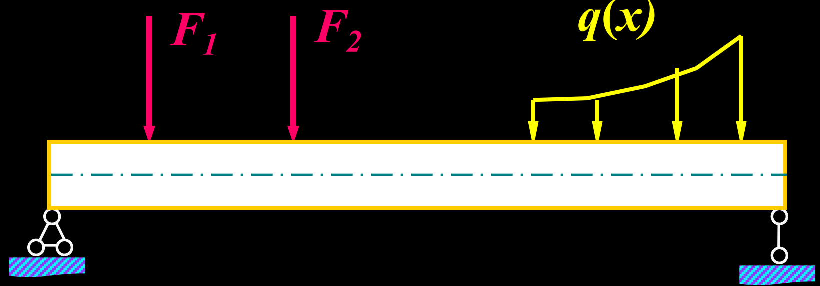
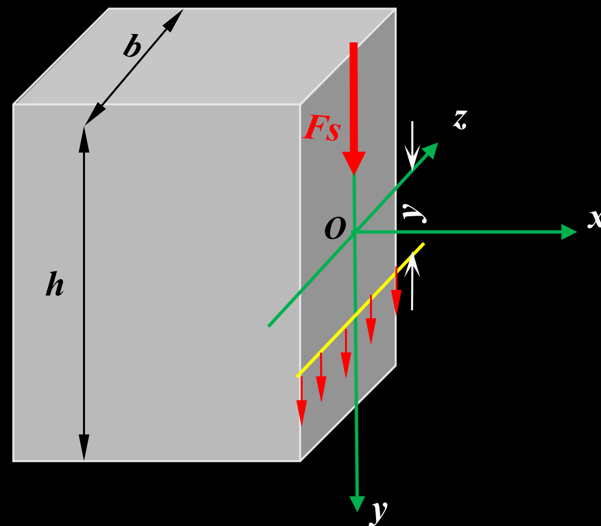
第二节

平面弯曲时梁横截面上的切应力分析

梁在横力弯曲的情况下，梁的横截面上既有弯矩也有剪力，相应地，横截面上既有正应力也有切应力。本节讨论弯曲切应力的计算公式。

一、矩形截面梁

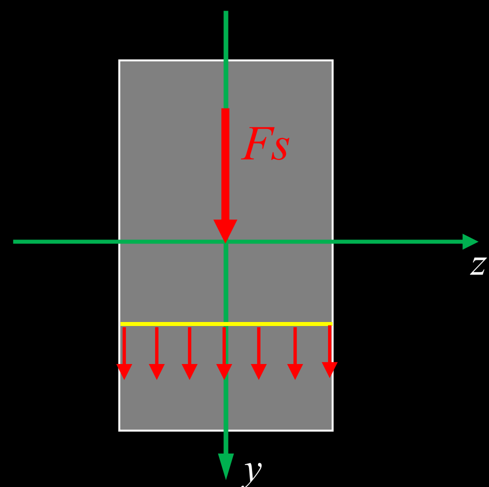


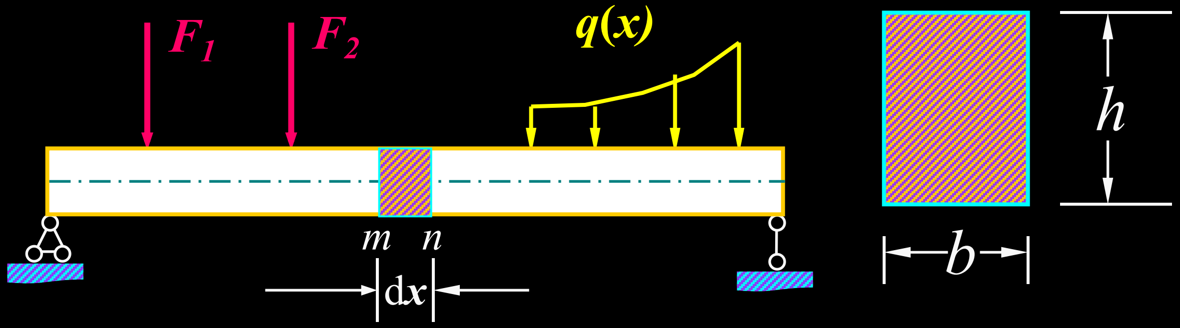


1、两点假设：

（1）*切应力与该截面上剪力的方向一致*

（2）*切应力沿截面宽度均匀分布*





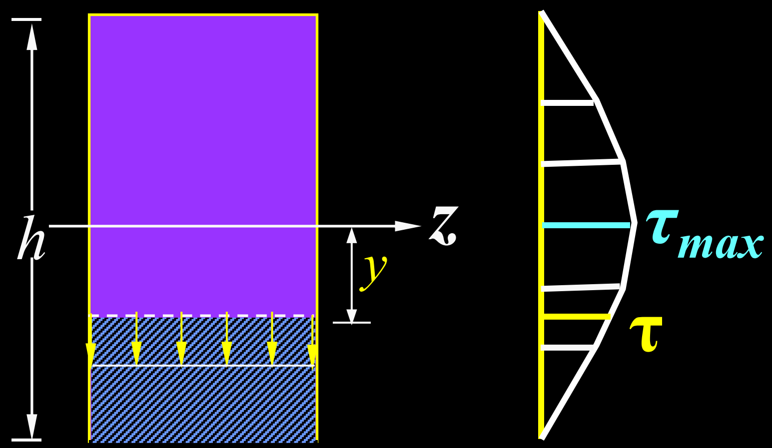
$$\tau = \tau' = \frac{F_s S_z^*}{I_z b}$$

