JOURNAL OF STATISTICAL COMPUTATION AND SIMULATION, 2017
https://doi.org/10.1080/00949655.2017.1392522

A Monte Carlo comparison of GMM and QMLE estimators for short dynamic panel data models with spatial errors

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ABSTRACT
We suggest a generalized spatial system GMM (SGMM) estimation for short dynamic panel data models with spatial errors and fixed effects when n is large and T is fixed (usually small). Monte Carlo studies are conducted to evaluate the finite sample properties with the quasi-maximum likelihood estimation (QMLE). The results show that, QMLE, with a proper approximation for initial observation, performs better than SGMM in general cases. However, it performs poorly when spatial dependence is large. QMLE and SGMM perform better for different parameters when there is unknown heteroscedasticity in the disturbances and the data are highly persistent. Both estimates are not sensitive to the treatment of initial values. Estimation of the spatial autoregressive parameter is generally biased when either the data are highly persistent or spatial dependence is large. Choices of spatial weights matrices and the sign of spatial dependence do affect the performance of the estimates, especially in the case of the heteroscedastic disturbance. We also give empirical guidelines for the model.

ARTICLE HISTORY
Received 25 December 2016
Accepted 11 October 2017

KEYWORDS
Short dynamic panel data models; Spatial errors; Generalized spatial system GMM estimation; Quasi-maximum likelihood estimation; Monte carlo studies

AMS SUBJECT CLASSIFICATION
62-07; 62J12; 62P20

1. Introduction

Recently, a vigorous literature has been developed on spatial panel data models and spatial error models (SEM). These methods have been widely used in many empirical areas, see, for instance, [1,2]. Glass et al. [3, p.2–3] state that 'SEM captures spatial dependence more fully' and with SEM 'the spatial dependence can be affected by other factors in addition to shocks to the spatially lagged dependent variable'. In addition, in the majority of research in empirical microeconomic and industrial organization with panel data, we have to deal with short panels with a large number of cross-sectional units and a small number of time periods [1] . It is well known that quasi-maximum likelihood estimation (QMLE) may not be consistent if the information in the first period is discarded when T is small [4,5] .

Su and Yang [6] extend the works of [4,5] to spatial settings, however, there might be some drawbacks:

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(i) QMLE is generally inconsistent for cross-sectional spatial autoregressive models in the presence of unknown heteroscedasticity [7,8].
(ii) QMLE may not perform well when the data are persistent.
(iii) The initial time period m has to be specified first. Therefore, misspecification of m may affect the finite sample results.

The above drawbacks motivate us to suggest a three-step generalized spatial SGMM estimation for this model and to conduct a series of Monte Carlo experiments to compare the finite sample results with the QMLE.

The rest of the paper is organised as follows. Section 2 introduces the model and gives a brief overview of QMLE suggested in [6]. Section 3 presents the three-step SGMM estimation of the model with regressors and fixed effects. Section 4 provides Monte Carlo experiments to verify finite sample results of QMLE and SGMM under different settings and analyzes the results. Section 5 gives guidelines for empirical researchers and concludes.

2. Model specification and QMLE

2.1. The model

Motivated by the empirical requirements, the model considered in this paper is

$$
\begin{aligned}
y_{it} &= \rho y_{i,t-1} + x_{it}'\beta + z_i'\gamma + u_{it},\\
u_{it} &= \mu_i + v_{it},\\
v_{it} &= \lambda \sum_{j=1,j\neq i}^{n} w_{ij}v_{jt} + \varepsilon_{it}, \quad i=1,\ldots,n,\ t=1,\ldots,T,
\end{aligned}
\tag{1}
$$

which refers to a short dynamic panel data models with an $AR(1)$ type spatial errors[1] and fixed effects.

Ordering the data first by time $(t = 1,\ldots,T)$ and then by individual units $(i = 1,\ldots,n)$, we have the model (1) in the stacked form:

$$y_t = \rho y_{t-1} + x_t\beta + z\gamma + u_t, \quad t = 1,\ldots,T, \tag{2}$$

where $y_t = (y_{1t},\ldots,y_{nt})'$, y_{t-1} is the observation of dependent variable in the previous period, the scalar $\rho(|\rho| < 1)$ is a parameter used to capture the dynamic effect, $x_t = (x_{1t},\ldots,x_{nt})'$ is an $n \times k_1$ matrix of observations of time-varying exogenous variables, $z = (z_1,\ldots,z_n)'$ is an $n \times k_2$ matrix of observations of time-invariant exogenous variables that may include the constant term, dummy variables and variables representing individual specified characters. Similarly, the disturbance term u_t is specified as

$$
\begin{aligned}
u_t &= \mu + v_t,\\
v_t &= \lambda W_n v_t + \varepsilon_t, \quad t = 1,\ldots,T,
\end{aligned}
\tag{3}
$$

where $u_t = (u_{1t},\ldots,u_{nt})'$, $v_t = (v_{1t},\ldots,v_{nt})'$, $\mu = (\mu_1,\ldots,\mu_n)'$ represents the individual fixed effect which are n free parameters and is correlated with x_t, W_n is an $n \times n$ known spatial weight matrix whose diagonal elements are zero, $\varepsilon_t = (\varepsilon_{1t},\ldots,\varepsilon_{nt})'$ is a purely idiosyncratic disturbance which is assumed to be independently distributed with zero mean and finite variance. Denoting $R_n = I_n - \lambda W_n$, the model has the reduced-form representation

$$\begin{aligned} y_t &= \rho y_{t-1} + x_t\beta + z\gamma + u_t, \quad u_t = \mu + R_n^{-1}\varepsilon_t, \\ t &= 1, \dots, T. \end{aligned} \tag{4}$$

We make the following assumptions:

Assumption 2.1 (Initialization): Data collection starts from the 0th period and the process starts from the $-m$th period. y_{-m} is treated as exogenous. The available observations are: (y_t, x_t, z), $t = 0, 1, \dots, T$.

Assumption 2.2 (Short panels): The number of cross-sectional units is large, $n \to \infty$, where the number of time periods is fixed, $T \geq 2$.

Assumption 2.3 (Error components): (i) The idiosyncratic disturbance $\{\varepsilon_{it}$, $i = 1, \dots, n,\ t = 1, \dots, T\}$ is independently distributed for all i and t with $E(\varepsilon_{it}) = 0$, $\mathrm{Var}(\varepsilon_{it}) = \sigma_\varepsilon^2$ and $E|\varepsilon_{it}|^{4+\epsilon_0} < \infty$ for some $\epsilon_0 > 0$; (ii) μ_i is independent of ε_{it}.

Assumption 2.4 (Spatial weighting matrix and spatial parameter): (i) W_n is a non-stochastic weighting matrix with zero diagonals; (ii) The parameter space for λ is a closed interval contained in $(-1/\lambda_{\max}, 1/\lambda_{\max})$, where $\lambda_{\max}$ is the largest absolute eigenvalue of W_n which is assumed to be bounded away from zero; (iii) W_n and R_n are invertible for all $\lambda \in (-1/\lambda_{\max}, 1/\lambda_{\max})$ and are uniformly bounded in both row and column sums as $n \to \infty$.

2.2. The method of quasi-Maximum likelihood estimation

To avoid incidental parameter problems, we have to eliminate the fixed effects parameters before proposing the QML estimation. We use first-differencing to eliminate μ:

$$\begin{aligned} \Delta y_t &= \rho \Delta y_{t-1} + \Delta x_t \beta + \Delta u_t, \\ \Delta u_t &= {R_n}^{-1} \Delta \varepsilon_t, \quad t = 2, \dots, T. \end{aligned} \tag{5}$$

Let $\Delta u = (\Delta u_2', \dots, \Delta u_T')'$, it is easy to see that

$$\mathrm{Var}(\Delta u) = \sigma_v^2 \{B \otimes [R_n' R_n]^{-1}\} = \sigma_v^2 \Omega,$$

where B is a $(T-1) \times (T-1)$ constant matrix

$$B = \begin{bmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & \ddots & \ddots & \ddots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 2 & -1 \\ 0 & \cdots & 0 & -1 & 2 \end{bmatrix}.$$

Let $\Delta y = (\Delta y_2', \dots, \Delta y_T')'$, $\Delta y_{-1} = (\Delta y_1', \dots, \Delta y_{T-1}')'$, $\Delta x = (\Delta x_2', \dots, \Delta x_T')'$, $\psi = (\beta', \sigma_v^2, \rho, \lambda)'$ be the $(k_1 + 3) \times 1$ vector of unknown parameters and $\theta = (\beta', \rho)'$, the Gaussian quasi-log-likelihood of ψ, which, when ignoring the constant term, has the form

$$\mathcal{L}^f(\psi) = -\frac{n(T-1)}{2}\log(\sigma_v^2) - \frac{1}{2}\log|\Omega| - \frac{1}{2\sigma_v^2}\Delta u(\theta)'\Omega^{-1}\Delta u(\theta). \tag{6}$$

The score function $\mathcal{S}^f(\psi) = \partial\mathcal{L}^f(\psi)/\partial\psi$ has elements

$$\frac{\partial\mathcal{L}^f(\psi)}{\partial\beta} = \frac{1}{\sigma_v^2}\Delta' x\Omega^{-1}\Delta u(\theta),$$

$$\frac{\partial\mathcal{L}^f(\psi)}{\partial\sigma_v^2} = \frac{1}{2\sigma_v^4}\Delta' u(\theta)\Omega^{-1}\Delta u(\theta) - \frac{n(T-1)}{2\sigma_v^2},$$

$$\frac{\partial\mathcal{L}^f(\psi)}{\partial\rho} = \frac{1}{\sigma_v^2}\Delta' u(\theta)\Omega^{-1}\Delta y_{-1},$$

$$\frac{\partial\mathcal{L}^f(\psi)}{\partial\lambda} = \frac{1}{2\sigma_v^2}\Delta' u(\theta)(B\otimes A_n)\Delta u(\theta) - (T-1)tr(G_n),$$

where $A_n = W_n' R_n + R_n' W_n$ and $G_n = W_n R_n^{-1}$.

Since the third element of the score function contains Δy_1, however, Equation (4) is not defined for $t=1$ as Δy_1 depends on Δy_0 and the latter is not observed. As shown in Yang [9], with endogenous y_0, the QMLE is inconsistent if we mistakenly treat y_0 as exogenous, since the third element of the score function contains Δy_{-1}. To solve this problem, Su and Yang [6] suggest an approximation method for Δy_1. They break Δy_1 into the endogenous and exogenous parts: $\Delta y_1 = \Delta\eta_1 + \Delta\xi_1$, $\Delta\eta_1$ and ξ_1 are given as

$$\Delta\eta_1 = \rho^m E(\Delta y_{-m+1}) + \Sigma_{j=1}^{m-1}\rho^j\Delta x_{1-j}\beta + \Delta x_1\beta, \tag{7}$$

$$\Delta\xi_1 = \rho^m[\Delta y_{-m+1} - E(\Delta y_{-m+1})] + \Sigma_{j=0}^{m-1}\rho^j {R_n}^{-1}\Delta\varepsilon_{1-j}. \tag{8}$$

Following [5], we have another fundamental assumption for predictors of unobservable terms.

Assumption 2.5: (i) The optimal predictors for the unobservable terms, $\{\Delta x_{1-j},\ j = 1, 2, \ldots,\}$ and $\{E(\Delta y_{-m+1}),\ m = 0, 1, \ldots,\}$, conditional on the observable terms, are $\Delta\dot{x}$, where $\Delta\dot{x} = (\Delta x_1', \ldots, \Delta x_T')'$;

(ii) The error term ϵ for the prediction of $\rho^m E(\Delta y_{-m+1}) + \Sigma_{j=1}^{m-1}\rho^j\Delta x_{1-j}\beta$ is independently distributed with zero mean and variance $\sigma_\epsilon^2 I_n$. It is also independent with e, where $e = y_{-m} - E(y_{-m})$ and e is also independently distributed with zero mean and variance $\sigma_e^2 I_n$.

Then, define $\Delta\tilde{x} = (\iota_n, \Delta\dot{x})'$ and

$$\Delta Y^+(\rho) = \begin{pmatrix} \Delta y_1 \\ \Delta y_2 - \rho\Delta y_1 \\ \vdots \\ \Delta y_T - \rho\Delta y_{T-1} \end{pmatrix}, \quad \Delta X^+ = \begin{pmatrix} \Delta x_1 & \Delta\tilde{x} \\ \Delta x_2 & 0 \\ \vdots & \vdots \\ \Delta x_T & 0 \end{pmatrix}, \tag{9}$$

the likelihood function is formulated by the following system of equations:

$$\begin{aligned}\Delta y_1 &= \Delta\tilde{x}\pi + \Delta x_1\beta + \Delta\tilde{u}_1,\\ \Delta y_t &= \rho\Delta y_{t-1} + \Delta x_t\beta + {R_n}^{-1}\Delta\varepsilon_t, \quad t = 2,\ldots,T,\end{aligned} \tag{10}$$

where $\Delta\tilde{u}_1 = \epsilon - \rho^m(1-\rho)e + \rho^m R_n^{-1}\varepsilon_{-m+1} + \Sigma_{j=0}^{m-1}\rho^j R_n^{-1}\Delta\varepsilon_{1-j}$. Then, we can form the new quasi-log-likelihood based on Equation (10). Su and Yang [6] also derive the asymptotic properties of QMLEs and give some computational notes for the determinants and inverse of the variance–covariance matrix in the likelihood function.[2]

An additional issue of the model is the allowance of cross-sectional heteroscedasticity. The homoscedasticity assumption in Assumption 2.3 may be too restrictive, as heteroscedastic variance can be empirically important.[3] If heteroscedasticity exists but homoscedasticity is imposed, QMLE is generally inconsistent. Under large N, the QMLE encounters incidental parameter problems again since the number of variance parameters grows with sample size n. In addition, if we assume heteroscedasticity, Assumption 2.5 in Section 2.2. may not hold. Thus, the predictive model for Δy_1 may be misspecified. We will conduct an extensive Monte Carlo study for the finite sample performance of QMLE if homoscedasticity is mistakenly imposed by the empirical researcher in Section 4.2.1.

2.3. Estimation of γ

In Section 2.2, we give a brief overview of QMLE proposed by [6] for the model (3). However, by first-differencing, the time-invariant variables z are wiped out and γ cannot be estimated. In this subsection, we give the estimation procedure for the parameters γ. Since $(\hat{\rho}, \hat{\lambda}, \hat{\beta}')'$ have been estimated in the first stage, in the second stage, γ can be estimated using the level relationship

$$y_t - \hat{\rho}y_{t-1} - x_t\hat{\beta} = z\gamma + u_t, \quad t = 1,\ldots,T. \tag{11}$$

Let $\hat{d} = y_t - \hat{\rho}y_{t-1} - x_t\hat{\beta}$, to filter the spatial correlation in u_t, a Cochrane–Orcutt type transformation of $R_n(\hat{\lambda})$ gives

$$R_n(\hat{\lambda})\hat{d} = R_n(\hat{\lambda})z\gamma + R_n(\hat{\lambda})\mu + \varepsilon_t, \quad t = 1,\ldots,T. \tag{12}$$

Then, estimator of γ is given by

$$\hat{\gamma} = (z'R_n'(\hat{\lambda})R_n(\hat{\lambda})z)^{-1}(z'R_n'(\hat{\lambda})R_n(\hat{\lambda})\hat{d}). \tag{13}$$

3. A generalized spatial system GMM procedure

Consider again the model in Equations (2) and (3). In this section, we follow [10] to propose a three step procedure. In the first step, the model is estimated by the traditional system GMM approach. In the second step, the spatial autoregressive parameter λ is estimated in terms of residuals obtained via the first step by the spatial GMM approach suggested in [8]. In the third step, the model is re-estimated by the system GMM approach after a Cochrane–Orcutt type transformation to both the variables and the instruments.

