Tutorial 2 - Q2

Monday, September 22, 2025 10:14 AM

Question 2 *Check that all of the following AR(2) processes are causal stationary:*

a)[S] $X_t = -1.4X_{t-1} - 0.65X_{t-2} + \epsilon_t$,

b)[S] $X_t = 0.45X_{t-1} + 0.25X_{t-2} + \epsilon_t$,

c) $X_t = 1.2X_{t-1} - 0.75X_{t-2} + \epsilon_t$,

where ϵ_t W.N. *with* $E(\epsilon_t) = 0$ *and* $Var(\epsilon_t) = \sigma_\epsilon^2$. *Calculate and display* $\rho(k)$, $k = 0, 1, 2, ..., 9$.

Usually we have to check whether the conditions of Theorem 3.3 are fulfilled or not. For an AR(2) model

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t$$

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these conditions are equivalent to all of the following simple conditions on the coefficients holding:

(i) $\phi_1 + \phi_2 < 1$,

(ii) $\phi_2 - \phi_1 < 1$,

(iii) $-1 < \phi_2 < 1$.

Using these conditions it is easy to check whether an AR(2) model is causal stationary or not. An AR(2) model is stationary if conditions (i)-(iii) all hold. It is not stationary if one (or,more) of these conditions does not hold.

(a) $X_t = -1.4X_{t-1} - 0.65X_{t-2} + \epsilon_t$

(i) $\phi_1 + \phi_2 = -1.4 - 0.65 = -2.05 < 1$ ✓

(ii) $\phi_2 - \phi_1 = -0.65 + 1.4 = 0.75 < 1$ ✓

(iii) $-1 < \phi_2 < 1$: $-1 < -0.65 < 1$ ✓

Since all 3 conditions hold, then X_t is causal stationary.

(b) $X_t = 0.45X_{t-1} + 0.25X_{t-2} + \epsilon_t$

(i) $\phi_1 + \phi_2 = 0.45 + 0.25 = 0.7 < 1$ ✓

(ii) $\phi_2 - \phi_1 = 0.25 - 0.45 = -0.2 < 1$ ✓

(iii) $-1 < (\phi_2 = 0.25) < 1$ ✓

Since all 3 conditions hold, then X_t is causal stationary.

(c) $X_t = 1.2X_{t-1} - 0.75X_{t-2} + \epsilon_t$

(i) $1.2 - 0.75 = 0.45 < 1$ ✓

(ii) $-0.75 - 1.2 = -1.95 < 1$ ✓

(iii) $-1 < (\phi_2 = -0.75) < 1$ ✓

Since all 3 conditions hold, then X_t is causal stationary.

To find the ACF at different lag k, we use the following for AR(k) processes:

$$\rho(0) = 1$$

$$\rho(\pm 1) = \frac{\phi_1}{1 - \phi_2}$$

$$\rho(\pm k) = \phi_1 \rho(k-1) + \phi_2 \rho(k-2) \text{ for } |k| \geq 2$$

(a)

Lag k	0	1	2	3	4	5	6	7	8	9
$\rho(k)$	1.000	-0.848	0.537	-0.201	-0.067	0.225	-0.271	0.233	-0.150	0.059

(b)

Lag k	0	1	2	3	4	5	6	7	8	9
$\rho(k)$	1.000	0.600	0.520	0.384	0.302	0.232	0.180	0.139	0.107	0.083

(c)

Lag k	0	1	2	3	4	5	6	7	8	9
$\rho(k)$	1.000	0.685	0.072	-0.426	-0.566	-0.360	-0.006	0.261	0.319	0.186