F_s — 横截面上剪力

b — 横截面宽度

I_z — 整个横截面对中性轴z轴的惯性矩

S_z^* — 过横截面上需求切应力的点的水平横线以外部分面积对中性轴的静矩。

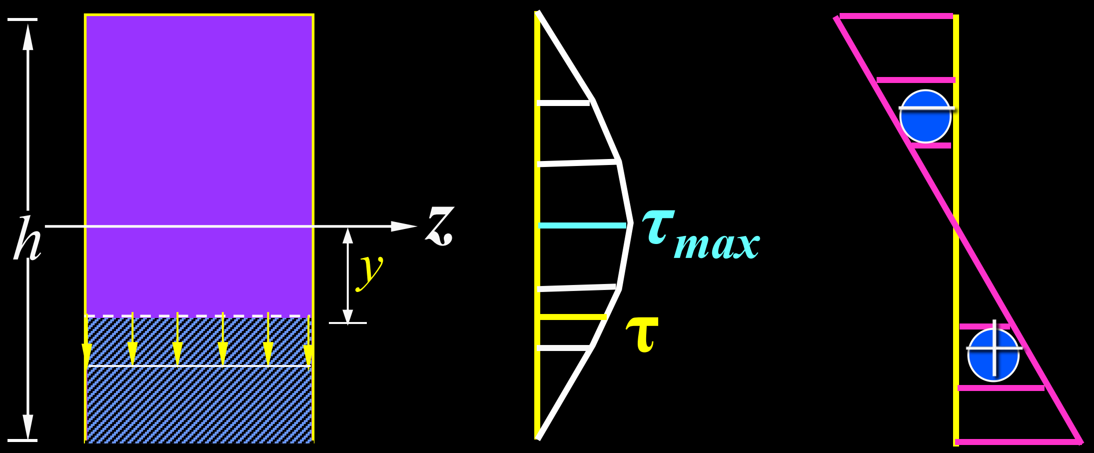


$$\tau = \tau' = \frac{F_s S_z^*}{I_z b}$$

$$y = \pm \frac{h}{2}, \quad \tau = 0$$

$$\tau = \frac{6F_s(x)}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

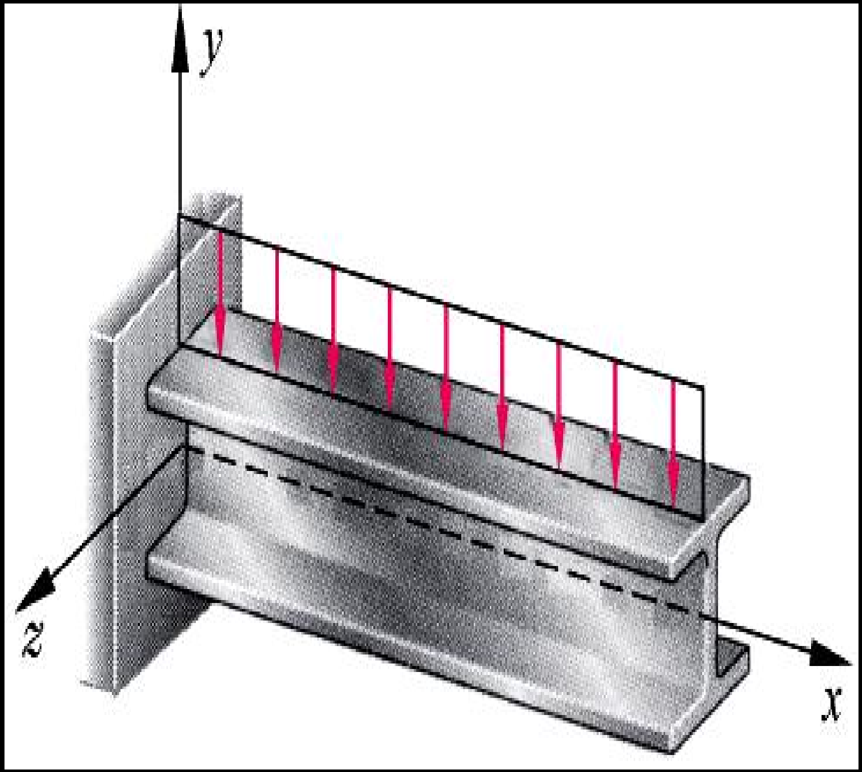
$$y = 0, \ \tau_{\max} = \frac{3F_s(x)}{2bh} = \frac{3F_s(x)}{2A}$$

