# Cassini State Separatrix Hopping

## Group Meeting Presentation

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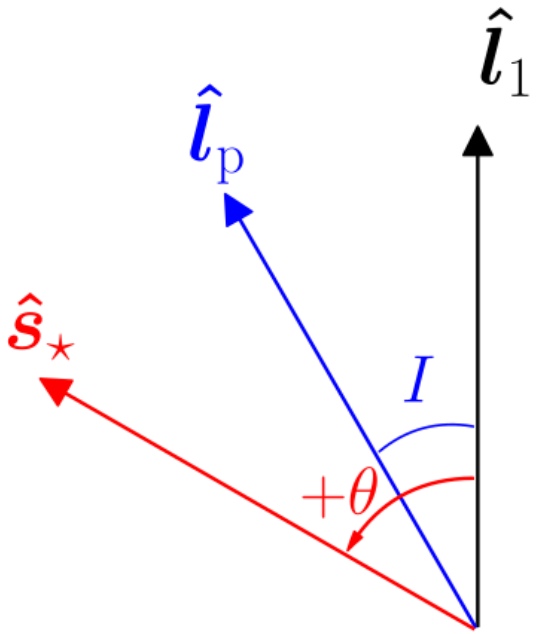



- In corotating ($\hat{l}_p$ fixed) frame,

$$\frac{d\hat{s}}{dt} = (\hat{s}\cdot\hat{l}_1)(\hat{s}\times\hat{l}_1) - \eta(\hat{s}\times\hat{l}_p).$$

  - $\eta = \frac{|g|}{\alpha}$: $g$ is $\hat{l}_1$ precession around total angular momentum axis, $\alpha$ spin precession.
- Hamiltonian

$$\mathcal{H} = \frac{(\hat{s}\times\hat{l}_1)^2}{2} - \eta(\hat{s}\cdot\hat{l}_p).$$

