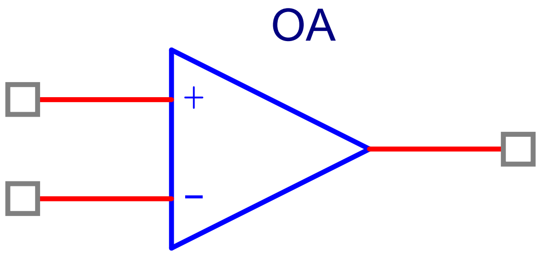
Operational Amplifiers

- Ideal and real voltage feedback amplifier (VFA) - parameter
- Negative feedback with VFA
  - Basic circuits (frequency independent)
  - Error sources – offset voltage, input bias current
  - Stability – Bode plot
  - Noise
  - Basic circuits (frequency dependent)
- Positive feedback
  - Schmidt trigger, oscillator
- Other (non-voltage-feedback) amplifiers

Operational Amplifier (VFA)



An ideal Op-Amp has:
- infinite large gain
- infinite high input impedance – no input current
- zero output impedance
- infinite bandwidth
- output voltage is 0 when the two inputs have the same voltage (shorted)

A real Op-Amp has
- large, but limited gain: 60-80-100-120 dB (10^3-10^6)
- very high input impedance (MΩ and more)
- very low input current (nA, pA)
- low output impedance (Ω)
- bandwidth in Hz to kHz (**without** feedback)
- offset input voltage (mV)

An Op-Amp is used almost always **with** feedback!!!

Operational Amplifier – more parameters

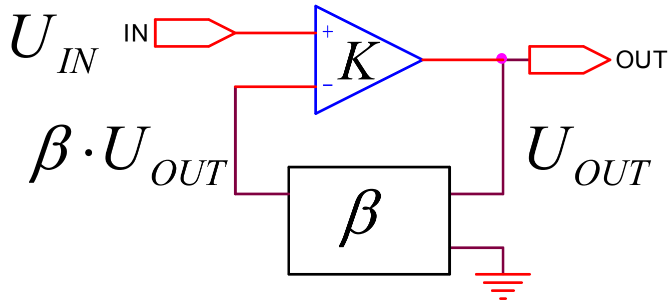
- min/max supply voltage
- min/max output voltage (depending on the supply voltage)
  - Rail to Rail Output (RRO)
- min/max input voltage, so that it
  - still works
    - Rail to Rail Input (RRI)
  - remains healthy
- temperature dependence of the
  - offset voltage
  - input current
- minimal gain with feedback (for non-inverting circuit), so that it remains stable (no oscillations)
  - most have 1 (unity gain stable), some 2, others 5 or 10 – be careful when selecting an OP!

- maximal output current
- maximal capacitive load for stable operation
- bandwidth at full amplitude
- maximal speed at the output (slew rate) in V/μs

Be patient: we will understand some of the parameters after considering the concept of negative feedback

Feedback

- Negative feedback (invented by Harold S. Black in 1927) – feed some part of the output signal back to the input



$$K \cdot \left(U_{IN} - \beta \cdot U_{OUT}\right) = U_{OUT}$$

$$K \cdot U_{IN} = \left(1 + \beta \cdot K\right) \cdot U_{OUT}$$

$$U_{OUT} = U_{IN} \frac{K}{1 + \beta \cdot K} = K_{\beta} \cdot U_{IN}$$

- Why do we need feedback? Obviously the gain will get smaller!
  – There is no known simple way to make an amplifier with well defined stable gain and bandwidth **without** using negative feedback!

Feedback – benefits

- The negative feedback reduces by a factor of

$$1+\beta\cdot K$$

any non-linearity in the transfer function

- Any variation of the gain *K* with time, temperature, production process, power supply etc. will be reduced by a factor of

$$1+\beta\cdot K$$

- The output impedance will be reduced by a factor of

$$1+\beta\cdot K$$

- The bandwidth will be extended by a factor of

$$1+\beta\cdot K$$

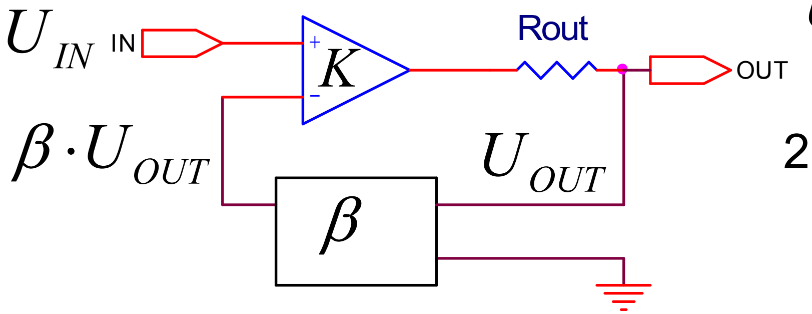
- The time constant of the step response will be reduced by a factor of

$$1+\beta\cdot K$$

… too nice to be true

Feedback - R_{OUT}

- The negative feedback reduces the output impedance:



1) Output open

$$U_{OUT} = U_{IN} \frac{K}{1+\beta \cdot K} = K_{\beta} \cdot U_{IN}$$

2) Output shorted to GND

$$I_{OUT} = U_{IN} \frac{K}{R_{OUT}}$$

3) For the effective output impedance with feedback we get

$$R'_{OUT} = \frac{R_{OUT}}{1+\beta \cdot K}$$

The feedback reduces the output impedance $1+\beta \cdot K$ times! This number is normally huge, but only as long as

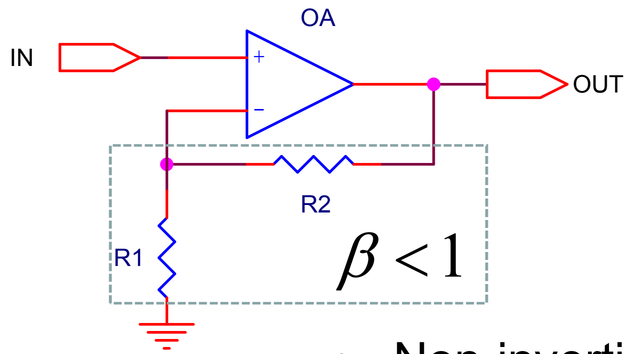
- β is not very low (the resulting gain is **not** very high)
- K is **high**, note that K is frequency dependent!

Non-inverting amplifier

Simplification rules used to analyse a feedback circuit with near-ideal amplifier:

• As the gain is near infinite, a very small voltage difference at the inputs is enough to create any output voltage. As the feedback is negative, the OA will try to minimize the input voltage difference → the input voltage difference is **zero**

• The input currents are very small, the input impedance is very high → any voltage source (or divider) feeding the inputs can be considered to be **unloaded**



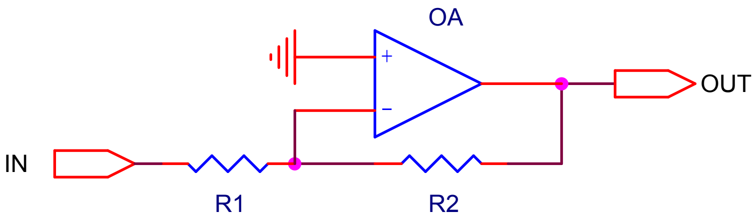
$$\frac{R_1}{R_1 + R_2} U_{OUT} = \beta \cdot U_{OUT} \approx U_{IN}$$

$$K_\beta = \frac{U_{OUT}}{U_{IN}} \approx 1 + \frac{R_2}{R_1} = \frac{1}{\beta} \geq 1$$

- Non-inverting
- Common mode not constant!

- *High* input impedance
- Minimal gain = 1 when R_1 disconnected

Inverting amplifier

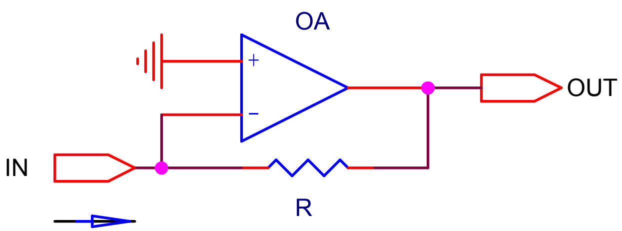


- Inverting
- Input impedance = R_1
- Common mode constant
- |Gain| can be < 1

The voltages at + and – inputs of the OP should be equal, the input currents are zero →

$$\frac{U_{IN}}{R_1} = -\frac{U_{OUT}}{R_2} \qquad K = \frac{U_{OUT}}{U_{IN}} = -\frac{R_2}{R_1}$$

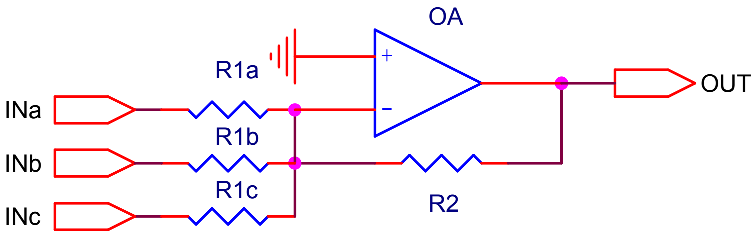
Special case:
current → voltage converter



$$U_{OUT} = -R \cdot I_{IN}$$

The negative input is a **virtual ground**, as long as the OP is active
- outputs not saturated
- signals not too fast
- gain sufficient high

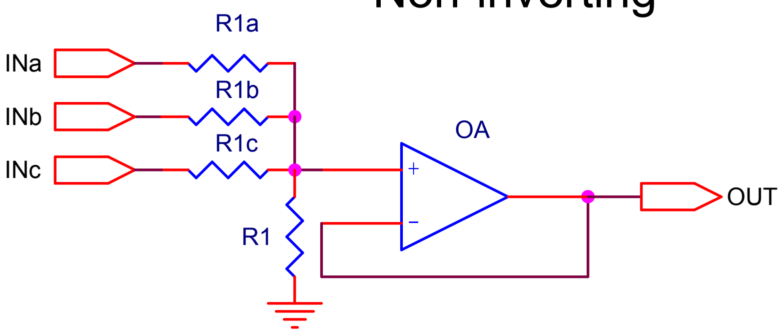
Summing Amplifier



Inverting

$$\frac{U_{OUT}}{R_2} = -\left(\frac{U_{INa}}{R_{1a}} + \frac{U_{INb}}{R_{1b}} + \frac{U_{INc}}{R_{1c}}\right)$$

Non-inverting



For $R_{1a} = R_{1b} = R_{1c} = R_1$

$$U_{OUT} = \left(U_{INa} + U_{INb} + U_{INc}\right)/4$$

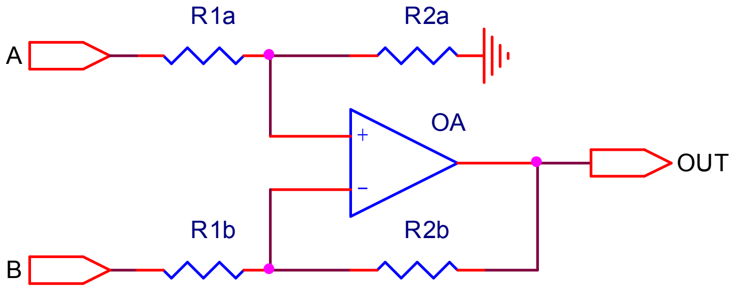
For $R_{1a} = R_{1b} = R_{1c},\ R_1 = \infty$

$$U_{OUT} = \left(U_{INa} + U_{INb} + U_{INc}\right)/3$$

Pros and cons!