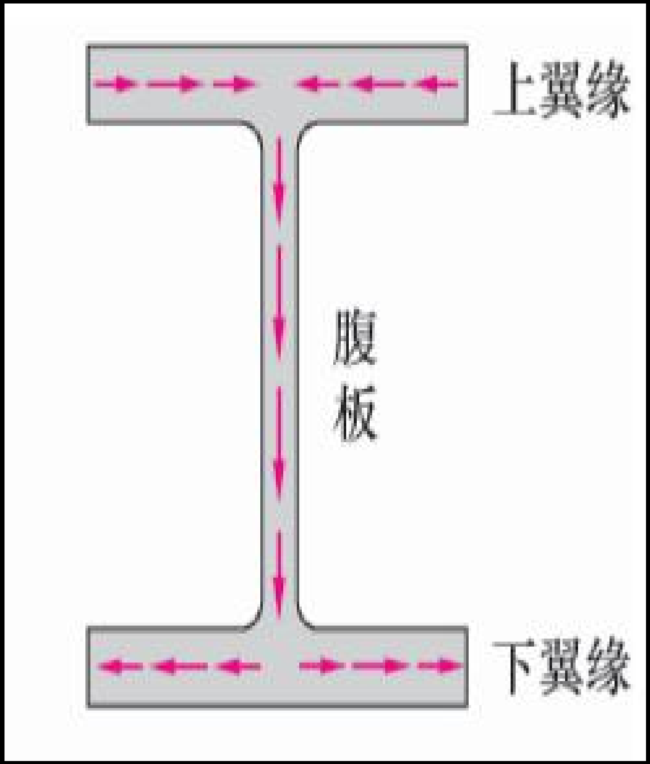


$$y=0,\ \tau_{\max}=\frac{3F_s(x)}{2bh}=\frac{3F_s(x)}{2A}$$

二、工字形截面梁的切应力

工字形截面由腹板和上、下翼缘组成。在横力弯曲条件下，翼缘和腹板上均有切应力存在。

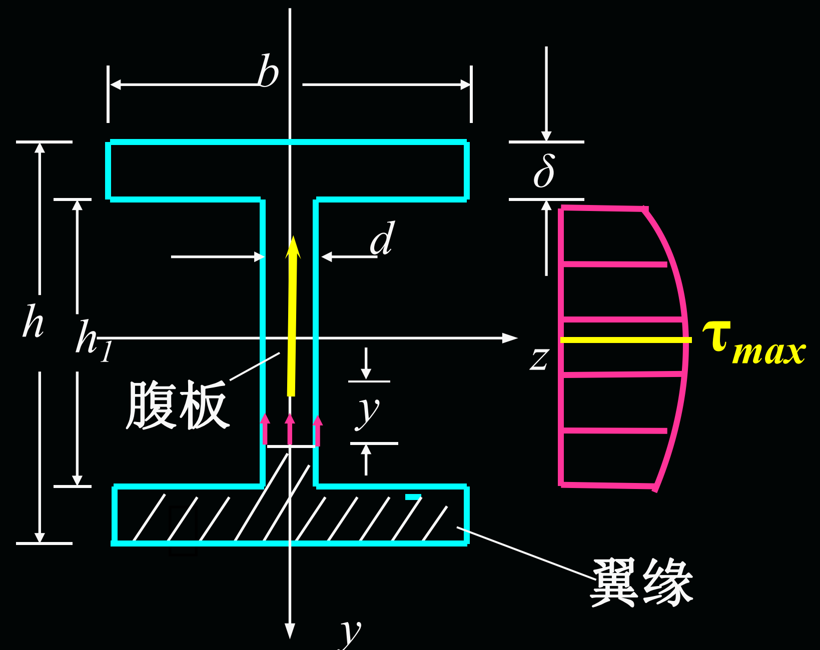




二、工字形截面梁

腹板上的切应力：

$$\tau = \frac{F_s S_z^*}{I_z d}$$



F_s—横截面上剪力

I_z—整个工字形截面对中性轴z的惯性矩

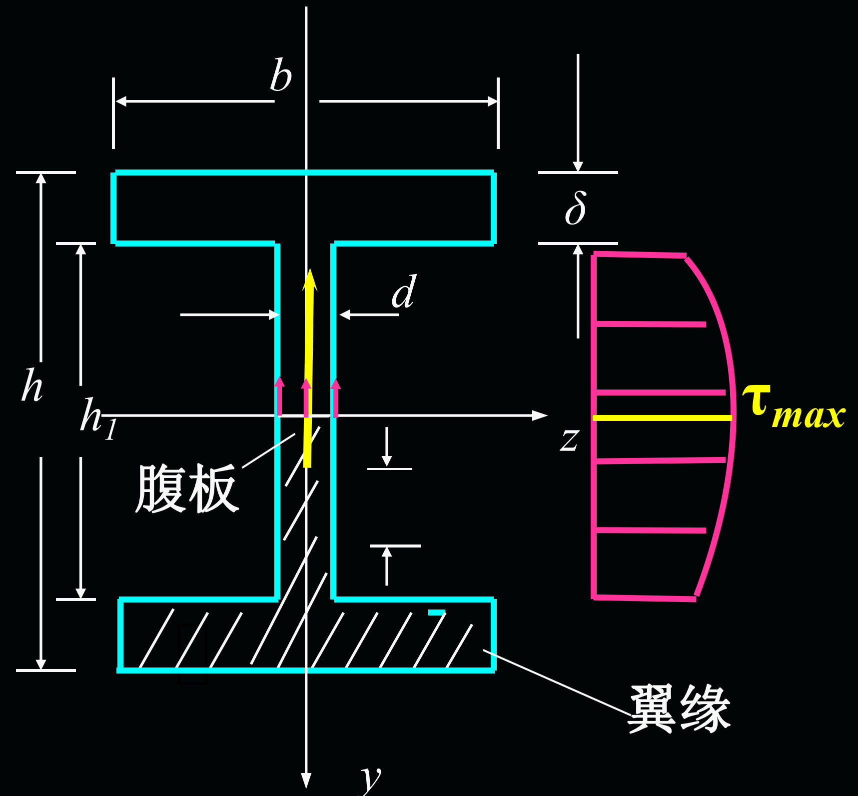
d—腹板宽度

S_z^*—距z轴y处横线一侧 阴影部分面积对z轴的静矩

二、工字形截面梁

腹板上切应力分布规律：

腹板上切应力沿腹板高度呈抛物线规律分布，中性轴上切应力最大，即：



$$\tau_{\max} = \frac{F_S S^*_{z\max}}{I_Z d}$$

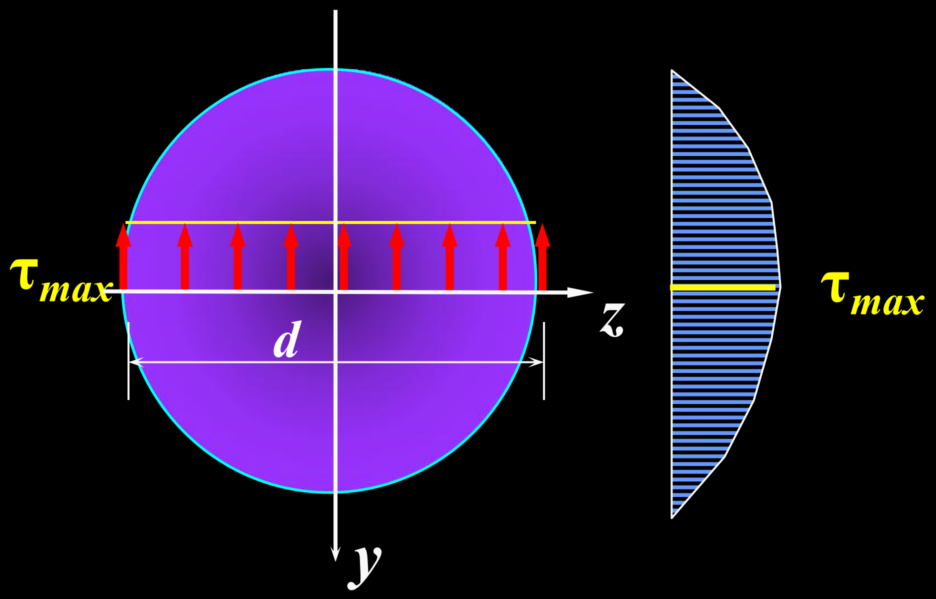
$\dfrac{I_Z}{S^*_{Z\max}}$ — 型钢表中可以查得

翼缘上的切应力很小，一般不必计算。

三、圆形截面梁

圆截面的最大切应力仍发生在中性轴上，方向平行于剪力F_s的方向

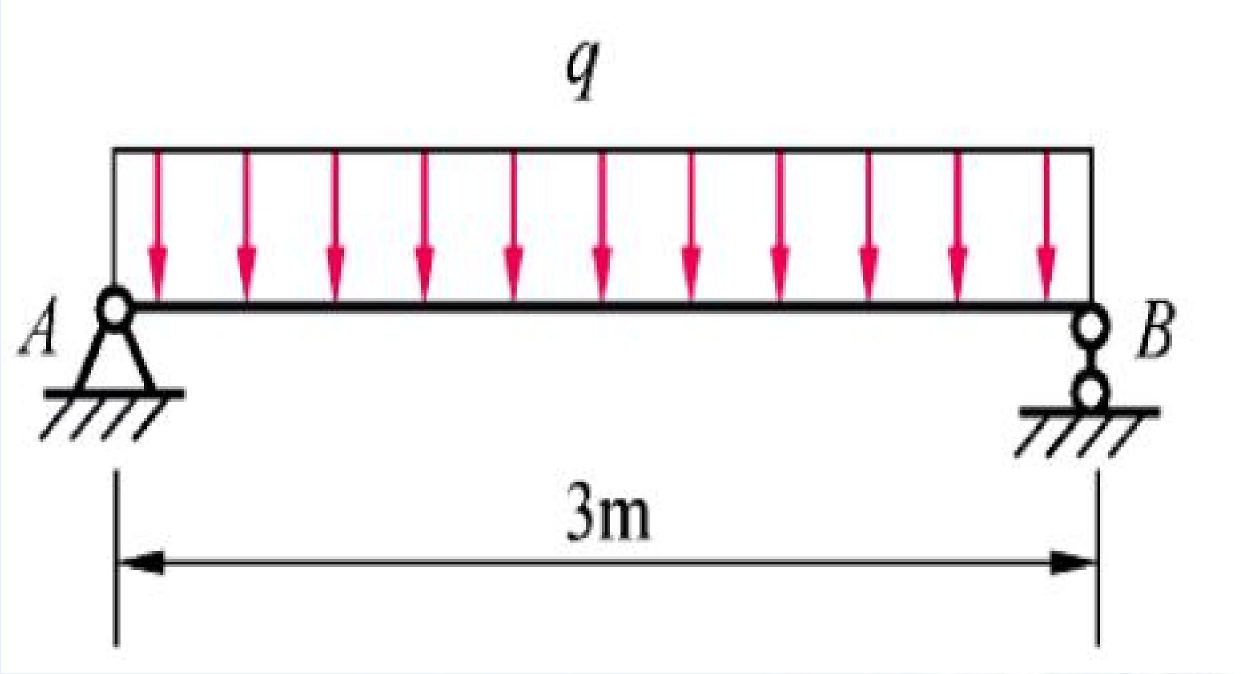
$$\tau_{max} = \frac{F_s S_z^*}{I_z b}$$

