ACF

Question 1 *Determine the autocorrelation function for:*

(*i*) *the MA(2) process* $Y_t = Z_t + \beta_1 Z_{t-1} + \beta_2 Z_{t-2}$

(*ii*) *the MA(3) process* $Y_t = Z_t + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_3 Z_{t-3}$

Plot the autocorrelation for:

(*i*) *MA(2):* $\beta_1 = 0.8, \beta_2 = 0.5$

(*ii*) *MA(3):* $\beta_1 = 0.8, \beta_2 = -0.4, \beta_3 = -0.3$

The general ACF of MA(p) process is:

$$\rho(k) = \begin{cases} 1, & \text{if } k=0 \\ \dfrac{\gamma(\pm k)}{\gamma(0)}, & \text{if } |k| \le p \\ 0, & \text{if } |k| > p \end{cases} \qquad (2.5)$$

with ACVF $\gamma(k)$ distributed as:

$$\gamma(k) = \begin{cases} \left(\sum_{i=0}^{p-k} \psi_i \psi_{i+k} \right) \sigma_\varepsilon^2, & \text{if } |k| \le p \\ 0, & \text{if } |k| > p \end{cases} \qquad (2.4)$$

(i) We have an MA(2) process,

$$Y_t = Z_t + \beta_1 Z_{t-1} + \beta_2 Z_{t-2}$$

$\underline{k=0:}$

$$\gamma(0) = \left(\sum_{i=0}^{2-0} \psi_i \psi_{i+0} \right) \sigma_z^2 = (\psi_0^2 + \psi_1^2 + \psi_2^2)\sigma_z^2 = \underline{(1 + \beta_1^2 + \beta_2^2)\sigma_z^2}$$

$\underline{k=\pm 1:}$

$$\gamma(\pm 1) = \left(\sum_{i=0}^{2-1} \psi_i \psi_{i+1} \right) \sigma_z^2 = (\psi_0 \psi_1 + \psi_1 \psi_2)\sigma_z^2 = ((1)(\beta_1) + (\beta_1)(\beta_2))\sigma_z^2 = \underline{(\beta_1 + \beta_1\beta_2)\sigma_z^2}$$

$\underline{k=\pm 2:}$

$$\gamma(\pm 2) = \left(\sum_{i=0}^{2-2} \psi_i \psi_{i+2} \right) \sigma_z^2 = (\psi_0 \psi_2)\sigma_z^2 = \underline{\beta_2 \sigma_z^2}$$

$\underline{|k|>2:}$ $\gamma(\pm k) = 0$ for $|k| > 2$

Therefore, we have the following ACF:

(ii) We have the following MA(3) process:

$$Y_t = Z_t + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_3 Z_{t-3}$$

ACVF:

$$\gamma(0) = (1 + \beta_1^2 + \beta_2^2 + \beta_3^2)\sigma_z^2$$

$$\gamma(\pm 1) = (\beta_1 + \beta_1\beta_2 + \beta_2\beta_3)\sigma_z^2$$

$$\gamma(\pm 2) = (\beta_2 + \beta_1\beta_3)\sigma_z^2$$

$$\gamma(\pm 3) = \beta_3 \sigma_z^2$$

$$\gamma(\pm k) = 0 \text{ for } |k| > 3$$

ACF:

$$\rho(k) = \begin{cases} \dfrac{\gamma(0)}{\gamma(0)} = 1, & \text{if } k=0 \\ \dfrac{\gamma(\pm 1)}{\gamma(0)} = \dfrac{\beta_1 + \beta_1\beta_2 + \beta_2\beta_3}{1 + \beta_1^2 + \beta_2^2 + \beta_3^2}, & \text{if } k = \pm 1 \\ \dfrac{\gamma(\pm 2)}{\gamma(0)} = \dfrac{\beta_2 + \beta_1\beta_3}{1 + \beta_1^2 + \beta_2^2 + \beta_3^2}, & \text{if } k = \pm 2 \\ \dfrac{\gamma(\pm 3)}{\gamma(0)} = \dfrac{\beta_3}{1 + \beta_1^2 + \beta_2^2 + \beta_3^3}, & \text{if } k = \pm 3 \\ 0, & \text{if } |k| > 3 \end{cases}$$

(i) ma2 = c(0.8, 0.5)   (ii) ma3 = c(0.8, -0.4, -0.3)

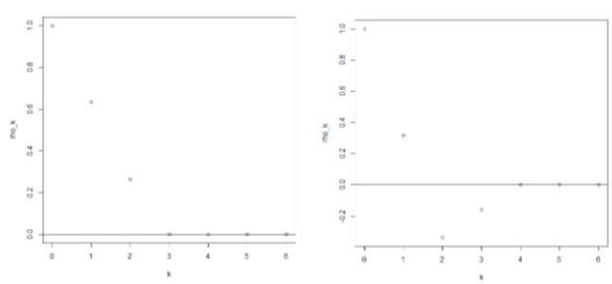

Figure 1: Autocorrelation functions for Q1(b).

$$
\rho(k) = \begin{cases} \dfrac{\gamma(0)}{\gamma(0)} = 1 , & \text{if } k = 0 \\ \dfrac{\gamma(\pm 1)}{\gamma(0)} = \dfrac{\beta_1 + \beta_1 \beta_2}{1 + \beta_1^2 + \beta_2^2} , & \text{if } |k| = 1 \\ \dfrac{\gamma(\pm 2)}{\gamma(0)} = \dfrac{\beta_2}{1 + \beta_1^2 + \beta_2^2} , & \text{if } |k| = 2 \\ 0 , & \text{if } |k| > 2 \end{cases}
$$