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Analytical Predictions of Explanet Obliquities Generated by Planet-Disk Interactions

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ABSTRACT

Abstract here

Key words: planet–star interactions

1 INTRODUCTION

Separatrix crossing was studied by (Henrard 1982).

2 THEORY AND EQUATIONS

We study an oblate planet orbiting around a star hosting a further out planet. The equations of motion in the absence of dissipation are:

We define s_c to be the critical spin where the precession frequencies ω_{s1} and ω_{21} are equal, or:

Throughout this paper, we often set the orbital frequency of the inner planet $\Omega_1 = 1$.

2.1 Cassini States

Equilibria of spin dynamics are *Cassini States* (CSs). Refer to Su & Lai 2020 for more detailed discussion. In the notation of the previous paper,

$$\eta = s_c/s. \tag{1}$$

Plots of CS locations as always.

Note that ϕ is conjugate to $\cos\theta$, and thus the spin dynamics exhibit Hamiltonian:

Plot of level curves of Hamiltonian, including $C_\pm$ notation.

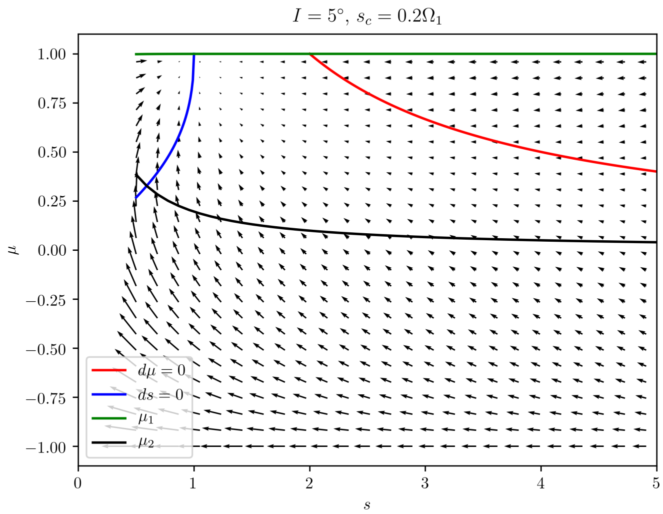

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Figure 1. Phase portrait.

2.2 Weak Tidal Friction

We use the weak tidal friction model (Lai 2012). The EOM for θ, s become:

The phase portrait for these EOM in (s, θ) space is shown in Fig. 1.

TODO overplot CS1, CS2 for *two* values of s_c. TODO show on plot the location where tides are so strong that CS2 ecomes unstable (is the only ϵ-dependent result of the paper).

2.3 Stable Equilibria of Tidal Friction

Generally, these EOM have equilibria at the points that are both a CS and satisfy $\dot{s} = 0$ under weak tidal friction. These are stable as $\epsilon \to 0$, see Appendix A. On Fig. 1, these are the intersection of the locations of CS1 and CS2 with the line $\dot{s} = 0$. It is clear that the locations of these equilibria depend on the value of s_c. Call these generalized equilibria *tidal Cassini States* (tCS), and number them tCS1 and tCS2 depending on whether they are CS1 or CS2 states.

Furthermore, if tides are too strong (large ϵ), CS2 can become unstable (cite Fabrycky). This can be quantified as:

3 PROBABILITY DISTRIBUTION OF OUTCOMES

The general question of the dynamics is then as follows: given problem parameters (including s_c) and an initial planet spin and obliquity (s, θ), what are the possible outcomes and their associated probabilities? We study this first at fixed s_c as a function of θ in Section 3.1, then as a function of s_c for an isotropic initial spin $\hat{s}$ in Section 3.2.