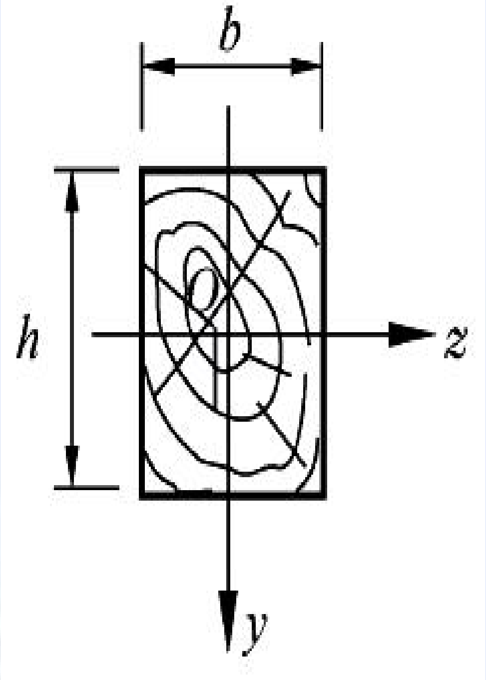


$$= \frac{F_s \cdot \frac{\pi d^2}{8} \cdot \frac{2d}{3\pi}}{\frac{\pi d^4}{64} \cdot d} = \frac{4}{3}\frac{F_s}{A}$$

$$A = \frac{\pi d^2}{4}$$

例题1 图示两端铰支矩形截面木梁，长度l=3m，受均布载荷作用，载荷集度q=10kN/m。已知木材的许用正应力$[\sigma]$=12MPa，顺纹许用切应力$[\tau]$=1.5MPa，设h/b=3/2试选择木梁截面尺寸，并进行切应力强度校核。



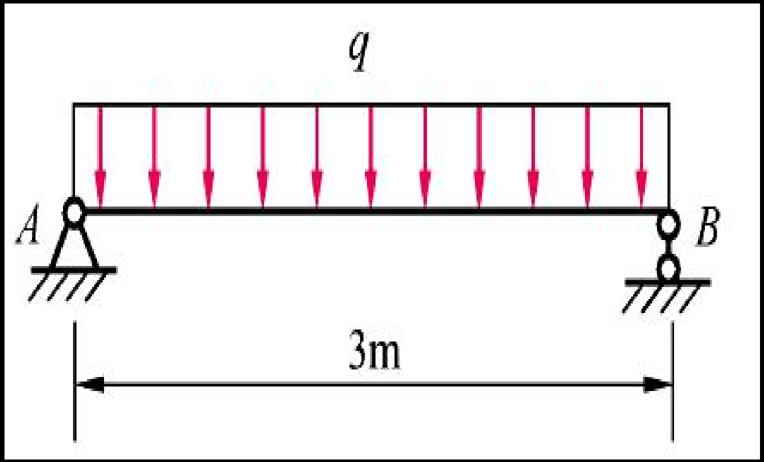


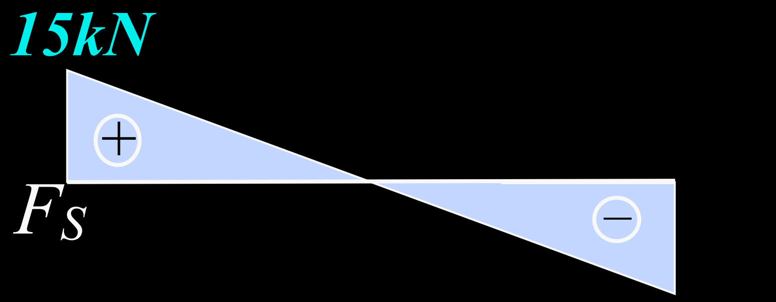
解（1）作梁的剪力图和弯矩图

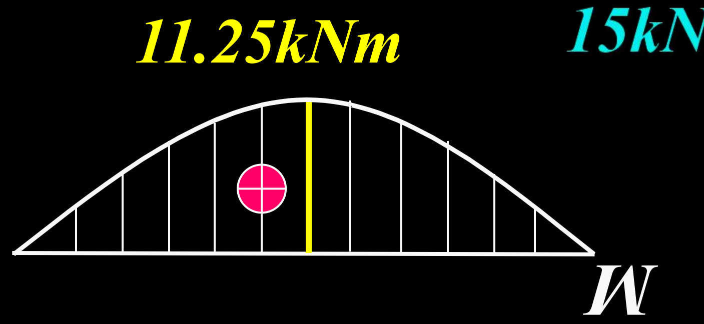
$$F_A = F_B = 15kN$$

$$F_{s\max} = 15kN$$

$$M_{\max} = 11.25kNm$$







(2) 按正应力强度条件选择截面

$$W_Z \geq \frac{M_{\max}}{[\sigma]}$$

$$\frac{bh^2}{6} \geq \frac{11.25\times10^3}{12\times10^6}$$

$$\frac{9b^3}{24} \geq 94\times10^{-5}$$

$$b \geq 135.8mm$$

取 $b = 136mm$

$$h = 204mm$$

$$h = \frac{3b}{2} = 1.5\times136$$

（3）校核梁的切应力强度

$$\tau_{\max}=\frac{3F_{s\max}}{2A}=\frac{3\times15\times10^{3}}{2\times136\times204\times10^{-6}}=0.811MPa$$