Instrumentational Amplifier(1)

(or differential amplifier)



Using the superposition principle, lets set first U_A=0:

$$U_{OUT}(U_A=0)=-U_B\frac{R_{2b}}{R_{1b}}$$ then U_B=0:

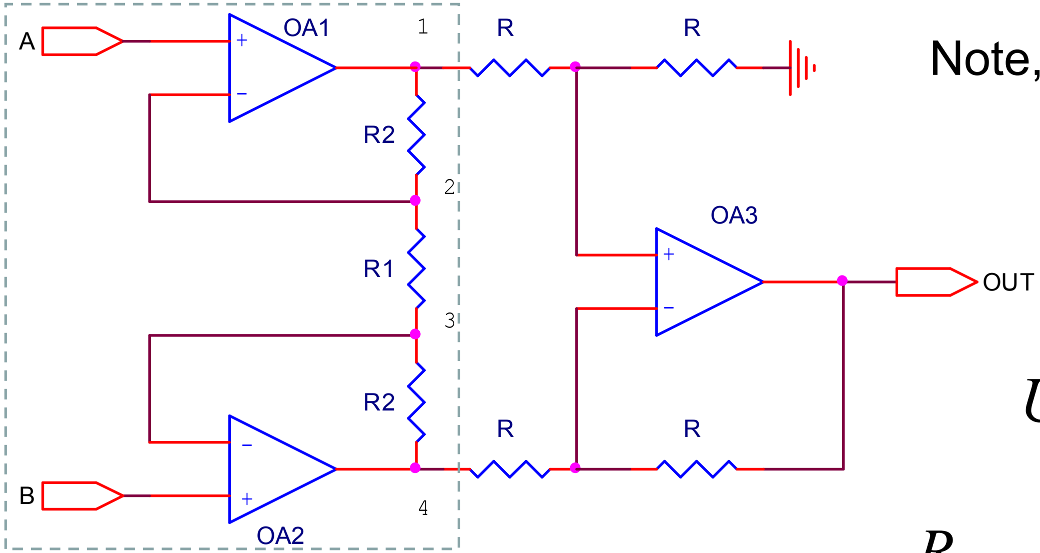
$$U_{OUT}(U_B=0)=U_A\frac{R_{2a}}{R_{1a}+R_{2a}}\cdot\left(1+\frac{R_{2b}}{R_{1b}}\right)=U_A\cdot\frac{R_{2a}}{R_{1a}}\cdot\frac{1+R_{2b}/R_{1b}}{1+R_{2a}/R_{1a}}$$

Now add both contributions together:

$$\text{if}\quad \frac{R_{2a}}{R_{1a}}=\frac{R_{2b}}{R_{1b}}=k\quad\text{then}\quad U_{OUT}=k\cdot U_A-k\cdot U_B=k(U_A-U_B)$$

Attention! The symmetry of the resistors is very important to suppress the amplification of the common mode signals! The input impedance of this circuit is not high and if using identical resistors, not equal on both inputs!

Instrumentational Amplifier(2)



Note, that because of the feedback:

$$U_2 = U_A, \; U_3 = U_B$$

Using the superposition principle, lets set first U_B=0:

$$U_1(U_B = 0) = U_A \cdot \left(1 + \frac{R_2}{R_1}\right)$$

Now lets set U_A=0: $U_1(U_A = 0) = -U_B \dfrac{R_2}{R_1}$

like non-inverting amplifier

like inverting amplifier

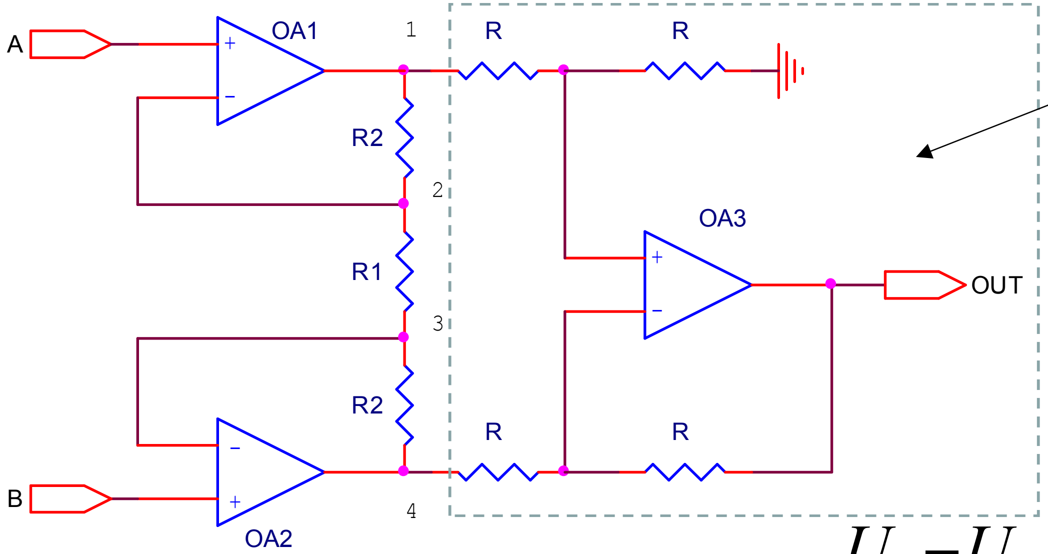
Add both contributions together:

$$U_1 = U_A \cdot \left(1 + \frac{R_2}{R_1}\right) - U_B \frac{R_2}{R_1} = U_A + (U_A - U_B)\frac{R_2}{R_1}$$

Similarly for U_4:

$$U_4 = U_B \cdot \left(1 + \frac{R_2}{R_1}\right) - U_A \frac{R_2}{R_1} = U_B + (U_B - U_A)\frac{R_2}{R_1}$$

Instrumentational Amplifier(3)



Already known simple version of the IA.
The inputs 1 and 4 get a common mode and a differential component:

$$U_1 + U_2 = U_A + U_B$$

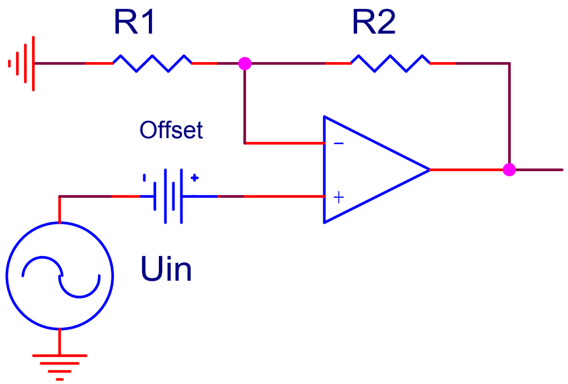
$$U_1 - U_2 = (U_A - U_B) \cdot \left(1 + 2 \cdot \frac{R_2}{R_1}\right)$$

So, the first stage of this IA consisting of OA1 and OA2 brings us high input impedance, high gain for differential and unity gain for common mode signals. If well balanced, the second stage around OA3 has unity gain for differential and near zero gain for common mode signals. Combined together, the 3-Op Ampl IA has very good properties!
Normally the commercially available IAs have pins for external resistor R1 to set the gain.

Non-ideal Op Amps – limitations and workarounds

- Offset voltage
- Input bias current
- Limited output current
- Limited input/output voltage range
- Limited bandwidth
  - Stability
- Limited slew rate
- Final time to settle
- Noise

Offset Voltage



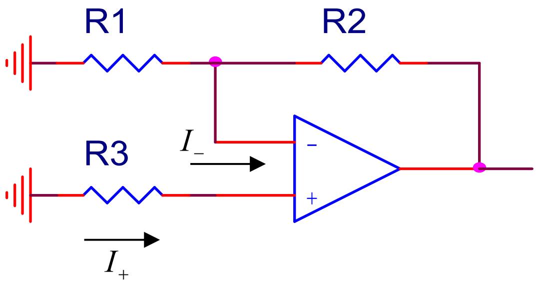
At U_{IN}=0

$$U_{OUT} = U_{OFFSET}\left(1+\frac{R_2}{R_1}\right)$$

$$U_{OUT} = \underbrace{U_{OFFSET}\left(1+\frac{R_2}{R_1}\right)}_{\text{Offset appears amplified at the output!}} + U_{IN}\left(1+\frac{R_2}{R_1}\right)$$

- The offset voltage is temperature and time dependent!
- When large gain necessary – select Op Amp with low and stable offset voltage!
    - chopper or auto zero stabilized OPs (slow)
- AC coupling
- Compensation according to the datasheet

Input (bias) current



$$U_{OUT} = \left(I_- \cdot R_1 \parallel R_2 - I_+ \cdot R_3\right) \cdot \left(1 + \frac{R_2}{R_1}\right)$$

The offset-current (the difference) $I_{offset} = I_+ - I_-$ is normally in BJT Op Amps smaller than each of the input currents.

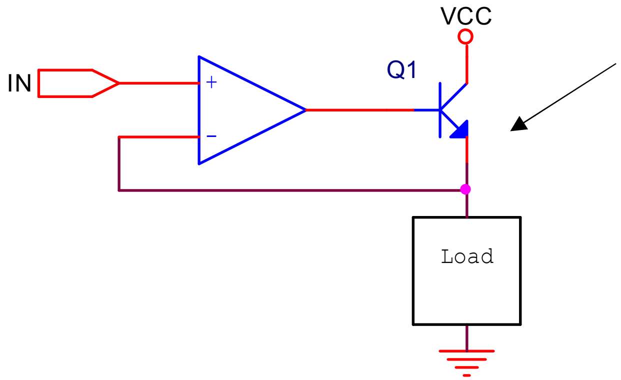
Only in this case : In order to minimize the described effect of the input currents on the output select $R_1 \parallel R_2 = R_3$

Note that R_3 will add its thermal noise! In all other Op Amps use R3=0!