3.1. The first step of the procedure

In the standard GMM literature,[4] where there is no error cross-sectional dependence ($\lambda = 0$), we typically assume the error components structure

$$E(\mu_i) = E(v_{it}) = E(\mu_i v_{it}) = 0, \quad i = 1, \ldots, n,\ t = 1, \ldots, T \tag{14}$$

and

$$E(v_{it} v_{is}) = 0, \quad i = 1, \ldots, n,\ t, s = 1, \ldots, T,\ t \neq s. \tag{15}$$

Following [11] , there is also a standard assumption on the initial conditions:

$$E(y_{i0} v_{it}) = 0, \quad i = 1, \ldots, n,\ t = 1, \ldots, T. \tag{16}$$

Under Equations (14)–(16), with further assumptions of strictly exogenous of regressors x_{it}, we have the following moment conditions on the differenced equations (5):

$$E(y_{is} \Delta u_{it}) = 0, \quad s = 0, \ldots, t-2,\ t = 2, \ldots, T, \tag{17}$$

$$E(x_{is} \Delta u_{it}) = 0, \quad s, t = 2, \ldots, T, \tag{18}$$

$$E(z_i \Delta u_{it}) = 0, \quad t = 2, \ldots, T. \tag{19}$$

If we further assume that

$$E(y_{is} \mu_i) = E(y_{it} \mu_i), \quad E(x_{is} \mu_i) = E(x_{it} \mu_i), \quad \forall s, t,$$

the following non-redundant moment conditions, based on level equations (4), become available:

$$E[\Delta y_{i,t-1}(\mu_i + v_{it})] = 0, \quad t = 2, \ldots, T, \tag{20}$$

$$E[\Delta x_{i,t-1}(\mu_i + v_{it})] = 0, \quad t = 2, \ldots, T. \tag{21}$$

Let

$$Z_i^D = \begin{bmatrix} (y_{i0}, x_{i1}, \ldots, x_{iT}, z_i) & & & 0 \\ & (y_{i0}, y_{i1}, x_{i1}, \ldots, x_{iT}, z_i) & & \\ & & \ddots & \\ 0 & & & (y_{i0}, y_{i1}, \ldots, y_{i,T-2}, x_{i1}, \ldots, x_{iT}, z_i) \end{bmatrix},$$

$$Z_i^L = \begin{bmatrix} \Delta y_{i1}, \Delta x_{i2} & & & 0 \\ & \Delta y_{i2}, \Delta x_{i3} & & \\ & & \ddots & \\ 0 & & & \Delta y_{i,T-1}, \Delta x_{iT} \end{bmatrix}$$

and

$$Z_i^S = \begin{bmatrix} Z_i^D & \\ & Z_i^L \end{bmatrix},$$

the moment conditions in Equations (17)–(21) can be written in the compact form

$$E[Z_i'^S U_i^S] = 0, \tag{22}$$

where

$$Y_i^S = \begin{bmatrix} \Delta y_i \\ y_i \end{bmatrix}, \quad W_i^S = \begin{bmatrix} \Delta y_{i,-1} & \Delta x_i & \mathbf{0}_{(T-1)\times 1} \\ y_{i,-1} & x_i & \iota_{T-1} \otimes z_i \end{bmatrix}, \quad U_i^S = \begin{bmatrix} \Delta u \\ u \end{bmatrix},$$

and $U_i^S = Y_i^S - W_i^S\delta$, $\delta = (\rho, \beta', \gamma')'$, $y_i = (y_{i2}, \ldots, y_{iT})'$, $y_{i,-1} = (y_{i1}, \ldots, y_{i,T-1})'$, $x_i = (x_{i2}, \ldots, x_{iT})'$, $u_i = (u_{i2}, \ldots, u_{iT})'$, $\Delta y_i = (\Delta y_{i2}, \ldots, \Delta y_{iT})'$, $\Delta y_{i,-1} = (\Delta y_{i1}, \ldots, \Delta y_{i,T-1})'$, $\Delta x_i = (\Delta x_{i2}, \ldots, \Delta x_{iT})'$, $\Delta u_i = (\Delta u_{i2}, \ldots, \Delta u_{iT})'$.

The SGMM estimator based on the moment condition (22) is given by

$$\hat{\delta}^S = [Q'_{zx} V^S Q_{zx}]^{-1} [Q'_{zx} V^S Q_{zy}], \tag{23}$$

where $Q_{zx} = (1/n)\sum_{i=1}^n Z_i'^S W_i^S$, $Q_{zy} = (1/n)\sum_{i=1}^n Z_i'^S Y_i^S$, and V^S is a positive-definite weighting matrix. Note that compared with the QMLE, although the initial observation y_0 is treated as exogenous, instruments in both first-differencing and levels are used for the estimation where only first-differencing variables are used to obtain the likelihood function.

The consistency of the SGMM estimator in Equation (23) requires the following conditions:

$$\frac{1}{n}\sum_{i=1}^n Z_i'^S U_i^S \xrightarrow{p} 0,$$

$$\frac{1}{n}\sum_{i=1}^n Z_i'^S W_i^S \xrightarrow{p} Q^S \neq 0.$$

For the proof, see Theorems 1 and 2 in Sarafidis [12].

The optimal weighting matrix is typically given by replacing V^S with $\hat{\Omega}_n^{-1}$, where $\hat{\Omega}_n = (1/n)\sum_{i=1}^n Z_i'^S \hat{U}_i^S \hat{U}_i'^S Z_i^S$, $U_i^S = Y_i^S - W_i^S \hat{\delta}^S$, and $\hat{\delta}^S$ is obtained by using the identity weighting matrix in Equation (23). However, optimality of using $\hat{\Omega}_n$ requires the condition $\hat{\Omega}_n \xrightarrow{p} \Omega$, where $\Omega = (1/n)E[Z_i'^S U_i^S U_i'^S Z_i^S]$. This convergence requirement is challenging because of the ignorance of spatial correlation [12]. Thus, in the first step, we use the suboptimal identity matrix as the weighting matrix in Equation (23).

Although $\hat{\delta}^S$ is consistent, the spatial autoregressive parameter λ cannot be estimated, and the spatial correlation is ignored, making $\hat{\delta}^S$ inefficient. We therefore turn to the second step of our procedure.

3.2. The second step of the procedure

Let $y = (y_1', \ldots, y_T')'$, $x = (x_1', \ldots, x_T')'$, $\dot{z} = \iota_T \otimes z$, $\mu = \iota_T \otimes \mu$, $\varepsilon = (\varepsilon_1', \ldots, \varepsilon_T')'$, where $y_s = (y_{1s}, \ldots, y_{ns})'$, $x_s = (x_{1s}, \ldots, x_{ns})'$, $\varepsilon_s = (\varepsilon_{1s}, \ldots, \varepsilon_{ns})'$ for $s = 1, \ldots, T$. The residuals from the first step are given by

$$\hat{u} = y - \hat{\rho} y_{-1} - x\hat{\beta} - \dot{z}\hat{\gamma} = \mu + (I_T \otimes R_n^{-1}(\lambda))\varepsilon. \tag{24}$$

The GMM estimator of spatial parameter λ is defined as the minimizer of the following quadratic form:

$$\hat{\lambda} = \arg\min_{\lambda} [m'_\kappa(\lambda) V_{nT} m_\kappa(\lambda)], \tag{25}$$

where m_κ is the sample counterpart of r quadratic moment conditions:

$$M_\kappa(\lambda) = \frac{1}{nT}E[u'^*(\lambda)(I_T \otimes A_\kappa)u^*(\lambda)], \quad \kappa = 1, \dots, r, \tag{26}$$

and V_{nT} is a positive-definite weighting matrix. Inner weighting matrices A_κ have bounded row and column sums and zero diagonal elements. $u^*(\lambda)$ is defined as

$$u^*(\lambda) = (J_T \otimes (I_N - \lambda W_n))\hat{u}, \tag{27}$$

where $J_T = I_T - (1/T)\iota_T\iota_T'$. For the selection of the inner weighting matrices, we follow [8], where

$$A_1 = W_n'W_n - \mathrm{diag}(W_n'W_n), \quad A_2 = W_n, \tag{28}$$

which are robust under heteroscedasticity. When A_1 and A_2 have been proposed, the optimal weighing matrix V_{nT} is defined as

$$V_{nT} = \frac{1}{n}\begin{pmatrix} \mathrm{tr}(\Sigma_n A_1(A_1'\Sigma_n + \Sigma_n A_1)) & \mathrm{tr}(\Sigma_n A_1(A_2'\Sigma_n + \Sigma_n A_2)) \\ \mathrm{tr}(\Sigma_n A_2(A_1'\Sigma_n + \Sigma_n A_1)) & \mathrm{tr}(\Sigma_n A_2(A_2'\Sigma_n + \Sigma_n A_2)) \end{pmatrix}, \tag{29}$$

where $\Sigma_n = \sigma_\varepsilon^2 I_n$ and consistent estimation of σ_ε^2 can be obtained by $\hat{\sigma}_\varepsilon^2 = \hat{u}'\hat{u}$.[5]

3.3. The third step of the procedure

A Cochrane–Orcutt type transformation by $R = (I_T \otimes (I_n - \hat{\lambda} W_n))$ of the regressors gives:

$$\tilde{y} = Ry; \tilde{y}_{-1} = Ry_{-1}, \quad \Delta\tilde{y} = R\Delta y, \quad \Delta\tilde{y}_{-1} = R\Delta y_{-1}, \quad \tilde{x} = Rx, \quad \Delta\tilde{x} = R\Delta x, \quad \tilde{z} = R\dot{z}.$$

The SGMM estimator based on the transformed regressors is given by

$$\hat{\delta}^{BS} = [\tilde{Q}_{zx}'V^{*S}\tilde{Q}_{zx}]^{-1}[\tilde{Q}_{zx}'V^{*S}\tilde{Q}_{zy}], \tag{30}$$

where $\tilde{Q}_{zx} = (1/n)\sum_{i=1}^n \tilde{Z}_i'^S \tilde{W}_i^S$, $\tilde{Q}_{zy} = (1/n)\sum_{i=1}^n \tilde{Z}_i'^S \tilde{Y}_i^S$, V^{*S} is a positive-definite weighting matrix and $\tilde{Z}_i^S$, $\tilde{W}_i^S$, $\tilde{Y}_i^S$ are defined similarly in Section 3.1 with regressors after Cochrane–Orcutt type transformation.

In this stage, there is nothing different from the standard GMM literature. Thus, usual consistency and asymptotic normality requirements apply directly to this model and the optimal weighting matrix is approximated by $\tilde{\hat{\Omega}}_n^{-1}$, where $\tilde{\hat{\Omega}}_n = (1/n)\sum_{i=1}^n \tilde{Z}_i'^S \tilde{\hat{U}}_i^S \tilde{\hat{U}}_i'^S \tilde{Z}_i^S$ and $\tilde{\hat{U}}_i^S = \tilde{Y}_i^S - \tilde{W}_i^S \hat{\delta}_1^{BS}$, $\hat{\delta}_1^{BS}$ is obtained by Equation (30) using the identity matrix as the weighting matrix.[6]

We now summarize our three-step procedure below:

Step 1: get the initial estimation for $\delta = (\rho, \beta', \gamma)'$ by Equation (23);

Step 2: get the estimation for spatial parameter λ by Equation (25);

Step 3: get the efficient estimation for $\delta = (\rho, \beta', \gamma)'$ by Equation (30).

Remark 3.1: Kripfganz and Schwarz [13] also consider the estimation of linear dynamic panel data models with time-invariant variables. They find that estimation of γ performs poorly by Equation (30) and suggests using Equation (13) to re-estimate γ after step 3. In the next section, we will report estimation results for γ by both Equations (13) and (30).

4. Simulation study

4.1. Experiment design

A series of Monte Carlo experiments are conducted to investigate the finite sample properties of the QMLE and SGMM. We investigate the consequence of unknown heteroscedasticity, highly persistent series and the misspecification of the initial time period m on the process. We follow [6] to use the following DGP for each i:

$$y_{it} = \rho y_{i,t-1} + x_{it}\beta + z_i\gamma + u_{it},\ \ u_{it} = \mu_i + v_{it},\ \ v_{it} = \lambda W_n v_{it} + \varepsilon_{it}\ \ \text{for } t = -m, \ldots, T.$$

Several spatial weights matrices are considered. We first consider group interaction spatial weights matrix and generate it as follows. First, group size m_r ($r = 1, \ldots, R$) is generated randomly from a $U(2, 6)$ variable (round to the closest integer). Then, for the ith row of W_r, we generate an integer k uniformly at random from the set of integers $[0, 1, 2, 3, 4]$. We set the $(i + 1)$th, ..., $(i + k)$th element of the ith row of W_r to be ones and others to be zeros if $i + k < m_r$; otherwise, the first entry of $m_r - k$ elements will be ones and other elements will be zero. The number of groups is $R = 15$ or $R = 30$ and the number of cross-sectional units are $n = \Sigma_{r=1}^{R} m_r$. The average number of cross-sectional units are $n = 60$ or $n = 120$.

We then define spatial weights matrix in a circular world as in [14], where the first and the last one third of cross-sectional units (round to the closest integer) have five neighbors in front and five in back, while the middle third only has one neighbor in front and one in back. The sample sizes of cross-sectional units are $n = 50{,}100$. We also consider a real-world spatial weights matrix that describes the spatial arrangement of 30 provinces in China[7] and sample size of $n = 90$ with the preceding 30×30 matrix as their diagonal blocks.

The elements of x_t are generated as $x_t = \mu_x + gtl_n + \zeta_t$, where ζ_t follows a stationary $ARMA(1, 1)$ process $(1 - \pi_1 L)\zeta_t = (1 + \pi_2 L)e_t$ with innovations $e_t \sim N(0, \sigma_1^2 I_n)$ and $\mu_x = e + (1/(T + m + 1))\Sigma_{t=-m}^{T} e_t$ with $e \sim N(0, \sigma_2^2)$. The parameters for generation of x_t are $\theta_x = (g, \pi_1, \pi_2, \sigma_1^2, \sigma_2^2)' = (0.1, 0.2, 0.4, 1, 1)'$. The elements of z are generated randomly from Bernoulli(0.5) and v_t are from $N(0, \sigma_v^2)$. We choose $(\beta, \gamma, \sigma_v^2)' = (2, 1, 1)'$.[8] For autoregressive parameters $(\rho, \lambda)'$, we choose $\rho = 0.5, \rho = 0.9$ to consider weak and strong persistence and we choose combinations of $(-0.8, -0.5, -0.2, 0.2, 0.5, 0.8)'$ for λ to allow for weak, medium and strong spatial dependence. Unlike most of the Monte Carlo results of theoretical papers, the consideration of negative spatial dependence is motivated by Kao and Bera [15] because the sign of spatial dependence parameter lead to different structure of the variance–covariance matrix of observations.[9]

For each case below, the results reported are based on 500 Monte Carlo replications[10] for QMLE and 2000 replications for SGMM. The number of time periods are $T = 3$ or $T = 7$ and $m = 6$, $m = 50$ or $m = 100$.[11] To summarize the simulation results, we report the mean (Mean),the standard deviation (SD) and the root mean square errors (RMSE) of the empirical distribution of the estimates.[12,13]

4.2. Simulation results

4.2.1. Unknown heteroscedasticity

In order to evaluate the estimator's relative performance under unknown heteroscedasticity, we follow the following procedures in [8] to generate heteroscedastic errors for circular world and real-world spatial weights matrices. First, the innovation vector ξ_{it} is generated from the standard normal distribution. Next, we let $\varepsilon_{it} = \omega_i \xi_{it}$, where $\omega_i^2 = d_i/4$ and d_i is the number of adjacent units for each $i, i = 1, \ldots, n$. Then, we follow [7] to generate heteroscedastic errors for group interaction spatial weights matrix by letting them relate to the group size for the group interaction spatial weights matrix. For each $r(r = 1, \ldots, R), t(t = -m, \ldots 0, \ldots, T)$, ε_{rt} are generated from $N(0, \sqrt{m_r})$.

Tables A1–A4 present the results for circular world and real-world spatial weights matrices when the spatial dependence is weak and moderate. QMLE performs generally better than SGMM in both homoscedasticity and heteroscedasticity cases except for the spatial parameter $\hat{\lambda}$. In a circular world spatial weights matrix, QMLE for $\hat{\lambda}$ is biased under heteroscedasticity. Although SGMM for $\hat{\lambda}$ is biased when n is small, $\hat{\lambda}$ converges to the true value as n increases. In a real-world spatial weights matrix, there is a slight increase in bias and corresponding SD's and RMSE's in QMLE under heteroscedastic disturbance. For negative spatial dependence, both estimators perform worse in the presence of heteroscedasticity under a circular world spatial weights matrix. On the other hand, there is no significant difference for heteroscedastic errors under a real-world spatial weights matrix.

Tables A5 and A6 present the results for the circular world and real-world spatial weights matrices when the spatial dependence is large. We find that QMLE can be quite biased and as n increases it does not show any sign of convergence for $\hat{\lambda}$ even in the homoscedasticity case. The QMLE for $\hat{\rho}$ and $\hat{\beta}$ converge to the true value as n increases, whereas the SGMM for $\hat{\gamma}$ and $\hat{\lambda}$ result in better estimates and improves significantly as n increases, although there is still a moderate bias in $\hat{\lambda}$ and the convergence rate for $\hat{\lambda}$ seems slow.

Tables A7 and A8 presents the results for the group interaction spatial weights matrix, which employs weak and moderate spatial dependence. QMLE results in better estimate generally, except for $\hat{\lambda}$ and the bias increases as $\hat{\lambda}$ increases, whereas SGMM is more robust. When there are both heteroscedastic disturbance and positive spatial dependence, QMLE performs worse in the case of small n and small T, however, it quickly converges to the true value as n increase. There is indeed an increase in bias and corresponding SD's and RMSE's in QMLE for $\hat{\rho}$, $\hat{\beta}$ and $\hat{\gamma}$ under negatively weak and moderate spatial dependence.