$$\tau_{\max}\leq[\tau]$$

所选木梁截面尺寸满足切应力强度要求。

小结

1、了解纯弯曲梁弯曲正应力的推导方法

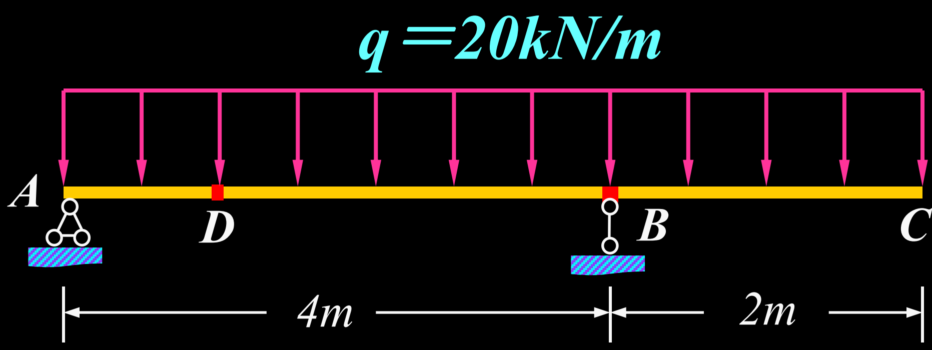
2、熟练掌握弯曲正应力的计算、弯曲正应力强度条件及其应用

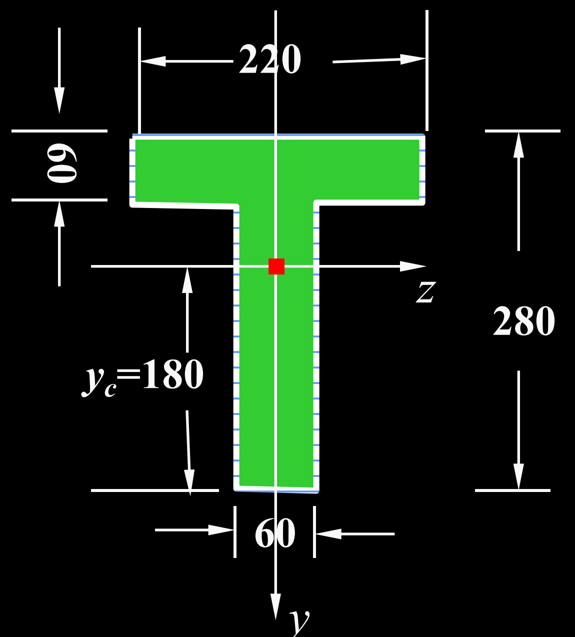
3、了解提高梁强度的主要措施


本节结束

本节结束

例2：一T形截面外伸梁及其所受荷载如图所示。试求最大拉应力及最大压应力，并求出最大剪力截面复板上的切应力分布图。



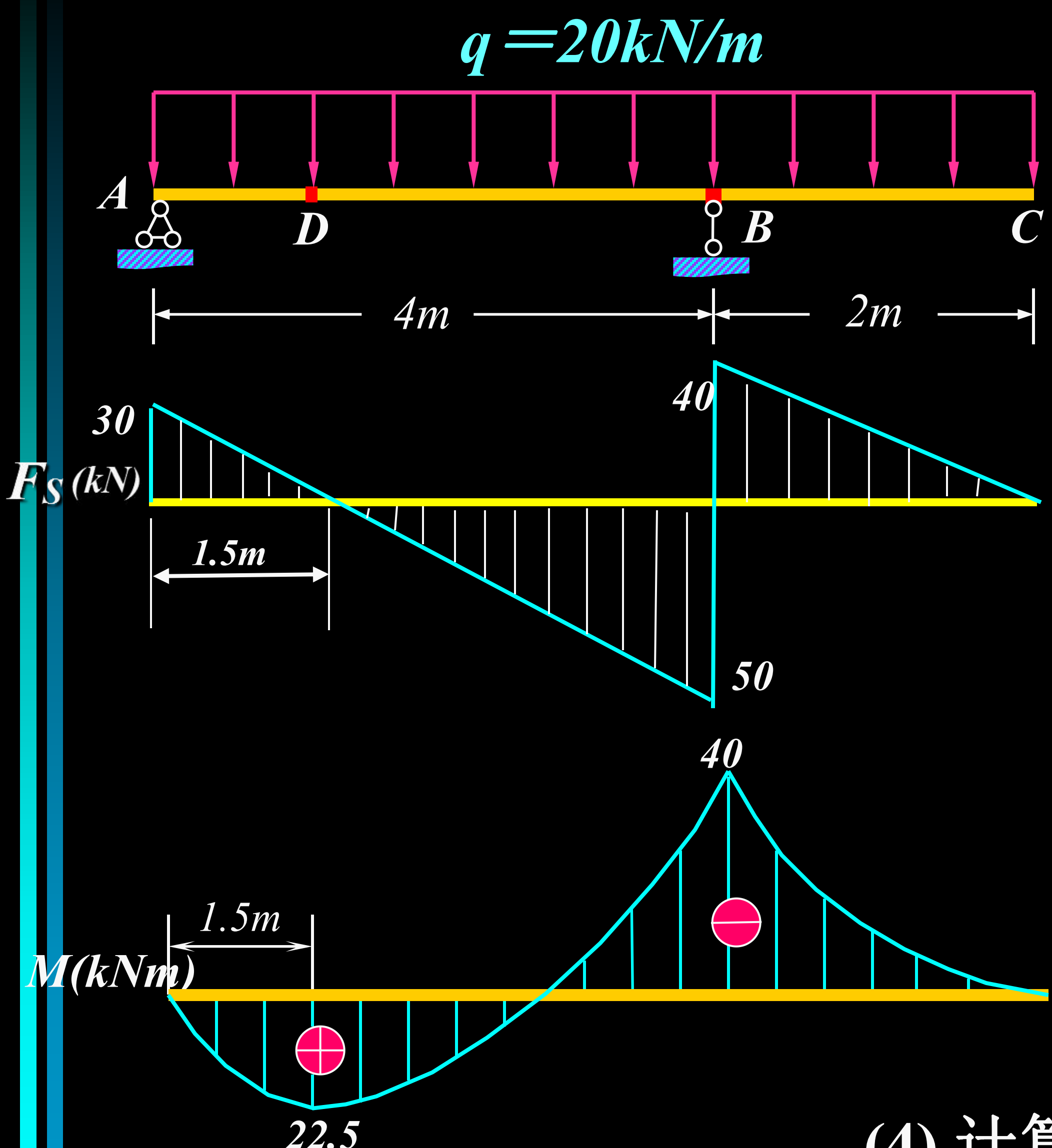