One can use lower values of the resistors, but not too low, as:

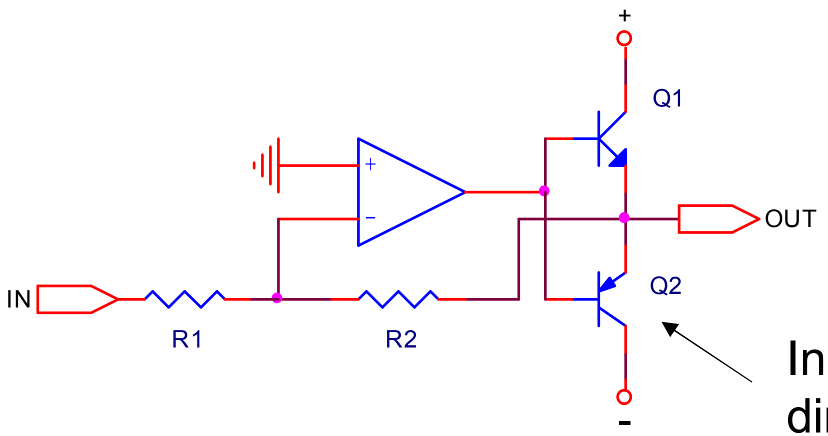
- the output will be loaded
- the current consumptions will rise
- the OP will be warmer – this brings temperature effects

Limited output current



Very simple circuit, good if only sourcing is enough

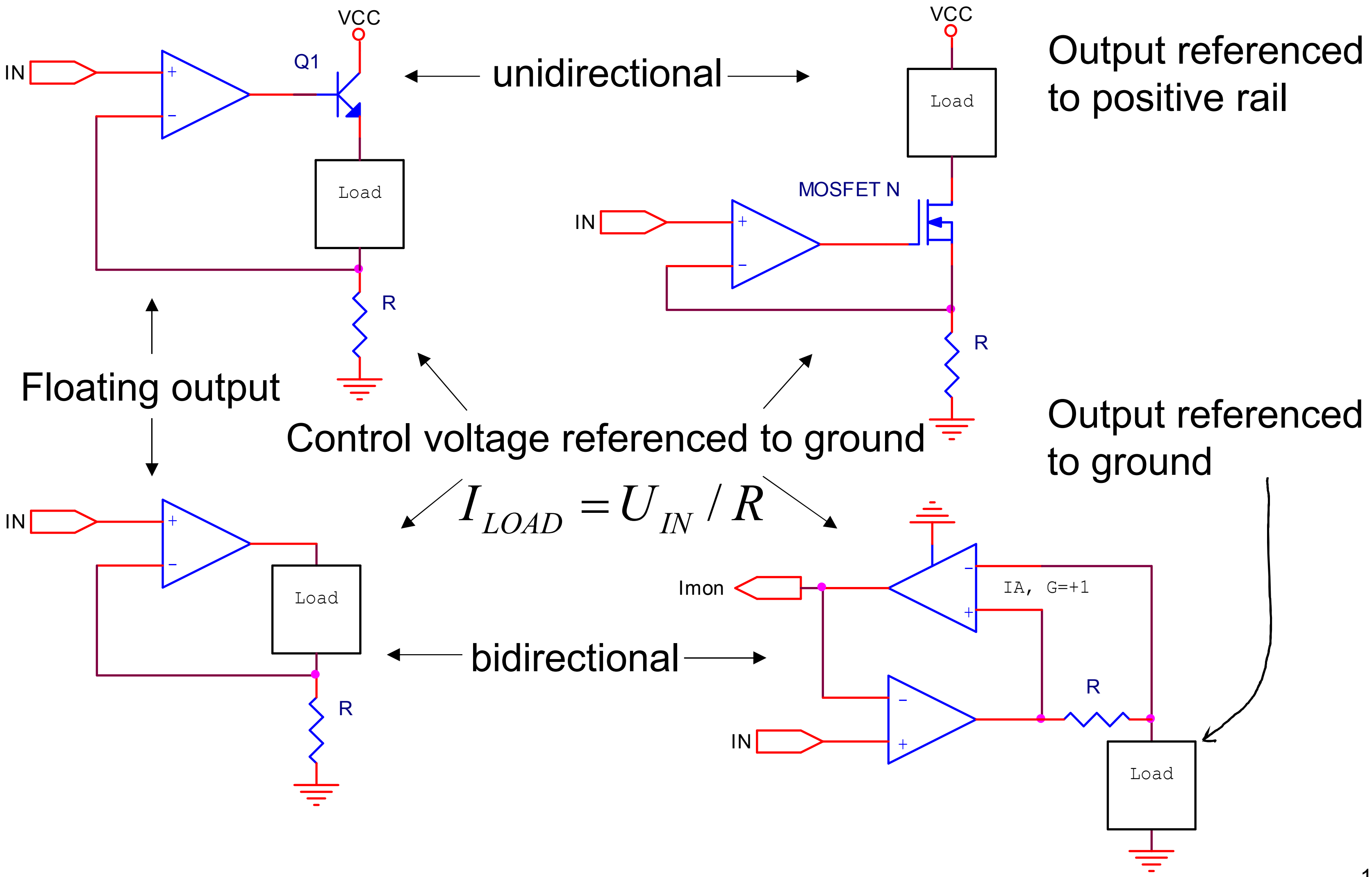
The negative feedback includes the additional transistor stage!
The Op Amp with its large gain compensates through the feedback any non-linearity (provided remains active and not saturated)



In order to drive the output in both directions (push-pull), two transistors are necessary

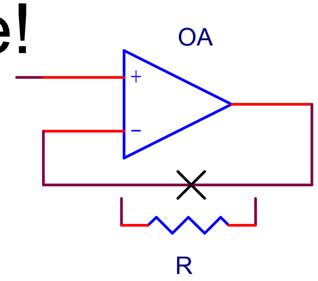
What happens when the output crosses the zero?

Voltage controlled current sources



Limited input/output voltage range

- Common mode input range
  - Rail to rail input (RRI)
  - Input to V- rail only (important for single supply)
  - all this nice features don't come for free!



- Maximal differential input voltage
- Output voltage range
  - Rail to rail output (RRO) – be careful with this specification
    - is always less than supply rails and depends on load
    - the output impedance rises near rails

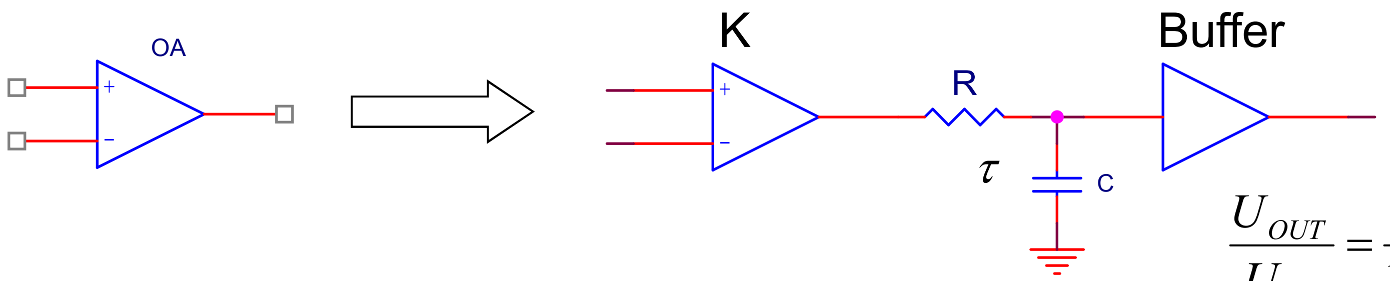
Limited bandwidth (1)

- We will consider the so called fully compensated operational amplifier
- Their gain frequency plot contains one dominant pole (or frequency), so that the next pole appears at gain < 1
- Remember the general expression for the gain with feedback:

$$U_{OUT} = U_{IN} \frac{K}{1+\beta \cdot K} = K_{\beta} \cdot U_{IN}$$

- If K is negative, the feedback becomes positive and if $|\beta \cdot K| > 1$, the circuit will oscillate!

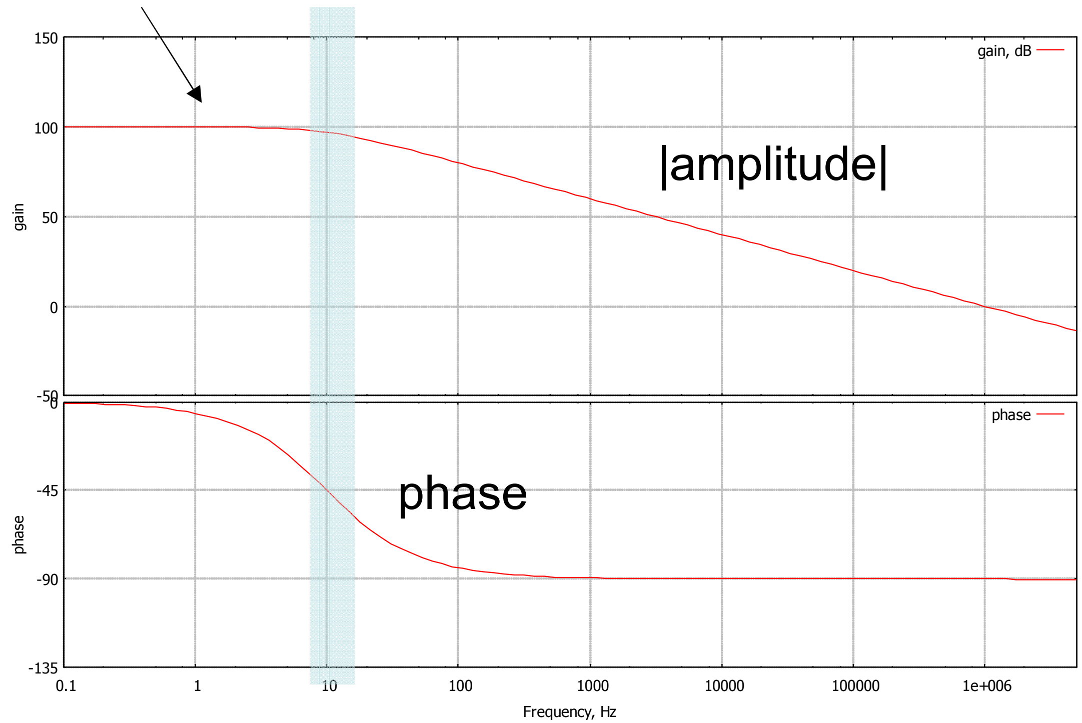
Limited bandwidth (2)