When spatial dependence is large, as demonstrated in Table A9, we find that QMLE is largely biased for positively strong spatial dependence when n and T are small and show signs of convergence as n increases, although $\hat{\lambda}$ is downward biased. The QMLE for heteroscedasticity case is quite confusing since the results become worse when n is large, except for $\hat{\beta}$. SGMM is more robust; however, there is still a moderate bias for $\hat{\gamma}$ and $\hat{\lambda}$, especially in the case of heteroscedasticity. Similar results are obtained when spatial dependence is strong and negative, although the QMLE performs well in the homoscedasticity case when n and T are small,except for $\hat{\lambda}$.

4.2.2. Persistent series

We consider cases when the time series y_{it} becomes highly persistent by choosing $\rho = -0.9$ and $\rho = 0.9$, corresponding to both the positively and the negatively strong dynamic effect, respectively. The results are displayed in Table A10–A18 and summarized as follows.

(i) When both the dynamic effect and spatial dependence are positively large, QMLE performs very poorly and does not show signs of convergence as n increases.

(ii) In most experiments, both QMLE and SGMM show signs of convergence as n increases, except for the case of the positively large dynamic effect and the negatively large spatial dependence under a circular world spatial weights matrix.
(iii) QMLE for $\hat{\beta}$ works better. The results for $\hat{\rho}$,$\hat{\gamma}$ and $\hat{\lambda}$ are generally better under SGMM procedure. $\hat{\lambda}$ shows a sign of convergence as n increases; however, there is a downside bias for $|\hat{\lambda}|$ and the bias for QMLE is larger than SGMM in most of the experiments. There is not a significant difference for $\hat{\gamma}$ between QMLE and SGMM as n increases, especially when the dynamic effect is negatively large.
(iv) SGMM is generally more robust than QMLE in the presence of persistence and strong spatial dependence. SGMM also has relatively better results when n and T are small, for instance, as demonstrated in Table A10, even the spatial dependence is weak. QMLE for $\hat{\rho}$ and $\hat{\gamma}$ are largely biased when dynamic effect is positively large and spatial dependence is positively moderate, however, SGMM for these estimates are nearly unbiased. Both estimates for $\hat{\lambda}$ have a downward bias and are more robust under a real-world spatial weights matrix.

4.2.3. Specification of initial time periods

As we have demonstrated in Section 4.1, the initial time period m must be specified before proposing the QMLE. Su and Yang [6] have considered the case when m has been specified much larger than the true initial time period. Here, we investigate the cases when true m is 50 or 100, but we specify $m = 6$, which means that there is some useful information for the approximation of y_0 which is largely ignored. The settings for parameters in this section are the same as in the homoscedastic case in Section 4.2.1.

Tables A19–A27 report the results of misspecification of m on both QMLE and SGMM. There is a slight efficiency loss when spatial dependence is weak and moderate. The differences of corresponding SD's and RMSE's are small under a real-world spatial weights matrix. When spatial dependence is positively large, the estimation results are worse than the correctly specified m under circular world and group interaction spatial weights matrices. The results are even worse when n and T are small. In general, the estimates under a wrong m are quite satisfactory and it seems that QMLE is not sensitive to the treatment of initial values when spatial dependence is moderate. The findings are consistent with [6]. SGMM is also not sensitive to the treatment of m and it seems that SGMM is more robust under large spatial dependence and different settings of spatial weights matrices.

5. Conclusion

This paper proposes a generalized spatial system GMM estimation procedure for short dynamic panel data models with spatial errors. The QMLE procedure uses an approximation procedure for the initial observation y_0, while the three-step SGMM procedure uses both level and difference relationship and y_0 is treated as exogenous. We design extensive Monte Carlo experiments to verify their finite sample properties under different cases. We also present a series of robustness checks for the degree of spatial dependence and different choices of spatial weights matrices.

The results are carefully discussed in Section 4. We find that QMLE, with a proper approximation for initial observation, performs generally better than SGMM. When the dynamic effect is large (persistent series), QMLE is better for $\hat{\beta}$, while SGMM is better for estimates of ρ, γ and λ. Both QMLE and SGMM are not sensitive to the treatment of initial values. When m is not correctly specified, the results for QMLE are better than SGMM. For the SGMM estimates of γ, we suggest to use Equation (12) after step 3.

QMLE performs poorly when spatial dependence is large, especially in the case of both the positively large dynamic effect and strong spatial dependence, while SGMM is generally more robust, except for the spatial parameter $\hat{\lambda}$. In the presence of heteroscedasticity, QMLE performs generally better for $\hat{\rho}$, $\hat{\beta}$, while SGMM performs better for $\hat{\gamma}$, $\hat{\lambda}$.

The above results are crucial for empirical researchers. We suggest to gather samples of a large number of cross-sectional units and use QMLE first. If the estimate of ρ is close to 1, re-estimate the model using SGMM to obtain more accurate estimates for the autoregressive parameters.

We note that estimates for $|\hat{\lambda}|$ is downward biased in most cases, especially when the dynamic effect and spatial dependence are large under heteroscedastic disturbances. Lee and Yu [16–18] have proposed a spatial differencing transformation method to deal with spatial unit root which can be applied to the model. Another interesting finding is that the choice of spatial weights matrices affects the finite sample performance and the prior setting of spatial interaction structure may lead to non-robustness empirical results. Therefore, specification test for spatial weights matrices and some new estimation methods for endogenous spatial weights matrices are strongly required for this model. Moreover, the results are different depending on the sign of spatial dependence, although the results are slightly better under negative spatial dependence. A detailed justification on the effects of the sign of spatial dependence is strongly required.

These further extensions are beyond the scope of the present paper and will certainly demand further studies.

Notes

1. Sarafidis [19] considers the model with an $MA(1)$ type spatial dependence which refers to the localized transmission of shocks to cross-sectional units. Here, we are interested in globalized transmission of shocks, which is described as an $AR(1)$ type spatial error dependence.
2. We follow these computational notes in Section 3.3. of [6] in simulation exercises.
3. For example, see the application in Lin and Lee [7].
4. See Arellano [20] for a recent review.
5. If there is unknown heteroscedasticity, $\Sigma_n = \mathrm{diag}(\sigma^2_{\varepsilon 1}, \ldots, \sigma^2_{\varepsilon n})$ and consistent estimation of Σ_n can be obtained similarly as before.
6. In the early version of the paper, we use the identity matrix as the weighting matrix for all the following three steps. From the simulation results, we find that using the optimal weighting matrix leads to efficiency gains when n is small, whereas there are no significant differences when n is large.
7. We generate the matrix for 30 provinces except Tibet, Hong Kong, Macao and Taiwan. We consider Guangdong proximity to Hainan province.
8. In the Monte Carlo, we focus on the case of homoscedasticity and $\rho = 0.5$ if not specified.
9. For instance, consider a simple spatial autoregressive model, $Y = \lambda WY + e$, where Y is an $N \times 1$ vector of observations, W is the spatial weight matrix. The variance–covariance matrix of Y can be written as $Var(Y) = (I - \lambda W)((I - \lambda W)')^{-1}\sigma^2_e$. The structure can be different for positive and negative spatial dependence because of the interaction term λW.
10. We run the simulations with MATLAB R2015a for Windows, using a PC with a Pentium 2.6 GHz processor and 8GB memory. The computational time for QMLE results in ranges from around 52 minutes to more than 2 hours.It is an extremely computational burdensome for the case the

dynamic effect and spatial dependence are large. Thus, we only consider 500 replications for QMLE.

11. We follow [6] to treat m as a known parameter here because in an empirical study we can get an approximate knowledge of what the m value is.
12. We report the results of SGMM for $\hat{\gamma}$ by both Equation (13) (with subscript 2) and Equation (30) (with subscript 1) in the tables. For all the Monte Carlo experiments, $\hat{\gamma}$ in the SGMM framework by Equation (13) after step 3 are much better than (30). We point out it here and sketch it in the main article.
13. All the simulation results are given in the Appendix.

Acknowledgments

We thank the associate editor and the referee for their valuable suggestions to improve the quality of the paper. The first author is also grateful to Isabella WolfsKeil, Ji Feng and Zichao Yang for help comments and discussions.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was funded by the Innovation Fund of Huazhong University of Science & Technology [Grant 2016AA001].

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Appendix. Simulation results

Table A1. Unknown heteroscedasticity (a circular and real-world spatial weights matrix, $\lambda = 0.2$).

		$m = 6$				Homoscedasticity			Heteroscedasticity		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.5022	0.0381	0.0381	0.5005	0.0442	0.0441
				β		2.0029	0.0590	0.0591	2.0033	0.0766	0.0766
				γ		0.9827	0.2896	0.2898	0.9752	0.3193	0.3200
				λ		0.1839	0.1588	0.1594	0.1448	0.1495	0.1592
		100	7	ρ		0.5018	0.0212	0.0213	0.5029	0.0281	0.0283
				β		2.0005	0.0219	0.0219	2.0015	0.0300	0.0300
				γ		1.0058	0.1619	0.1618	0.9983	0.1913	0.1911
				λ		0.2047	0.0612	0.0614	0.1610	0.0650	0.0758
	SGMM	50	3	ρ		0.4996	0.0403	0.0403	0.4951	0.0511	0.0513
				β		2.0037	0.0814	0.0815	2.0010	0.1035	0.1035
				γ	1	0.9772	2.5980	2.5974	1.0057	2.8170	2.8163
					2	0.9998	0.3137	0.3136	1.0090	0.3764	0.3764
				λ		0.1743	0.2182	0.2196	0.1786	0.2491	0.2499
		100	7	ρ		0.4951	0.0225	0.0230	0.4892	0.0293	0.0312
				β		1.9953	0.0780	0.0781	1.9877	0.1047	0.1054
				γ	1	1.0901	0.9612	0.9652	1.1842	1.2070	1.2207
					2	1.0179	0.1871	0.1879	1.0258	0.2134	0.2156
				λ		0.1917	0.0623	0.0629	0.1896	0.0739	0.0746
Real world	QMLE	30	3	ρ		0.5047	0.0464	0.0465	0.5047	0.0471	0.0473
				β		2.0052	0.0808	0.0809	2.0048	0.0809	0.0809
				γ		0.9702	0.3743	0.3751	0.9846	0.4033	0.4032
				λ		0.2015	0.1872	0.1871	0.2001	0.1684	0.1682
		90	7	ρ		0.5019	0.0213	0.0214	0.5019	0.0244	0.0245
				β		2.0014	0.0247	0.0247	2.0015	0.0261	0.0261
				γ		0.9954	0.1779	0.1778	1.0113	0.1877	0.1879
				λ		0.2035	0.0704	0.0704	0.2027	0.0643	0.0643
	SGMM	30	3	ρ		0.4981	0.0594	0.0594	0.4976	0.0587	0.0588
				β		2.0097	0.1219	0.1222	2.0078	0.1210	0.1212
				γ	1	1.0160	2.9135	2.9128	1.1747	2.8046	2.8094
					2	0.9936	0.4514	0.4514	1.0098	0.4521	0.4521
				λ		0.1751	0.2588	0.2599	0.1800	0.2314	0.2322
		90	7	ρ		0.4949	0.0248	0.0254	0.4944	0.0252	0.0259
				β		1.9945	0.0855	0.0857	1.9942	0.0884	0.0886
				γ	1	1.0803	1.0333	1.0362	1.0761	1.0195	1.0221
					2	1.0103	0.2039	0.2046	1.0153	0.2105	0.2110
				λ		0.1919	0.0622	0.0627	0.1929	0.0557	0.0561

Table A2. Unknown heteroscedasticity (a circular and real-world spatial weights matrix, $\lambda = -0.2$).

W_n		$m=6$				Homoscedasticity			Heteroscedasticity		
		N	T			Mean	SD	RMSE	Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.5017	0.0346	0.0346	0.5017	0.0470	0.0470
				β		2.0032	0.0563	0.0564	2.0049	0.0763	0.0764
				γ		0.9862	0.2776	0.2776	0.9748	0.3000	0.3008
				λ		−0.2071	0.1532	0.1532	−0.1388	0.1321	0.1455
		100	7	ρ		0.5036	0.0277	0.0280	0.5045	0.0338	0.0340
				β		2.0015	0.0232	0.0232	2.0020	0.0306	0.0306
				γ		0.9996	0.1662	0.1661	0.9881	0.1878	0.1880
				λ		−0.1952	0.0672	0.0673	−0.1344	0.0613	0.0897
	SGMM	50	3	ρ		0.4997	0.0403	0.0403	0.4959	0.0509	0.0511
				β		2.0035	0.0806	0.0807	2.0018	0.1046	0.1045
				γ	1	1.0600	1.6797	1.6803	0.9835	2.1424	2.1419
					2	0.9977	0.2970	0.2970	1.0055	0.3515	0.3515
				λ		−0.1765	0.2248	0.2260	−0.1526	0.2627	0.2669
		100	7	ρ		0.4949	0.0231	0.0237	0.4885	0.0302	0.0323
				β		1.9944	0.0796	0.0798	1.9859	0.1064	0.1073
				γ	1	1.0859	0.6349	0.6405	1.1795	0.8009	0.8206
					2	1.0182	0.1917	0.1925	1.0292	0.2250	0.2303
				λ		−0.1897	0.0676	0.0684	−0.1788	0.0805	0.0832
Real world	QMLE	30	3	ρ		0.5064	0.0515	0.0519	0.5064	0.0512	0.0515
				β		2.0063	0.0803	0.0804	2.0063	0.0814	0.0816
				γ		0.9597	0.3552	0.3571	1.0003	0.3665	0.3662
				λ		−0.2061	0.1942	0.1941	−0.2084	0.1748	0.1748
		90	7	ρ		0.5026	0.0248	0.0250	0.5036	0.0276	0.0278
				β		2.0020	0.0243	0.0244	2.0025	0.0253	0.0254
				γ		0.9874	0.1735	0.1738	0.9827	0.1895	0.1901
				λ		−0.1967	0.0672	0.0672	−0.1963	0.0629	0.0630
	SGMM	30	3	ρ		0.4979	0.0589	0.0589	0.4974	0.0586	0.0587
				β		2.0083	0.1190	0.1193	2.0056	0.1211	0.1212
				γ	1	0.9780	2.0611	2.0607	1.0467	1.9141	1.9142
					2	0.9966	0.4246	0.4245	1.0058	0.4284	0.4284
				λ		−0.1651	0.2796	0.2817	−0.1650	0.2528	0.2552
		90	7	ρ		0.4947	0.0256	0.0261	0.4942	0.0267	0.0273
				β		1.9934	0.0859	0.0861	1.9938	0.0893	0.0895
				γ	1	1.0587	0.6586	0.6611	1.0674	0.6934	0.6965
					2	1.0197	0.2071	0.2080	1.0197	0.2128	0.2140
				λ		−0.1897	0.0709	0.0716	−0.1901	0.0631	0.0639

Table A3. Unknown heteroscedasticity (a circular and real-world spatial weights matrix, $\lambda = 0.5$).

W_n		$m=6$				Homoscedasticity			Heteroscedasticity		
		N	T			Mean	SD	RMSE	Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.5006	0.0343	0.0343	0.4999	0.0449	0.0448
				β		2.0015	0.0560	0.0559	2.0026	0.0767	0.0766
				γ		0.9787	0.3171	0.3175	0.9691	0.3495	0.3505
				λ		0.4944	0.1085	0.1085	0.4106	0.1208	0.1502
		100	7	ρ		0.5052	0.0323	0.0327	0.5040	0.0306	0.0308
				β		2.0020	0.0243	0.0243	2.0020	0.0305	0.0305
				γ		0.9870	0.2152	0.2153	0.9978	0.2092	0.2090
				λ		0.4932	0.0923	0.0924	0.4229	0.0718	0.1054
	SGMM	50	3	ρ		0.4993	0.0397	0.0397	0.4956	0.0505	0.0507
				β		2.0020	0.0796	0.0796	2.0023	0.1015	0.1015
				γ	1	1.0632	3.1487	3.1485	0.9658	4.4448	4.4438
					2	1.0056	0.3571	0.3571	1.0047	0.4171	0.4170
				λ		0.4459	0.1875	0.1951	0.4375	0.2165	0.2253

(*continued*)

Table A3. Continued.

