ECON 425 HW3 Solutions

January 24, 2024

Problem 1

Let $f(x) = x^4 - 6x^2 + 4x + 18$. Use pen and paper to complete (i) and (ii).

(i) Set the initial value $x^{(0)} = 1$ and the learning rate $\alpha = 0.1$. Perform three steps of gradient descent (GD) on f.

(ii) Repeat (i) starting from the point $x^{(0)} = 0$.

Use a computer for the rest of this problem.

(iii) Plot the function f over $x \in [-2.5, 2.5]$. How many local/global minima do you see? What are their approximate values? Can there be other local minima?

(iv) Write your own code for S steps of GD on this function (do not use built-in or third-party GD codes).

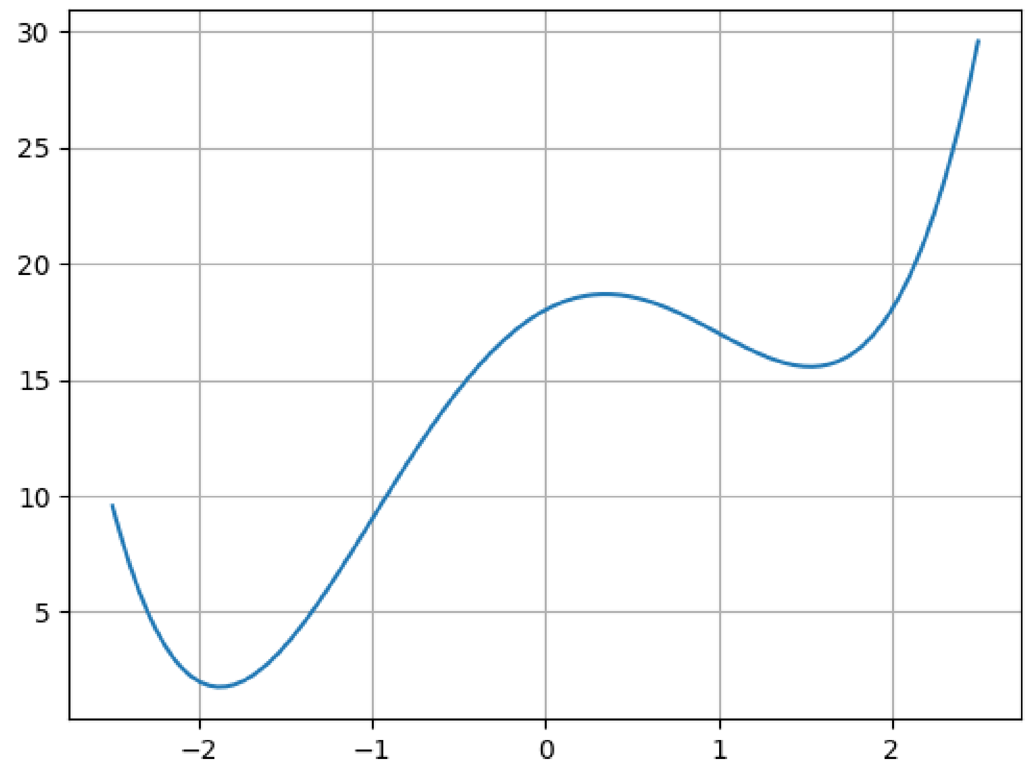
(v) Repeat (i) and (ii) using your code with $S = 20$ steps. Do you observe convergence in both cases?

(vi) Now set the learning rate to $\alpha = 0.01$ and repeat (v). Explain why GD performs differently from (v).

[10]:
```
# SOLUTION TO PROBLEM 1

import numpy as np
import matplotlib.pyplot as plt

S = 20 # number of steps
alpha = 0.01 # learning rate
f = lambda x: x**4 - 6*x**2 + 4*x + 18
df = lambda x: 4*x**3 - 12*x + 4

# plot the function:
x_range = np.linspace(-2.5,2.5,100)
plt.plot(x_range, f(x_range))
plt.grid(True)

# approximate local minima are -1.88 (also global) and 1.53

x = np.empty((S+1,))
# initial point:
```

```python
x[0] = 1
print('>>> Running GD with starting value x =', x[0], 'and learning rate alpha␣
↪=', alpha)

for i in range(1,S+1):
    x[i] = x[i-1] - alpha*df(x[i-1])
    print('Step', i, '  x =', x[i], '  f(x) =', f(x[i]))
```

```
>>> Running GD with starting value x = 1.0 and learning rate alpha = 0.01
Step 1   x = 1.04   f(x) = 16.84025856
Step 2   x = 1.0798054400000001   f(x) = 16.682851897094334
Step 3   x = 1.119020840068549   f(x) = 16.530861492173784
Step 4   x = 1.1572534830439267   f(x) = 16.387152244708915
Step 5   x = 1.1941304973887767   f(x) = 16.254162806484764
Step 6   x = 1.229315454262303   f(x) = 16.13373816834498
Step 7   x = 1.262522837530415   f(x) = 16.027028821639956
Step 8   x = 1.2935289442241535   f(x) = 15.93446856222769
Step 9   x = 1.3221782260494936   f(x) = 15.855828153659868
Step 10   x = 1.3483847002782723   f(x) = 15.790328788009905
Step 11   x = 1.372128707840095   f(x) = 15.736791016005489
Step 12   x = 1.3934498428239785   f(x) = 15.693793045012892
Step 13   x = 1.4124372242060033   f(x) = 15.659816283499957
Step 14   x = 1.4292183920198969   f(x) = 15.633363464226218
Step 15   x = 1.4439480114680723   f(x) = 15.613042922596875
Step 16   x = 1.4567973253950453   f(x) = 15.597619534954735
Step 17   x = 1.4679449872507762   f(x) = 15.586037367258307
Step 18   x = 1.4775696023631308   f(x) = 15.577421161028575
Step 19   x = 1.4858440513255176   f(x) = 15.571063925776986
Step 20   x = 1.4929314867366945   f(x) = 15.566406869204114
```



