CHAPTER - 4

THEORY OF PRODUCTION

Producer Equilibrium in Long Run (In Terms of Minimization of Cost and Maximization of Output):

Producer Equilibrium:

Producer equilibrium is a situation at where, producer produces maximum output from given resources or inputs. It is a condition of profit maximization of the producer from given level of inputs (i.e. labor and capital). Two conditions must be fulfilled form producer equilibrium and they are:

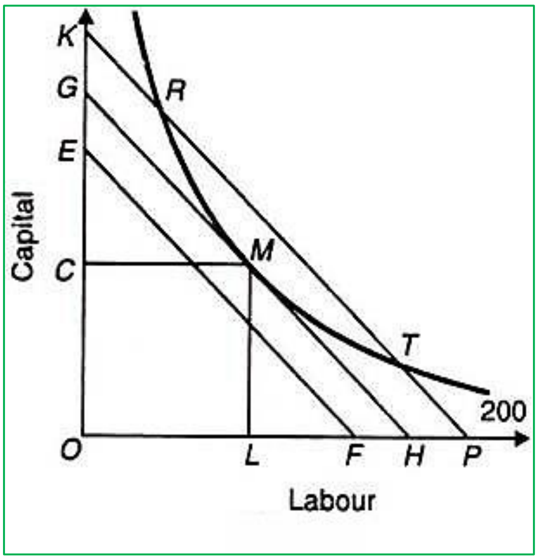
1. **Necessary Condition:** The slope of ISO-QUANT must be equal to the ISO-COST line i.e. $MRTS_{LK}$ = W/R.
2. **Sufficient Condition:** ISO-QUANT must be convex to the origin at the point of tangency to the ISO-COST line.

Producer equilibrium can be explained in terms of cost minimization and output maximization with the help of diagram.

These analysis are based on the following assumptions:

1. There are two factors, labor and capital.
2. All units of labor and capital are homogeneous.
3. The prices of units of labor (w) and that of capital (r) are given and constant.
4. The cost outlay is given.
5. The firm produces a single product.
6. The price of the product is given and constant.
7. The firm aims at profit maximization.
8. There is perfect competition in the factor market.

A. **Cost Minimization:**

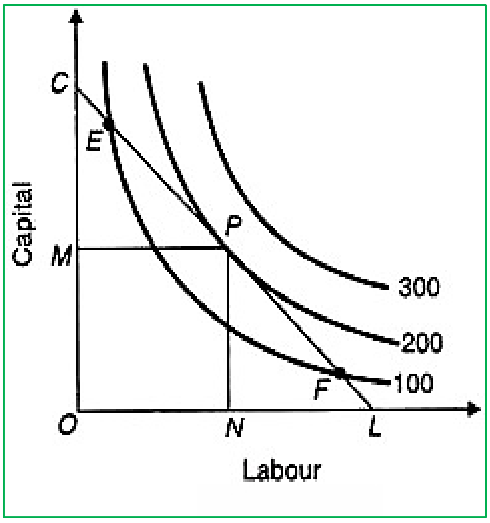


Given these assumptions, the point of least-cost combination of factors for a given level of output is where the isoquant curve is tangent to an iso-cost line. In Figure at right, the iso-cost line GH is tangent to the isoquant 200 at point M.

The firm employs the combination of OC of capital and OL of labor to produce 200 units of output at point M with the given cost-outlay GH. At this point, the firm is minimizing its cost for producing 200 units.

Any other combination on the isoquant 200, such as R or T, is on the higher iso-cost line KP which shows higher cost of production. The iso-cost line EF shows lower cost but output 200 cannot be attained with it. Therefore, the firm will choose the minimum cost point M which is the least-cost factor combination for producing 200 units of output.

B. Output Maximization:



The firm also maximizes its profits by maximizing its output, given its cost outlay and the prices of the two factors. This analysis is based on the same assumptions, as given above. The firm is in equilibrium at point P where the isoquant curve 200 is tangent to the iso-cost line CL in Figure at right.

At this point, the firm is maximizing its output level of 200 units by employing the optimal combination of OM of capital and ON of labor, given its cost outlay CL. But it cannot be at points E or F on the iso-cost line CL, since both points give a smaller quantity of output, being on the iso-quant 100, than on the iso-quant 200.

The firm can reach the optimal factor combination level of maximum output by moving along the iso-cost line CL from either point E or F to point P. This movement involves no extra cost because the firm remains on the same iso-cost line.

The firm cannot attain a higher level of output such as isoquant 300 because of the cost constraint. Thus the equilibrium point has to be P with optimal factor combination OM + ON. At point P, the slope of the isoquant curve 200 is equal to the slope of the iso-cost line CL. It implies that $w/r = MP_L/MPC = MRTS_{LC.}$

Cobb-Douglas Production Function:

The Cobb-Douglas production function is based on the empirical study of the American manufacturing industry made by Paul H. Douglas and Charles W. Cobb. It is a linear homogeneous production function of degree one, which takes into account two inputs, labor and capital, for the entire output of the manufacturing industry.

The Cobb-Douglas production function is expressed as: $= A * L^{\alpha} * C^{\beta}$, where Q is output, L and C are inputs of labor and capital respectively. A, α and β are positive parameters where, $\alpha > 0$, $\beta > 0$. The equation tells that output depends directly on L and C, and that part of output which cannot be explained by L and C is explained by A which is the 'residual', often called technical change.

