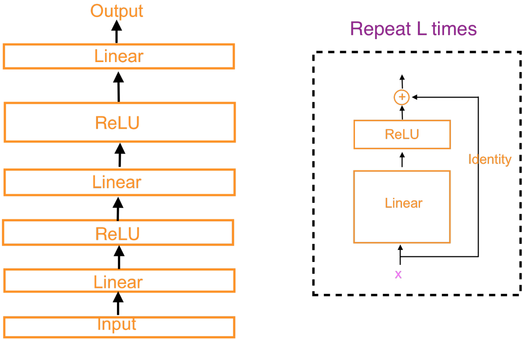

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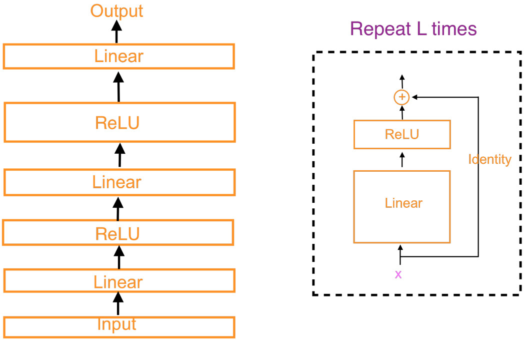
- 

- Message: depth provably matters.

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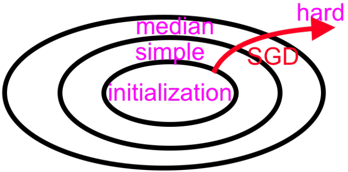

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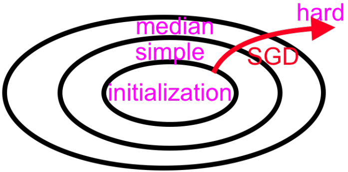

  -
- Question: Does this bias always lead to best generalization? Can we do better?

SGD with learning rate decay

- When we train a neural network (VGG, ResNet, Dense Net, etc. for image classification etc.)

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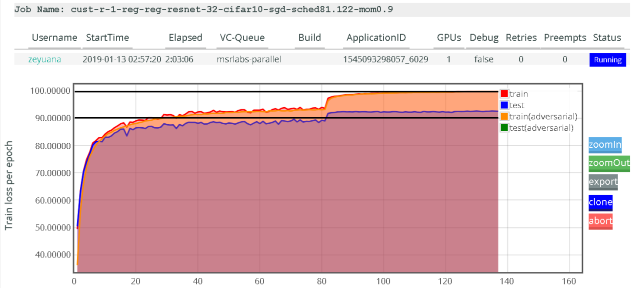
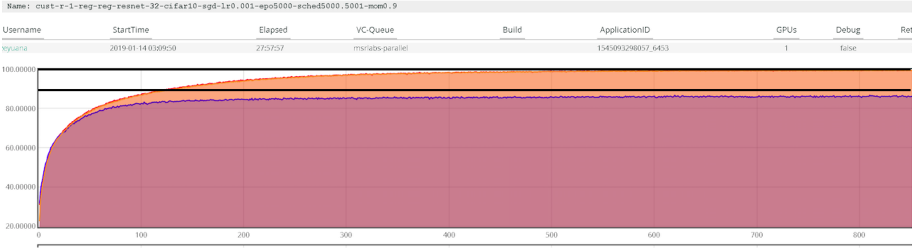
SGD with different learning rates





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- Training loss of using large learning rate + learning rate decay and using small learning rate are all close to zero.

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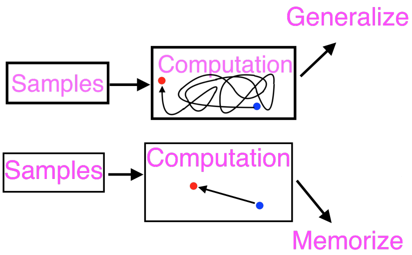

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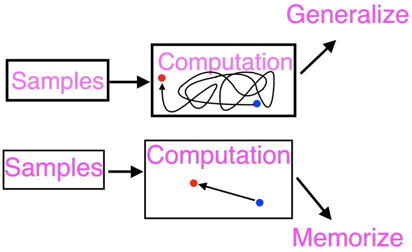

-
- But why?

Our current work

- The principle of "noisy computation":

Our current work

- The principle of "noisy computation":
- On certain data sets, when training a neural network using SGD, large learning rate + learning rate decay provably generalizes better than small learning rate.

