Spin-Orbit Resonances in 3+ Planet Systems

Dong Lai Group Meeting Presentation

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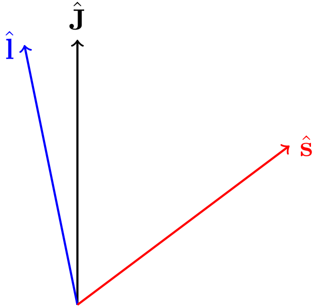
- In 1 + 1 system:

$$\frac{\mathrm{d}\hat{\mathbf{s}}}{\mathrm{d}t} = \underbrace{\omega_{\mathrm{sl}}}_{\alpha} \left(\hat{\mathbf{s}} \cdot \hat{\mathbf{l}}\right) \left(\hat{\mathbf{s}} \times \hat{\mathbf{l}}_{\mathrm{p}}\right),$$

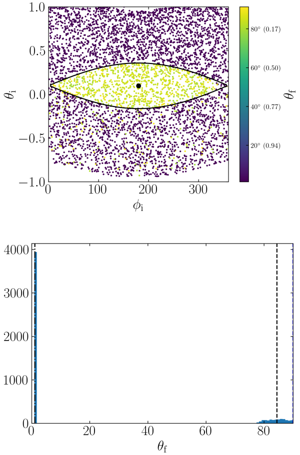
$$\frac{\mathrm{d}\hat{\mathbf{l}}}{\mathrm{d}t} = \underbrace{\omega_{\mathrm{lp}} \left(\hat{\mathbf{l}} \cdot \hat{\mathbf{J}}\right)}_{-g} \left(\hat{\mathbf{l}} \times \hat{\mathbf{J}}\right).$$

  Here, $\mathbf{J} = \mathbf{l} + \mathbf{l}_{\mathrm{p}}$, or $\hat{\mathbf{J}}$ is the invariable plane.

- Can be a resonance when $\alpha \sim -g$.



- Tides drive $\hat{\mathbf{s}} \to \hat{\mathbf{l}}$.
- Two stable Cassini States: $\theta_{\rm sl} \approx 0$, $\theta_{\rm sl} \approx 90^\circ$.
- Choose random $\hat{\mathbf{s}}$, where does it go?
  - If very prograde, $\to$ CS1.
  - If inside Cassini State resonance, $\to$ CS2.
  - If very retrograde, *probabilistic*.



- In $1 + n$ system?

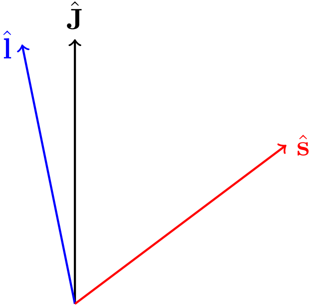
$$\frac{\mathrm{d}\hat{\mathbf{s}}}{\mathrm{d}t} = \underbrace{\omega_{\mathrm{sl}}}_{\alpha} \left(\hat{\mathbf{s}} \cdot \hat{\mathbf{l}}\right) \left(\hat{\mathbf{s}} \times \hat{\mathbf{l}}_{\mathrm{p}}\right),$$

$$\mathcal{I}(t) = \sum_{k=1}^{n} \mathcal{I}_k \exp\left[i g_k t + \phi_k\right],$$

$$\hat{\mathbf{l}}(t) = \mathrm{Re}\left(\mathcal{I}\right)\hat{\mathbf{x}} + \mathrm{Im}\left(\mathcal{I}\right)\hat{\mathbf{y}} + \sqrt{1 - \mathcal{I}^2}\hat{\mathbf{J}}.$$

Laplace-Lagrange.

- We focus on $n = 2$, so two modes. Likely two CSs?
- *Chaos* when resonance overlap (existing work by Laskar).



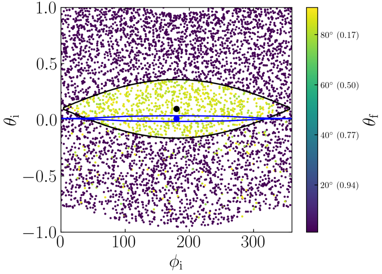
More Planets

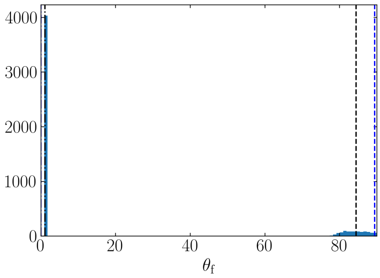
Plus Tides

- NEW: with tides, where are stable / long-lived equilibria?
- Naively: at best, each of CS1/CS2 (two stable CSs) for each g_i?
- Chaos can likely change which ICs converge to which equilbria.