解：(1) 确定截面形心位置

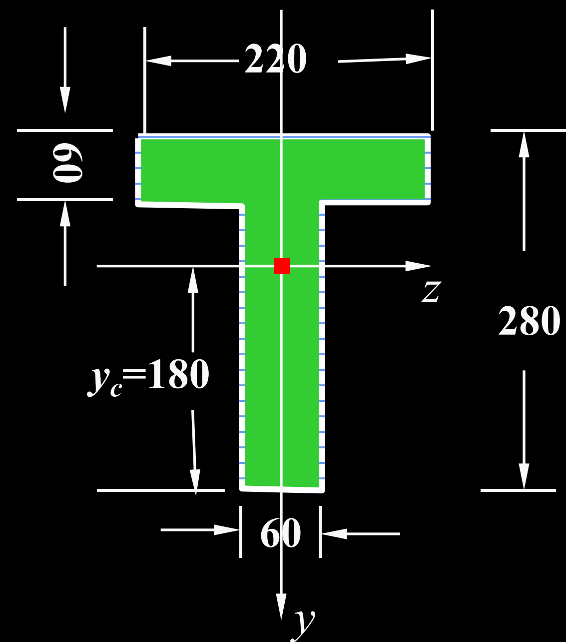
(2) 计算横截面的惯性矩I_z

(3) 画剪力图和弯矩图

$$I_z=186.6\times10^6 mm^4$$

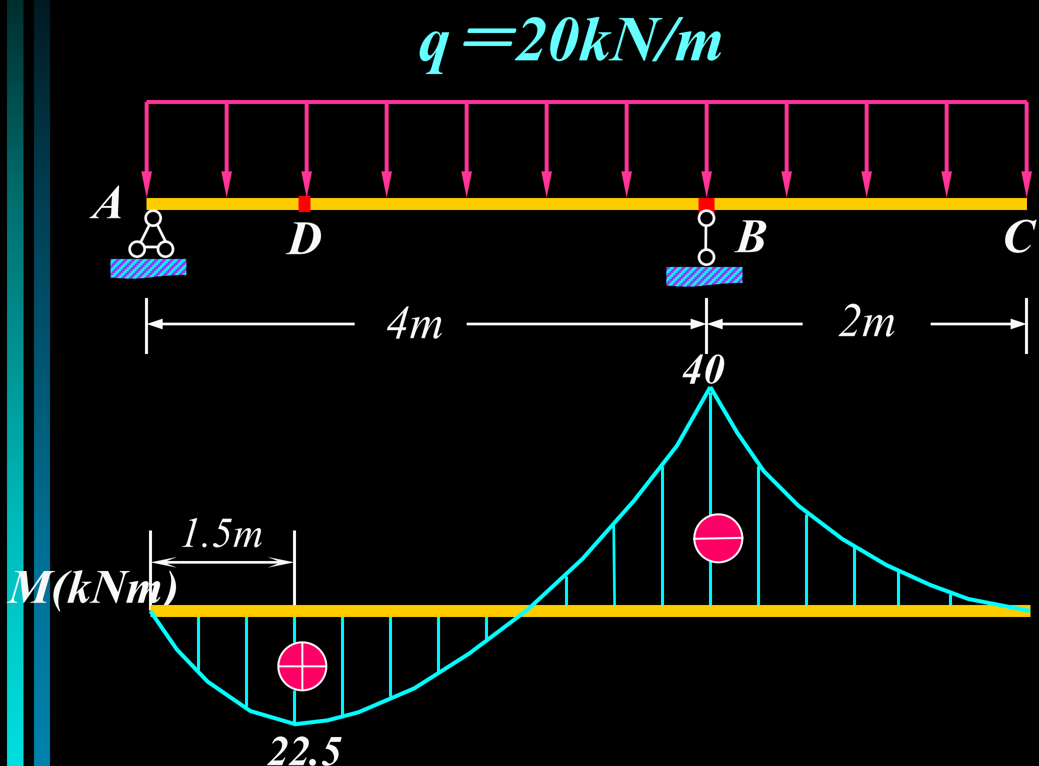


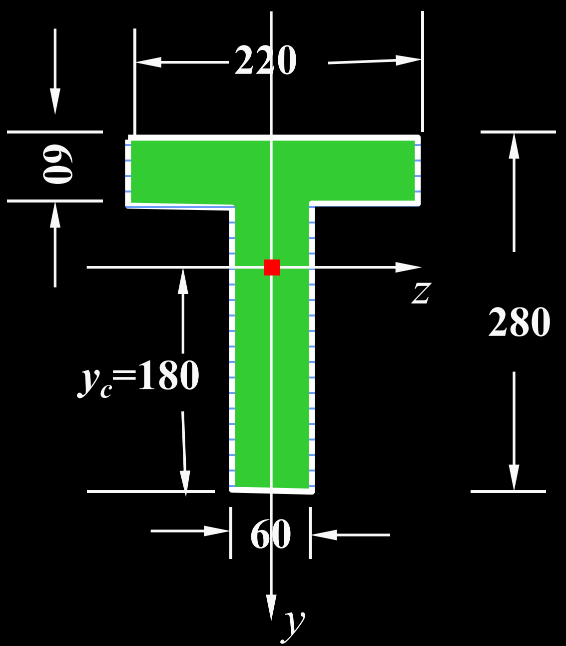
q＝20kN/m
A
D
B
C
4m
2m
30
40
50
Fs (kN)
1.5m
40
1.5m
M(kNm)
22.5



220
60
280
z
yc=180
60
y

(4) 计算最大拉、压应力.





