Tutorial 1 - Q1

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Question 1 Let $\{\epsilon_t\} \sim WN(0, \sigma_\epsilon^2)$ (i.e. white noise). Remember, that $\{\epsilon_t\}$ may not be independent, so nowhere in your arguments should you assume that they are independent. What you do know is they are uncorrelated.

Show by calculation that for all t,

(i) $E(\epsilon_t^2) = \sigma_\epsilon^2$.

(ii) For all $s \neq t$, $E(\epsilon_s \epsilon_t) = 0$.

(iii) For all $s \neq t$ and for real-valued constants α and β,

$$\text{Var}(\alpha\epsilon_s + \beta\epsilon_t) = (\alpha^2 + \beta^2)\sigma_\epsilon^2.$$

(i)

$E(\epsilon_t^2) = \sigma_\epsilon^2 \qquad \epsilon_t \sim WN(0, \sigma_\epsilon^2)$

$Var(\epsilon_t) = \sigma_\epsilon^2$

$Var(\epsilon_t) = E(\epsilon_t^2) - \underbrace{\cancel{E(\epsilon_t)^2}}_{0}$

$= \sigma_\epsilon^2$

(ii) $E(\epsilon_s \epsilon_t) = 0 \quad s \neq t$

White noise terms are uncorrelated.

$0 = Corr(\epsilon_s, \epsilon_t) = \dfrac{Cov(\epsilon_s, \epsilon_t) = 0}{\underbrace{\sqrt{Var(\epsilon_s)}\sqrt{Var(\epsilon_t)}}_{\neq 0}}$

$Cov(\epsilon_s, \epsilon_t)$

$= E((\epsilon_s - \cancelto{0}{E(\epsilon_s)})(\epsilon_t - \cancelto{0}{E(\epsilon_t)}))$ WN$(0, \sigma_\epsilon^2)$

$= E((\epsilon_s)(\epsilon_t)) = E(\epsilon_s \epsilon_t)$

$= \underbrace{E(\epsilon_s)}_{0}\underbrace{E(\epsilon_t)}_{0}$

(iii)

$Var(\alpha\epsilon_s + \beta\epsilon_t) = (\alpha^2 + \beta^2)\sigma_\epsilon^2$

$\alpha^2 \underbrace{Var(\epsilon_s)}_{\sigma_\epsilon^2} + \beta^2 \underbrace{Var(\epsilon_t)}_{\sigma_\epsilon^2} + \underbrace{2\alpha\beta Cov(\epsilon_s, \epsilon_t)}_{\text{from (ii) } 0}$

$(\alpha^2 + \beta^2)\sigma_\epsilon^2$