# Cassini States

## Separatrix

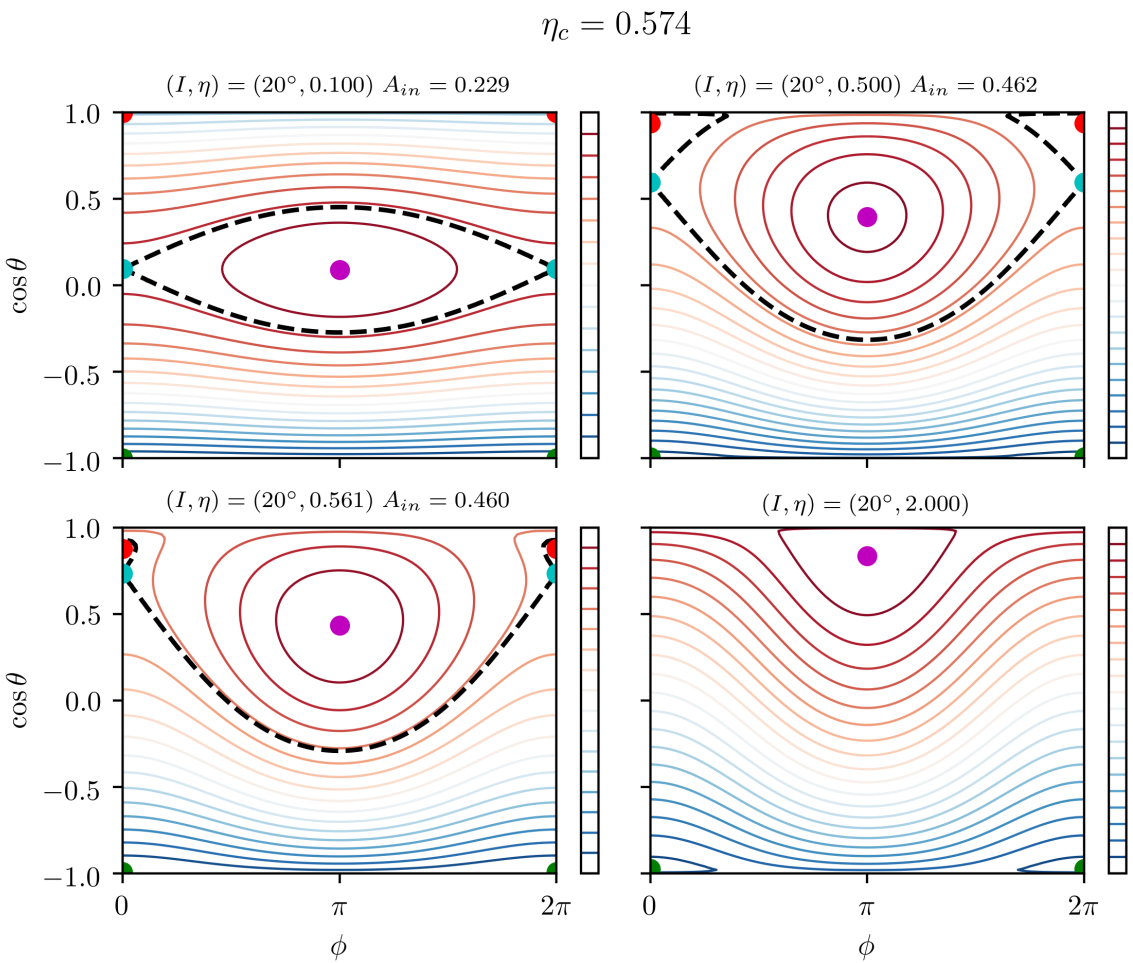



**Figure:** Black line corresponds to *separatrix*. Equipotential surface joining two saddle points, *flows cannot cross*. Area can be numerically estimated.

# Cassini States

## Separatrix

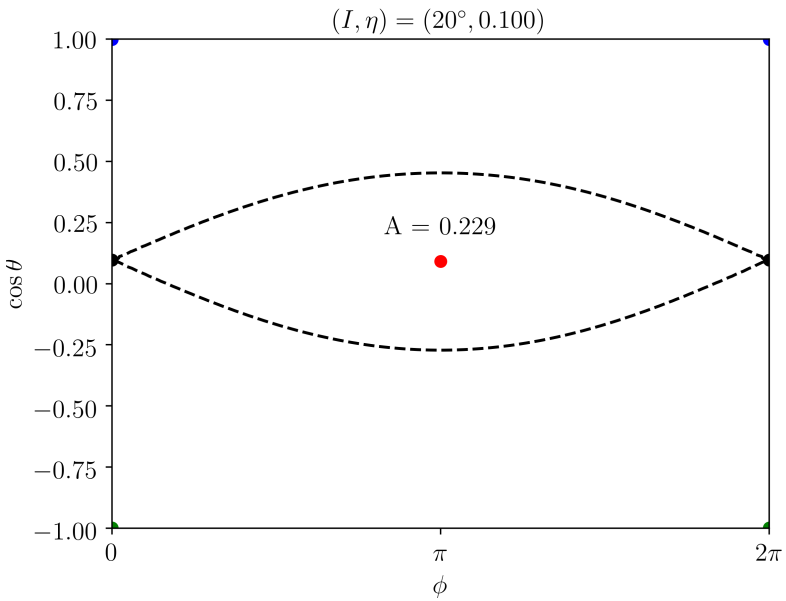



**Figure:** Red, purple Cassini states are stable, *attracting* with dissipation.

- Zoom in on $\eta = 0.1$ case (left).
- **If weak dissipation & random IC, what is fate of system?**
- *Hypothesis:*

  Outside Alignment (77.1%)

  Inside High obliquity (22.9%)

# Cassini States

## Simulations

Introduce tidal force $\left(\frac{\mathrm{d}\hat{s}}{\mathrm{d}t}\right)_{tide} = \epsilon\hat{s}\times\left(\hat{l}_1\times\hat{s}\right) = -\epsilon\sin\theta\hat{\theta}$. Fate?

