Tutorial 2 - Q3

Monday, September 22, 2025 10:14 AM

Question 3 *Consider the AR(2) process* $Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + Z_t$. *Determine* ρ_1 *and* ρ_2 *in terms of* α_1 *and* α_2 *and vice–versa.*

We have the following AR(2) process

$$Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + Z_t$$

where Z_t is white noise.

From the notes, we have $\rho(\pm 1) = \frac{\alpha_1}{1-\alpha_2}$

and $\rho(\pm 2) = \alpha_1 \rho(\pm 1) + \alpha_2 \rho(0)$

$$= \frac{\alpha_1^2}{1-\alpha_2} + \alpha_2$$

Let's start with $\rho(\pm 1) = \rho_1$,

$$\rho_1(1-\alpha_2) = \alpha_1$$

$$\Rightarrow \alpha_1 = \rho_1(1-\alpha_2)$$

Note we still have α_2 contained within α_1.

Now we have $\rho(\pm 2) = \rho_2$.

$$\rho_2 = \frac{(\rho_1(1-\alpha_2))^2}{1-\alpha_2} + \alpha_2$$

$$\rho_2 = \frac{\rho_1^2(1-\alpha_2)^2}{1-\alpha_2} + \alpha_2$$

$$\rho_2 = \rho_1^2(1-\alpha_2) + \alpha_2$$

$$\rho_2 = \rho_1^2 - \rho_1^2\alpha_2 + \alpha_2$$

$$\rho_2 = \rho_1^2 + \alpha_2(1-\rho_1^2)$$

$$\Rightarrow \alpha_2 = \frac{\rho_2 - \rho_1^2}{1-\rho_1^2}$$

$$\Rightarrow \quad \alpha_2 = \frac{r_2 - r_1}{1 - p_1^2}$$

$$\alpha_1 = p_1 \left(1 - \left(\frac{p_2 - p_1^2}{1 - p_1^2} \right) \right)$$

$$= p_1 \left(\frac{(1 - p_1^2) - (p_2 - p_1^2)}{1 - p_1^2} \right)$$

$$= p_1 \left(\frac{1 - p_2}{1 - p_1^2} \right)$$

$$= \frac{p_1 (1 - p_2)}{1 - p_1^2}$$