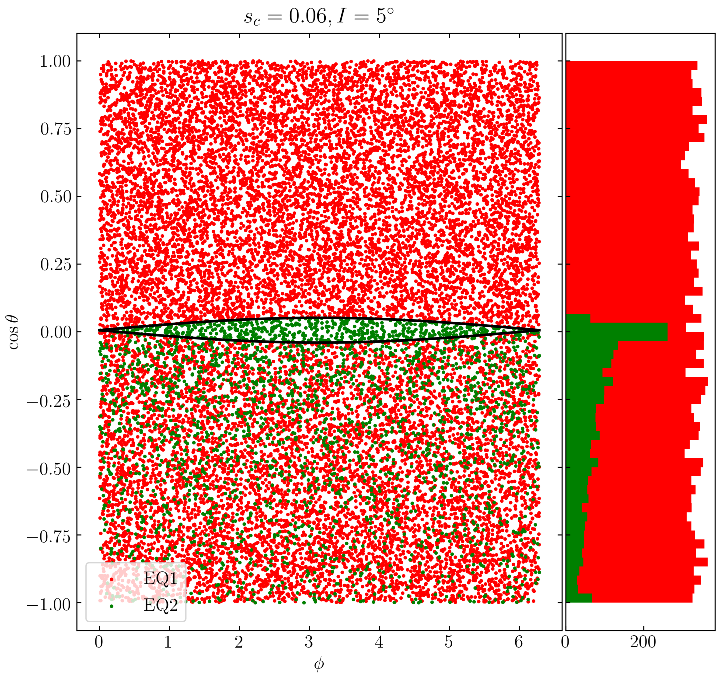

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Figure 2. $s_c = 0.06$. Note that it is difficult to reach tCS2.

3.1 Distribution As a Function of θ

As a result of subsection 2.3, the tCS are the only possible final outcomes. We generate initial spin vectors $\hat{s}$ for isotropic initial distribution $(\cos\theta, \phi)$, and marginalize over ϕ (which generally cannot be fixed by any formation mechanism) in histograms. This gives histograms in Figs. **??**

The governing mechanism for these probabilities is probabilistic separatrix crossing, which is covered in Appendix B3. The calculation is difficult analytically, but can be performed semi-analytically (describe procedure here, calculate $\eta_\star$ numerically etc.). The agreement of this curve with the histograms is examined for the same three s_c values in Figs. **??**.

3.2 Distribution As a Function of s_c

Assuming that $\hat{s}$ is drawn from an isotropic distribution, we may then calculate the probabilities of going to either tCS as a function of s_c. This is shown below for $I = 5°$

Note that while only the results for an isotropic distribution of initial $\hat{s}$ are shown, in principle arbitrary distributions $P(\theta_i)$ can be convolved against the $P_{tCS2}(\theta_i)$ distributions shown in subsection 3.1.

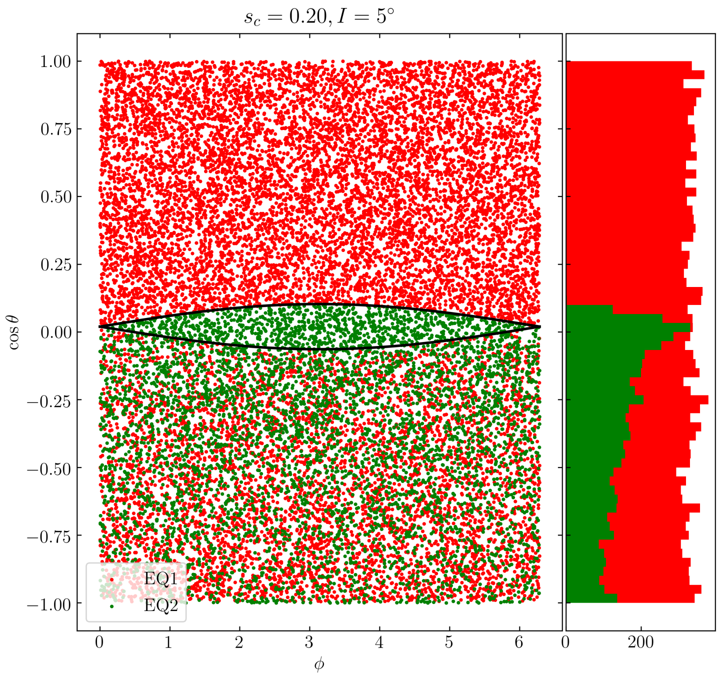

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Figure 3. $s_c = 0.2$. Note that tCS2 is both reached with substantial probability and has substantial obliquity.