Example of the data set

- Texts labeled as happy:

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- Texts labeled as happy:
  - I am so happy ☺

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- Texts labeled as happy:
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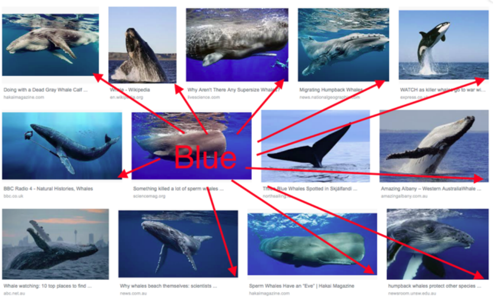

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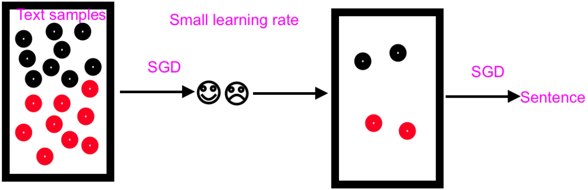

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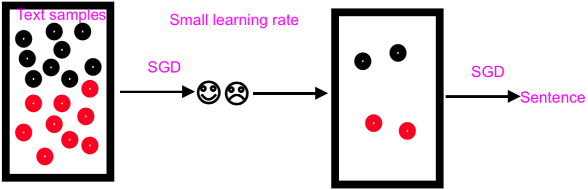
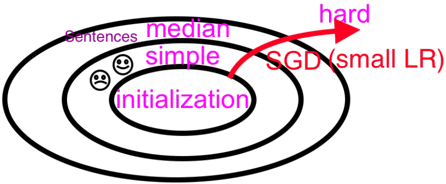


- 

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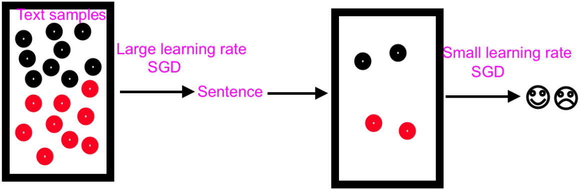

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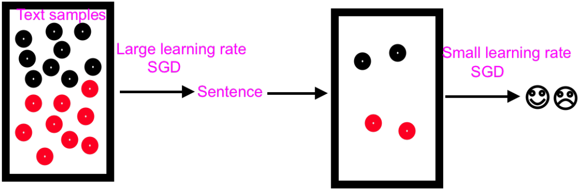
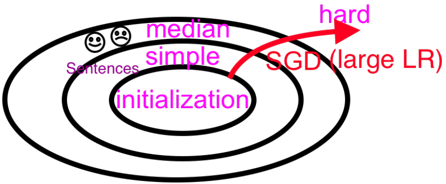


- 

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$$x_2 = \begin{cases} z & \text{if } y(x) = 0; \\ z \pm \delta & \text{if } y(x) = 1. \end{cases}$$

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("special symbol" signal)
- Remark: $\langle w, x\rangle$ can be replaced by two layer networks.

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1. *SGD with large learning rate + learning rate decay achieves generalization error $\leq \tau^{1+\Omega(1)}$.*
2. *SGD with small learning rate has generalization error $\approx \tau$.*

- When $N \gg p$, they both generalize (as shown in the principle of over-parameterization), but SGD with large learning rate + learning rate decay generalizes even better.

Intuition

Generalization:

```
def TTI_talk_example(x):
    x = x^2
    x = x + 10
    x = x*3
    x = x^(1/3)
    x = x - 3
    return x
```

Memorization:

```
def TTI_talk_example(x):
    if x = 1, return 0.2
    if x = 2, return 0.48
    if x = 3, return 0.84
```

SGD with
small learning rate:
I like this one!

Intuition

Generalization:

```
def  TTI_talk_example(x):
        x = x^2.03
      x = x + 10.04
        x = x*3.02
    x = x^(1/3.01)
       x = x - 3.05
       return x
```

SGD with
large learning rate
(noisy)

almost correct

Memorization:

```
def  TTI_talk_example(x):
  if x = 1.02 , return 0.2
  if x = 2.04,return 0.48
  if x = 3.01, return 0.84
```

?????

x = 1

Principle of “noisy computation”

- Memorization in neural network: when $m \gg N$, by chance there will be N-neurons in the network that maps all data to linear independent positions. (Geometry of ReLU Lemma in [LL’18, ALS’18(a,b)])

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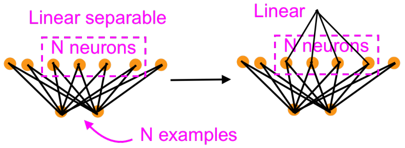

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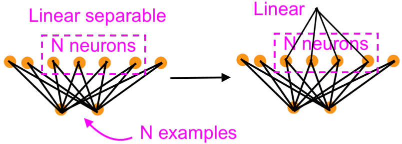
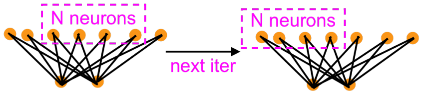


- SGD with large noise will make this set of N-neurons changing rapidly, and destroy the linear separator to prevent memorization.