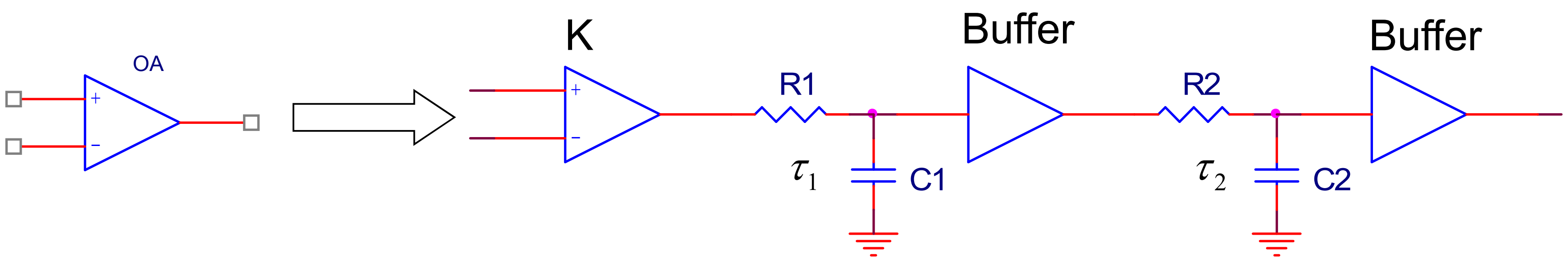
$$\frac{U_{OUT}}{U_{IN}} = \frac{K_0}{1 + j\omega\tau}$$

Bode plot (Hendrik Bode in 1935)

- In reality there are more equivalent low pass filters (poles) in a OP and each of them rotates the phase at some higher frequency further!
- Phase margin is the distance of the phase to -180^0 at $|\beta \cdot K| = 1$. Larger phase margin → better stability and less oscillations

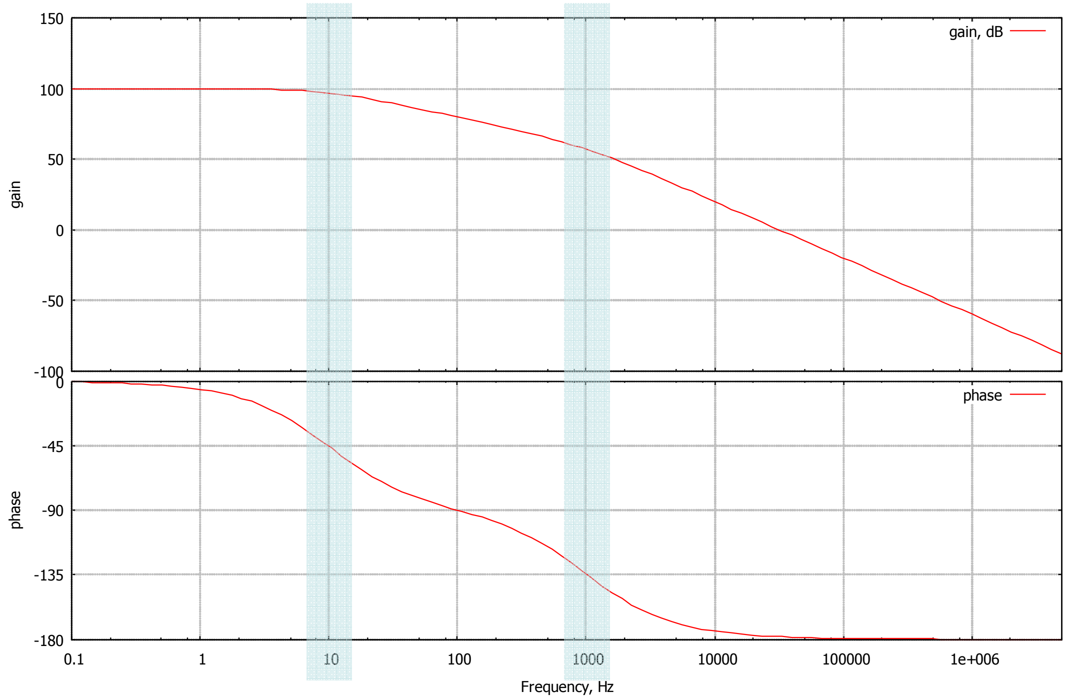


Limited bandwidth (3)



$$\frac{U_{OUT}}{U_{IN}} = \frac{K_0}{(1 + j\omega\tau_1)\cdot(1 + j\omega\tau_2)}$$

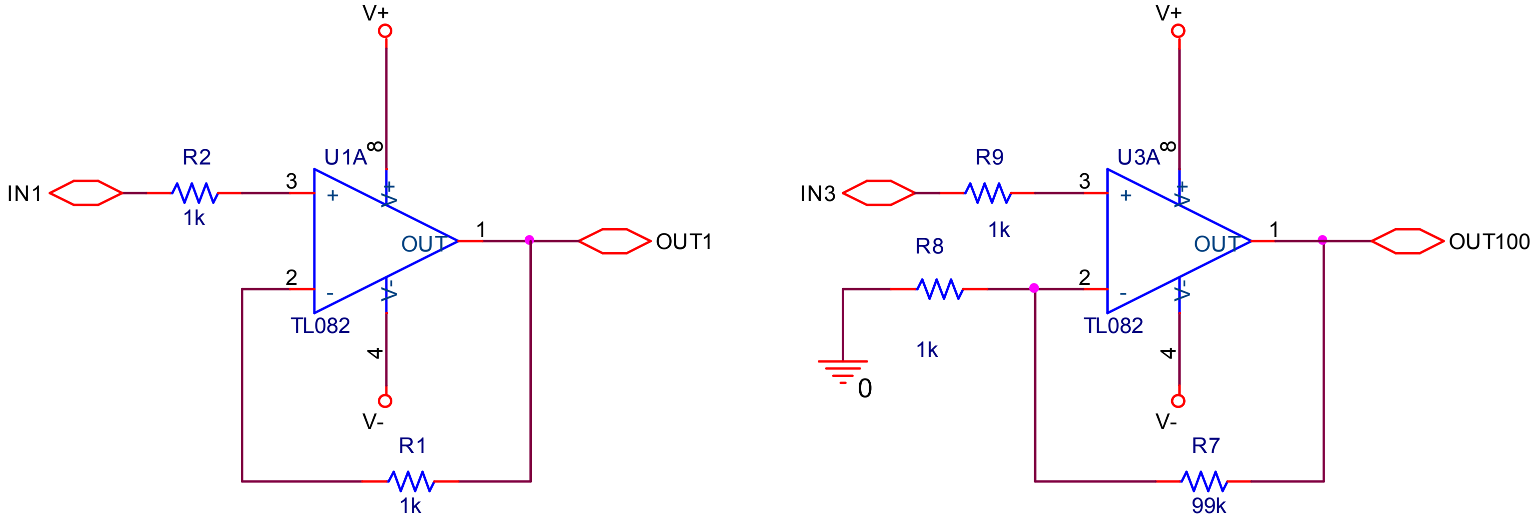
- With two poles, the phase shift can be dangerous large and with deep feedback (G=+1) the amplifier will tend to oscillate
- This is the case when the Op Amp is **not** fully compensated!
- Pay attention to the parameter **min gain** in the datasheet (with its **sign**)!



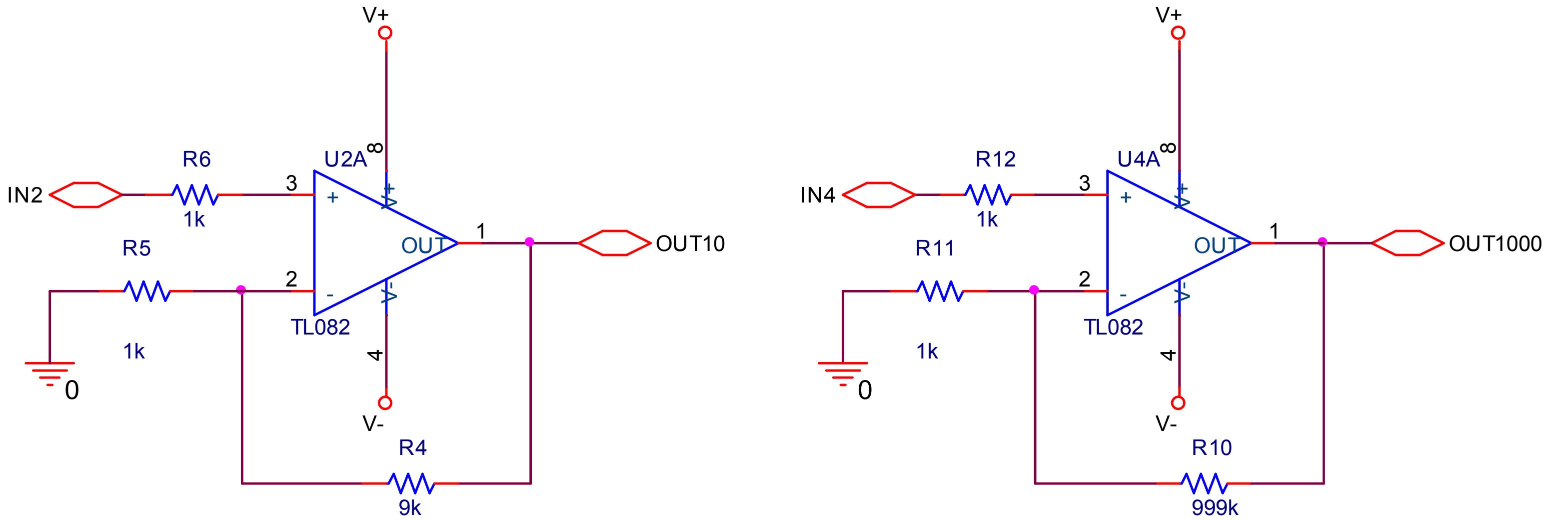
Stability with feedback

- At frequencies before the first pole, the transfer function is constant (horizontal)
- At the first pole the output amplitude is reduced by 3dB, the phase is shifted by 45^0 and becomes -45^0
- After the first pole, the transfer function has a slope of −20db/decade, the phase asymptotically approaches -90^0 (phase margin 90^0)
- At the second pole the phase shift is about -135^0(phase margin 45^0), at higher frequencies approaches -180^0
- After the second pole the slope of the transfer function is -40dB/decade