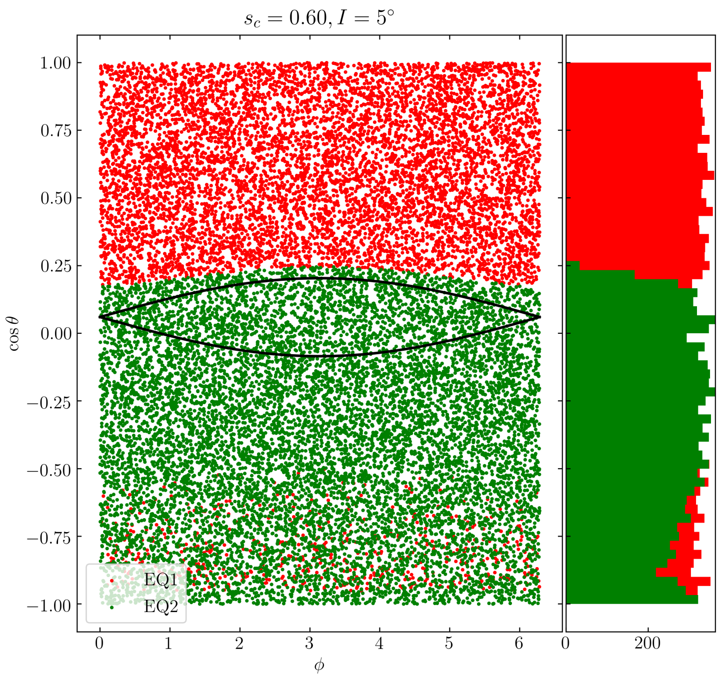

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Figure 4. $s_c = 0.6$. Note that tCS2 becomes attracting over tCS1, but will have obliquity $\approx I$ and is uninteresting.

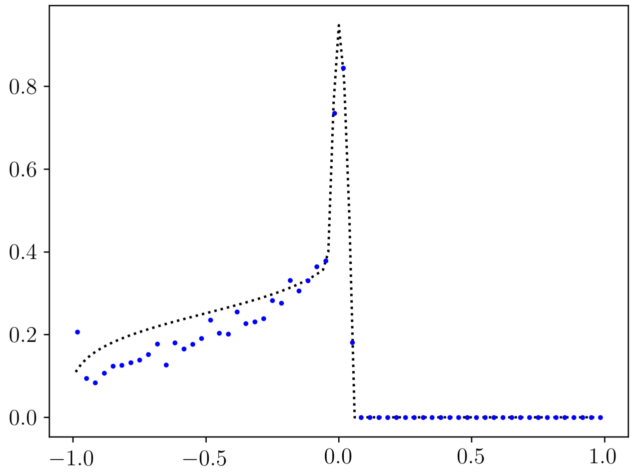

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Figure 5. $s_c = 0.06$ fit to histogram.

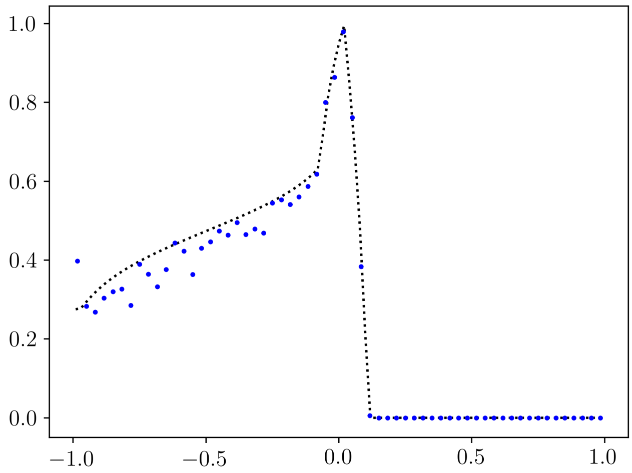

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Figure 6. $s_c = 0.20$ fit to histogram.

