High Spin-Orbit Misalignment is Sometimes Attracting: Cassini State Systems with Weak Tidal Friction

Group Meeting

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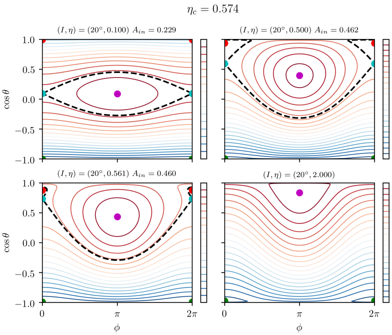
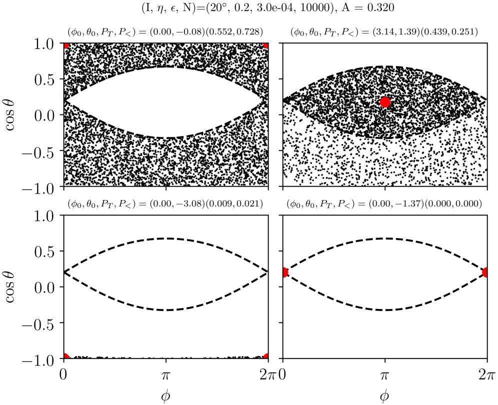
- Close-in planet to a star w/ $\vec{l}$ gstarts with random spin $\hat{s}$ (e.g. collision). Evolves under tides + precession around perturber $\hat{l}_p$.
- **Toy Problem:** Assume constant tidal dissipation, fate?
- Cassini States: $H^{(0)} = \frac{\left(\hat{s}\cdot\hat{l}\right)^2}{2} + \eta\hat{s}\cdot\hat{l}_p$. CS4 is saddle point, *separatrix*.



- Constant tides $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \epsilon \sin\theta$, EOM ($\mu = \cos\theta$):

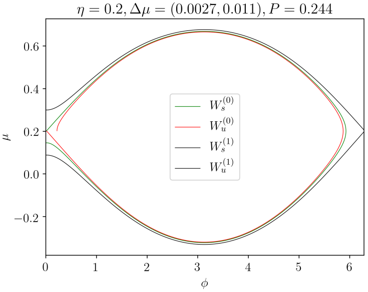
$$\frac{\mathrm{d}\hat{s}}{\mathrm{d}\tau} = \left(\hat{s}\cdot\hat{l}\right)\left(\hat{s}\times\hat{l}\right) - \eta\hat{s}\times\hat{l}_p + \epsilon\hat{s}\times\left(\hat{l}\times\hat{s}\right).$$

- *Review:* Last meeting, found $P_{hop} \propto \eta^{3/2}\epsilon^0$.



Problem 1

Flow Boundaries (optional)



- Key result:

$$P_{hop} = \frac{16\eta^{3/2}\cos I\sqrt{\sin I}}{\pi}.$$

- Analytical, similar to MMR capture probability.

Problem 2

Realistic Tides

- In realistic tides, η can evolve as s spins down.
- $\eta \equiv \frac{s_c}{s}$, so s_c is *critical spin at which perturber strength is of order spin-orbit coupling*.

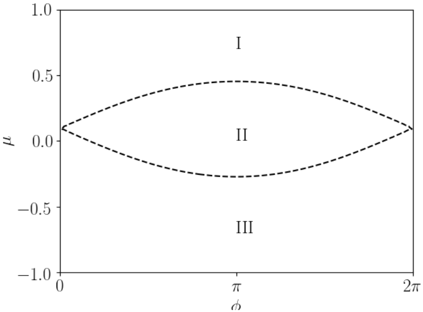
$$\frac{\mathrm{d}\hat{s}}{\mathrm{d}\tau} = \frac{s}{s_c}\left(\hat{s}\cdot\hat{l}\right)\left(\hat{s}\times\hat{l}\right) - \hat{s}\times\hat{l}_p + \frac{\epsilon 2\Omega}{s}\left(1 - \frac{s}{2\Omega}\left(\hat{l}\cdot\hat{s}\right)\right)\hat{s}\times\left(\hat{l}\times\hat{s}\right), \tag{1}$$

$$\frac{\mathrm{d}s}{\mathrm{d}\tau} = \epsilon 2\Omega\left(\hat{s}\cdot\hat{l} - \frac{s}{2\Omega}\left(1 + \left(\hat{s}\cdot\hat{l}\right)^2\right)\right). \tag{2}$$