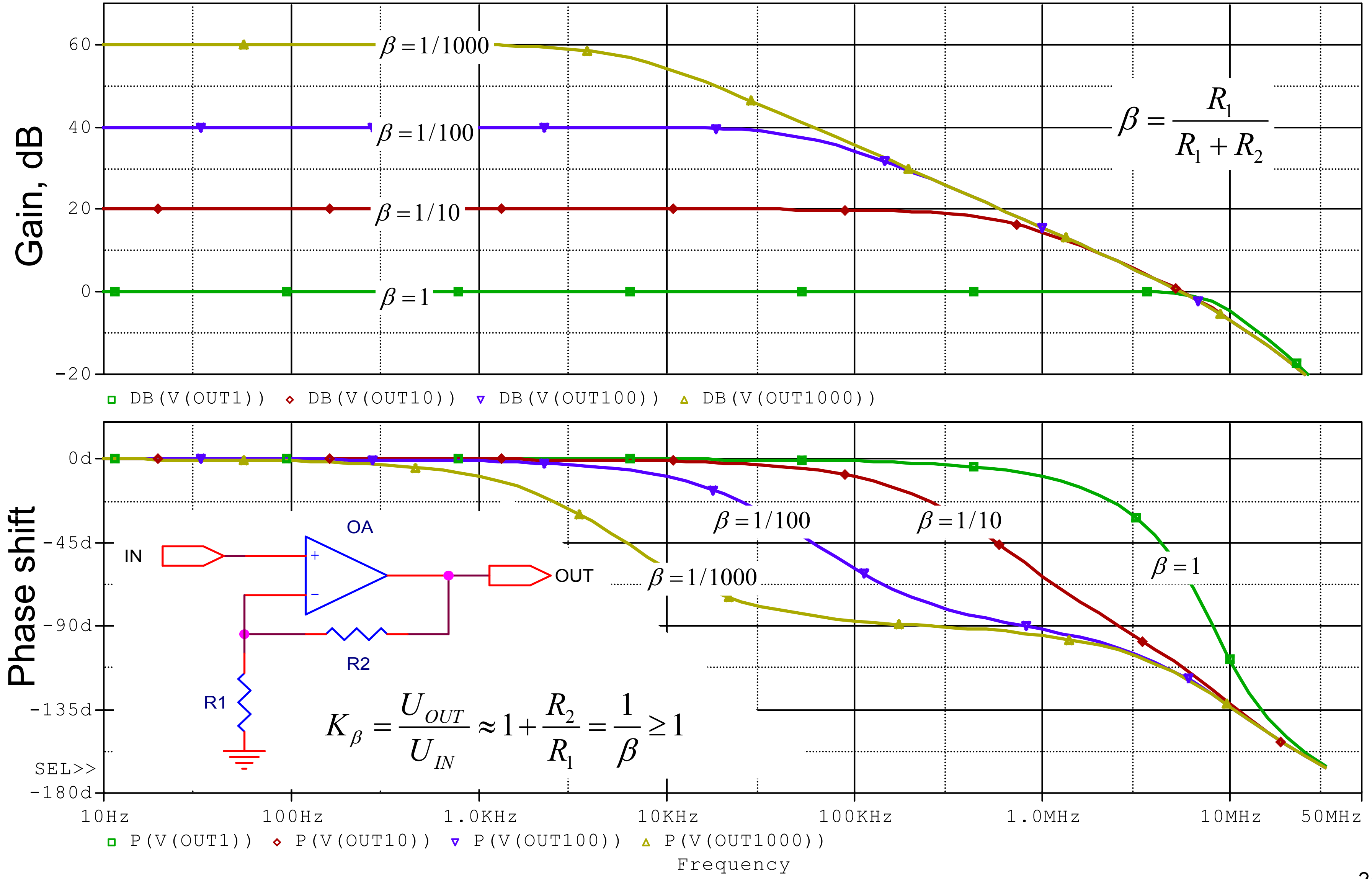
Example: bandwidth and feedback(1)



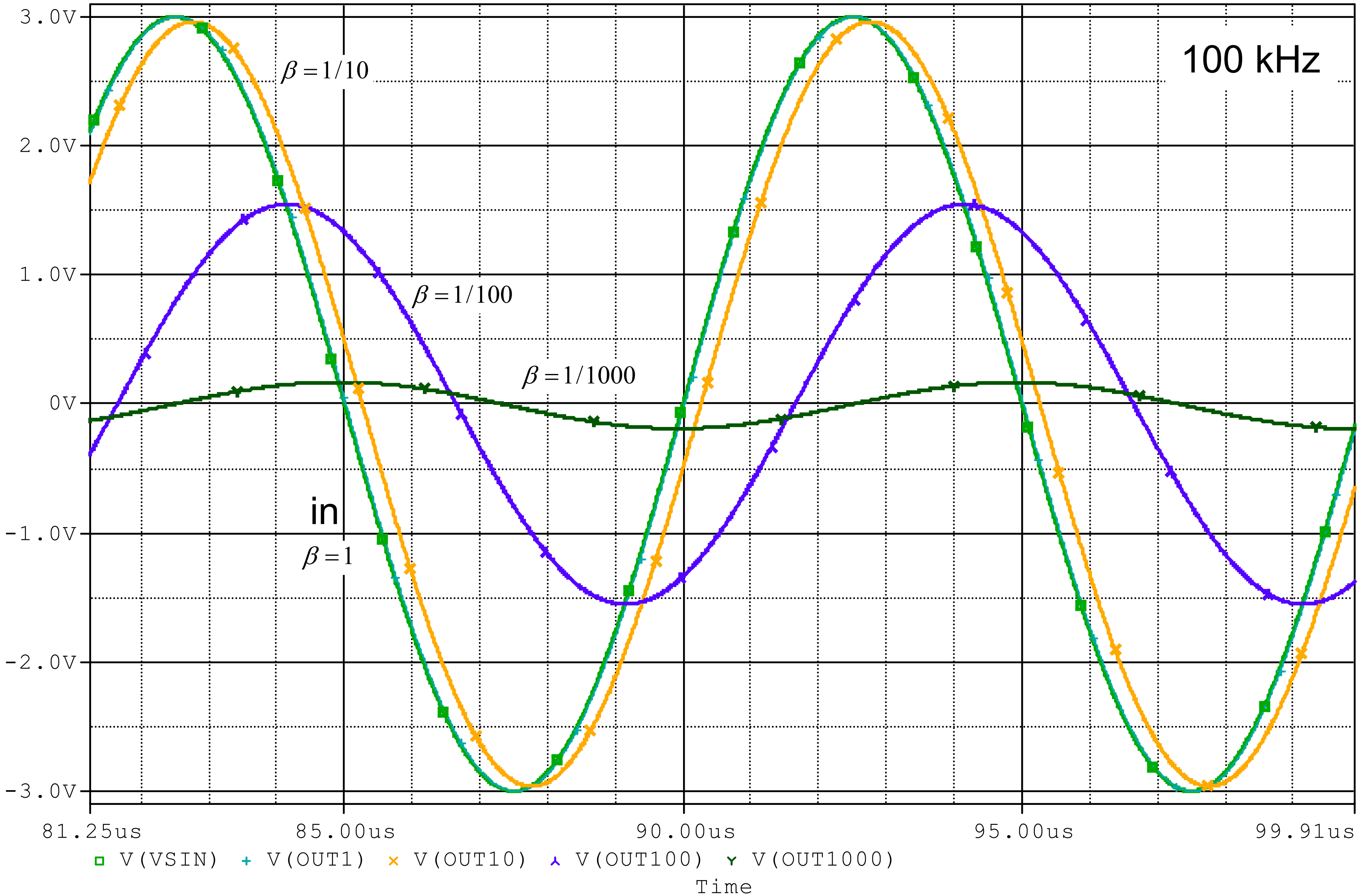
IN1=IN2=IN3=IN4 in AC and IN1=10*IN2=100*IN3=1000*IN4 in transient analysis



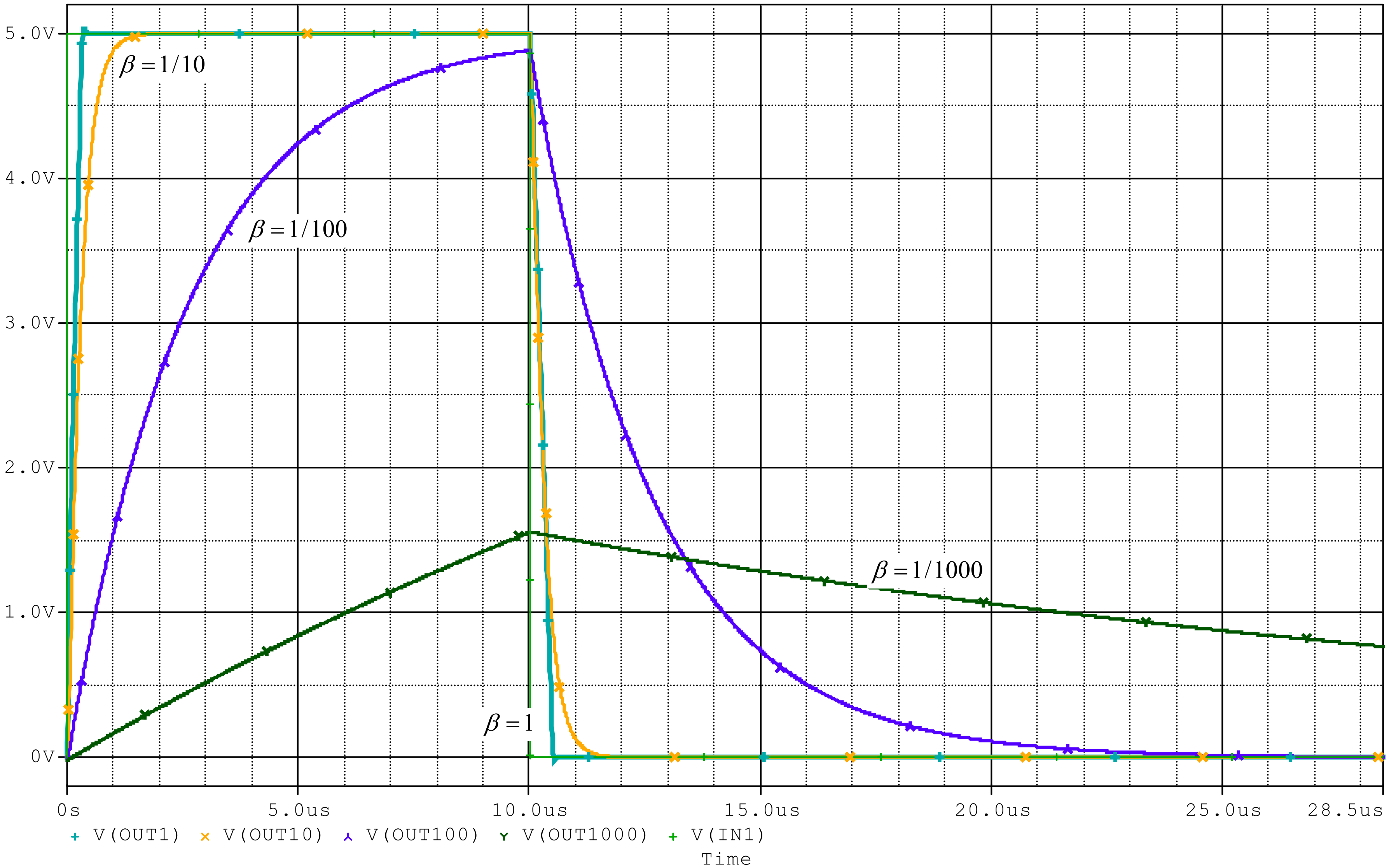
Example: bandwidth and feedback(2)



