Cassini States and Tidal Dissipation

Group Meeting Presentation

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- Zhu & Wu 2018 find that many SEs have CJ companions.
- SE may have experienced giant impacts, giving it a nontrivial initial obliquity (spin-orbit misalignment angle).
- Spin of SE evolves under tidal interactions with host star.
- SE also experiences Cassini State dynamics (spin-orbit and orbit-orbit precession).
- *What is the final outcome of the spin of the SE?*

- Central star $M_\star$, inner planet m, and perturber $m_{\rm p}$ mildly inclined by I.
- Two precession effects on inner planet:
  - Spin-orbit coupling:

$$\frac{\mathrm{d}\hat{\mathbf{s}}}{\mathrm{d}t} = \omega_{\rm sl}\left(\hat{\mathbf{s}}\cdot\hat{\mathbf{l}}\right)\left(\hat{\mathbf{s}}\times\hat{\mathbf{l}}\right),$$

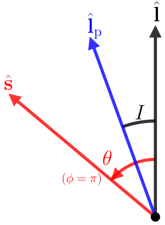
$$\omega_{\rm sl} \equiv \frac{3GJ_2mR^2M_\star}{2a^3I}\Omega_{\rm s}.$$

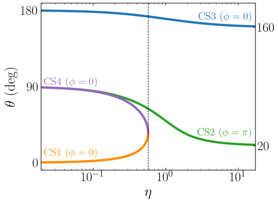
  - Orbit-orbit coupling

$$\frac{\mathrm{d}\hat{\mathbf{l}}}{\mathrm{d}t} = \omega_{\rm lp}\left(\hat{\mathbf{l}}\cdot\hat{\mathbf{l}}_{\rm p}\right)\left(\hat{\mathbf{l}}\times\hat{\mathbf{l}}_{\rm p}\right),$$

$$\omega_{\rm lp} = \frac{3m_{\rm p}}{4M_\star}\left(\frac{a}{a_{\rm p}}\right)^3 n.$$

- Equilibria (Cassini States) depend on $\eta \equiv \omega_{\rm lp}/\omega_{\rm sl} \propto \Omega_{\rm s}^{-1}$.

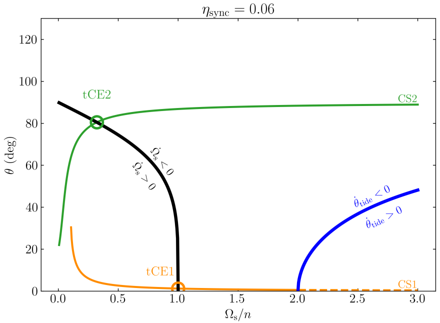




- **Objective:** Introduce dissipation into the system. For a given $\mathbf{s}_{\mathrm{i}}$ and $\Omega_{\mathrm{s,i}}$, what is the final outcome?
- Weak tidal friction: encourages (i) spin-orbit alignment, and (ii) spin-orbit synchronization:

$$\left(\frac{\mathrm{d}\hat{\mathbf{s}}}{\mathrm{d}t}\right)_{\mathrm{tide}} = \frac{1}{t_{\mathrm{s}}}\left[\frac{2n}{\Omega_{\mathrm{s}}} - (\hat{\mathbf{s}}\cdot\hat{\mathbf{l}})\right]\hat{\mathbf{s}}\times(\hat{\mathbf{l}}\times\hat{\mathbf{s}}),$$

$$\frac{1}{\Omega_{\mathrm{s}}}\left(\frac{\mathrm{d}\Omega_{\mathrm{s}}}{\mathrm{d}t}\right)_{\mathrm{tide}} = \frac{1}{t_{\mathrm{s}}}\left[\frac{2n}{\Omega_{\mathrm{s}}}(\hat{\mathbf{s}}\cdot\hat{\mathbf{l}}) - 1 - (\hat{\mathbf{s}}\cdot\hat{\mathbf{l}})^2\right].$$