B截面： $\sigma_{t\,\max} = 21.4\ MPa,$ $\sigma_{c\,\max} = 38.6\ MPa$

D截面： $\sigma_{t\,\max} = 21.7\ MPa,$ $\sigma_{c\,\max} = 12.1\ MPa$

$\therefore \sigma_{t\,\max} = 21.7\ MPa$，

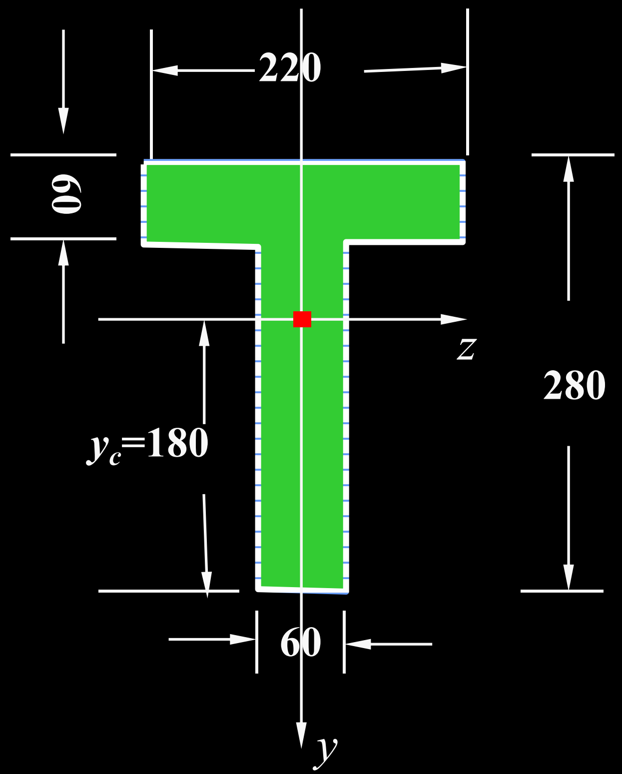
发生在D截面的下边缘各点处。

$\sigma_{c\,\max} = 38.6\ Mpa$，

发生在B截面的下边缘各点处。

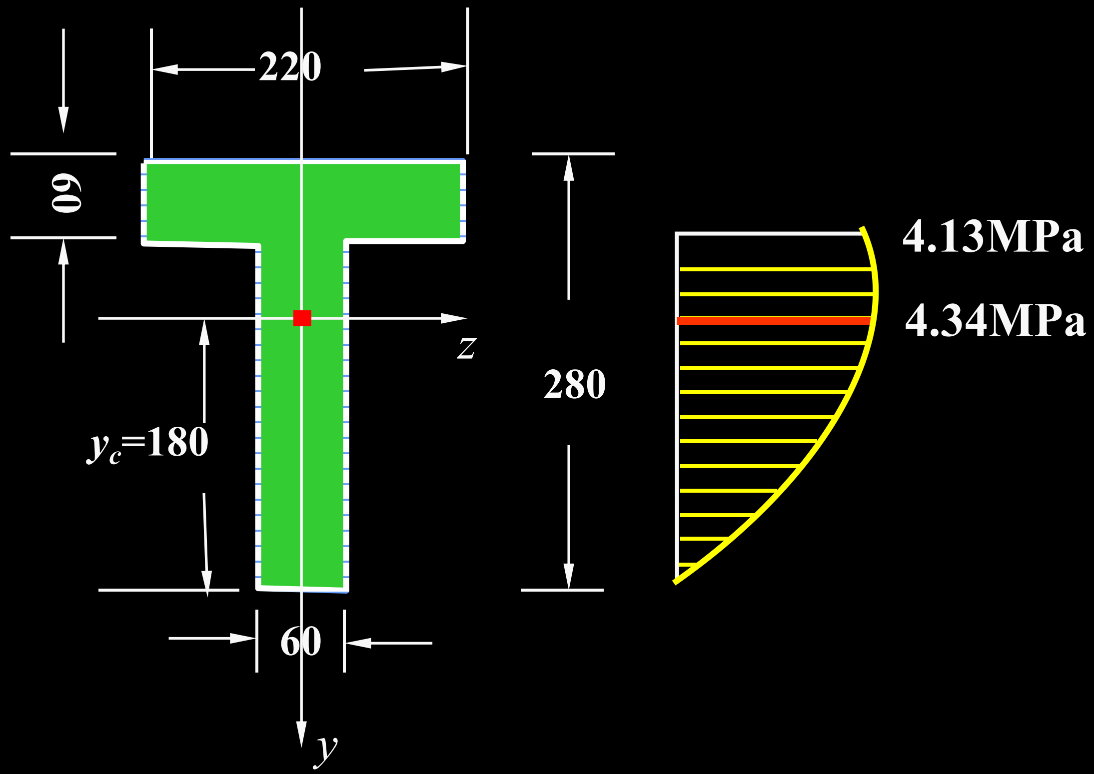
（5）求出剪力最大截面腹板上的切应力分布。

$F_{s\,\max} = 50\ kN$，B截面左侧



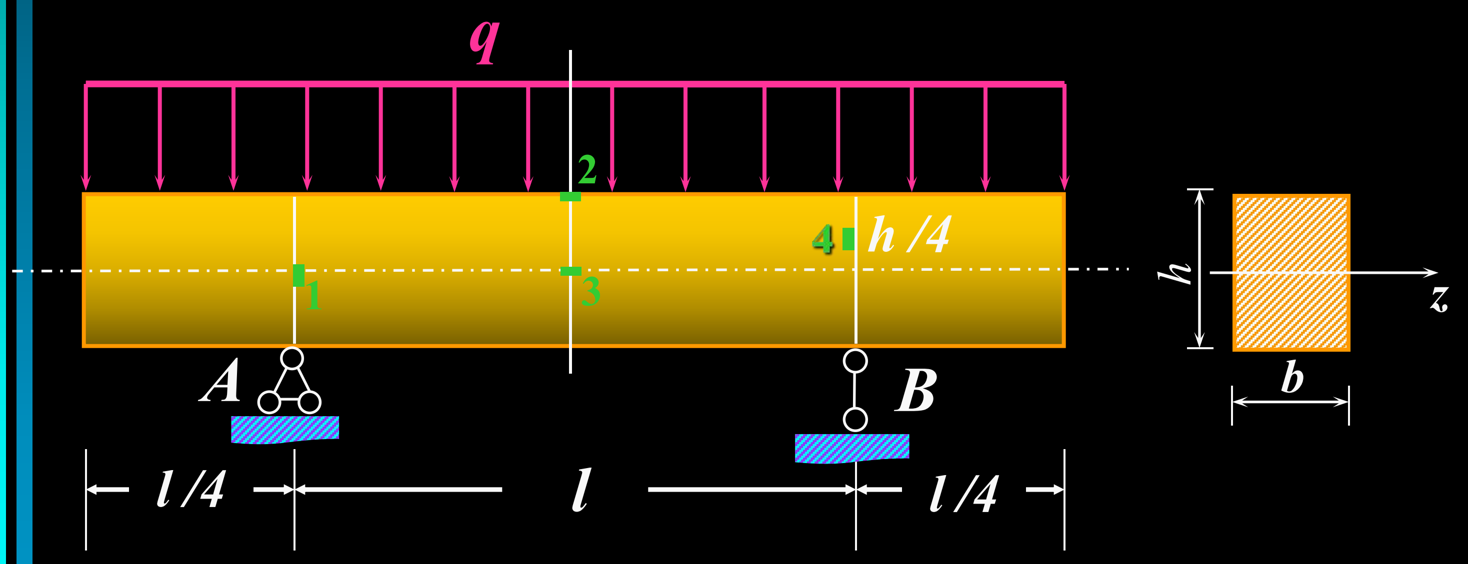
$$\tau_{\max} = \frac{F_{s\max} S^*_{z\max}}{I_Z d} = \frac{50\times10^3\times180\times60\times90\times10^{-9}}{186.6\times10^{-6}\times60\times10^{-3}} = 4.34MPa$$

$$\tau_1 = \frac{F_{s\max} S^*_Z}{I_z d} = \frac{50\times10^3\times220\times60\times70\times10^{-9}}{186.6\times10^{-6}\times60\times10^{-3}} = 4.13MPa$$