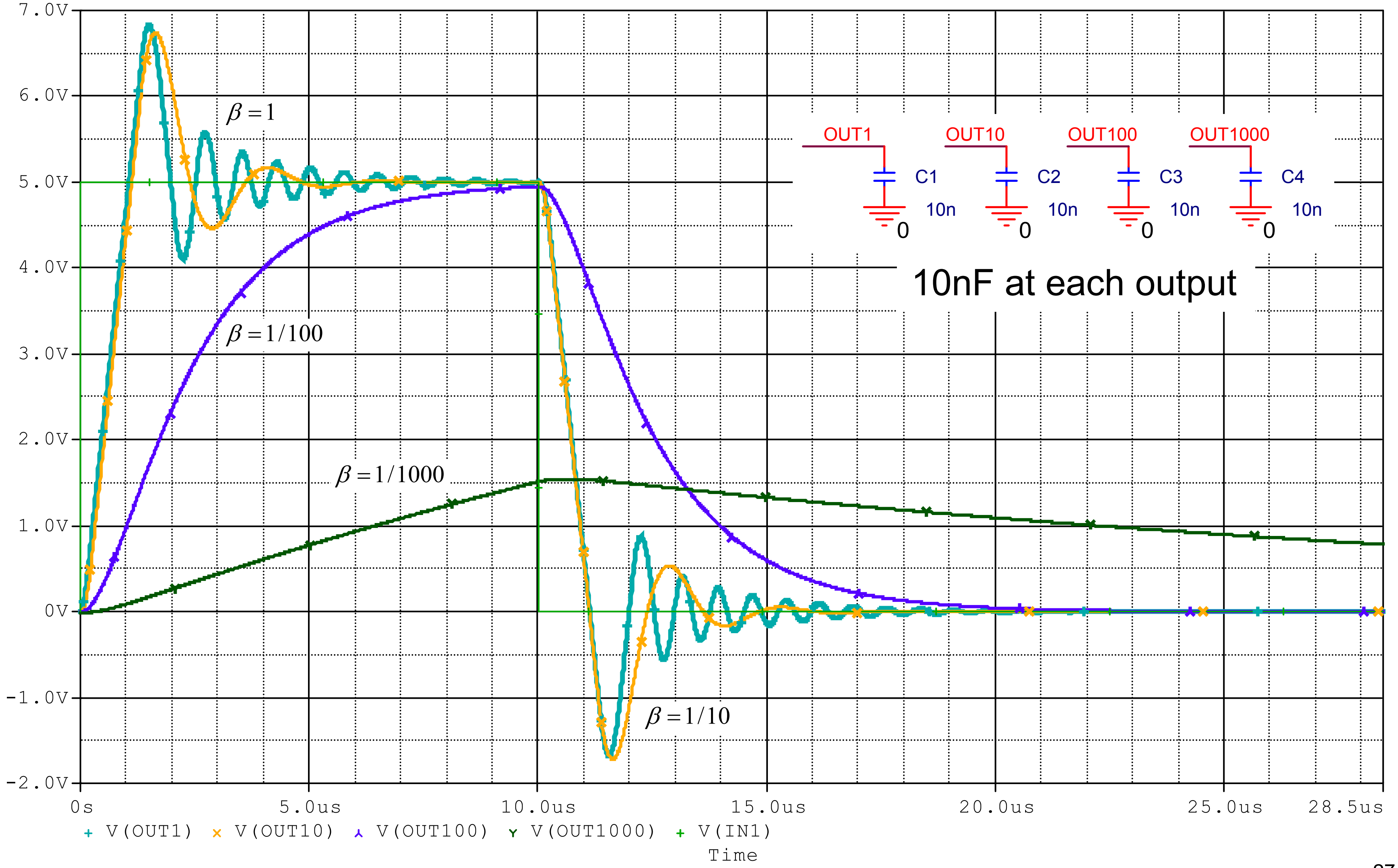
Example: bandwidth and feedback(3)



Example: bandwidth and feedback(4)



Example: bandwidth and feedback(5)

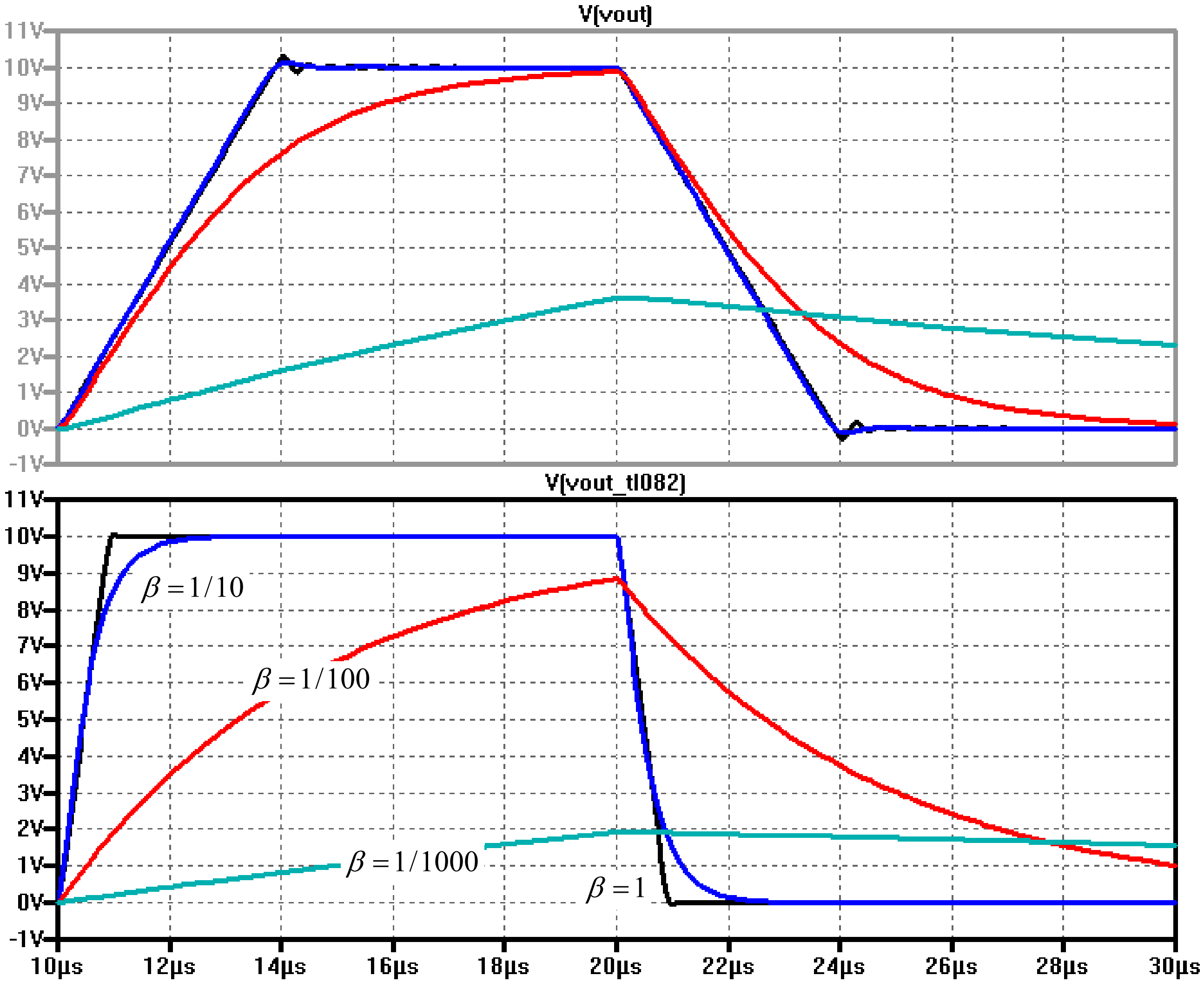


Limited slew rate (1)

- Slew rate is the max "speed" of the output voltage, measured in $V/\mu s$
- The bandwidth considered so far was "small signal bandwidth", for larger signals the slew rate limits additionally the bandwidth (depending on the signal amplitude)

$$\text{For } V = A \cdot \sin(2\pi \cdot f \cdot t), \text{ Slew rate max} = \left|\frac{dV}{dt}\right|_{MAX} = 2\pi \cdot f \cdot A$$

Limited slew rate (2)