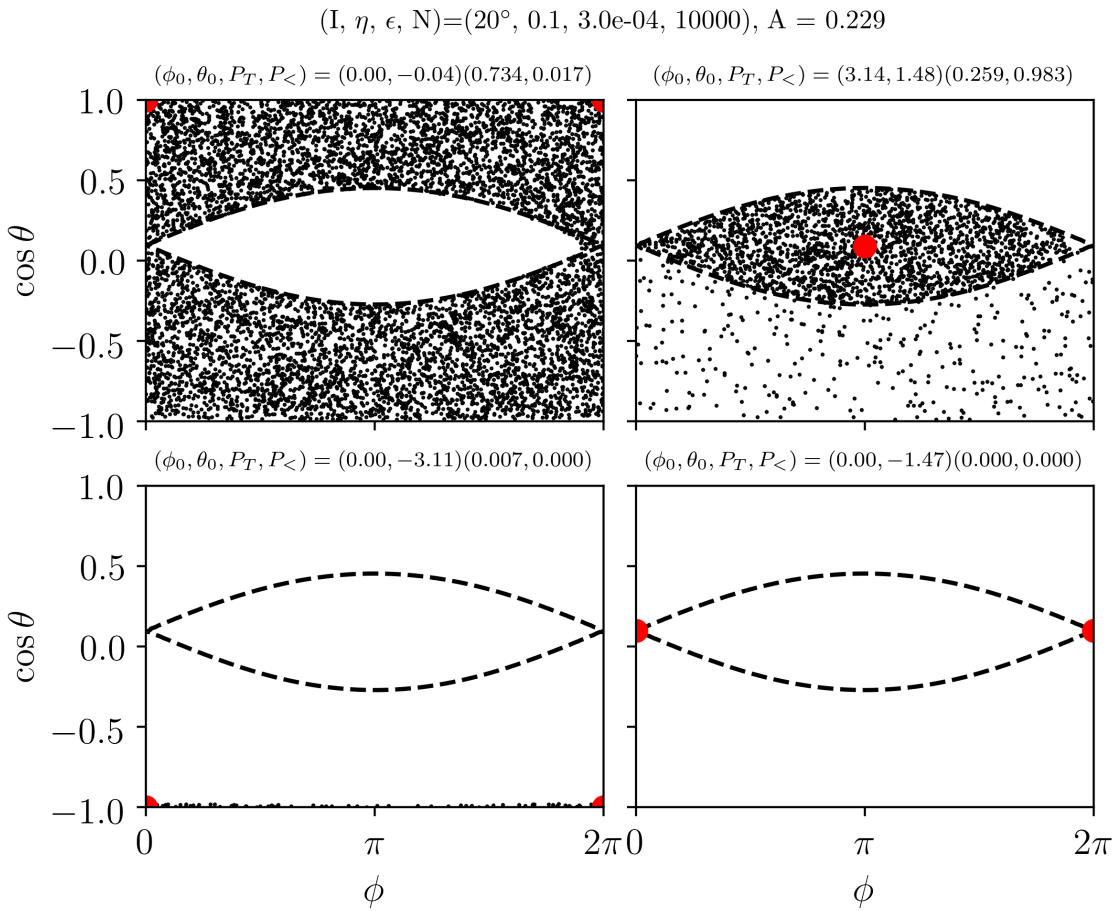



**Figure:** In other sims $\epsilon \to 0$, $P_{hop} \to 0.08$, *nonzero*!

- *Revised hypothesis*: **74%** align, **26%** high obliquity,
  - Above: Goes to alignment (38.55%)
  - Inside: High obliquity (22.9%)
  - Below: High-obliquity 8% (3.1%), rest align (35.55).
- Data: **73.4%** align, **25.9%** high obliquity!