4 SUMMARY AND DISCUSSION

REFERENCES

Henrard J., 1982, Celestial Mechanics and Dynamical Astronomy, 27, 3

Lai D., 2012, Monthly Notices of the Royal Astronomical Society, 423, 486

APPENDIX A: PROOF OF STABILITY OF CASSINI STATES 1 & 2

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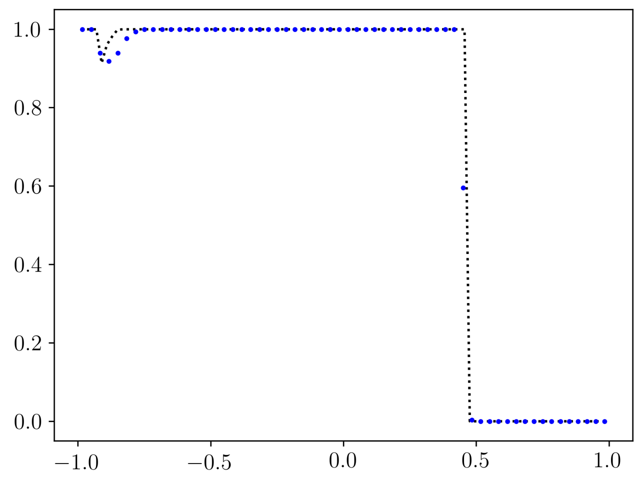

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Figure 7. $s_c = 0.70$ fit to histogram.

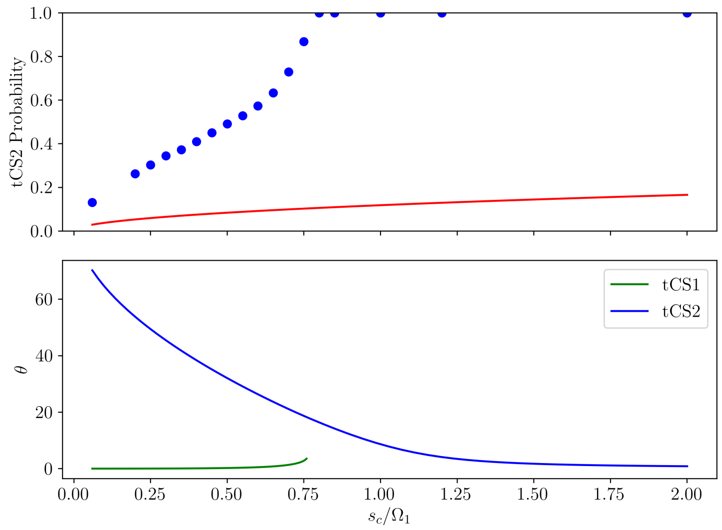

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Figure 8. Top: total probability of ending up in tCS2 (blue dots) and the prediction ignoring separatrix capture (red line). Bottom: obliquities of the tCSs.

APPENDIX B: SEPARATRIX CROSSING DYNAMICS

B1 Theory

B1.1 Application of Adiabatic Invariance: Henrard Theory

Review Henrard result, is already very successful e.g. MMR capture.

B1.2 Melnikov Integral

The first substantial new result.

B2 Example: Constant s

"Toy problem 1", the nice $P_c \propto \eta^{3/2}$ result. Application of Section B1.2.

B3 Separatrix Crossing Probability: Tidal Friction

Application of the full formula presented in Section B1. The key result is that one integrates

The capture probabilities are then

An analytical form that holds when $s \gg s_c$ is:

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