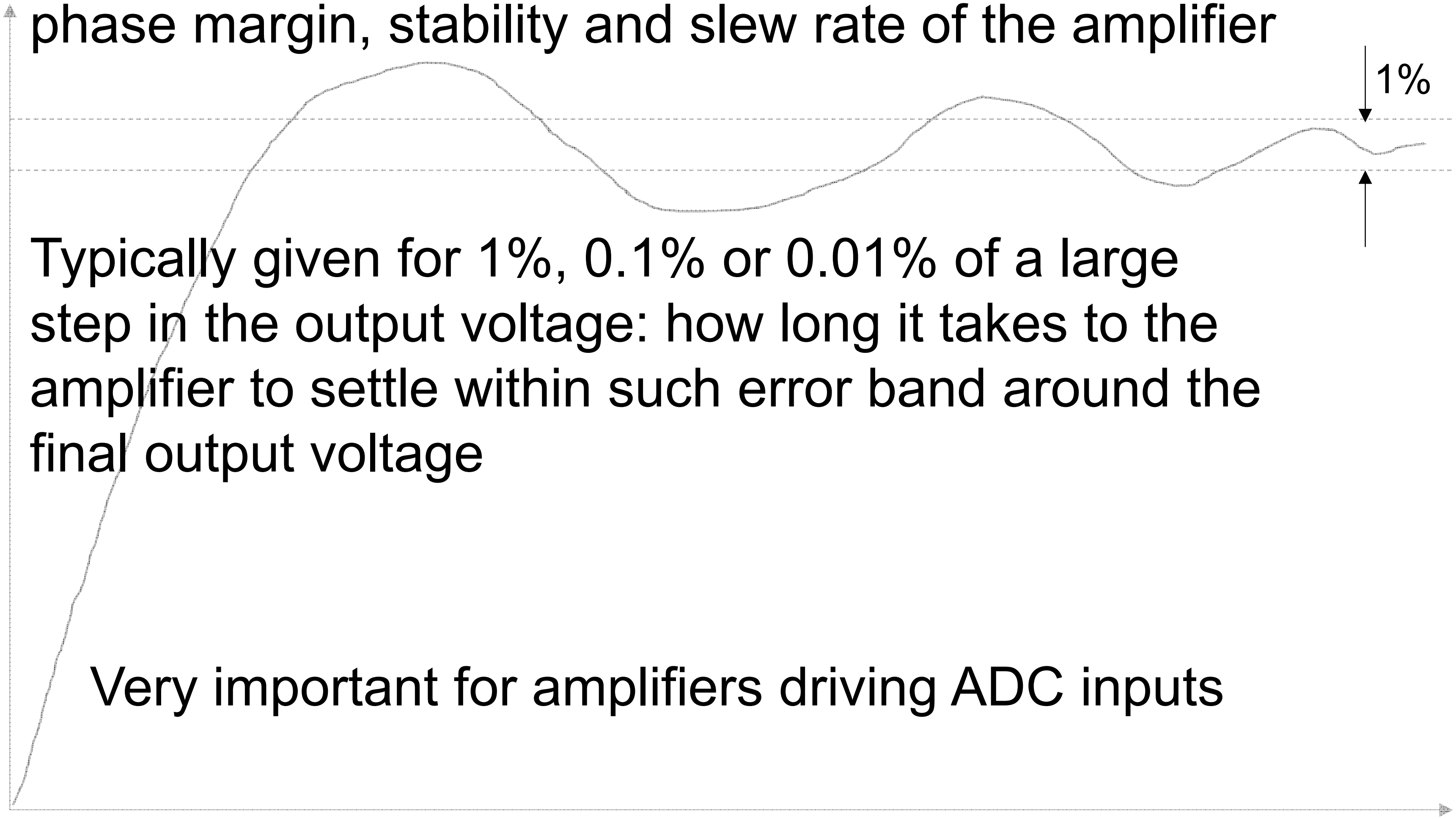


OPA227:
8 MHz, 2.3V/µs

TL082:
3 MHz, 13V/µs

Final time to settle (step response)

- This parameter actually depends on the bandwidth, phase margin, stability and slew rate of the amplifier



- Typically given for 1%, 0.1% or 0.01% of a large step in the output voltage: how long it takes to the amplifier to settle within such error band around the final output voltage

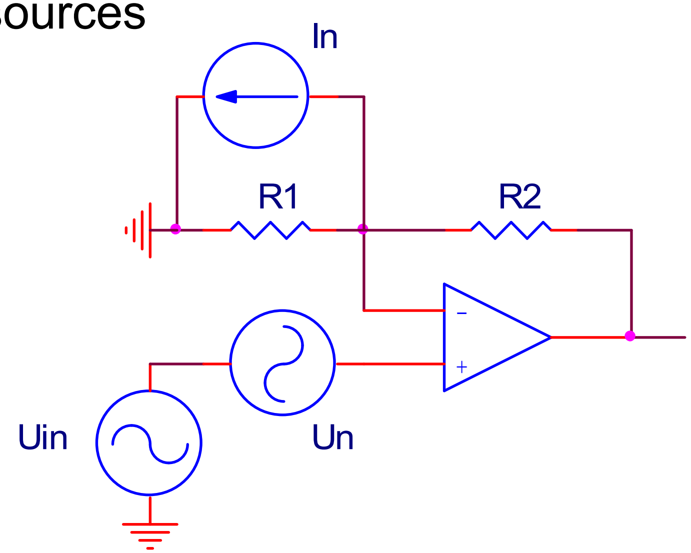
- Very important for amplifiers driving ADC inputs

Noise

- Noise origin
  - Outside the Op Amp circuit
    - 50 Hz (cable)
    - strong signals coupled capacitively or inductively
    - digital currents (bad layout, bad separation)
  - Components around the Op Amp
    - resistors (especially feedback resistor, keep them smaller)
    - supply voltages (voltage regulators, decoupling)
    - voltage or current references
  - **Op Amp itself**
    - Equivalent voltage source between the two inputs
    - Equivalent current sources at each input to GND

Noise - non-inverting amplifier (1)

Equivalent schematic of non-inverting amplifier with voltage and current noise sources



- Current noise, each input to GND

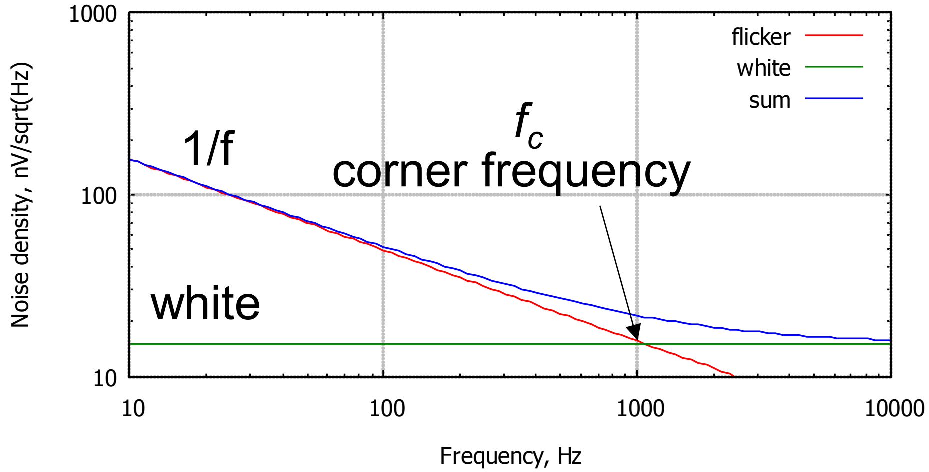
$$I_N(f), \text{in } pA/\sqrt{Hz}$$

- Voltage noise, serial to the inputs

$$U_N(f), \text{in } nV/\sqrt{Hz}$$

- Corner frequency, Hz

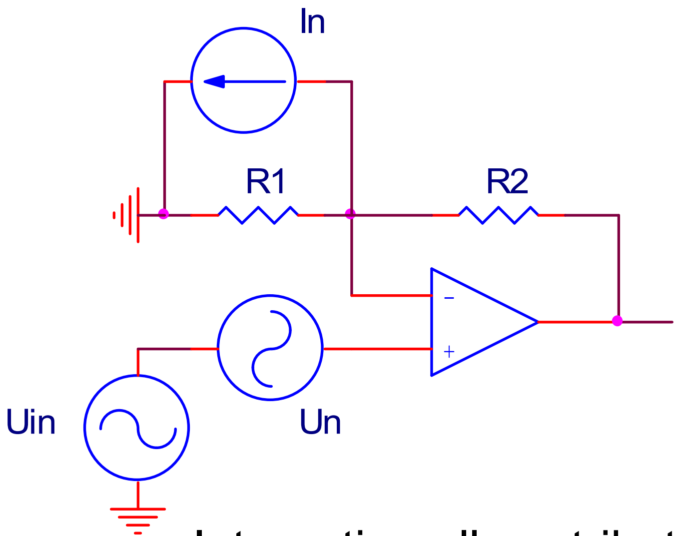
Typical spectral noise density (s. datasheets)

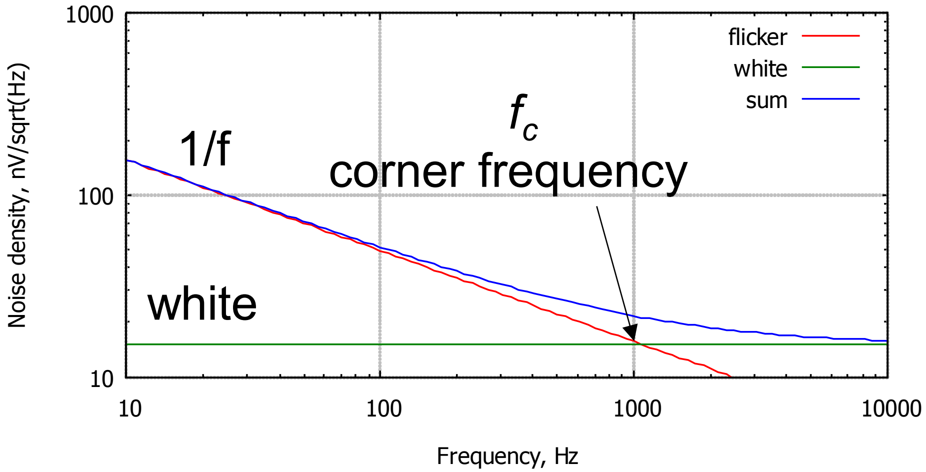


At the output – two noise contributions:

$$R_2 \cdot I_N \quad \text{and} \quad \left(1+\frac{R_2}{R_1}\right) \cdot U_N$$

Noise - non-inverting amplifier (2)





Integrating all contributions in the operating bandwidth (from f_1 to f_2):

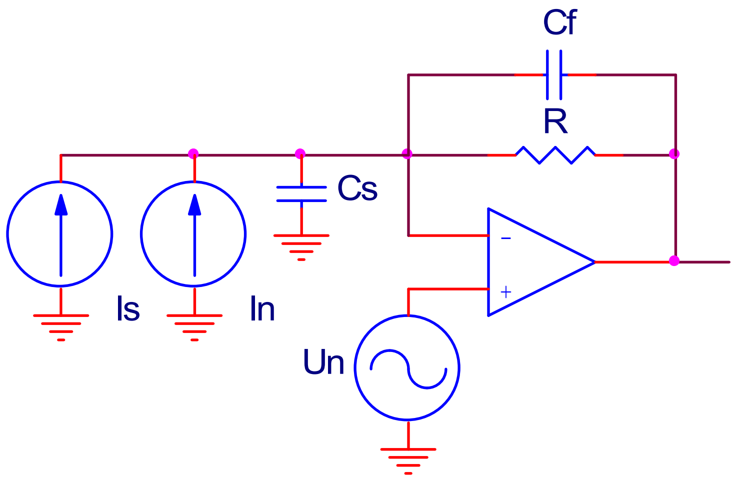
$$U_{OUT} = \sqrt{\int_{f_1}^{f_2} \left\{ \left(1 + \frac{R_2}{R_1}\right)^2 U_N^2 + \left(R_2 \cdot I_N\right)^2 \right\} df}$$

voltage → white, 1/f
current → white, 1/f

- For high frequencies the 1/f can be omitted, select amplifiers with lower f_c
- Use lower values of the feedback resistors
- Use amplifiers with low input current (JFET) for high impedance signal sources (like photodiodes), else use amplifiers with BJT input stages