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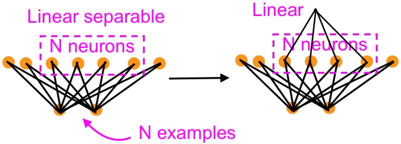

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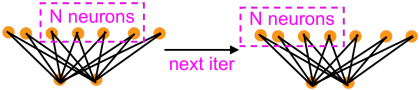

- This is what behind the principle of "noisy computation".

Summary of my works

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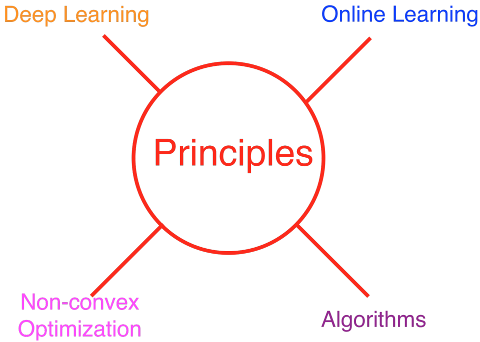

-

In each area

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Over-para: Training/Generalization [LL'18, ALS'18, ALL'18]

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Open problem since 2004

Open problem since 1991

Main open problem in contextual bandit (raised in COLT open problem session)

Best student paper(ALT 18)

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One step towards derandomizing PIT (complexity)

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Connected by principles

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Over-parameterization

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Inductive bias of SGD

Noisy computation

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Ensemble(Multiple Objectives)

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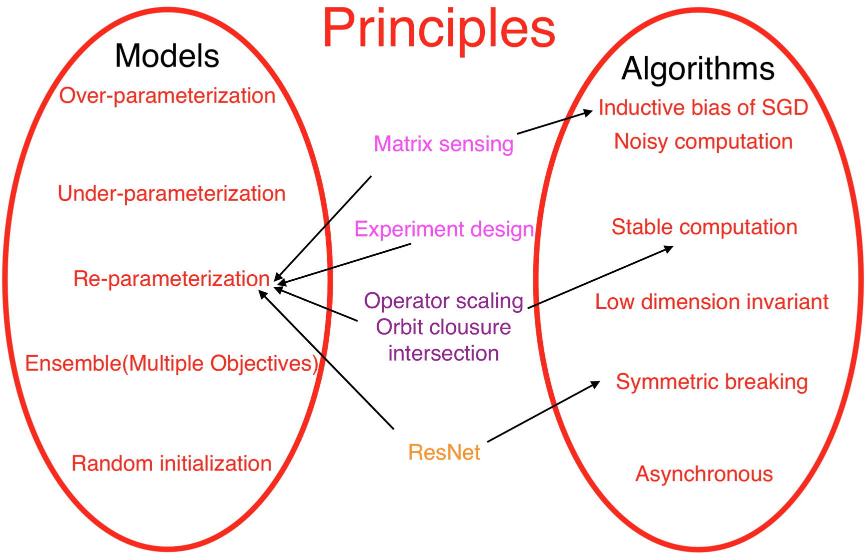

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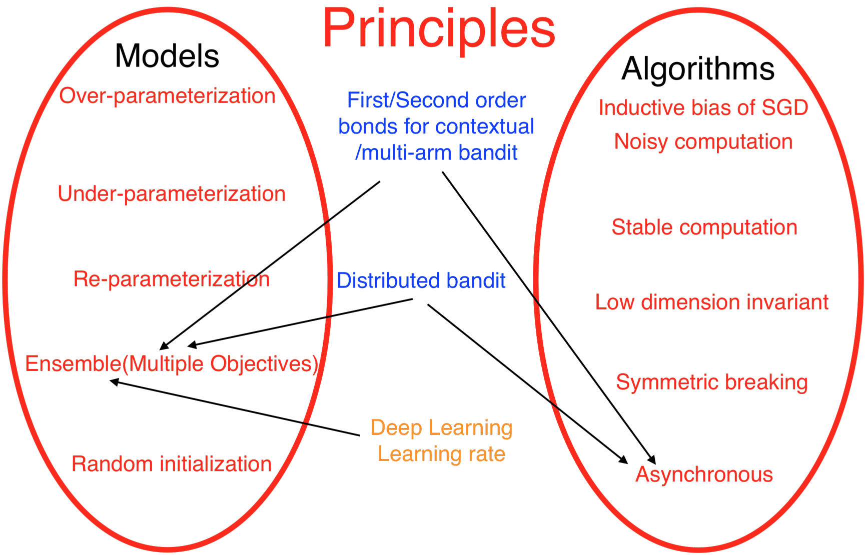

