High Spin-Orbit Misalignment as an Attractor in Cassini State Systems with Weak Tidal Friction

Group Meeting XX/XX/XXXX

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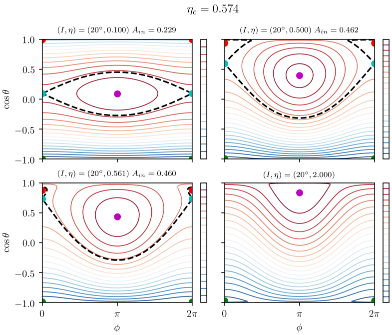
Some day

Problem 1

Introduction

- Close-in planet to a star w/ $\vec{l}$ gstarts with random spin $\hat{s}$ (e.g. collision). Evolves under tides + precession around perturber $\hat{l}_p$.
- **Toy Problem:** Assume constant tidal dissipation, fate?
- Cassini States: $H^{(0)} = \frac{\left(\hat{s}\cdot\hat{l}\right)^2}{2} + \eta\hat{s}\cdot\hat{l}_p$. CS4 is saddle point, *separatrix*.



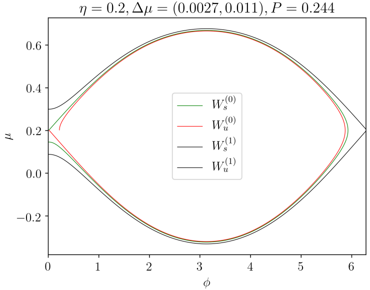
Problem 1

Constant Tides

- Constant tides $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \epsilon \sin\theta$, EOM ($\mu = \cos\theta$):

$$
\begin{aligned}
\frac{\mathrm{d}\hat{s}}{\mathrm{d}\tau} &= \left(\hat{s}\cdot\hat{l}\right)\left(\hat{s}\times\hat{l}\right) - \eta\hat{s}\times\hat{l}_p + \epsilon\hat{s}\times\left(\hat{l}\times\hat{s}\right),\\
\frac{\partial\phi}{\partial t} &= \mu - \eta\left(\cos I + \sin I\frac{\mu}{\sqrt{1-\mu^2}}\cos\phi\right),\\
\frac{\partial\mu}{\partial t} &= -\eta\sin I\sin\phi + \epsilon\left(1-\mu^2\right),
\end{aligned}
$$

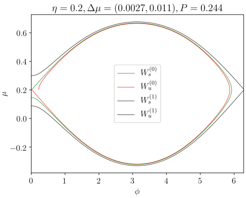
- *Review:* Last meeting, found probability that initial state $\cos\theta < 0$ jumps into separatrix $\propto \eta^{3/2}\epsilon^0$.



- Notation: subscripts *stable*, *unstable* manifolds, superscripts are left/right copy of CS4.
  - Below black, continues orbit $\mu < \mu_4$.
  - Black/green, escapes.
  - Green/red, captures.
- Evaluate $\Delta\mu$ at $\phi = \pi$ reproduces probability (numerical was $P_{hop}(\eta = 0.2) = 0.251$).

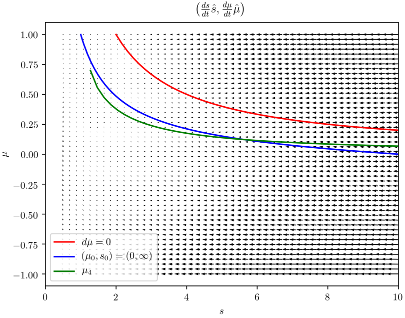
- **Goal:** Under weak perturbation, how much does trajectory deviate from level curve of H?
- Procedure:
  - Compute $\Delta H^{(0)}$ unperturbed over trajectory.
  - Locate new level curve to find Δq along coordinate: $\Delta H^{(0)} = \frac{\partial H^{(0)}}{\partial q}\Delta q$.
- So looks something like:

$$\frac{\mathrm{d}H^{(0)}}{\mathrm{d}t} = \underbrace{\cancel{\frac{\partial H^{(0)}}{\partial \mu}\frac{\mathrm{d}\mu^{(0)}}{\mathrm{d}t} + \frac{\partial H^{(0)}}{\partial \phi}\frac{\mathrm{d}\phi^{(0)}}{\mathrm{d}t}}}_{\dot\phi^{(0)}\dot\mu^{(0)} - \dot\mu^{(0)}\dot\phi^{(0)}} + \frac{\partial H^{(0)}}{\partial \mu}\frac{\mathrm{d}\mu^{(1)}}{\mathrm{d}t}.$$



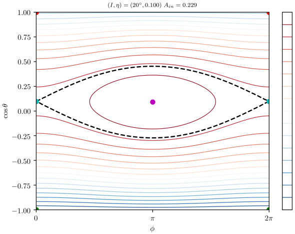
- Gives rise to Melnikov distance:
  - $\Delta\mu(\phi) = \frac{1}{\dot\phi(\phi)}\oint \dot\phi^{(0)}\epsilon\left(1-\mu^2\right)\mathrm{d}t$.
  - $\mu(\phi) \approx \eta\cos I \pm \mathcal{O}\left(\sqrt{\eta}\right)$.
  - $W_s^{(1)}, W_u^{(0)}$: dominated by 1.
  - $W_s^{(0)}, W_u^{(0)}$: dominated by $\eta^{3/2}$!

- In realistic tides, η can evolve as s spins down.
- $\eta = \frac{s_c}{s}$, full expression below.
- First, consider tides alone, phase portrait at right:
  - Like ignoring $\frac{\mathrm{d}\mu}{\mathrm{d}\phi}$ over precession orbit.
  - Breaks down near μ_4!



$$\frac{\mathrm{d}\hat{s}}{\mathrm{d}\tau} = \frac{s}{s_c}\left(\hat{s}\cdot\hat{l}\right)\left(\hat{s}\times\hat{l}\right) - \hat{s}\times\hat{l}_p + \frac{\epsilon 2\Omega}{s}\left(1 - \frac{s}{2\Omega}\left(\hat{l}\cdot\hat{s}\right)\right)\hat{s}\times\left(\hat{l}\times\hat{s}\right), \tag{1}$$

$$\frac{\mathrm{d}s}{\mathrm{d}\tau} = \epsilon 2\Omega\left(\hat{s}\cdot\hat{l} - \frac{s}{2\Omega}\left(1 + \left(\hat{s}\cdot\hat{l}\right)^2\right)\right). \tag{2}$$



- Reason 2

- Actually, $\mu_0 \equiv \mu(\phi = 0)$ sets when encounters CS4.
  - $|\mu_0 - \mu_4| \gtrsim \sqrt{\eta \sin I}$, then $\mu(\phi) \approx \mu_0$.