W_n		$m=6$ N	T			Homoscedasticity Mean	SD	RMSE	Heteroscedasticity Mean	SD	RMSE
		100	7	ρ		0.4961	0.0219	0.0222	0.4911	0.0280	0.0293
				β		1.9966	0.0768	0.0769	1.9913	0.1017	0.1020
				γ	1	1.0697	1.2170	1.2187	1.0832	1.4071	1.4092
					2	1.0144	0.2138	0.2142	1.0201	0.2332	0.2348
				λ		0.4810	0.0516	0.0550	0.4727	0.0644	0.0699
Real world	QMLE	30	3	ρ		0.5029	0.0416	0.0416	0.5068	0.0596	0.0599
				β		2.0034	0.0765	0.0765	2.0072	0.0904	0.0906
				γ		0.9923	0.4482	0.4478	1.0058	0.4634	0.4630
				λ		0.5087	0.1410	0.1412	0.5054	0.1313	0.1313
		90	7	ρ		0.5046	0.0341	0.0343	0.5032	0.0286	0.0287
				β		2.0019	0.0258	0.0259	2.0016	0.0258	0.0258
				γ		0.9830	0.2448	0.2451	0.9987	0.2226	0.2224
				λ		0.5013	0.0785	0.0785	0.5028	0.0735	0.0735
	SGMM	30	3	ρ		0.4974	0.0576	0.0576	0.4977	0.0587	0.0588
				β		2.0068	0.1240	0.1241	2.0073	0.1223	0.1225
				γ	1	1.0503	3.5791	3.5785	1.2730	3.5274	3.5370
					2	1.0065	0.5039	0.5039	1.0073	0.5027	0.5026
				λ		0.4357	0.2248	0.2338	0.4415	0.2005	0.2088
		90	7	ρ		0.4959	0.0238	0.0242	0.4957	0.0242	0.0245
				β		1.9961	0.0828	0.0828	1.9959	0.0855	0.0856
				γ	1	1.0694	1.2666	1.2682	1.0538	1.2669	1.2677
					2	0.9971	0.2213	0.2213	1.0109	0.2309	0.2311
				λ		0.4810	0.0531	0.0564	0.4817	0.0487	0.0520

Table A4. Unknown heteroscedasticity (a circular and real-world spatial weights matrix, $\lambda = -0.5$).

W_n		$m=6$ N	T			Homoscedasticity Mean	SD	RMSE	Heteroscedasticity Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.5034	0.0422	0.0423	0.5047	0.0567	0.0568
				β		2.0050	0.0594	0.0595	2.0075	0.0803	0.0805
				γ		0.9867	0.2886	0.2886	0.9770	0.3206	0.3211
				λ		−0.4904	0.1289	0.1292	−0.3364	0.1100	0.1971
		100	7	ρ		0.5051	0.0294	0.0298	0.5069	0.0374	0.0380
				β		2.0023	0.0239	0.0240	2.0033	0.0320	0.0322
				γ		0.9849	0.1951	0.1955	0.9842	0.2092	0.2096
				λ		−0.4878	0.0672	0.0683	−0.3426	0.0441	0.1635
	SGMM	50	3	ρ		0.5006	0.0392	0.0392	0.4971	0.0501	0.0501
				β		2.0041	0.0791	0.0792	2.0032	0.1012	0.1012
				γ	1	1.0653	1.4510	1.4521	0.9703	2.5146	2.5141
					2	0.9960	0.2970	0.2970	1.0137	0.3444	0.3445
				λ		−0.4392	0.2087	0.2174	−0.4022	0.2508	0.2692
		100	7	ρ		0.4958	0.0233	0.0237	0.4899	0.0304	0.0320
				β		1.9954	0.0790	0.0791	1.9872	0.1055	0.1063
				γ	1	1.0611	0.5028	0.5064	1.1289	0.6412	0.6539
					2	1.0169	0.2068	0.2074	1.0383	0.2412	0.2441
				λ		−0.4754	0.0644	0.0689	−0.4565	0.0825	0.0933
Real world	QMLE	30	3	ρ		0.5092	0.0612	0.0618	0.5092	0.0612	0.0618
				β		2.0081	0.0828	0.0831	2.0086	0.0829	0.0832
				γ		0.9721	0.3901	0.3907	0.9792	0.3881	0.3883
				λ		−0.5049	0.1839	0.1838	−0.5032	0.1650	0.1648
		90	7	ρ		0.5074	0.0367	0.0374	0.5084	0.0400	0.0408
				β		2.0040	0.0256	0.0259	2.0045	0.0266	0.0270
				γ		0.9789	0.2087	0.2096	0.9782	0.2143	0.2152
				λ		−0.4824	0.0899	0.0916	−0.4790	0.0825	0.0851

(*continued*)

Table A4. Continued.

W_n		$m=6$ N	T			Homoscedasticity Mean	SD	RMSE	Heteroscedasticity Mean	SD	RMSE
	SGMM	30	3	ρ		0.4992	0.0568	0.0568	0.4983	0.0569	0.0570
				β		2.0078	0.1177	0.1180	2.0068	0.1172	0.1173
				γ	1	0.9904	1.7870	1.7866	1.0071	1.4979	1.4975
					2	0.9896	0.4157	0.4157	0.9977	0.4290	0.4289
				λ		−0.4069	0.2799	0.2949	−0.4124	0.2570	0.2714
		90	7	ρ		0.4960	0.0260	0.0263	0.4958	0.0268	0.0271
				β		1.9957	0.0844	0.0845	1.9959	0.0867	0.0868
				γ	1	1.0409	0.5299	0.5313	1.0372	0.5623	0.5634
					2	1.0147	0.2164	0.2169	1.0102	0.2209	0.2212
				λ		−0.4713	0.0773	0.0724	−0.4731	0.0692	0.0743

Table A5. Unknown heteroscedasticity (a circular and real-world spatial weights matrix, $\lambda = 0.8$)

W_n		$m=6$ N	T			Homoscedasticity Mean	SD	RMSE	Heteroscedasticity Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.5583	0.1530	0.1636	0.5543	0.1566	0.1656
				β		2.0508	0.1459	0.1544	2.0487	0.1571	0.1643
				γ		0.9049	0.6571	0.6633	0.8964	0.5869	0.5954
				λ		0.7162	0.1616	0.1819	0.6565	0.1423	0.2020
		100	7	ρ		0.5049	0.0620	0.0622	0.5167	0.0776	0.0793
				β		2.0021	0.0353	0.0353	2.0082	0.0479	0.0486
				γ		0.9997	0.3259	0.3256	0.9543	0.3752	0.3776
				λ		0.7376	0.1851	0.1951	0.6603	0.2073	0.2498
	SGMM	50	3	ρ		0.4991	0.0380	0.0380	0.4966	0.0495	0.0496
				β		2.0020	0.0780	0.0780	2.0010	0.1015	0.1015
				γ	1	1.0294	3.5900	3.5893	1.0778	4.4024	4.4020
					2	1.0084	0.4092	0.4091	1.0050	0.4620	0.4619
				λ		0.7230	0.1306	0.1516	0.7077	0.1538	0.1794
		100	7	ρ		0.4972	0.0218	0.0220	0.4930	0.0269	0.0278
				β		1.9983	0.0744	0.0744	1.9934	0.0972	0.0974
				γ	1	1.0034	1.3630	1.3627	0.9784	1.5110	1.5108
					2	0.9998	0.2502	0.2501	1.0112	0.2731	0.2733
				λ		0.7673	0.0412	0.0526	0.7594	0.0504	0.0647
Real world	QMLE	30	3	ρ		0.5134	0.0839	0.0849	0.5266	0.1127	0.1156
				β		2.0132	0.0974	0.0982	2.0221	0.1181	0.1201
				γ		1.0033	0.6060	0.6054	0.9425	0.6677	0.6695
				λ		0.7656	0.1354	0.1396	0.7479	0.1537	0.1621
		90	7	ρ		0.5167	0.0616	0.0638	0.5231	0.0775	0.0808
				β		2.0074	0.0373	0.0380	2.0102	0.0425	0.0437
				γ		0.9580	0.3173	0.3197	0.9351	0.3330	0.3389
				λ		0.7398	0.2033	0.2119	0.7191	0.2307	0.2442
	SGMM	30	3	ρ		0.4968	0.0579	0.0580	0.4953	0.0596	0.0597
				β		2.0087	0.1289	0.1292	2.0087	0.1290	0.1293
				γ	1	0.9243	3.4265	3.4264	1.0919	3.3893	3.3897
					2	1.0220	0.5873	0.5876	1.0162	0.5979	0.5980
				λ		0.6874	0.1773	0.2100	0.6872	0.1647	0.1995
		90	7	ρ		0.4972	0.0231	0.0233	0.4971	0.0229	0.0231
				β		1.9970	0.0804	0.0805	1.9977	0.0812	0.0812
				γ	1	1.0029	1.3034	1.3031	0.9813	1.3524	1.3521
					2	0.9976	0.2727	0.2726	1.0054	0.2718	0.2718
				λ		0.7633	0.0492	0.0614	0.7598	0.0499	0.0641

Table A6. Unknown heteroscedasticity (a circular and real-world spatial weights matrix, $\lambda = -0.8$).

W_n		$m=6$ N	T			Homoscedasticity Mean	SD	RMSE	Heteroscedasticity Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.5220	0.0844	0.0871	0.5127	0.0728	0.0738
				β		2.0193	0.0796	0.0818	2.0134	0.0842	0.0852
				γ		0.9498	0.3615	0.3646	0.9707	0.3395	0.3404
				λ		−0.7631	0.1133	0.1191	−0.5903	0.0975	0.2312
		100	7	ρ		0.5193	0.0610	0.0639	0.5144	0.0540	0.0558
				β		2.0077	0.0316	0.0324	2.0057	0.0342	0.0346
				γ		0.9420	0.2492	0.2556	0.9568	0.2481	0.2516
				λ		−0.7539	0.1242	0.1324	−0.5992	0.0818	0.2168
	SGMM	50	3	ρ		0.5020	0.0380	0.0380	0.4990	0.0484	0.0484
				β		2.0056	0.0773	0.0775	2.0041	0.0996	0.0996
				γ	1	1.0756	1.1688	1.1709	1.0382	1.3941	1.3943
					2	0.9916	0.3007	0.3007	0.9951	0.3320	0.3320
				λ		−0.7077	0.1692	0.1927	−0.6753	0.2047	0.2396
		100	7	ρ		0.4971	0.0235	0.0237	0.4923	0.0297	0.0307
				β		1.9965	0.0762	0.0763	1.9901	0.0985	0.0989
				γ	1	1.0391	0.4369	0.4385	1.0844	0.5297	0.5362
					2	1.0105	0.2141	0.2146	1.0371	0.2448	0.2475
				λ		−0.7637	0.0554	0.0662	−0.7488	0.0725	0.0887
Real world	QMLE	30	3	ρ		0.5136	0.0685	0.0698	0.5141	0.0702	0.0715
				β		2.0117	0.0803	0.0811	2.0128	0.0819	0.0828
				γ		0.9627	0.4396	0.4407	0.9671	0.4101	0.4110
				λ		−0.7864	0.1781	0.1785	−0.7813	0.1633	0.1642
		90	7	ρ		0.5143	0.0489	0.0509	0.5142	0.0472	0.0492
				β		2.0068	0.0278	0.0286	2.0066	0.0261	0.0269
				γ		0.9680	0.2319	0.2339	0.9583	0.2307	0.2343
				λ		−0.7435	0.1571	0.1668	−0.7348	0.1537	0.1668
	SGMM	30	3	ρ		0.4998	0.0557	0.0557	0.4996	0.0544	0.0544
				β		2.0081	0.1173	0.1175	2.0097	0.1138	0.1142
				γ	1	1.0081	1.3160	1.3157	0.9922	1.6369	1.6365
					2	0.9931	0.4334	0.4334	0.9987	0.4311	0.4310
				λ		−0.6311	0.2677	0.2765	−0.6418	0.2475	0.2937
		90	7	ρ		0.4974	0.0259	0.0260	0.4976	0.0263	0.0264
				β		1.9969	0.0809	0.0809	1.9978	0.0817	0.0817
				γ	1	1.0275	0.4785	0.4792	1.0227	0.4943	0.4947
					2	1.0102	0.2282	0.2283	1.0086	0.2323	0.2324
				λ		−0.7458	0.0893	0.1044	−0.7485	0.0817	0.0965

Table A7. Unknown heteroscedasticity (group interaction spatial weights matrix, $\lambda = \pm 0.2$).

		$m=6$ R	T			Homoscedasticity Mean	SD	RMSE	Heteroscedasticity Mean	SD	RMSE
$\lambda = 0.2$	QMLE	15	3	ρ		0.5021	0.0291	0.0292	0.5017	0.0647	0.0647
				β		2.0037	0.0488	0.0489	2.0002	0.1108	0.1107
				γ		1.0029	0.2378	0.2375	1.0114	0.3753	0.3751
				λ		0.1847	0.1059	0.1069	0.2051	0.1122	0.1122
		30	7	ρ		0.5046	0.0340	0.0343	0.5047	0.0374	0.0377
				β		2.0027	0.0234	0.0235	2.0056	0.0404	0.0407
				γ		0.9890	0.1792	0.1794	0.9961	0.2068	0.2066
				λ		0.1885	0.0563	0.0574	0.1919	0.0415	0.0422
	SGMM	15	3	ρ		0.5003	0.0314	0.0314	0.4899	0.0666	0.0674
				β		2.0040	0.0602	0.0603	1.9970	0.1310	0.1310
				γ	1	1.0027	2.6441	2.6435	1.1657	4.6482	4.6500
					2	0.9875	0.2511	0.2514	1.0305	0.4299	0.4309
				λ		0.1773	0.1490	0.1507	0.1831	0.1677	0.1685

(*continued*)

Table A7. Continued.

		$m=6$				Homoscedasticity			Heteroscedasticity		
		R	T			Mean	SD	RMSE	Mean	SD	RMSE
		30	7	ρ		0.4960	0.0200	0.0204	0.4809	0.0366	0.0413
				β		1.9985	0.0693	0.0693	1.9802	0.1321	0.1335
				γ	1	1.0971	0.9196	0.9245	1.3773	1.4775	1.5246
					2	1.0151	0.1709	0.1715	1.0642	0.2464	0.2546
				λ		0.1930	0.0499	0.0503	0.1934	0.0564	0.0568
$\lambda=-0.2$	QMLE	15	3	ρ		0.5019	0.0298	0.0298	0.5036	0.0612	0.0612
				β		2.0028	0.0506	0.0506	1.9999	0.1044	0.1043
				γ		0.9961	0.2189	0.2187	0.9945	0.3413	0.3410
				λ		−0.1633	0.0961	0.1028	−0.1807	0.1071	0.1087
		30	7	ρ		0.5019	0.0207	0.0208	0.5049	0.0412	0.0415
				β		1.9997	0.0189	0.0188	2.0010	0.0399	0.0399
				γ		0.9997	0.1560	0.1559	0.9794	0.2133	0.2141
				λ		−0.1807	0.0491	0.0527	−0.1962	0.0621	0.0622
	SGMM	15	3	ρ		0.4996	0.0336	0.0336	0.4902	0.0663	0.0670
				β		2.0017	0.0647	0.0647	1.9957	0.1345	0.1345
				γ	1	1.0130	1.5283	1.5279	1.1767	2.9474	2.9520
					2	0.9989	0.2516	0.2516	1.0257	0.4208	0.4215
				λ		−0.1850	0.1609	0.1616	−0.1970	0.1844	0.1844
		30	7	ρ		0.4958	0.0213	0.0217	0.4806	0.0373	0.0420
				β		1.9968	0.0671	0.0671	1.9801	0.1268	0.1283
				γ	1	1.0813	0.6635	0.6683	1.3059	1.0560	1.0992
					2	1.0106	0.1821	0.1824	1.0651	0.2506	0.2588
				λ		−0.1930	0.0650	0.0654	−0.1935	0.0541	0.0545

Table A8. Unknown heteroscedasticity (group interaction spatial weights matrix, $\lambda=\pm0.5$).