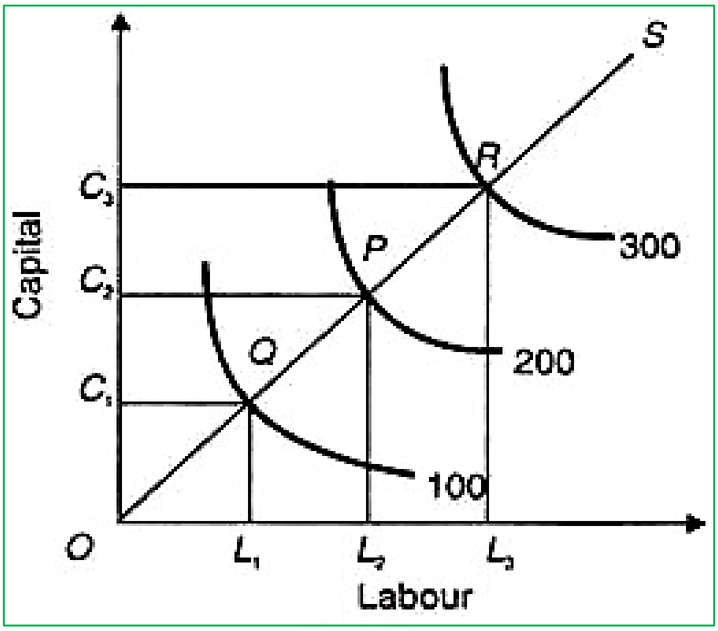
The production function solved by Cobb-Douglas had 1/4 contribution of capital to the increase in manufacturing industry and 3/4 of labor so that the C-D production function is: $Q = AL^{3/4} C1^{/4}$ which shows constant returns to scale because the total of the values of L and C is equal to one: (3/4 + 1/4), i.e. (a + β = 1). The coefficient of laborer in the C-D function measures the percentage increase in (Q that would result from a 1 per cent increase in L, while holding C as constant.

Similarly, B is the percentage increase in Q that would result from a 1 per cent increase in C, while holding L as constant. The C-D production function showing constant returns to scale is depicted in Figure below. Labor input is taken on the horizontal axis and capital on the vertical axis.

To produce 100 units of output, OC, units of capital and OL units of labor are used. If the output were to be doubled to 200, the inputs of labor and capital would have to be doubled. OC is exactly double of OC_1 and of OL_2 is double of OL_2.

Similarly, if the output is to be raised three-fold to 300, the units of labor and capital will have to be increased three-fold. OC_3 and OL_3 are three times larger than OC_1, and OL_1, respectively. Another method is to take the scale line or expansion path connecting the equilibrium points Q, P and R. OS is the scale line or expansion path joining these points.

It shows that the isoquants 100, 200 and 300 are equidistant. Thus, on the OS scale line OQ = QP = PR which shows that when capital and labor are increased in equal proportions, the output also increases in the same proportion.



Properties:

1. **Factor Intensity:**

The factor intensity can be measured by taking the ration between α and β.

a. If α/β>1, there is operation of labor intensive production technique.
b. If α/β<1, there is operation of capital intensive production technique.

2. **Efficiency of Production:**

The efficiency of production can be measured by the coefficient A.

a. If the value of A is higher, there is higher degree of efficiency of production.
b. If the value of A is lower, there is lower degree of efficiency of production.

3. **Returns to Scale:**

The various degrees of returns to scale can be measured by taking the sum of α and β.

Let, α+β = V

a. If V > 1, there is operation of increasing returns to scale.
b. If V = 1, there is operation of constant returns to scale.
c. If V < 1, there is operation of decreasing returns to scale.

4. **Average Productive of Inputs:**

a. Average productivity of labor $(AP_L) = \frac{Q}{L} = \frac{AL^{\alpha}K^{\beta}}{L}$

b. Average productivity of capital $(AP_K) = \frac{Q}{K} = \frac{AL^{\alpha}K^{\beta}}{K}$

5. **Marginal Productivities of Inputs:**

a. Marginal productivity of labor

$$(MP_L) = \frac{\Delta Q}{\Delta L} = \frac{\partial Q}{\partial L} = \frac{\partial(AL^{\alpha}K^{\beta})}{\partial L} = AK^{\beta} * \alpha L^{\alpha-1} = \frac{\alpha(AL^{\alpha}K^{\beta})}{L} = \alpha(AP_L)$$

b. Marginal productivity of capital

$$(MP_K) = \frac{\Delta Q}{\Delta K} = \frac{\partial Q}{\partial K} = \frac{\partial(AL^{\alpha}K^{\beta})}{\partial K} = AL^{\alpha} * \beta K^{\beta-1} = \frac{\beta\left(AL^{\alpha}K^{\beta}\right)}{K} = \beta(AP_K)$$

6. **The Marginal Rate of Technical Substitution:**

$$MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{\beta(AP_L)}{(AP_K)} = \frac{\alpha(Q/L)}{\beta(Q/K)} = \frac{\alpha}{\beta} * \frac{K}{L}$$

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7. **The Elasticity of Technical Substitution:**

$$\sigma = \frac{d(K/L)/(K/L)}{d(MRTS)/(MRTS)} = 1$$

Importance:

1. It has been used widely in empirical studies of manufacturing industries and in inter-industry comparisons.
2. It is used to determine the relative shares of labor and capital in total output.
3. It is used to prove Euler's Theorem.
4. Its parameters a and b represent elasticity coefficients that are used for inter-sectorial comparisons.
5. This production function is linear homogeneous of degree one which shows constant returns to scale, If α + β = 1, there are increasing returns to scale and if α + β < 1, there are diminishing returns to scale.
6. Economists have extended this production function to more than two variables.