- Compare to Problem 1:

$$\frac{\mathrm{d}\hat{s}}{\mathrm{d}\tau} = \left(\hat{s}\cdot\hat{l}\right)\left(\hat{s}\times\hat{l}\right) - \eta\hat{s}\times\hat{l}_p + \epsilon\hat{s}\times\left(\hat{l}\times\hat{s}\right).$$

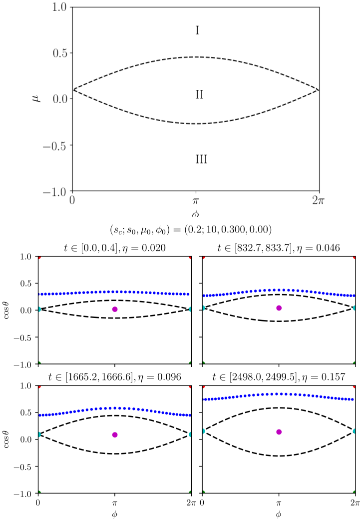
- Three zones:

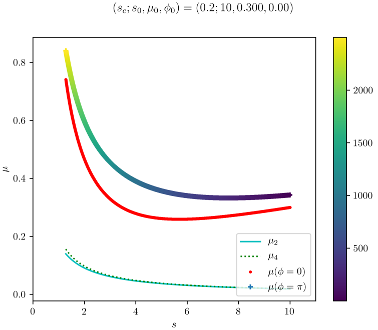


- According to work from Problem 1, expect:
  - I Goes to CS1/alignment.
  - II Goes to CS2/misalignment.
  - III P_{hop} to CS2 or CS1.

Problem 2

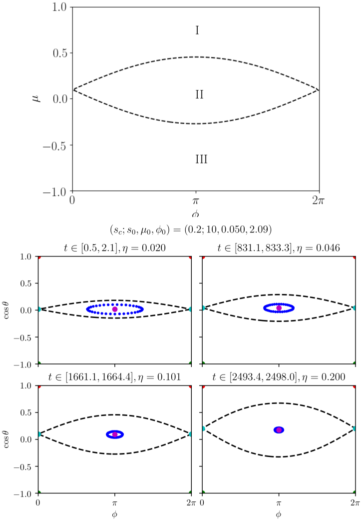
$s_c = 0.2$, Zone I

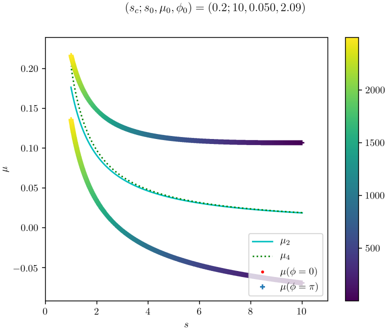




Problem 2

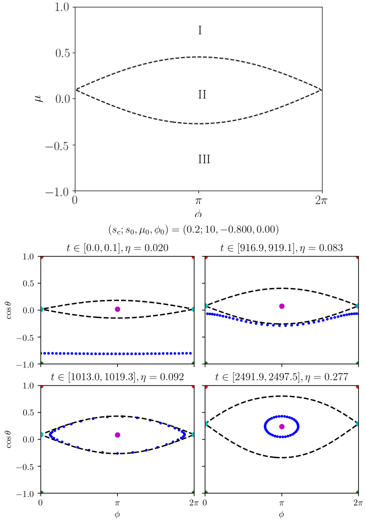
$s_c = 0.2$, Zone II

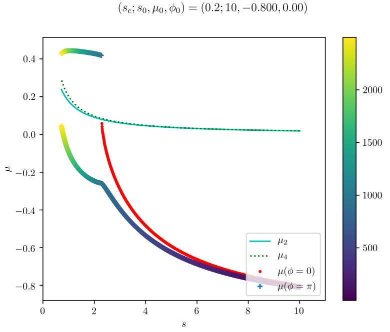




Problem 2

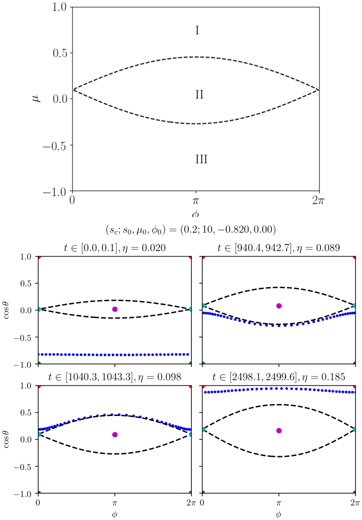
$s_c = 0.2$, Zone III, Separatrix Hopping