		$m=6$				Homoscedasticity			Heteroscedasticity		
		R	T			Mean	SD	RMSE	Mean	SD	RMSE
$\lambda=0.5$	QMLE	15	3	ρ		0.4990	0.0344	0.0344	0.5286	0.1230	0.1262
				β		2.0026	0.0542	0.0542	2.0233	0.1396	0.1413
				γ		0.9971	0.2761	0.2759	0.9419	0.5170	0.5197
				λ		0.4900	0.0962	0.0966	0.4794	0.0782	0.0808
		30	7	ρ		0.5028	0.0215	0.0216	0.5043	0.0363	0.0365
				β		2.0018	0.0204	0.0205	2.0018	0.0392	0.0392
				γ		0.9908	0.1792	0.1792	0.9949	0.2191	0.2190
				λ		0.4897	0.0635	0.0642	0.4877	0.0629	0.0640
	SGMM	15	3	ρ		0.5005	0.0342	0.0342	0.4917	0.0631	0.0636
				β		1.9998	0.0672	0.0671	1.9962	0.1275	0.1275
				γ	1	1.0398	2.6501	2.6497	1.1893	4.6835	4.6861
					2	0.9965	0.2901	0.2901	1.0172	0.4580	0.4582
				λ		0.4529	0.1697	0.1760	0.4522	0.1424	0.1501
		30	7	ρ		0.4983	0.0175	0.0176	0.4845	0.0332	0.0366
				β		1.9994	0.0611	0.0611	1.9829	0.1220	0.1231
				γ	1	1.0673	1.0915	1.0933	1.2859	1.6781	1.7019
					2	1.0064	0.1792	0.1792	1.0312	0.2516	0.2563
				λ		0.4852	0.0452	0.0450	0.4842	0.0416	0.0445
$\lambda=-0.5$	QMLE	15	3	ρ		0.5025	0.0344	0.0345	0.5040	0.0659	0.0660
				β		2.0010	0.0506	0.0506	2.0087	0.1065	0.1068
				γ		1.0018	0.2488	0.2485	0.9914	0.3301	0.3299
				λ		−0.4325	0.1010	0.1214	−0.3962	0.0857	0.1346
		30	7	ρ		0.5024	0.0223	0.0224	0.5087	0.0433	0.0441
				β		2.0003	0.0193	0.0193	2.0005	0.0398	0.0398
				γ		0.9833	0.1625	0.1632	0.9827	0.2138	0.2142
				λ		−0.4613	0.0543	0.0666	−0.4445	0.0423	0.0698

(*continued*)

Table A8. Continued.

	$m=6$				Homoscedasticity			Heteroscedasticity		
	R	T			Mean	SD	RMSE	Mean	SD	RMSE
SGMM	15	3	ρ		0.5007	0.0326	0.0326	0.4921	0.0660	0.0664
			β		2.0023	0.0628	0.0628	1.9948	0.1355	0.1356
			γ	1	1.0098	1.2947	1.2944	1.0716	2.8767	2.8769
				2	1.0001	0.2549	0.2548	1.0239	0.3991	0.3997
			λ		−0.4651	0.1766	0.1800	−0.4617	0.2344	0.2375
	30	7	ρ		0.4979	0.0179	0.0180	0.4825	0.0366	0.0406
			β		1.9983	0.0582	0.0582	1.9830	0.1293	0.1304
			γ	1	1.0410	0.4819	0.4835	1.2836	0.9250	0.9673
				2	1.0104	0.1706	0.1711	1.0652	0.2579	0.2660
			λ		−0.4803	0.0637	0.0667	−0.4768	0.0679	0.0717

Table A9. Unknown heteroscedasticity (group interaction spatial weights matrix, $\lambda = \pm 0.8$).

		$m=6$				Homoscedasticity			Heteroscedasticity		
		R	T			Mean	SD	RMSE	Mean	SD	RMSE
$\lambda = 0.8$	QMLE	15	3	ρ		0.5643	0.1585	0.1709	0.4893	0.1609	0.1611
				β		2.0521	0.1381	0.1475	1.9883	0.1612	0.1615
				γ		0.8588	0.6339	0.6489	1.0017	0.7485	0.7478
				λ		0.6874	0.1663	0.2006	0.7786	0.1848	0.1859
		30	7	ρ		0.5080	0.0424	0.0431	0.5883	0.1557	0.1789
				β		2.0042	0.0255	0.0258	2.0400	0.0803	0.0897
				γ		0.9852	0.2307	0.2309	0.7703	0.5060	0.5552
				λ		0.7521	0.1698	0.1763	0.5728	0.1634	0.2797
	SGMM	15	3	ρ		0.5013	0.0306	0.0307	0.4915	0.0632	0.0637
				β		2.0040	0.0587	0.0588	2.0033	0.1274	0.1274
				γ	1	0.8644	3.0672	3.0694	0.9376	7.3236	7.3220
					2	0.9887	0.3246	0.3247	1.0130	0.4989	0.4989
				λ		0.7311	0.0986	0.1202	0.7155	0.1098	0.1385
		30	7	ρ		0.4986	0.0159	0.0159	0.4870	0.0323	0.0348
				β		1.9988	0.0533	0.0533	1.9890	0.1202	0.1207
				γ	1	1.0398	1.2064	1.2068	1.1397	1.6770	1.6823
					2	1.0030	0.1969	0.1968	1.0358	0.2781	0.2804
				λ		0.7671	0.0416	0.0530	0.7550	0.0501	0.0674
$\lambda = -0.8$	QMLE	15	3	ρ		0.5077	0.0502	0.0507	0.5061	0.0751	0.0752
				β		2.0082	0.0550	0.0555	2.0091	0.1111	0.1114
				γ		0.9865	0.2492	0.2493	0.9868	0.3647	0.3646
				λ		−0.6756	0.1029	0.1614	−0.7335	0.1024	0.1220
		30	7	ρ		0.5055	0.0295	0.0300	0.5264	0.0681	0.0729
				β		2.0007	0.0196	0.0196	2.0110	0.0481	0.0493
				γ		0.9886	0.1822	0.1824	0.9191	0.2908	0.3016
				λ		−0.6916	0.0858	0.1382	−0.7327	0.0976	0.1185
	SGMM	15	3	ρ		0.5040	0.0292	0.0295	0.4948	0.0636	0.0638
				β		2.0045	0.0549	0.0550	1.9997	0.1393	0.1393
				γ	1	0.9867	1.1385	1.1383	1.1799	2.7779	2.7831
					2	0.9776	0.2397	0.2407	1.0188	0.4040	0.4044
				λ		−0.7206	0.1584	0.1771	−0.6933	0.1773	0.2069
		30	7	ρ		0.4995	0.0178	0.0179	0.4866	0.0352	0.0376
				β		2.0010	0.0554	0.0554	1.9872	0.1183	0.1189
				γ	1	1.0086	0.4414	0.4413	1.1888	0.7202	0.7443
					2	1.0012	0.1761	0.1760	1.0566	0.2604	0.2664
				λ		−0.7585	0.0697	0.0811	−0.7512	0.0762	0.0905

Table A10. Persistent series (a circular and real-world spatial weights matrix, $\lambda = 0.2$).

		$m = 6$				$\rho = 0.9$			$\rho = -0.9$		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.9376	0.1081	0.1143	−0.9179	0.0312	0.0360
				β		2.0365	0.1190	0.1244	1.9960	0.0549	0.0550
				γ		0.8391	0.8568	0.8709	0.9869	0.2799	0.2799
				λ		0.1610	0.2835	0.2859	0.1816	0.1489	0.1498
		100	7	ρ		0.9011	0.0293	0.0292	−0.9143	0.0177	0.0228
				β		2.0008	0.0312	0.0312	1.9979	0.0210	0.0211
				γ		0.9964	0.2504	0.2502	1.0216	0.1543	0.1556
				λ		0.1803	0.1564	0.1575	0.1818	0.0766	0.0787
	SGMM	50	3	ρ		0.9079	0.0266	0.0278	−0.8999	0.0121	0.0121
				β		2.0175	0.1104	0.1118	2.0114	0.0638	0.0648
				γ	1	1.6651	5.1479	5.1894	1.1264	4.0524	4.0534
					2	0.9638	0.3374	0.3392	0.9939	0.2822	0.2822
				λ		0.1618	0.2186	0.2219	0.1772	0.2183	0.2194
		100	7	ρ		0.9015	0.0139	0.0140	−0.8999	0.0052	0.0052
				β		1.9995	0.1732	0.1731	2.0229	0.0643	0.0682
				γ	1	2.2486	2.1087	2.4501	1.0664	1.9940	1.9946
					2	1.0048	0.2192	0.2192	0.9895	0.1594	0.1597
				λ		0.1783	0.0647	0.0682	0.1929	0.0623	0.0627
Real world	QMLE	30	3	ρ		0.9301	0.0963	0.1008	−0.9174	0.0318	0.0362
				β		2.0299	0.1149	0.1186	1.9930	0.0735	0.0737
				γ		0.8930	0.8964	0.9019	1.0097	0.3277	0.3275
				λ		0.1987	0.1969	0.1967	0.1901	0.1753	0.1754
		90	7	ρ		0.9035	0.0388	0.0390	−0.9141	0.0175	0.0224
				β		2.0025	0.0360	0.0360	1.9988	0.0217	0.0217
				γ		0.9581	0.3698	0.3718	1.0051	0.1607	0.1606
				λ		0.1813	0.1351	0.1363	0.1839	0.0715	0.0733
	SGMM	30	3	ρ		0.9068	0.0400	0.0406	−0.8995	0.0155	0.0155
				β		2.0247	0.1757	0.1774	2.0155	0.0985	0.0997
				γ	1	1.4816	7.1461	7.1605	0.9556	3.3084	3.3079
					2	0.9924	0.5476	0.5475	0.9965	0.3649	0.3648
				λ		0.1613	0.2599	0.2627	0.1740	0.2616	0.2629
		90	7	ρ		0.9017	0.0157	0.0157	−0.9000	0.0055	0.0055
				β		2.0047	0.1934	0.1934	2.0226	0.0743	0.0776
				γ	1	2.1336	2.1179	2.4017	1.0012	2.0140	2.0134
					2	0.9952	0.2349	0.2349	0.9913	0.1746	0.1747
				λ		0.1786	0.0661	0.0695	0.1928	0.0616	0.0620

Table A11. Persistent series (a circular and real-world spatial weights matrix, $\lambda = -0.2$).

		$m = 6$				$\rho = 0.9$			$\rho = -0.9$		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.9266	0.0925	0.0961	−0.9162	0.0306	0.0346
				β		2.0278	0.1023	0.1059	1.9970	0.0538	0.0538
				γ		0.8970	0.7347	0.7411	0.9908	0.2479	0.2478
				λ		−0.2038	0.2714	0.2712	−0.1184	0.2188	0.2333
		100	7	ρ		0.9037	0.0305	0.0307	−0.9115	0.0172	0.0207
				β		2.0024	0.0291	0.0291	1.9986	0.0210	0.0210
				γ		0.9834	0.2647	0.2650	1.0136	0.1536	0.1540
				λ		−0.2118	0.1440	0.1443	−0.1103	0.1634	0.1863
	SGMM	50	3	ρ		0.9082	0.0258	0.0271	−0.8999	0.0119	0.0119
				β		2.0179	0.1071	0.1086	2.0104	0.0631	0.0640
				γ	1	1.4762	5.0409	5.0620	1.0787	2.2332	2.2340
					2	0.9701	0.3258	0.3271	0.9931	0.2541	0.2542
				λ		−0.1491	0.2269	0.2325	−0.1836	0.2286	0.2291

(*continued*)

Table A11. Continued.

W_n		$m=6$ N	T			$\rho=0.9$ Mean	SD	RMSE	$\rho=-0.9$ Mean	SD	RMSE
		100	7	ρ		0.9013	0.0140	0.0140	−0.8999	0.0052	0.0052
				β		1.9992	0.1735	0.1735	2.0197	0.0654	0.0683
				γ	1	1.7865	1.4055	1.6103	1.0245	1.1484	1.1484
					2	0.9986	0.2233	0.2233	0.9893	0.1613	0.1616
				λ		−0.1693	0.0740	0.0801	−0.1945	0.0664	0.0666
Real world	QMLE	30	3	ρ		0.9330	0.1012	0.1064	−0.9168	0.0316	0.0358
				β		2.0313	0.1208	0.1247	1.9945	0.0723	0.0725
				γ		0.8311	0.8891	0.9041	1.0134	0.3152	0.3151
				λ		−0.1945	0.2188	0.2186	−0.1180	0.2547	0.2673
		90	7	ρ		0.9017	0.0291	0.0291	−0.9096	0.0163	0.0189
				β		2.0019	0.0298	0.0298	1.9997	0.0218	0.0218
				γ		0.9788	0.2914	0.2919	0.9995	0.1703	0.1701
				λ		−0.2046	0.1095	0.1095	−0.1277	0.1527	0.1688
	SGMM	30	3	ρ		0.9070	0.0382	0.0388	−0.8995	0.0156	0.0156
				β		2.0285	0.1737	0.1760	2.0165	0.0938	0.0952
				γ	1	1.2850	4.4795	4.4875	0.9674	2.2871	2.2868
					2	0.9792	0.4579	0.4583	0.9912	0.3382	0.3383
				λ		−0.1355	0.2857	0.2929	−0.1777	0.2789	0.2797
		90	7	ρ		0.9014	0.0158	0.0158	−0.8999	0.0055	0.0055
				β		1.9992	0.1918	0.1917	2.0226	0.0733	0.0767
				γ	1	1.7459	1.4769	1.6542	0.9490	1.1899	1.1906
					2	0.9950	0.2349	0.2349	0.9881	0.1719	0.1722
				λ		−0.1667	0.0794	0.0861	−0.1949	0.0697	0.0699

Table A12. Persistent series (a circular and real-world spatial weights matrix, $\lambda = 0.5$).

W_n		$m=6$ N	T			$\rho=0.9$ Mean	SD	RMSE	$\rho=-0.9$ Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.9569	0.1300	0.1418	−0.9198	0.0316	0.0373
				β		2.0555	0.1340	0.1449	1.9952	0.0535	0.0536
				γ		0.7400	1.0490	1.0797	0.9894	0.3121	0.3119
				λ		0.4564	0.1994	0.2039	0.4056	0.1659	0.1907
		100	7	ρ		0.9181	0.0706	0.0729	−0.9152	0.0171	0.0229
				β		2.0143	0.0592	0.0609	1.9977	0.0211	0.0212
				γ		0.8889	0.5830	0.5929	1.0221	0.1788	0.1800
				λ		0.4643	0.1602	0.1640	0.4027	0.1326	0.1644
	SGMM	50	3	ρ		0.9078	0.0265	0.0276	−0.9000	0.0118	0.0118
				β		2.0165	0.1125	0.1137	2.0096	0.0621	0.0628
				γ	1	1.6172	5.7297	5.7615	0.9662	4.1848	4.1839
					2	0.9703	0.3733	0.3743	1.0050	0.3209	0.3209
				λ		0.4045	0.2028	0.2241	0.4589	0.1842	0.1887
		100	7	ρ		0.9025	0.0131	0.0133	−0.8998	0.0051	0.0051
				β		2.0047	0.1706	0.1706	2.0230	0.0646	0.0686
				γ	1	2.3126	2.4887	2.8131	1.0304	2.4700	2.4695
					2	0.9848	0.2342	0.2346	0.9973	0.1893	0.1893
				λ		0.4479	0.0728	0.0895	0.4864	0.0507	0.0525
Real world	QMLE	30	3	ρ		0.9434	0.1136	0.1215	−0.9199	0.0318	0.0374
				β		2.0427	0.1332	0.1398	1.9940	0.0741	0.0743
				γ		0.6876	1.2274	1.2654	1.0137	0.4055	0.4053
				λ		0.4731	0.2088	0.2103	0.4031	0.2045	0.2261
		90	7	ρ		0.9143	0.0652	0.0667	−0.9134	0.0166	0.0213
				β		2.0109	0.0539	0.0549	1.9981	0.0216	0.0216
				γ		0.8999	0.5866	0.5945	1.0063	0.2159	0.2157
				λ		0.4638	0.1617	0.1655	0.4129	0.1324	0.1584

(continued)

Table A12. Continued.

		$m=6$				$\rho=0.9$			$\rho=-0.9$		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
	SGMM	30	3	ρ		0.9063	0.0379	0.0384	−0.8994	0.0153	0.0153
				β		2.0225	0.1902	0.1915	2.0170	0.0974	0.0988
				γ	1	1.3519	7.1870	7.1938	1.0043	3.7081	3.7071
					2	0.9822	0.5833	0.5834	1.0079	0.4141	0.4141
				λ		0.3921	0.2396	0.2627	0.4428	0.2275	0.2345
		90	7	ρ		0.9026	0.0146	0.0148	−0.9000	0.0055	0.0054
				β		2.0082	0.1907	0.1909	2.0223	0.0729	0.0762
				γ	1	2.1949	2.4699	2.7432	1.0388	2.5443	2.5439
					2	0.9845	0.2521	0.2525	0.9890	0.2043	0.2045
				λ		0.4445	0.0776	0.0954	0.4861	0.0512	0.0531

Table A13. Persistent series (a circular and real-world spatial weights matrix, $\lambda=-0.5$).