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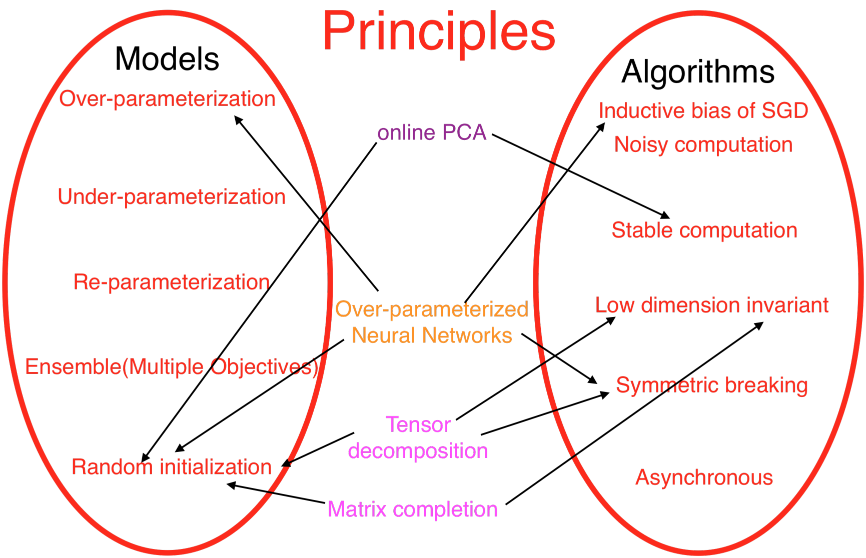

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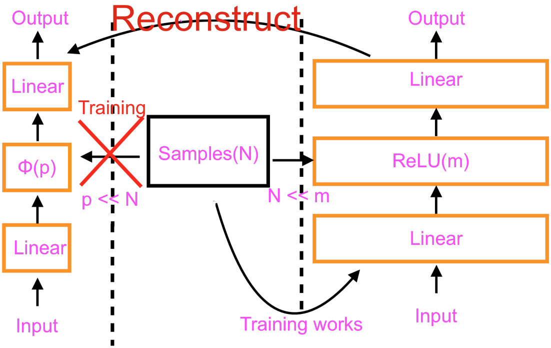

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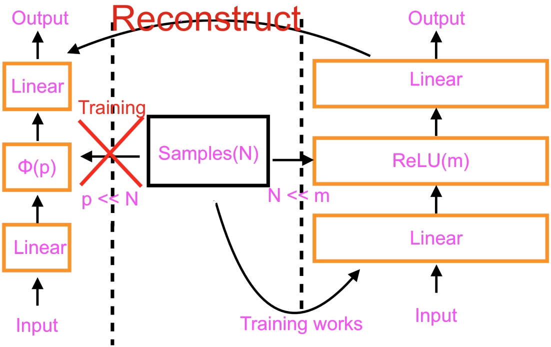

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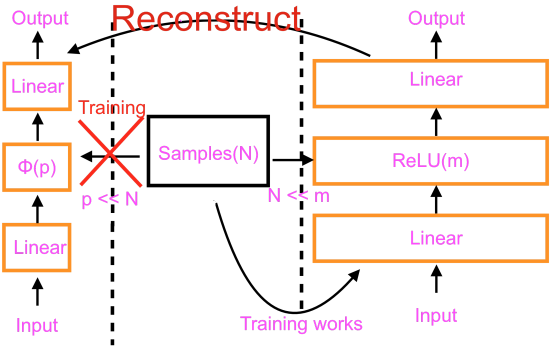

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- Much more exciting directions.

Yelp: Local Food & Services

Restaurant & Delivery Finder

4.1 ★★★★☆

4.42K Ratings

#10

Travel

12+

Age

What's New

Version History

Version 12.27.0

2d ago

We apologize to anyone who had problems with the app this week. We trained a neural net to eliminate all the bugs in the app and it deleted everything. We had to roll everything back. To be fair, we were 100% bug-free... briefly.