220
60
280
z
y_c=180
60
y
4.13MPa
4.34MPa

例3：一矩形截面外伸梁，如图所示。现自梁中1.2.3.4点处分别取四个单元体，试画出单元体上的应力，并写出应力的表达式。

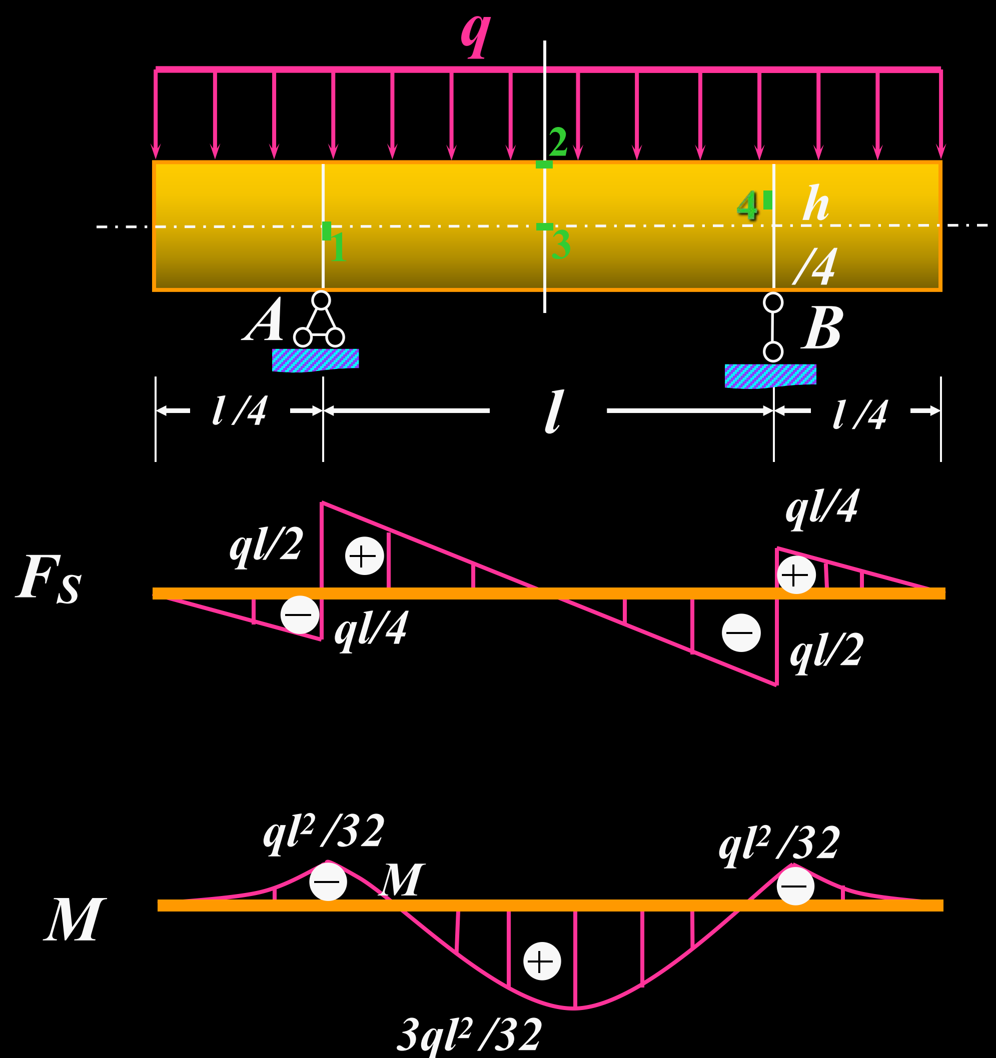


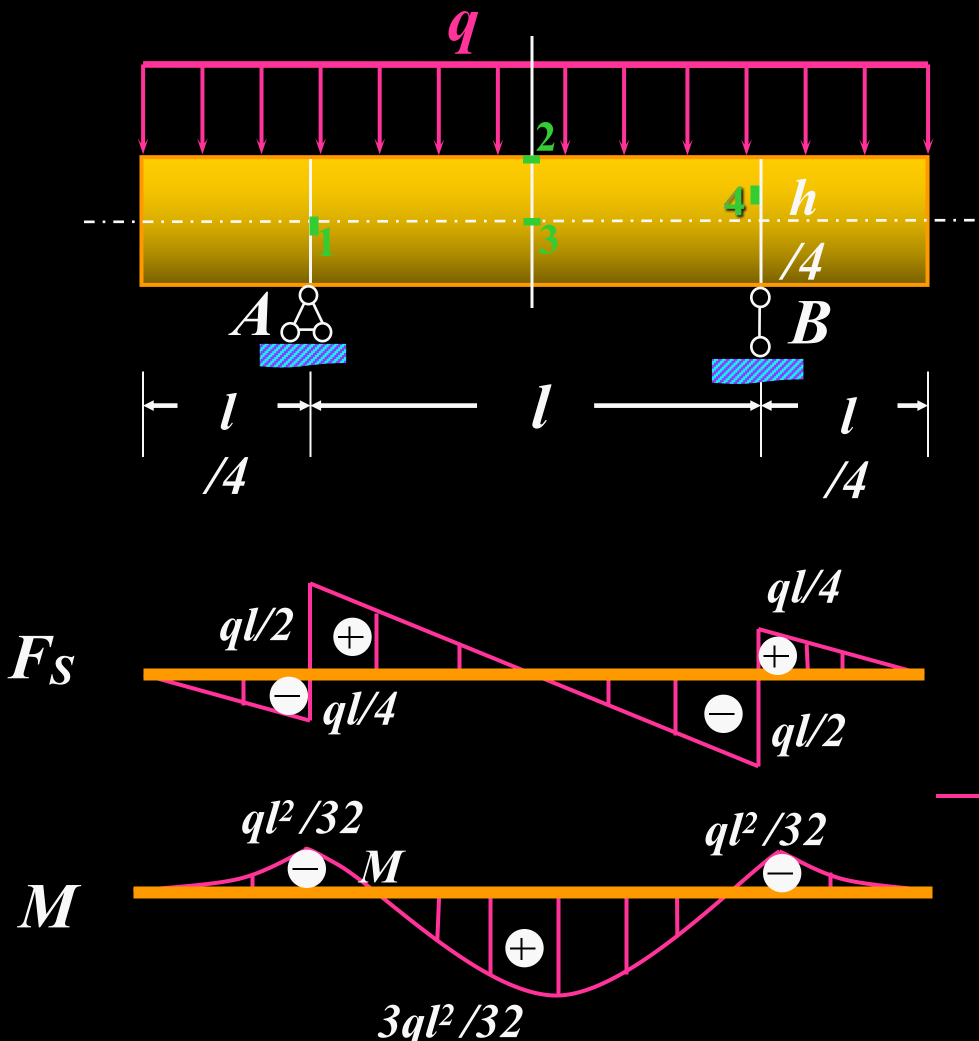
解：(1)求支座反力：

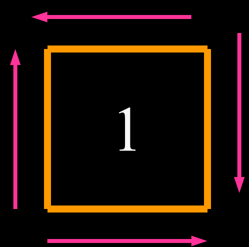
$$R_A = \frac{3}{4}ql$$

$$R_B = \frac{3}{4}ql$$

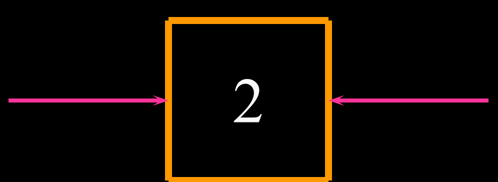
(2)画F_S图和M图



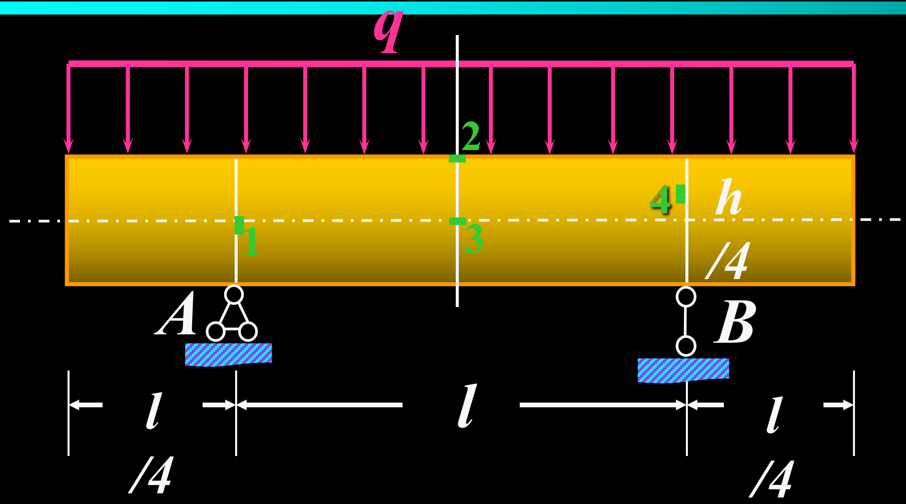


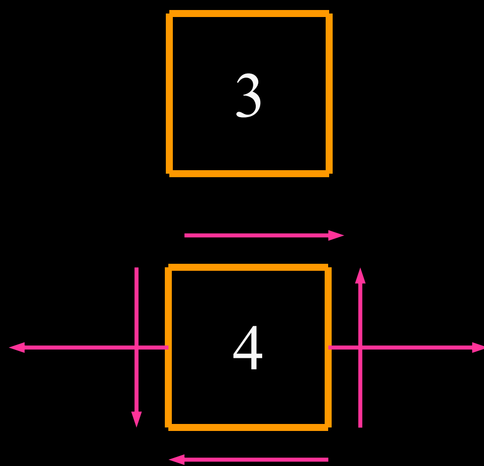


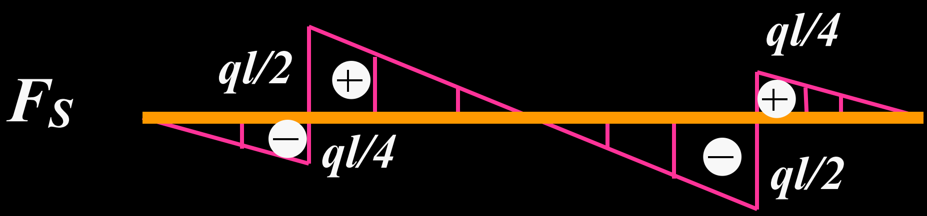
$$\tau = \tau_{\max} = \frac{3}{2}\frac{F_S}{bh} = \frac{3ql}{4bh}$$



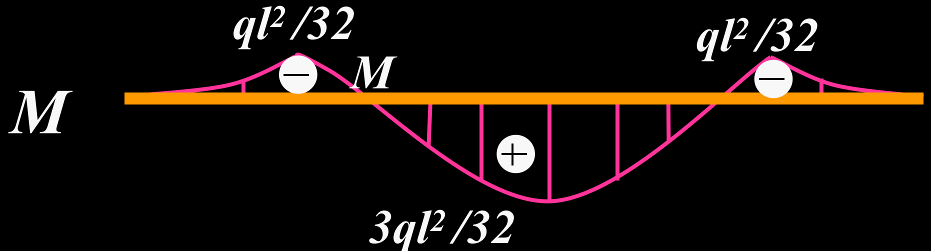
$$\sigma = \sigma_{\max}^{c} = \frac{9ql^2}{16bh^2}$$







$$\sigma = = \frac{My}{I_z} = \frac{3ql^2}{32bh^2}$$



$$\tau = \frac{F_S S_z^*}{I_z b} = \frac{-9ql}{16bh}$$