Problem 2

The College.csv dataset contains admissions data for a sample of 777 universities. We want to predict the number of applications received (‘Apps’) using the other variables in the dataset.

(i) Let the first 600 observations be the training set and the remaining 177 observations be the test set.

(ii) Fit the OLS regression on the training set, and report the test error obtained.

For the rest of the problem, let the penalization parameter λ vary on the 1000-point grid from 0.01 to 60.

(iii) Fit the LASSO regression on the training set, with the penalization parameter chosen by 20-fold cross-validation. Report the test error obtained.

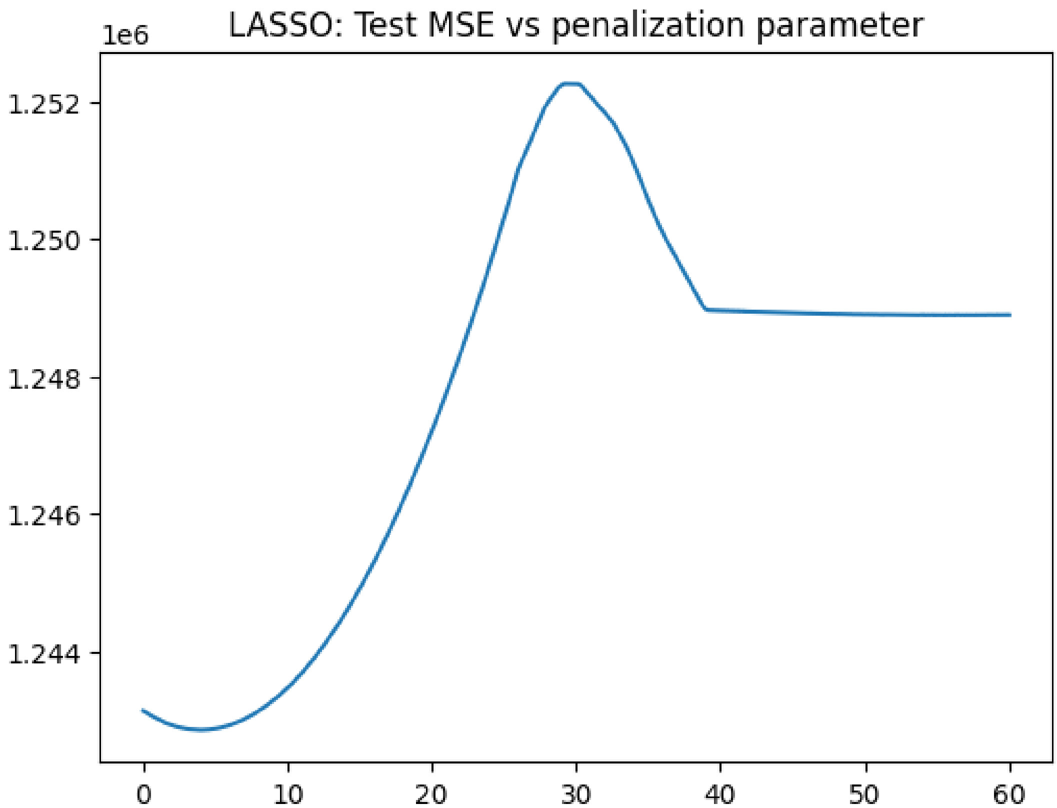
(iv) Fit the ridge regression on the training set, with the penalization parameter chosen by leave-one-out cross-validation. Report the test error obtained.

(v) Which of the three models do you prefer? Is there much difference among the test errors?