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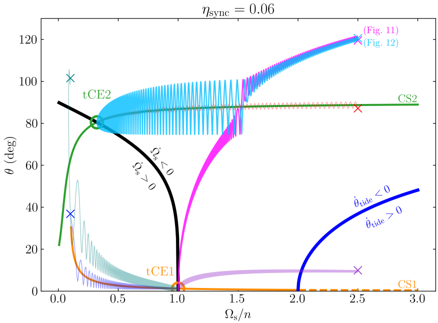
- Weak tidal friction: encourages (i) spin-orbit alignment, and (ii) spin-orbit synchronization:

$$\left(\frac{\mathrm{d}\hat{\mathbf{s}}}{\mathrm{d}t}\right)_{\mathrm{tide}} = \frac{1}{t_\mathrm{s}}\left[\frac{2n}{\Omega_\mathrm{s}} - \left(\hat{\mathbf{s}}\cdot\hat{\mathbf{l}}\right)\right]\hat{\mathbf{s}}\times\left(\hat{\mathbf{l}}\times\hat{\mathbf{s}}\right),$$

$$\frac{1}{\Omega_\mathrm{s}}\left(\frac{\mathrm{d}\Omega_\mathrm{s}}{\mathrm{d}t}\right)_{\mathrm{tide}} = \frac{1}{t_\mathrm{s}}\left[\frac{2n}{\Omega_\mathrm{s}}\left(\hat{\mathbf{s}}\cdot\hat{\mathbf{l}}\right) - 1 - \left(\hat{\mathbf{s}}\cdot\hat{\mathbf{l}}\right)^2\right].$$

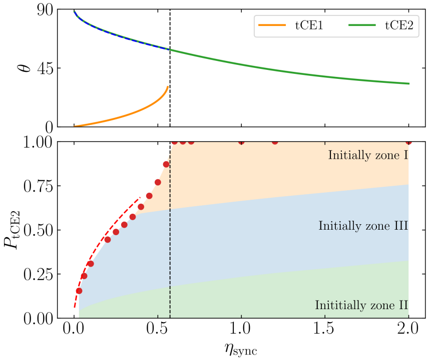


- Choose the initial conditions $\Omega_{\mathrm{s,i}} = 10n$, and $\hat{\mathbf{s}}_{\mathrm{i}}$ isotropically distributed.
- *What is the tCE2 probability?*
- Depends on the parameter

$$\eta_{\mathrm{sync}} \equiv \eta \frac{\Omega_{\mathrm{s}}}{n} \propto \Omega_{\mathrm{s}}^{0}$$

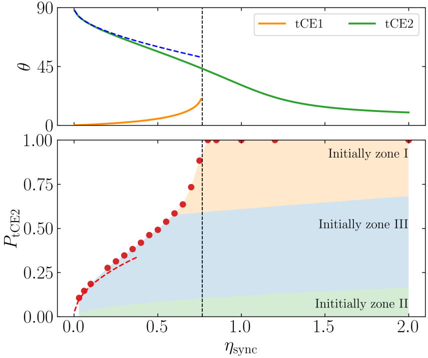
- Analytically, is approximately

$$P_{\mathrm{tCE2}} \simeq \frac{4\sqrt{\eta_{\mathrm{sync}} \sin I}}{\pi} f\left(\frac{\Omega_{\mathrm{s,i}}}{n}\right). \quad (1)$$



- Zhu & Wu 2018 find that many SEs have CJ companions.
- SE may have experienced giant impacts, $\sim$ isotropic $\hat{\mathbf{s}}$.
- Spin of SE evolves under tidal interactions with host star ($t_{\rm s} \sim 3 \times 10^7\,{\rm yr}$).
- SE also experiences Cassini State dynamics:

$$\eta_{\rm sync} = 0.303 \cos I \left(\frac{k}{k_{\rm q}}\right)\left(\frac{m_{\rm p}}{M_{\rm J}}\right) \times \left(\frac{m}{4M_\oplus}\right)\left(\frac{M_\star}{M_\odot}\right)^{-2}\left(\frac{a}{0.4\,{\rm AU}}\right)^6 \times \left(\frac{a_{\rm p}}{5\,{\rm AU}}\right)^{-3}\left(\frac{R}{2R_\oplus}\right)^{-3}.$$



- Maybe many high-obliquity SEs!