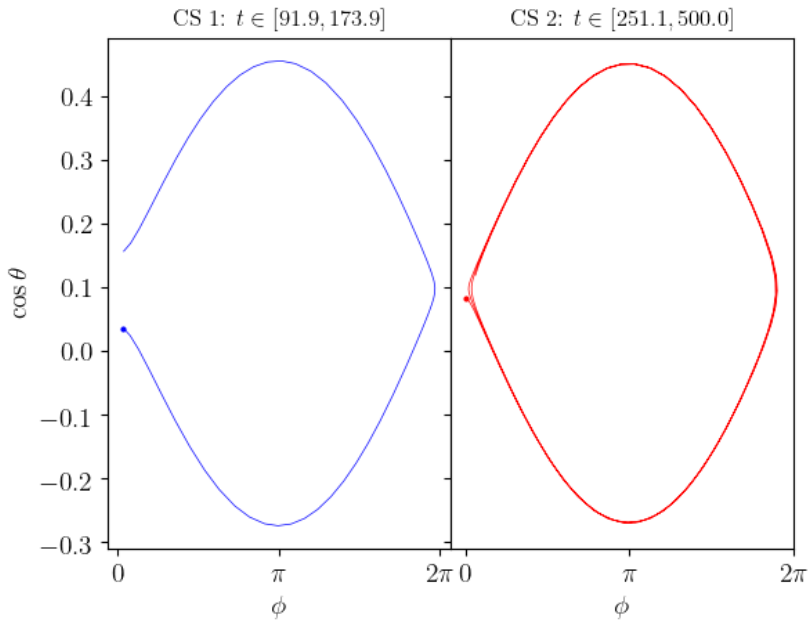
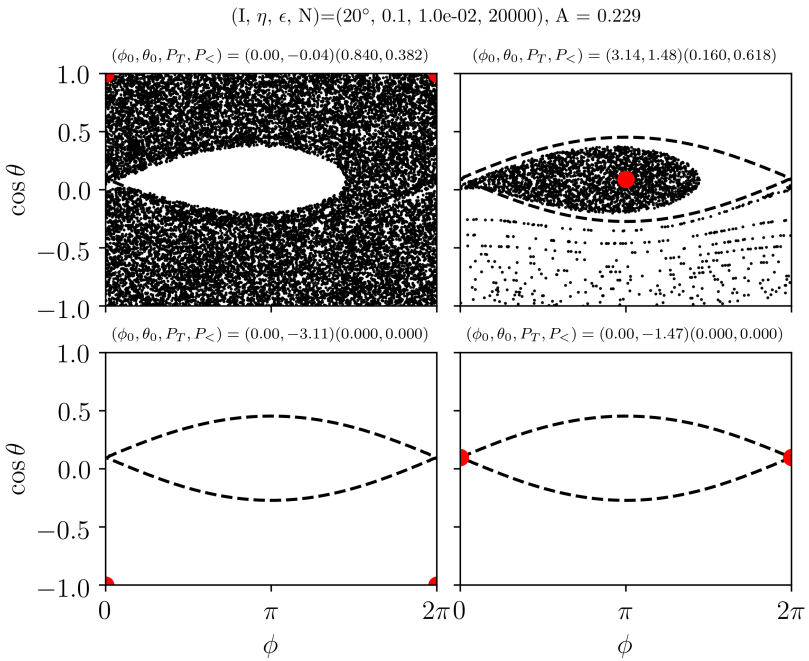