More Planets

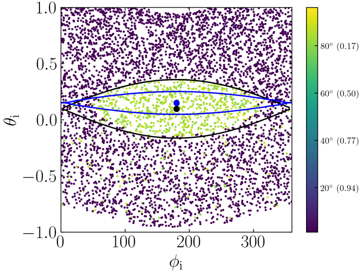
$I_1 = 10^\circ$, $I_2 = 1^\circ$, $g_1 = 0.1\alpha$, $g_2 = 0.1g_1$

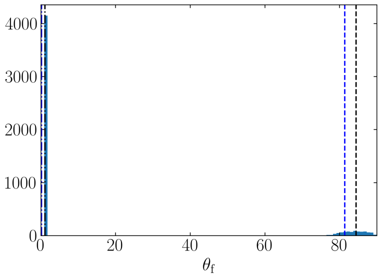




More Planets

$I_1 = 10°$, $I_2 = 1°$, $g_1 = 0.1\alpha$, $g_2 = 1.5 g_1$





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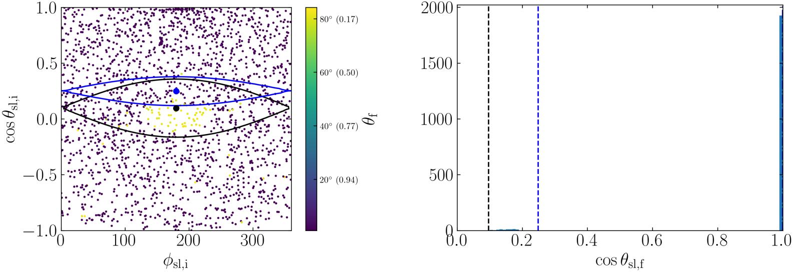
$I_1 = 10°$, $I_2 = 1°$, $g_1 = 0.1\alpha$, $g_2 = 2.5g_1$



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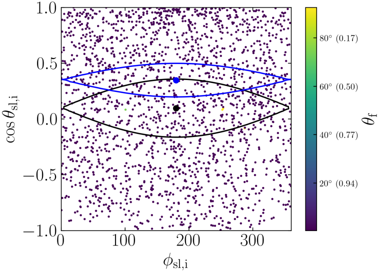
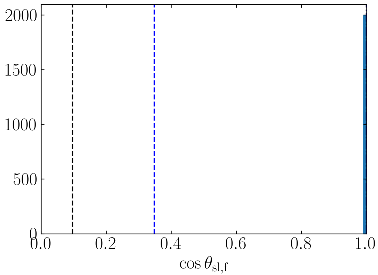
$I_1 = 10°$, $I_2 = 1°$, $g_1 = 0.1\alpha$, $g_2 = 3.5 g_1$





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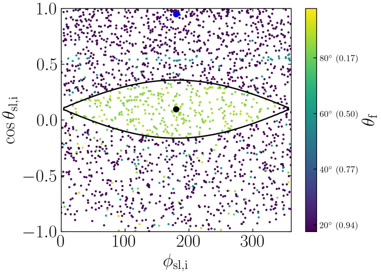
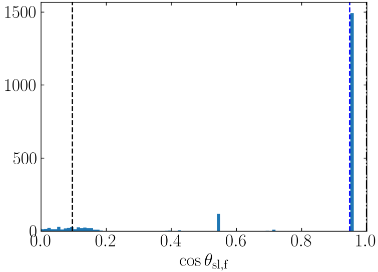
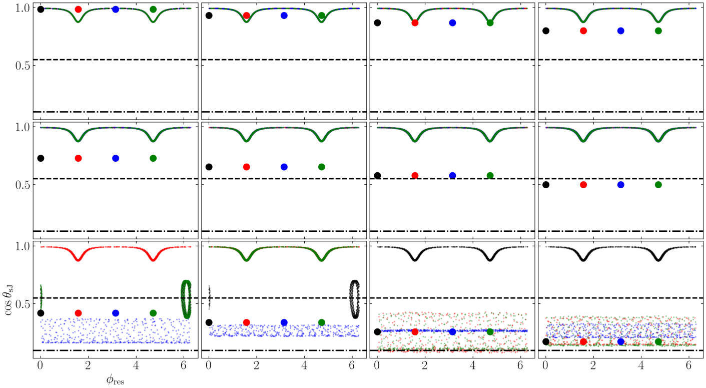
$I_1 = 10^\circ$, $I_2 = 1^\circ$, $g_1 = 0.1\alpha$, $g_2 = 10g_1$

Mixed mode? Frequency is none of CSs!





Resonance angle is $\phi_{\rm res} = \phi_{\rm sJ} + (g_1 + g_2)\,t/2$. $I_1 = 10^\circ$, $I_2 = 1^\circ$, $g_1 = 0.1\alpha$, and $g_2 = 10 g_1$.



More Planets

Mixed Mode

Resonance angle is $\phi_{\rm res} = \phi_{\rm sJ} + (g_1 + g_2)\,t/2$. $I_1 = 10^\circ$, $I_2 = 2^\circ$, $g_1 = 0.1\alpha$, and $g_2 = 15 g_1$.