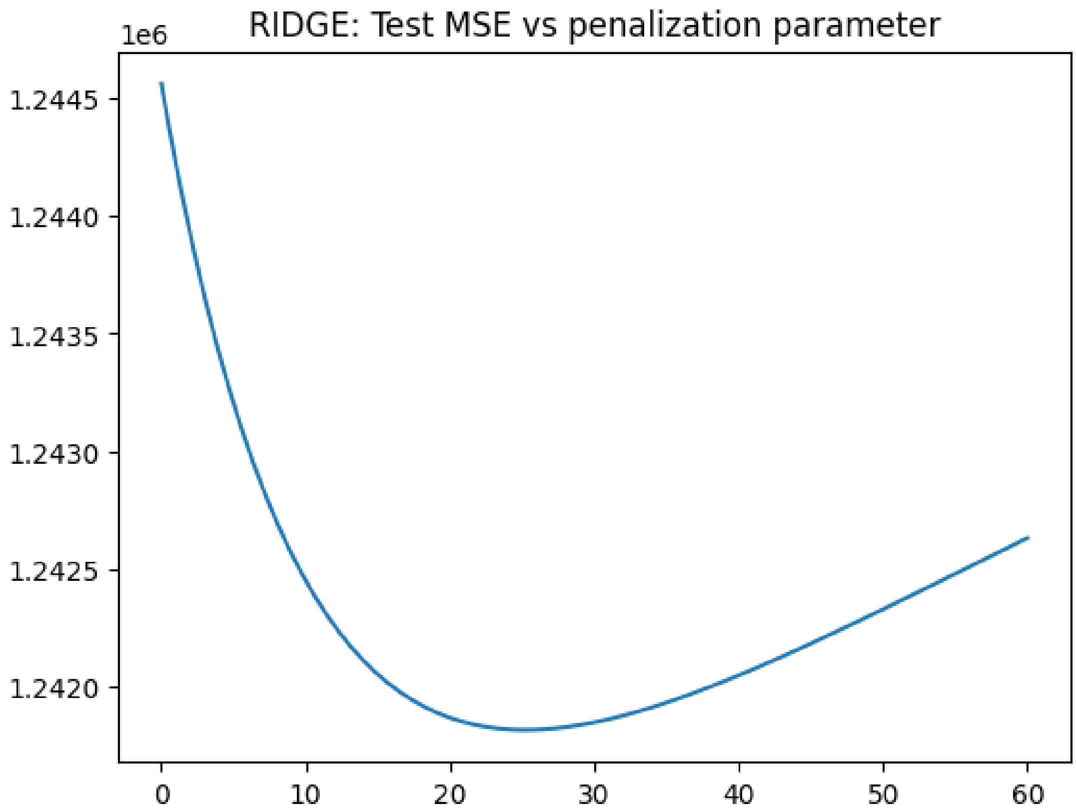
[11]:
```
import matplotlib.pyplot as plt
import pandas as pd
from sklearn.linear_model import LassoCV, RidgeCV, LinearRegression
from sklearn.metrics import mean_squared_error
```

```
datapath = '/Users/franguri/Library/CloudStorage/Dropbox/_TEACHING_/Econ 425 ML␣
 ↪UCLA Winter 2024/WEEK 3 Regularization in linear models/College.csv'
data = pd.read_csv(datapath)

# set data and grid:
Y = data['Apps']
X = data.drop('Apps', axis=1)
X['Private'] = X['Private'] == 'Yes'

Y_train = Y[:600]
X_train = X[:600]
Y_test = Y[600:]
X_test = X[600:]

alphas = np.linspace(0.01,60,1000)

# OLS:
reg = LinearRegression().fit(X_train, Y_train)
print('OLS: test MSE = ', mean_squared_error(Y_test, reg.predict(X_test)))

# LASSO:
reg = LassoCV(cv=20, random_state=0, alphas=alphas).fit(X_train, Y_train)
print('LASSO: test MSE = ', mean_squared_error(Y_test, reg.predict(X_test)))
plt.plot(reg.alphas_, np.mean(reg.mse_path_, axis=1))
plt.title('LASSO: Test MSE vs penalization parameter')
plt.show()

# RIDGE:
reg = RidgeCV(cv=None, alphas=alphas, store_cv_values=True).fit(X_train,␣
 ↪Y_train)
print('RIDGE: test MSE = ', mean_squared_error(Y_test, reg.predict(X_test)))
plt.plot(reg.alphas, np.mean(reg.cv_values_, axis=0))
plt.title('RIDGE: Test MSE vs penalization parameter')
plt.show()
```

```
OLS: test MSE =  1502077.4348215673
LASSO: test MSE =  1502356.4555399297
```




```
RIDGE: test MSE =  1504968.4737527145
```


RIDGE: Test MSE vs penalization parameter
1e6
1.2445
1.2440
1.2435
1.2430
1.2425
1.2420
0
10
20
30
40
50
60