Noise – current to voltage

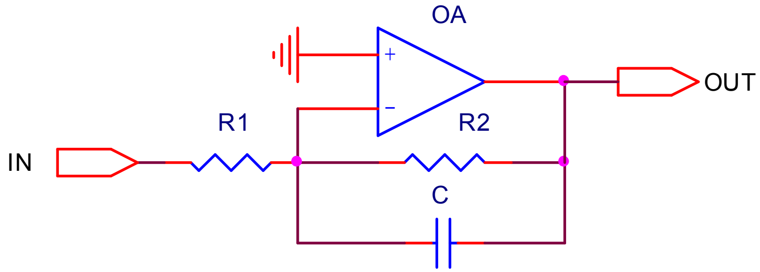
- Transimpendance amplifier – the gain has the dimension of resistance
- I_N and U_N are the equivalent noise sources
- C_f is an optional feedback capacitor



- C_S is the total parasitic capacitance at the input
  - for low frequencies, U_N appears at the output unchanged, **but** for higher frequencies U_N is amplified $1 + j\omega RC_S$ times!
  - it brings instability to the amplifier, as typically R is large – the solution is to add C_f ; it is easier to use fully compensated Op Amp
  - I_N is added directly to the input signal I_s – use low noise amplifier with FET input stage
- More tips can be found in “The Art of Electronics, 3rd edition”

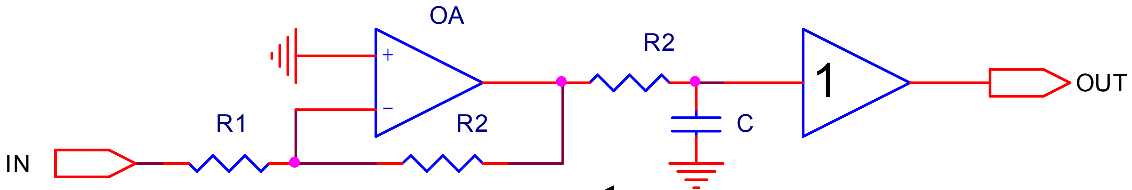
Frequency dependent feedback - integrators

- Low pass filter: $\dfrac{U_{IN}}{R_1} = -\dfrac{U_{OUT}}{R_2 \| Z_C}$



$$K = \frac{U_{OUT}}{U_{IN}} = -\frac{R_2 \| Z_C}{R_1} = -\frac{R_2}{R_1} \cdot \frac{Z_C}{R_2 + Z_C} = -\frac{R_2}{R_1} \cdot \frac{1}{1 + R_2 / Z_C} = -\frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega R_2 C}$$

- The equivalent circuit consists of an inverting amplifier, passive low pass filter and buffer



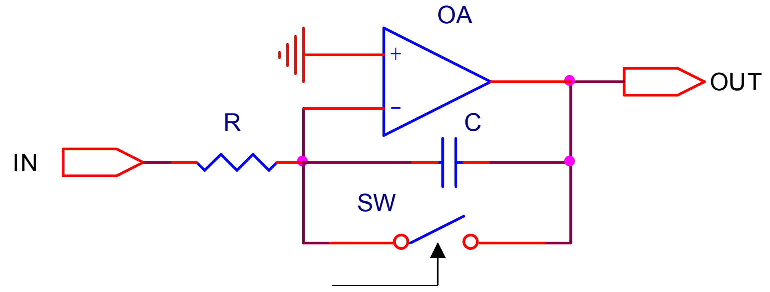
- The cutting frequency is $f_C = \dfrac{1}{2\pi R_2 C}$
- Usually more than one stage is needed, to get stronger attenuation at higher frequencies (above f_C)

Frequency dependent feedback - integrators

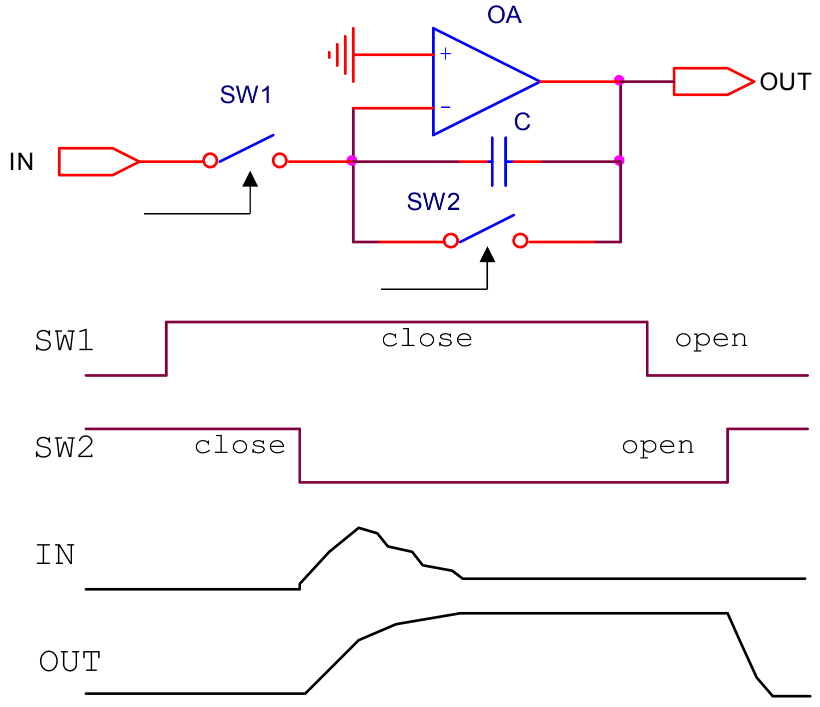
- Integrator circuit

If the switch is opened at t_1, the output voltage at t_2 is:

$$I_{IN} = \frac{U_{IN}}{R}, \quad U_{OUT} = -\frac{1}{RC}\int_{t_1}^{t_2} U_{IN}\,dt$$



If the input is a current source, R is not necessary!



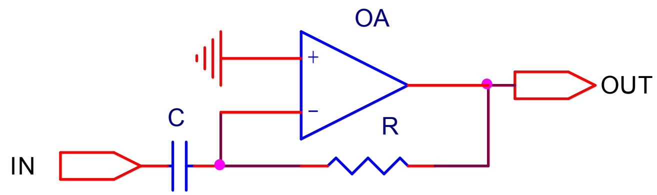
Select Op Amp with low input bias current!

There is a parasitic coupling from the control input to the switch terminals!

Frequency dependent feedback - differentiator

- The negative input of the Op Amp is a virtual ground
- The input current is

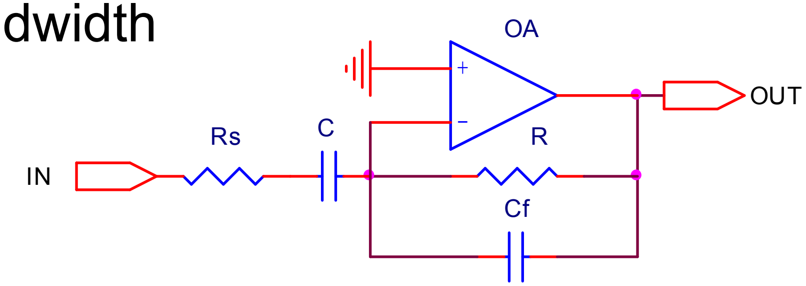
$$I_{IN} = C\frac{dU_{IN}}{dt}$$



- The output voltage is

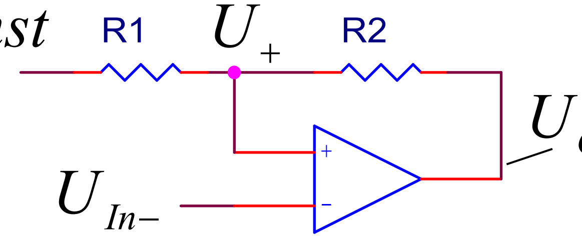
$$U_{OUT} = -RC\frac{dU_{IN}}{dt}$$

- Add a serial resistor R_S for better stability of the Op Amp and small C_f to limit the bandwidth



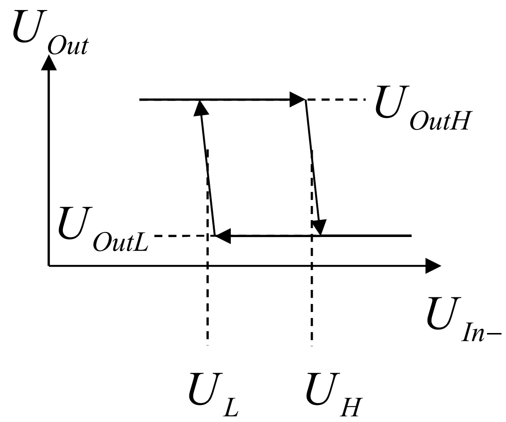
Positive Feedback - Comparator with Hysteresis

$$U_{In+} = const$$



$$U_{Out} = \begin{cases} U_{OutH} \\ U_{OutL} \end{cases}$$

saturation voltages



$$\begin{cases} U_H = U_{In+} \dfrac{R_2}{R_1 + R_2} + U_{OutH} \dfrac{R_1}{R_1 + R_2} \\ U_L = U_{In+} \dfrac{R_2}{R_1 + R_2} + U_{OutL} \dfrac{R_1}{R_1 + R_2} \end{cases}$$

$$Hysteresis = U_H - U_L = \left(U_{OutH} - U_{OutL}\right)\frac{R_1}{R_1 + R_2}$$

Example $R_1 = 1\text{k}, R_2 = 50\text{k}, U_{Out} = 0 \,|\, 5\text{V},\ Hyst. = (1/51)\cdot 5\text{V} = 98\text{mV} \approx 100\text{mV}$

Comparator or Op Ampl?

- Some Op Ampl don’t like large differential input signals
- In most cases the Op Ampl output can not be feed directly to a logic gate
- The Op Ampl needs typically more time to recover from saturation state

⇒ Use a dedicated comparator IC, except you have a very good reason to take an Op Ampl!

Op Amps – final remarks

- Great variety of Op Amps
  - Texas Instruments: 1427 OAs, 43 IAs
  - Analog Devices: 358 OAs, 43 IAs; LTC 413 OAs
- Other types (up to now only voltage feedback amplifier considered):
  - Fully differential operational amplifier – with differential outputs
  - Current feedback operational amplifier
    - high speed, bandwidth (ideally) independent of the gain
    - low impedance negative input acting as current summing point
    - less precise than the voltage feedback