		$m=6$				$\rho=0.9$			$\rho=-0.9$		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.9224	0.0848	0.0876	−0.9117	0.0279	0.0303
				β		2.0245	0.0966	0.0995	1.9986	0.0537	0.0537
				γ		0.9002	0.6474	0.6544	0.9895	0.2603	0.2602
				λ		−0.4861	0.2528	0.2530	−0.3868	0.2703	0.2928
		100	7	ρ		0.9038	0.0274	0.0276	−0.9068	0.0143	0.0158
				β		2.0026	0.0281	0.0282	1.9992	0.0202	0.0202
				γ		0.9732	0.2913	0.2923	1.0148	0.1571	0.1577
				λ		−0.5075	0.1504	0.1504	−0.4219	0.1877	0.2032
	SGMM	50	3	ρ		0.9080	0.0249	0.0262	−0.9000	0.0116	0.0116
				β		2.0144	0.1075	0.1085	2.0111	0.0605	0.0615
				γ	1	1.4135	2.5754	2.6078	1.0441	2.2792	2.2791
					2	0.9650	0.3085	0.3104	0.9938	0.2561	0.2561
				λ		−0.3793	0.2300	0.2597	−0.4544	0.2065	0.2114
		100	7	ρ		0.9015	0.0135	0.0136	−0.8998	0.0051	0.0051
				β		2.0025	0.1722	0.1722	2.0196	0.0645	0.0674
				γ	1	1.5300	1.0809	1.2036	0.9891	0.8553	0.8551
					2	0.9958	0.2226	0.2226	0.9910	0.1677	0.1679
				λ		−0.4298	0.0945	0.1177	−0.4848	0.0604	0.0622
Real world	QMLE	30	3	ρ		0.9248	0.0869	0.0903	−0.9146	0.0316	0.0348
				β		2.0233	0.1030	0.1055	1.9961	0.0721	0.0722
				γ		0.9232	0.7069	0.7104	1.0119	0.3281	0.3280
				λ		−0.4891	0.2309	0.2309	−0.3661	0.3270	0.3530
		90	7	ρ		0.9014	0.0262	0.0262	−0.9064	0.0142	0.0156
				β		2.0018	0.0268	0.0269	2.0004	0.0204	0.0204
				γ		0.9835	0.2715	0.2717	1.0121	0.1723	0.1726
				λ		−0.4912	0.1015	0.1018	−0.4175	0.2000	0.2161
	SGMM	30	3	ρ		0.9073	0.0386	0.0393	−0.8996	0.0151	0.0151
				β		2.0292	0.1794	0.1817	2.0187	0.0938	0.0957
				γ	1	1.3480	3.2908	3.3083	1.0296	2.1316	2.1313
					2	0.9821	0.4817	0.4819	0.9858	0.3231	0.3233
				λ		−0.3423	0.3063	0.3445	−0.4331	0.2751	0.2831
		90	7	ρ		0.9015	0.0154	0.0155	−0.8999	0.0053	0.0053
				β		1.9991	0.1915	0.1914	2.0198	0.0712	0.0739
				γ	1	1.5509	1.1596	1.2835	0.9682	0.9080	0.9083
					2	0.9951	0.2405	0.2404	0.9886	0.1759	0.1762
				λ		−0.4128	0.1126	0.1424	−0.4836	0.0724	0.0742

Table A14. Persistent series (a circular and real-world spatial weights matrix, $\lambda = 0.8$).

		$m = 6$				$\rho = 0.9$			$\rho = -0.9$		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		1.0410	0.1690	0.2200	−0.9159	0.0301	0.0340
				β		2.1400	0.1789	0.2271	1.9956	0.0528	0.0529
				γ		0.3264	1.8540	1.9708	0.9895	0.3564	0.3562
				λ		0.6874	0.1598	0.1954	0.7004	0.1808	0.2063
		100	7	ρ		1.0915	0.1120	0.2218	−0.9077	0.0138	0.0158
				β		2.1489	0.0956	0.1769	1.9980	0.0198	0.0199
				γ		−0.1616	1.1849	1.6585	0.9839	0.2296	0.2300
				λ		0.3090	0.2655	0.5580	0.7479	0.1042	0.1164
	SGMM	50	3	ρ		0.9081	0.0261	0.0273	−0.9000	0.0115	0.0115
				β		2.0164	0.1251	0.1261	2.0095	0.0617	0.0624
				γ	1	1.7137	7.2399	7.2732	1.2086	5.1222	5.1252
					2	0.9624	0.4611	0.4625	0.9968	0.3723	0.3722
				λ		0.6580	0.1745	0.2249	0.7400	0.1212	0.1352
		100	7	ρ		0.9035	0.0121	0.0126	−0.8998	0.0050	0.0050
				β		2.0088	0.1626	0.1628	2.0229	0.0646	0.0685
				γ	1	2.0367	2.5438	2.7464	1.0274	2.6025	2.6020
					2	0.9997	0.2559	0.2559	1.0000	0.2382	0.2381
				λ		0.7188	0.0897	0.1210	0.7768	0.0363	0.0430
Real world	QMLE	30	3	ρ		0.9600	0.1283	0.1415	−0.9162	0.0294	0.0335
				β		2.0604	0.1488	0.1604	1.9930	0.0690	0.0693
				γ		0.6066	1.7080	1.7510	1.0490	0.4794	0.4814
				λ		0.7551	0.1531	0.1594	0.6862	0.2101	0.2387
		90	7	ρ		1.0750	0.1257	0.2154	−0.9101	0.0150	0.0181
				β		2.1386	0.1082	0.1758	1.9983	0.0208	0.0208
				γ		−0.0366	1.2575	1.6287	1.0180	0.2403	0.2407
				λ		0.3750	0.2925	0.5158	0.7266	0.1239	0.1439
	SGMM	30	3	ρ		0.9060	0.0428	0.0432	−0.8993	0.0152	0.0152
				β		2.0276	0.2325	0.2341	2.0153	0.1029	0.1040
				γ	1	1.5081	7.1021	7.1185	1.0095	3.7932	3.7923
					2	0.9828	0.7530	0.7531	0.9946	0.4853	0.4852
				λ		0.6194	0.2344	0.2959	0.7005	0.1757	0.2019
		90	7	ρ		0.9035	0.0138	0.0142	−0.9000	0.0053	0.0053
				β		2.0098	0.1852	0.1854	2.0238	0.0718	0.0756
				γ	1	1.8231	2.6108	2.7369	1.1107	2.5219	2.5237
					2	0.9879	0.2891	0.2893	1.0061	0.2462	0.2462
				λ		0.7108	0.0993	0.1335	0.7735	0.0419	0.0495

Table A15. Persistent series (a circular and real-world spatial weights matrix, $\lambda = -0.8$).

		$m = 6$				$\rho = 0.9$			$\rho = -0.9$		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.9070	0.0512	0.0516	−0.9100	0.0259	0.0277
				β		2.0087	0.0687	0.0692	1.9995	0.0506	0.0506
				γ		0.9657	0.4352	0.4361	0.9883	0.2548	0.2548
				λ		−0.8237	0.1979	0.1991	−0.6904	0.2582	0.2803
		100	7	ρ		0.9154	0.0462	0.0487	−0.9032	0.0104	0.0109
				β		2.0116	0.0413	0.0428	2.0000	0.0187	0.0187
				γ		0.8975	0.4352	0.4467	1.0056	0.1694	0.1693
				λ		−0.7169	0.2872	0.2987	−0.7747	0.0864	0.0899
	SGMM	50	3	ρ		0.9081	0.0254	0.0266	−0.9000	0.0113	0.0113
				β		2.0158	0.1140	0.1151	2.0109	0.0592	0.0601
				γ	1	1.3401	2.0037	2.0319	1.0369	1.1273	1.1276
					2	0.9766	0.3100	0.3108	0.9963	0.2553	0.2553
				λ		−0.6228	0.2253	0.2866	−0.7257	0.1579	0.1745

(*continued*)

Table A15. Continued.

		$m=6$				$\rho=0.9$			$\rho=-0.9$		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
		100	7	ρ		0.9016	0.0133	0.0134	−0.8999	0.0049	0.0049
				β		2.0011	0.1664	0.1664	2.0178	0.0638	0.0662
				γ	1	1.4118	0.8709	0.9631	1.0024	0.7127	0.7125
					2	0.9940	0.2209	0.2209	0.9873	0.1726	0.1730
				λ		−-0.6991	0.1139	0.1522	−0.7759	0.0448	0.0509
Real world	QMLE	30	3	ρ		0.9266	0.0879	0.0917	−0.9134	0.0295	0.0323
				β		2.0246	0.0987	0.1016	1.9956	0.0703	0.0704
				γ		0.9366	0.6929	0.6951	0.9979	0.3430	0.3426
				λ		−0.7694	0.2394	0.2411	−0.6128	0.4019	0.4430
		90	7	ρ		0.9006	0.0248	0.0248	−0.9037	0.0115	0.0121
				β		2.0013	0.0272	0.0272	2.0010	0.0194	0.0194
				γ		0.9938	0.2464	0.2462	0.9994	0.1908	0.1906
				λ		−0.7816	0.1518	0.1528	−0.7373	0.1961	0.2057
	SGMM	30	3	ρ		0.9066	0.0380	0.0386	−0.8995	0.0148	0.0148
				β		2.0293	0.1863	0.1886	2.0179	0.0899	0.0917
				γ	1	1.2491	3.2121	3.2209	1.0269	1.4986	1.4985
					2	0.9805	0.5058	0.5061	0.9957	0.3337	0.3337
				λ		−0.5236	0.3217	0.4241	−0.6695	0.2487	0.2807
		90	7	ρ		0.9017	0.0149	0.0150	−0.8999	0.0051	0.0051
				β		1.9982	0.1846	0.1845	2.0196	0.0688	0.0715
				γ	1	1.4302	0.9636	1.0550	0.9557	0.7701	0.7712
					2	0.9944	0.2463	0.2463	0.9905	0.1824	0.1826
				λ		−0.6473	0.1592	0.2205	−0.7676	0.0771	0.0836

Table A16. Persistent series (group interaction spatial weights matrix, $\lambda = \pm 0.2$).

		$m=6$				$\rho=0.9$			$\rho=-0.9$		
		R	T			Mean	SD	RMSE	Mean	SD	RMSE
$\lambda=0.2$	QMLE	15	3	ρ		0.9289	0.0993	0.1033	−0.9120	0.0268	0.0293
				β		2.0256	0.1053	0.1082	1.9958	0.0479	0.0480
				γ		0.8742	0.6868	0.6975	0.9939	0.2332	0.2330
				λ		0.1600	0.2055	0.2091	0.1644	0.1049	0.1106
		30	7	ρ		0.9036	0.0380	0.0381	−0.9137	0.0174	0.0222
				β		2.0030	0.0364	0.0365	1.9983	0.0183	0.0184
				γ		0.9870	0.2570	0.2570	1.0176	0.1537	0.1545
				λ		0.1788	0.0865	0.0890	0.1741	0.0661	0.0709
	SGMM	15	3	ρ		0.9077	0.0249	0.0260	−0.8997	0.0112	0.0112
				β		2.0175	0.0939	0.0955	2.0086	0.0594	0.0600
				γ	1	1.6779	4.4706	4.5207	0.9646	3.0933	3.0927
					2	0.9808	0.3154	0.3159	0.9983	0.2555	0.2554
				λ		0.1574	0.1907	0.1953	0.1759	0.1684	0.1701
		30	7	ρ		0.9024	0.0124	0.0126	−0.9001	0.0048	0.0048
				β		2.0079	0.1460	0.1461	2.0172	0.0507	0.0535
				γ	1	2.1266	1.8571	2.1717	1.0256	1.7644	1.7642
					2	0.9873	0.1902	0.1906	0.9902	0.1475	0.1478
				λ		0.1811	0.0561	0.0592	0.1936	0.0481	0.0485
$\lambda=-0.2$	QMLE	15	3	ρ		0.9177	0.0807	0.0825	−0.9114	0.0281	0.0302
				β		2.0124	0.0884	0.0892	1.9967	0.0475	0.0476
				γ		0.9273	0.5773	0.5813	1.0126	0.2192	0.2193
				λ		−0.1607	0.1267	0.1325	−0.1258	0.1679	0.1834
		30	7	ρ		0.9023	0.0253	0.0254	−0.9083	0.0153	0.0174
				β		2.0014	0.0264	0.0265	1.9996	0.0186	0.0186
				γ		0.9936	0.1956	0.1955	1.0017	0.1400	0.1399
				λ		−0.2075	0.1481	0.1481	−0.1487	0.1051	0.1169

(*continued*)

Table A16. Continued.

	$m=6$				$\rho=0.9$			$\rho=-0.9$		
	R	T			Mean	SD	RMSE	Mean	SD	RMSE
SGMM	15	3	ρ		0.9078	0.0227	0.0240	−0.9002	0.0105	0.0105
			β		2.0091	0.0854	0.0858	2.0078	0.0537	0.0543
			γ	1	1.6965	3.3904	3.4603	1.0098	2.6326	2.6319
				2	0.9700	0.2713	0.2729	0.9980	0.2226	0.2225
			λ		−0.1736	0.1937	0.1955	−0.1963	0.1743	0.1743
	30	7	ρ		0.9020	0.0119	0.0120	−0.9000	0.0049	0.0049
			β		2.0054	0.1467	0.1467	2.0179	0.0577	0.0604
			γ	1	1.7946	1.2724	1.4998	1.0140	1.2782	1.2780
				2	0.9890	0.1866	0.1869	0.9921	0.1473	0.1475
			λ		−0.1744	0.0645	0.0693	−0.1935	0.0585	0.0589

Table A17. Persistent series (group interaction spatial weights matrix, $\lambda=\pm0.5$).

		$m=6$				$\rho=0.9$			$\rho=-0.9$		
		R	T			Mean	SD	RMSE	Mean	SD	RMSE
$\lambda=0.5$	QMLE	15	3	ρ		0.9396	0.1086	0.1155	−0.9127	0.0280	0.0307
				β		2.0374	0.1161	0.1218	1.9947	0.0438	0.0441
				γ		0.8822	0.7338	0.7425	1.0079	0.2531	0.2530
				λ		0.4402	0.1777	0.1873	0.4182	0.1521	0.1726
		30	7	ρ		0.9109	0.0647	0.0655	−0.9135	0.0172	0.0219
				β		2.0086	0.0524	0.0531	1.9975	0.0181	0.0183
				γ		0.9751	0.3946	0.3950	1.0154	0.1648	0.1654
				λ		0.4537	0.1440	0.1511	0.3990	0.1253	0.1609
	SGMM	15	3	ρ		0.9079	0.0232	0.0245	−0.8999	0.0101	0.0101
				β		2.0127	0.0855	0.0865	2.0082	0.0525	0.0532
				γ	1	1.9871	5.2926	5.3826	1.1449	4.4902	4.4914
					2	0.9741	0.3117	0.3127	1.0031	0.2704	0.2704
				λ		0.4247	0.1585	0.1754	0.4653	0.1425	0.1466
		30	7	ρ		0.9031	0.0109	0.0113	−0.9000	0.0043	0.0043
				β		2.0095	0.1322	0.1325	2.0149	0.0458	0.0481
				γ	1	2.3000	2.1478	2.5101	1.0435	2.2314	2.2312
					2	0.9857	0.1946	0.1951	0.9981	0.1578	0.1578
				λ		0.4495	0.0686	0.0851	0.4920	0.0394	0.0402
$\lambda=-0.5$	QMLE	15	3	ρ		0.9157	0.0675	0.0692	−0.9097	0.0257	0.0275
				β		2.0166	0.0795	0.0811	1.9953	0.0432	0.0434
				γ		0.9626	0.4242	0.4254	1.0215	0.2151	0.2159
				λ		−0.3907	0.1619	0.1952	−0.3716	0.1839	0.2241
		30	7	ρ		0.8995	0.0256	0.0256	−0.9067	0.0145	0.0160
				β		1.9984	0.0239	0.0239	1.9982	0.0176	0.0177
				γ		1.0091	0.2361	0.2361	0.9931	0.1535	0.1535
				λ		−0.4313	0.1230	0.1408	−0.3920	0.1678	0.1994
	SGMM	15	3	ρ		0.9090	0.0254	0.0269	−0.8998	0.0098	0.0098
				β		2.0153	0.0966	0.0978	2.0059	0.0558	0.0561
				γ	1	1.4775	2.9206	2.9587	0.9741	1.5227	1.5226
					2	0.9783	0.2985	0.2992	1.0012	0.2300	0.2300
				λ		−0.4079	0.2313	0.2489	−0.4850	0.2132	0.2137
		30	7	ρ		0.9027	0.0118	0.0121	−0.8997	0.0047	0.0047
				β		2.0078	0.1503	0.1504	2.0178	0.0536	0.0565
				γ	1	1.5610	1.0649	1.2034	0.9288	0.9938	0.9961
					2	0.9781	0.1947	0.1959	0.9880	0.1507	0.1511
				λ		−0.4234	0.0939	0.1211	−0.4871	0.0604	0.0618

Table A18. Persistent series (group interaction spatial weights matrix, $\lambda = \pm 0.8$).