**Figure:** Trajectories at moment of crossing $\theta_4$, converging to two attracting CSs.

# Separatrix Hopping

## Heteroclinic Orbits



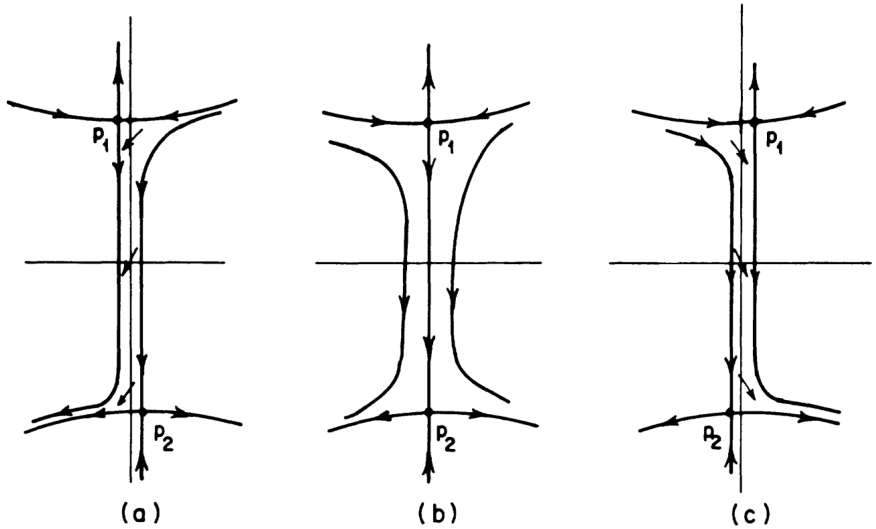




**Figure:** Heteroclinic orbit (*separatrix*) under perturbation.

# Separatrix Hopping

## Hypothesis

- Modified Cassini state under tides, $\delta\phi_4 = +\frac{\epsilon}{\eta \sin I}$.
- Opens gap $\propto \epsilon$, but alignment strength also $\propto \epsilon$ (draw).
- Thus, hopping probability $\propto \mathcal{O}(\epsilon^2)$.

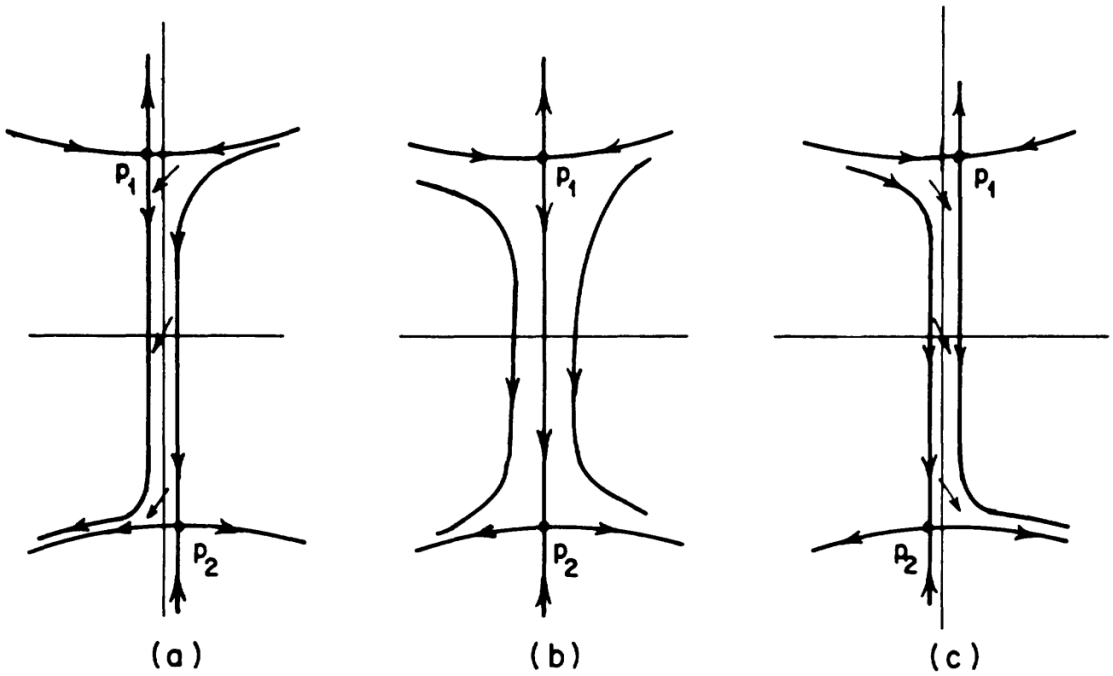



**Figure:** Heteroclinic orbit (*separatrix*) under perturbation.

# Separatrix Hopping

## Varying $\eta$

- Tried varying $\eta$, fixed $\epsilon$. Maybe $P_{hop} \propto \eta A$?:

| $\eta$ | 0.025 | 0.05 | 0.1 | 0.2 |
|---|---|---|---|---|
| $P_{hop}$ | 0.01 | 0.028 | 0.08 | 0.25 |
| $A$ | 0.115 | 0.163 | 0.229 | 0.320 |