		$m = 6$				$\rho = 0.9$			$\rho = -0.9$		
		R	T			Mean	SD	RMSE	Mean	SD	RMSE
$\lambda = 0.8$	QMLE	15	3	ρ		0.9753	0.1373	0.1565	−0.9083	0.0223	0.0238
				β		2.0706	0.1395	0.1562	1.9957	0.0406	0.0408
				γ		0.6311	1.0415	1.1039	1.0037	0.2947	0.2944
				λ		0.7213	0.1532	0.1720	0.7578	0.1080	0.1159
		30	7	ρ		1.0863	0.1114	0.2170	−0.9061	0.0127	0.0141
				β		2.1413	0.0933	0.1693	1.9990	0.0167	0.0168
				γ		−0.0545	1.0649	1.4979	1.0093	0.1963	0.1963
				λ		0.3076	0.2603	0.5569	0.7573	0.0914	0.1008
	SGMM	15	3	ρ		0.9081	0.0218	0.0233	−0.9002	0.0100	0.0100
				β		2.0126	0.0886	0.0894	2.0068	0.0517	0.0521
				γ	1	1.6842	6.0860	6.1228	1.1704	4.2732	4.2755
					2	0.9739	0.3372	0.3381	1.0007	0.3140	0.3139
				λ		0.6720	0.1805	0.2212	0.7467	0.0957	0.1095
		30	7	ρ		0.9042	0.0103	0.0111	−0.8997	0.0043	0.0044
				β		2.0059	0.1322	0.1323	2.0212	0.0579	0.0616
				γ	1	2.1793	2.2481	2.5381	1.0413	2.3422	2.3419
					2	0.9753	0.2060	0.2074	0.9959	0.2021	0.2021
				λ		0.7119	0.0972	0.1312	0.7688	0.0416	0.0520
$\lambda = -0.8$	QMLE	15	3	ρ		0.9116	0.0567	0.0578	−0.9086	0.0238	0.0253
				β		2.0123	0.0652	0.0663	1.9963	0.0413	0.0414
				γ		0.9592	0.4083	0.4099	1.0204	0.2189	0.2196
				λ		−0.6851	0.1833	0.2162	−0.5896	0.2289	0.3107
		30	7	ρ		0.9016	0.0374	0.0374	−0.9038	0.0116	0.0122
				β		1.9999	0.0333	0.0333	1.9989	0.0166	0.0166
				γ		0.9962	0.2713	0.2710	0.9898	0.1599	0.1601
				λ		−0.6851	0.1580	0.1952	−0.6983	0.1682	0.1964
	SGMM	15	3	ρ		0.9093	0.0219	0.0238	−0.8999	0.0099	0.0099
				β		2.0134	0.0899	0.0908	2.0089	0.0537	0.0545
				γ	1	1.3874	3.2039	3.2264	0.9594	1.7843	1.7844
					2	0.9765	0.2765	0.2774	0.9971	0.2397	0.2397
				λ		−0.6286	0.2157	0.2755	−0.7523	0.1692	0.1757
		30	7	ρ		0.9025	0.0119	0.0122	−0.8998	0.0042	0.0043
				β		2.0078	0.1401	0.1403	2.0171	0.0503	0.0531
				γ	1	1.4945	0.8905	1.0184	0.9787	0.8498	0.8499
					2	0.9860	0.1995	0.1999	0.9858	0.1509	0.1516
				λ		−0.6723	0.1289	0.1814	−0.7645	0.0587	0.0686

Table A19. Initial time period (a circular and real-world spatial weights matrix, $\lambda = 0.2$).

						$m = 50$			$m = 100$		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.5000	0.0344	0.0343	0.5020	0.0331	0.0332
				β		2.0010	0.0562	0.0562	2.0051	0.0543	0.0545
				γ		0.9972	0.2847	0.2845	1.0008	0.2659	0.2656
				λ		0.2063	0.1551	0.1551	0.2108	0.1541	0.1544
		100	7	ρ		0.5011	0.0208	0.0208	0.5021	0.0210	0.0211
				β		2.0007	0.0228	0.0228	2.0006	0.0232	0.0231
				γ		0.9921	0.1810	0.1810	0.9971	0.1732	0.1730
				λ		0.2024	0.0617	0.0617	0.2030	0.0775	0.0775
	SGMM	50	3	ρ		0.4987	0.0402	0.0402	0.5003	0.0404	0.0403
				β		2.0005	0.0802	0.0802	2.0040	0.0779	0.0780
				γ	1	0.9705	2.4241	2.4237	1.0341	2.6726	2.6721
					2	0.9972	0.3084	0.3083	1.0031	0.2984	0.2983
				λ		0.1757	0.2195	0.2208	0.1751	0.2117	0.2131

(*continued*)

Table A19. Continued.

W_n		N	T			m = 50 Mean	SD	RMSE	m = 100 Mean	SD	RMSE
		100	7	ρ		0.4952	0.0233	0.0238	0.4948	0.0235	0.0241
				β		1.9960	0.0770	0.0771	1.9950	0.0786	0.0788
				γ	1	1.0709	0.9772	0.9796	1.0933	1.0094	1.0034
					2	1.0107	0.1918	0.1920	1.0141	0.1939	0.1944
				λ		0.1954	0.0599	0.0600	0.1919	0.0624	0.0629
Real world	QMLE	30	3	ρ		0.5008	0.0458	0.0458	0.5068	0.0493	0.0497
				β		1.9977	0.0735	0.0734	2.0053	0.0762	0.0763
				γ		0.9738	0.3835	0.3840	0.9555	0.3436	0.3461
				λ		0.2035	0.1908	0.1906	0.1971	0.1958	0.1956
		90	7	ρ		0.5035	0.0277	0.0279	0.5026	0.0293	0.0293
				β		2.0024	0.0251	0.0252	2.0026	0.0256	0.0257
				γ		0.9836	0.1898	0.1903	0.9952	0.1850	0.1849
				λ		0.1995	0.0709	0.0708	0.2021	0.0607	0.0607
	SGMM	30	3	ρ		0.5004	0.0574	0.0574	0.5000	0.0593	0.0592
				β		2.0037	0.1225	0.1225	1.9987	0.1230	0.1229
				γ	1	0.9677	2.9350	2.9344	1.1247	2.7657	2.7678
					2	0.9910	0.4601	0.4601	0.9939	0.4546	0.4545
				λ		0.1580	0.2656	0.2688	0.1652	0.2648	0.2670
		90	7	ρ		0.4949	0.0244	0.0249	0.4946	0.0248	0.0254
				β		1.9973	0.0852	0.0853	1.9951	0.0905	0.0906
				γ	1	1.0374	1.0121	1.0125	1.0653	1.0124	1.0143
					2	1.0067	0.2006	0.2007	1.0204	0.2097	0.2106
				λ		0.1921	0.0619	0.0624	0.1901	0.0597	0.0605

Table A20. Initial time period (a circular and real-world spatial weights matrix, $\lambda = -0.2$).

W_n		N	T			m = 50 Mean	SD	RMSE	m = 100 Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.4994	0.0349	0.0349	0.5036	0.0374	0.0375
				β		2.0003	0.0552	0.0551	2.0074	0.0578	0.0582
				γ		1.0107	0.2599	0.2598	0.9827	0.2644	0.2647
				λ		−0.1896	0.1569	0.1570	−0.1895	0.1570	0.1572
		100	7	ρ		0.5042	0.0301	0.0303	0.5024	0.0198	0.0199
				β		2.0017	0.0241	0.0241	2.0008	0.0230	0.0230
				γ		0.9776	0.1930	0.1941	0.9944	0.1597	0.1596
				λ		−0.2012	0.0678	0.0677	−0.1980	0.0658	0.0657
	SGMM	50	3	ρ		0.4997	0.0403	0.0403	0.5003	0.0400	0.0400
				β		2.0009	0.0789	0.0788	2.0050	0.0770	0.0771
				γ	1	1.0060	1.8044	1.8040	1.0689	1.8210	1.8218
					2	1.0004	0.2962	0.2961	0.9987	0.2855	0.2854
				λ		−0.1731	0.2296	0.2312	−0.1749	0.2207	0.2221
		100	7	ρ		0.4953	0.0240	0.0245	0.4944	0.0240	0.0246
				β		1.9943	0.0770	0.0772	1.9951	0.0796	0.0797
				γ	1	1.0694	0.6283	0.6320	1.0759	0.6506	0.6549
					2	1.0170	0.1936	0.1943	1.0253	0.1913	0.1929
				λ		−0.1861	0.0652	0.0666	−0.1895	0.0674	0.0682
Real world	QMLE	30	3	ρ		0.5044	0.0523	0.0524	0.5063	0.0483	0.0487
				β		1.9999	0.0746	0.0745	2.0063	0.0755	0.0757
				γ		0.9603	0.3761	0.3778	0.9768	0.3619	0.3623
				λ		−0.2120	0.2006	0.2008	−0.2156	0.2079	0.2083
		90	7	ρ		0.5055	0.0322	0.0326	0.5033	0.0295	0.0297
				β		2.0030	0.0262	0.0264	2.0029	0.0239	0.0241
				γ		0.9797	0.1916	0.1925	0.9941	0.1898	0.1898
				λ		−0.1984	0.0735	0.0734	−0.1958	0.0700	0.0700

(continued)

Table A20. Continued.

					$m = 50$			$m = 100$		
W_n		N	T		Mean	SD	RMSE	Mean	SD	RMSE
SGMM		30	3	ρ	0.5007	0.0574	0.0573	0.4996	0.0566	0.0566
				β	2.0027	0.1247	0.1247	1.9998	0.1184	0.1183
				γ 1	0.9492	2.0014	2.0016	0.9988	2.5522	2.5516
				2	0.9906	0.4287	0.4287	0.9781	0.4151	0.4156
				λ	−0.1819	0.2827	0.2832	−0.1747	0.2823	0.2833
		90	7	ρ	0.4946	0.0251	0.0257	0.4946	0.0259	0.0265
				β	1.9961	0.0858	0.0858	1.9943	0.0926	0.0927
				γ 1	1.0519	0.6621	0.6640	1.0749	0.6669	0.6709
				2	1.0168	0.2048	0.2054	1.0241	0.2063	0.2076
				λ	−0.1898	0.0697	0.0704	−0.1897	0.0674	0.0682

Table A21. Initial time period (a circular and real-world spatial weights matrix, $\lambda = 0.5$).

						$m = 50$			$m = 100$		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.5003	0.0323	0.0323	0.5026	0.0377	0.0378
				β		2.0012	0.0541	0.0541	2.0050	0.0587	0.0589
				γ		0.9912	0.3223	0.3221	0.9999	0.3519	0.3516
				λ		0.5061	0.1179	0.1179	0.5112	0.1146	0.1150
		100	7	ρ		0.5025	0.0258	0.0259	0.5022	0.0239	0.0240
				β		2.0013	0.0235	0.0235	2.0008	0.0236	0.0236
				γ		0.9815	0.2044	0.2050	0.9790	0.2130	0.2138
				λ		0.5004	0.0700	0.0699	0.5056	0.0684	0.0686
	SGMM	50	3	ρ		0.4993	0.0392	0.0392	0.5006	0.0405	0.0405
				β		2.0010	0.0782	0.0781	2.0041	0.0756	0.0757
				γ	1	1.0792	3.0267	3.0270	0.8645	3.2571	3.2591
					2	0.9905	0.3504	0.3504	0.9916	0.3505	0.3505
				λ		0.4464	0.1902	0.1976	0.4469	0.1820	0.1895
		100	7	ρ		0.4959	0.0226	0.0230	0.4956	0.0227	0.0231
				β		1.9967	0.0764	0.0765	1.9959	0.0780	0.0780
				γ	1	1.0469	1.3043	1.3048	1.0787	1.3125	1.3145
					2	1.0137	0.2175	0.2179	1.0089	0.2166	0.2167
				λ		0.4842	0.0500	0.0525	0.4809	0.0526	0.0560
Real world	QMLE	30	3	ρ		0.5009	0.0456	0.0456	0.5043	0.0417	0.0419
				β		1.9980	0.0707	0.0706	2.0023	0.0722	0.0722
				γ		0.9645	0.4332	0.4342	0.9974	0.4321	0.4317
				λ		0.5060	0.1605	0.1605	0.5090	0.1488	0.1490
		90	7	ρ		0.5046	0.0318	0.0321	0.5037	0.0328	0.0330
				β		2.0028	0.0254	0.0255	2.0030	0.0258	0.0259
				γ		0.9932	0.2174	0.2173	0.9953	0.2261	0.2259
				λ		0.4975	0.0805	0.0805	0.4987	0.0776	0.0776
	SGMM	30	3	ρ		0.5006	0.0563	0.0563	0.5008	0.0584	0.0584
				β		2.0044	0.1222	0.1222	2.0001	0.1243	0.1243
				γ	1	1.1570	3.3767	3.3795	1.0786	2.9615	2.9618
					2	0.9975	0.4958	0.4957	1.0039	0.5021	0.5020
				λ		0.4216	0.2311	0.2440	0.4283	0.2323	0.2430
		90	7	ρ		0.4957	0.0237	0.0241	0.4954	0.0243	0.0247
				β		1.9983	0.0836	0.0836	1.9972	0.0869	0.0869
				γ	1	1.0724	1.2609	1.2626	1.0412	1.2953	1.2956
					2	1.0094	0.2253	0.2254	1.0161	0.2292	0.2297
				λ		0.4808	0.0527	0.0561	0.4777	0.0526	0.0571

Table A22. Initial time period (a circular and real-world spatial weights matrix, $\lambda = -0.5$).

W_n		N	T			$m = 50$ Mean	SD	RMSE	$m = 100$ Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.5026	0.0462	0.0463	0.5030	0.0356	0.0357
				β		2.0030	0.0606	0.0607	2.0070	0.0537	0.0541
				γ		0.9980	0.2636	0.2634	0.9930	0.2726	0.2724
				λ		−0.4770	0.1338	0.1356	−0.4792	0.1324	0.1339
		100	7	ρ		0.5073	0.0363	0.0369	0.5044	0.0272	0.0276
				β		2.0026	0.0247	0.0248	2.0016	0.0240	0.0240
				γ		0.9733	0.2145	0.2160	0.9914	0.1769	0.1769
				λ		−0.4884	0.0660	0.0669	−0.4875	0.0698	0.0709
	SGMM	50	3	ρ		0.4998	0.0394	0.0394	0.5006	0.0390	0.0390
				β		2.0019	0.0774	0.0774	2.0042	0.0774	0.0775
				γ	1	1.0379	1.8253	1.8252	1.0367	1.2566	1.2568
					2	0.9967	0.2887	0.2886	0.9999	0.2872	0.2871
				λ		−0.4348	0.2115	0.2213	−0.4365	0.2054	0.2149
		100	7	ρ		0.4961	0.0244	0.0247	0.4954	0.0246	0.0250
				β		1.9950	0.0750	0.0751	1.9962	0.0784	0.0785
				γ	1	1.0479	0.4988	0.5009	1.0591	0.5185	0.5217
					2	1.0186	0.2033	0.2041	1.0205	0.2044	0.2054
				λ		−0.4733	0.0630	0.0684	−0.4755	0.0642	0.0687
Real world	QMLE	30	3	ρ		0.5090	0.0611	0.0617	0.5089	0.0539	0.0546
				β		2.0036	0.0793	0.0793	2.0086	0.0767	0.0771
				γ		0.9877	0.4016	0.4014	0.9604	0.3847	0.3864
				λ		−0.5071	0.1906	0.1905	−0.5141	0.1974	0.1977
		90	7	ρ		0.5063	0.0306	0.0312	0.5051	0.0310	0.0314
				β		2.0032	0.0255	0.0257	2.0031	0.0233	0.0235
				γ		0.9739	0.1943	0.1959	0.9873	0.2061	0.2063
				λ		−0.4888	0.0875	0.0881	−0.4903	0.0838	0.0843
	SGMM	30	3	ρ		0.5000	0.0560	0.0560	0.5002	0.0573	0.0573
				β		2.0037	0.1232	0.1233	2.0005	0.1199	0.1198
				γ	1	1.0487	1.4569	1.4574	1.0326	1.6808	1.6807
					2	1.0014	0.4297	0.4296	0.9849	0.4384	0.4385
				λ		−0.4237	0.2799	0.2901	−0.4190	0.2803	0.2918
		90	7	ρ		0.4958	0.0256	0.0259	0.4957	0.0261	0.0264
				β		1.9972	0.0852	0.0852	1.9959	0.0902	0.0903
				γ	1	1.0268	0.5371	0.5376	1.0463	0.5176	0.5196
					2	1.0081	0.2141	0.2142	1.0241	0.2222	0.2235
				λ		−0.4714	0.0751	0.0804	−0.4695	0.0745	0.0805

Table A23. Initial time period (a circular and real-world spatial weights matrix, $\lambda = 0.8$).

W_n		N	T			$m = 50$ Mean	SD	RMSE	$m = 100$ Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.5654	0.1606	0.1732	0.5730	0.1685	0.1835
				β		2.0589	0.1530	0.1638	2.0633	0.1504	0.1630
				γ		0.8548	0.7143	0.7283	0.8966	0.6192	0.6271
				λ		0.7056	0.1639	0.1890	0.7065	0.1637	0.1884
		100	7	ρ		0.5062	0.0573	0.0575	0.5117	0.0653	0.0663
				β		2.0033	0.0347	0.0348	2.0056	0.0388	0.0392
				γ		0.9788	0.3057	0.3061	0.9570	0.3263	0.3288
				λ		0.7332	0.1913	0.2024	0.7268	0.2054	0.2178
	SGMM	50	3	ρ		0.5004	0.0384	0.0384	0.5007	0.0387	0.0387
				β		2.0022	0.0781	0.0781	2.0034	0.0750	0.0751
				γ	1	0.8029	4.1299	4.1336	1.1035	3.2418	3.2426
					2	0.9817	0.4108	0.4111	0.9985	0.4021	0.4020
				λ		0.7224	0.1306	0.1519	0.7234	0.1262	0.1475

(*continued*)

Table A23. Continued.

						$m=50$			$m=100$		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
		100	7	ρ		0.4970	0.0219	0.0221	0.4968	0.0222	0.0224
				β		1.9987	0.0739	0.0739	1.9971	0.0757	0.0757
				γ	1	0.9795	1.3730	1.3728	1.0094	1.3469	1.3465
					2	1.0012	0.2517	0.2517	1.0127	0.2503	0.2505
				λ		0.7689	0.0416	0.0519	0.7674	0.0448	0.0554
Real world	QMLE	30	3	ρ		0.5194	0.0990	0.1008	0.5228	0.1003	0.1028
				β		2.0126	0.1008	0.1015	2.0204	0.1071	0.1089
				γ		0.9039	0.6183	0.6251	0.9167	0.6507	0.6554
				λ		0.7520	0.1476	0.1550	0.7465	0.1480	0.1573
		90	7	ρ		0.5173	0.0638	0.0661	0.5189	0.0708	0.0732
				β		2.0087	0.0365	0.0375	2.0098	0.0394	0.0405
				γ		0.9459	0.2996	0.3041	0.9432	0.3372	0.3416
				λ		0.7293	0.2146	0.2257	0.7358	0.2056	0.2152
	SGMM	30	3	ρ		0.5009	0.0567	0.0567	0.4995	0.0590	0.0589
				β		2.0052	0.1292	0.1293	2.0007	0.1351	0.1351
				γ	1	0.9747	3.1328	3.1321	0.9978	3.4502	3.4493
					2	1.0032	0.6022	0.6020	1.0231	0.6053	0.6056
				λ		0.6782	0.1853	0.2218	0.6847	0.1857	0.2186
		90	7	ρ		0.4971	0.0226	0.0227	0.4962	0.0232	0.0235
				β		1.9991	0.0810	0.0810	1.9981	0.0824	0.0824
				γ	1	1.0370	1.2665	1.2667	0.9994	1.3704	1.3701
					2	1.0099	0.2753	0.2754	1.0174	0.2756	0.2760
				λ		0.7623	0.0481	0.0611	0.7592	0.0512	0.0654

Table A24. Initial time period (a circular and real-world spatial weights matrix, $\lambda = -0.8$).

						$m=50$			$m=100$		
W_n		N	T			Mean	SD	RMSE	Mean	SD	RMSE
Circular world	QMLE	50	3	ρ		0.5136	0.0688	0.0701	0.5152	0.0677	0.0693
				β		2.0118	0.0737	0.0746	2.0177	0.0756	0.0776
				γ		0.9666	0.3399	0.3411	0.9741	0.3220	0.3227
				λ		−0.7615	0.1091	0.1156	−0.7671	0.1074	0.1123
		100	7	ρ		0.5204	0.0589	0.0623	0.5183	0.0561	0.0590
				β		2.0070	0.0292	0.0300	2.0067	0.0299	0.0306
				γ		0.9301	0.2651	0.2739	0.9440	0.2442	0.2504
				λ		−0.7470	0.1301	0.1403	−0.7485	0.1310	0.1406
	SGMM	50	3	ρ		0.5013	0.0388	0.0389	0.5015	0.0376	0.0376
				β		2.0021	0.0754	0.0755	2.0057	0.0761	0.0763
				γ	1	1.0080	1.0482	1.0480	1.0091	0.9574	0.9572
					2	1.0012	0.3037	0.3036	0.9936	0.2931	0.2931
				λ		−0.7047	0.1693	0.1942	−0.7030	0.1660	0.1922
		100	7	ρ		0.4980	0.0239	0.0239	0.4968	0.0243	0.0245
				β		1.9971	0.0711	0.0712	1.9976	0.0734	0.0734
				γ	1	1.0176	0.4181	0.4183	1.0369	0.4508	0.4522
					2	1.0053	0.2054	0.2054	1.0159	0.2113	0.2119
				λ		−0.7627	0.0540	0.0657	−0.7634	0.0539	0.0651
Real world	QMLE	30	3	ρ		0.5131	0.0710	0.0721	0.5112	0.0597	0.0607
				β		2.0076	0.0871	0.0873	2.0114	0.0794	0.0802
				γ		0.9540	0.4311	0.4331	0.9715	0.3961	0.3967
				λ		−0.7897	0.1819	0.1820	−0.8017	0.1804	0.1803
		90	7	ρ		0.5155	0.0477	0.0501	0.5101	0.0403	0.0415
				β		2.0067	0.0292	0.0299	2.0051	0.0254	0.0259
				γ		0.9416	0.2456	0.2522	0.9606	0.2236	0.2269
				λ		−0.7442	0.1689	0.1777	−0.7608	0.1439	0.1490

(*continued*)

Table A24. Continued.

W_n		N	T			m = 50 Mean	SD	RMSE	m = 100 Mean	SD	RMSE
	SGMM	30	3	ρ		0.5004	0.0538	0.0538	0.5012	0.0560	0.0560
				β		2.0046	0.1188	0.1189	2.0035	0.1177	0.1177
				γ	1	0.9981	1.3300	1.3296	0.9726	1.4417	1.4416
					2	0.9997	0.4289	0.4288	0.9775	0.4266	0.4271
				λ		−0.6457	0.2610	0.3032	−0.6417	0.2644	0.3081
		90	7	ρ		0.4972	0.0251	0.0253	0.4971	0.0257	0.0259
				β		1.9984	0.0795	0.0795	1.9970	0.0854	0.0854
				γ	1	1.0154	0.4718	0.4719	1.0353	0.4722	0.4734
					2	1.0034	0.2205	0.2205	1.0201	0.2309	0.2317
				λ		−0.7470	0.0847	0.0999	−0.7430	0.0876	0.1045

Table A25. Initial time period (group interaction spatial weights matrix, $\lambda = \pm 0.2$).

		N	T			m = 50 Mean	SD	RMSE	m = 100 Mean	SD	RMSE
$\lambda = 0.2$	QMLE	15	3	ρ		0.5023	0.0362	0.0362	0.4987	0.0283	0.0283
				β		1.9992	0.0545	0.0545	1.9959	0.0528	0.0529
				γ		0.9882	0.2497	0.2497	1.0023	0.2522	0.2519
				λ		0.1964	0.1229	0.1228	0.1969	0.1097	0.1096
		30	7	ρ		0.5031	0.0248	0.0249	0.5021	0.0233	0.0233
				β		2.0008	0.0223	0.0223	1.9999	0.0205	0.0205
				γ		0.9792	0.1730	0.1741	1.0025	0.1543	0.1542
				λ		0.1920	0.0768	0.0772	0.1859	0.0487	0.0507
	SGMM	15	3	ρ		0.5018	0.0322	0.0322	0.4999	0.0339	0.0339
				β		2.0022	0.0629	0.0629	2.0043	0.0639	0.0641
				γ	1	1.0158	2.7823	2.7817	0.9446	2.3186	2.3186
					2	0.9930	0.2543	0.2543	0.9968	0.2584	0.2584
				λ		0.1843	0.1582	0.1589	0.1793	0.1658	0.1671
		30	7	ρ		0.4964	0.0174	0.0177	0.4951	0.0218	0.0223
				β		1.9963	0.0585	0.0586	1.9965	0.0714	0.0715
				γ	1	1.0662	0.8379	0.8403	1.0708	0.8562	0.8589
					2	1.0102	0.1571	0.1574	1.0152	0.1816	0.1821
				λ		0.1941	0.0469	0.0473	0.1947	0.0537	0.0539
$\lambda = -0.2$	QMLE	15	3	ρ		0.5018	0.0296	0.0297	0.4998	0.0306	0.0306
				β		1.9995	0.0489	0.0489	1.9968	0.0519	0.0520
				γ		0.9929	0.2048	0.2047	0.9839	0.2511	0.2513
				λ		−0.1655	0.0943	0.1004	−0.1780	0.1278	0.1296
		30	7	ρ		0.5006	0.0138	0.0138	0.5015	0.0191	0.0191
				β		2.0001	0.0190	0.0190	2.0007	0.0188	0.0188
				γ		0.9989	0.1361	0.1359	0.9941	0.1462	0.1462
				λ		−0.1811	0.0457	0.0494	−0.1780	0.0484	0.0531
	SGMM	15	3	ρ		0.5013	0.0359	0.0359	0.5007	0.0347	0.0347
				β		2.0042	0.0691	0.0693	2.0018	0.0670	0.0670
				γ	1	0.9945	1.5267	1.5263	1.0401	1.6303	1.6304
					2	0.9881	0.2611	0.2613	1.0065	0.2509	0.2509
				λ		−0.1936	0.1880	0.1881	−0.1981	0.1705	0.1704
		30	7	ρ		0.4963	0.0192	0.0196	0.4965	0.0186	0.0189
				β		1.9991	0.0622	0.0622	1.9974	0.0622	0.0623
				γ	1	1.0580	0.6092	0.6118	1.0812	0.6221	0.6272
					2	1.0075	0.1633	0.1635	1.0105	0.1643	0.1646
				λ		−0.1917	0.0587	0.0593	−0.1919	0.0528	0.0534

Table A26. Initial time period (group interaction spatial weights matrix, $\lambda = \pm 0.5$).

						$m = 50$			$m = 100$		
		R	T			Mean	SD	RMSE	Mean	SD	RMSE
$\lambda = 0.5$	QMLE	15	3	ρ		0.5029	0.0415	0.0416	0.5018	0.0315	0.0315
				β		2.0011	0.0544	0.0544	2.0002	0.0449	0.0449
				γ		0.9857	0.2624	0.2625	0.9878	0.2675	0.2675
				λ		0.4851	0.0773	0.0787	0.4916	0.0699	0.0703
		30	7	ρ		0.5004	0.0105	0.0105	0.5022	0.0170	0.0171
				β		2.0006	0.0185	0.0185	2.0012	0.0194	0.0194
				γ		0.9991	0.1601	0.1599	0.9866	0.1715	0.1718
				λ		0.4893	0.0481	0.0492	0.4889	0.0623	0.0632
	SGMM	15	3	ρ		0.5011	0.0366	0.0367	0.5013	0.0357	0.0357
				β		2.0040	0.0722	0.0723	2.0048	0.0703	0.0704
				γ	1	0.9724	2.5533	2.5528	1.0554	2.9332	2.9330
					2	0.9981	0.3000	0.2999	0.9906	0.3038	0.3039
				λ		0.4484	0.1864	0.1934	0.4641	0.1589	0.1629
		30	7	ρ		0.4972	0.0179	0.0181	0.4978	0.0158	0.0159
				β		1.9978	0.0618	0.0619	1.9991	0.0528	0.0528
				γ	1	1.0594	1.0717	1.0731	1.0260	1.0936	1.0936
					2	1.0058	0.1880	0.1881	0.9975	0.1729	0.1729
				λ		0.4841	0.0426	0.0455	0.4853	0.0391	0.0418
$\lambda = -0.5$	QMLE	15	3	ρ		0.5034	0.0326	0.0327	0.5009	0.0313	0.0313
				β		2.0007	0.0480	0.0480	1.9987	0.0490	0.0490
				γ		0.9924	0.2083	0.2082	1.0025	0.2192	0.2190
				λ		−0.4041	0.0887	0.1306	−0.4266	0.0989	0.1231
		30	7	ρ		0.5026	0.0212	0.0213	0.5041	0.0266	0.0269
				β		2.0006	0.0190	0.0190	2.0021	0.0201	0.0202
				γ		0.9935	0.1525	0.1525	0.9906	0.1796	0.1797
				λ		−0.4388	0.0507	0.0795	−0.4586	0.0580	0.0712
	SGMM	15	3	ρ		0.5002	0.0426	0.0426	0.5029	0.0322	0.0323
				β		2.0014	0.0851	0.0851	2.0040	0.0662	0.0663
				γ	1	1.0286	1.6621	1.6619	1.0594	1.3071	1.3081
					2	0.9953	0.3196	0.3195	0.9955	0.2503	0.2502
				λ		−0.4479	0.2554	0.2606	−0.4720	0.2102	0.2120
		30	7	ρ		0.4975	0.0191	0.0193	0.4971	0.0194	0.0196
				β		1.9981	0.0611	0.0611	2.0008	0.0623	0.0623
				γ	1	1.0322	0.5059	0.5068	1.0346	0.5009	0.5020
					2	1.0097	0.1794	0.1796	1.0081	0.1794	0.1796
				λ		−0.4769	0.0633	0.0674	−0.4782	0.0596	0.0635

Table A27. Initial time period (group interaction spatial weights matrix, $\lambda = \pm 0.8$).

						$m = 50$			$m = 100$		
		R	T			Mean	SD	RMSE	Mean	SD	RMSE
$\lambda = 0.8$	QMLE	15	3	ρ		0.5651	0.1541	0.1671	0.5701	0.1569	0.1718
				β		2.0531	0.1364	0.1462	2.0553	0.1337	0.1446
				γ		0.8417	0.6093	0.6289	0.8026	0.6462	0.6750
				λ		0.6826	0.1608	0.1990	0.6843	0.1603	0.1976
		30	7	ρ		0.5085	0.0465	0.0472	0.5098	0.0502	0.0511
				β		2.0048	0.0286	0.0290	2.0046	0.0273	0.0277
				γ		0.9765	0.2313	0.2323	0.9662	0.2540	0.2560
				λ		0.7503	0.1697	0.1767	0.7602	0.1519	0.1569
	SGMM	15	3	ρ		0.5014	0.0331	0.0331	0.5011	0.0308	0.0308
				β		2.0039	0.0633	0.0634	2.0038	0.0580	0.0581
				γ	1	0.9522	3.1958	3.1954	1.0228	2.8365	2.8359
					2	0.9948	0.3318	0.3317	0.9983	0.3032	0.3031
				λ		0.7229	0.1113	0.1353	0.7370	0.1097	0.1265

(*continued*)

Table A27. Continued.

						m = 50			m = 100		
		R	T			Mean	SD	RMSE	Mean	SD	RMSE
		30	7	ρ		0.4984	0.0158	0.0159	0.4981	0.0174	0.0175
				β		1.9999	0.0520	0.0520	2.0007	0.0584	0.0584
				γ	1	0.9980	1.1987	1.1984	1.0565	1.1651	1.1661
					2	1.0033	0.2083	0.2083	1.0117	0.1994	0.1997
				λ		0.7633	0.0446	0.0578	0.7681	0.0424	0.0530
$\lambda = -0.8$	QMLE	15	3	ρ		0.5081	0.0472	0.0478	0.5031	0.0400	0.0401
				β		2.0045	0.0528	0.0529	2.0009	0.0521	0.0521
				γ		0.9837	0.2335	0.2338	0.9989	0.2257	0.2254
				λ		−0.6731	0.0999	0.1614	−0.6646	0.0970	0.1665
		30	7	ρ		0.5077	0.0365	0.0373	0.5077	0.0358	0.0366
				β		2.0026	0.0226	0.0227	2.0033	0.0219	0.0221
				γ		0.9788	0.1884	0.1894	0.9790	0.2072	0.2081
				λ		−0.6849	0.0951	0.1493	−0.7238	0.1192	0.1414
	SGMM	15	3	ρ		0.5029	0.0318	0.0319	0.5015	0.0330	0.0330
				β		2.0061	0.0637	0.0640	2.0052	0.0636	0.0638
				γ	1	0.9509	1.1050	1.1058	1.0026	1.3479	1.3475
					2	0.9809	0.2503	0.2599	0.9920	0.2698	0.2698
				λ		−0.7069	0.1797	0.2023	−0.7247	0.1871	0.2016
		30	7	ρ		0.4986	0.0191	0.0192	0.4992	0.0162	0.0162
				β		1.9987	0.0602	0.0602	2.0010	0.0503	0.0503
				γ	1	1.0200	0.4832	0.4835	1.0185	0.4257	0.4260
					2	1.0143	0.1835	0.1840	0.9969	0.1668	0.1668
				λ		−0.7555	0.0661	0.0797	−0.7611	0.0650	0.0758