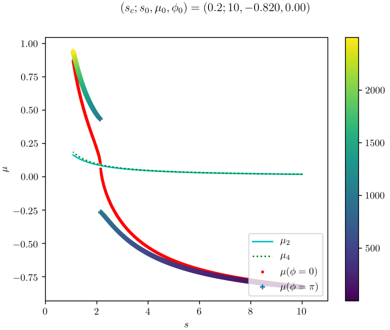




Problem 2

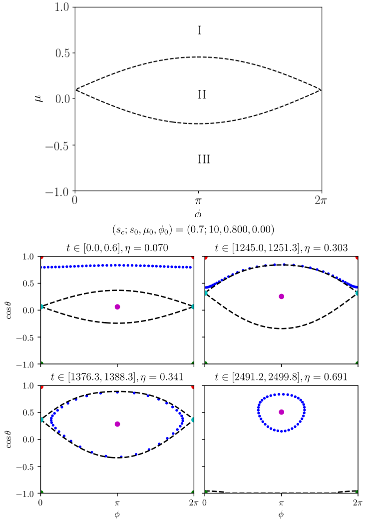
$s_c = 0.2$, Zone III, Separatrix Crossing

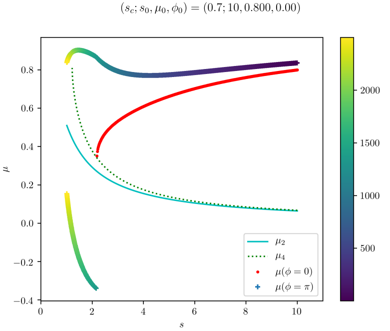




Problem 2

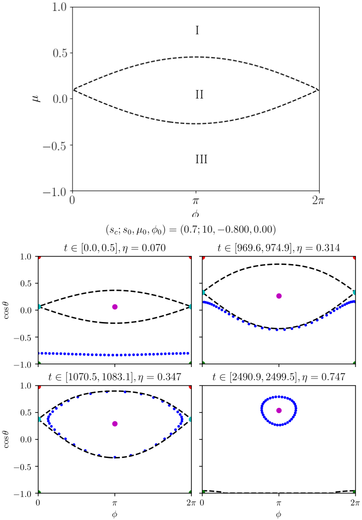
$s_c = 0.7$, Zone I, Separatrix Crossing!

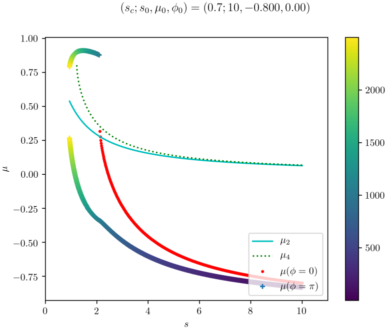




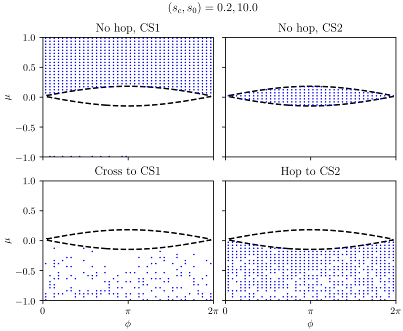
Problem 2

$s_c = 0.7$, Zone III, Always Separatrix Crossing!



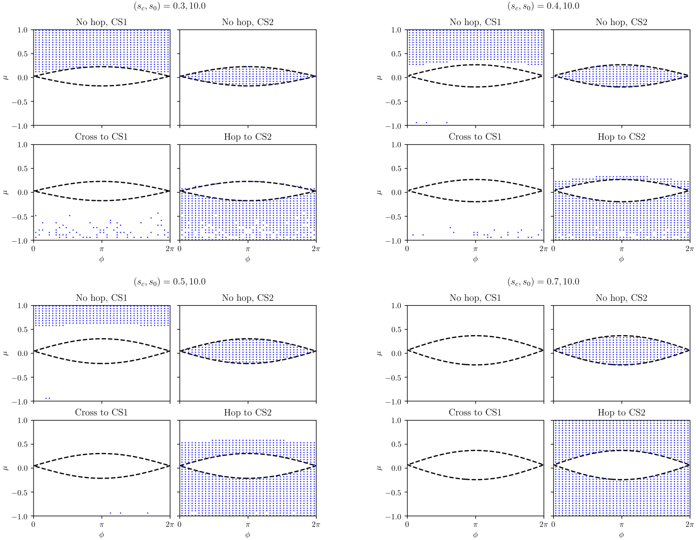


- In summary, going from $s_c = 0.2$ to $s_c = 0.7$ makes CS2 attracting in zone I, sets $P_{hop} = 1$ for zone III.
- Interesting histories
  - CS1, no sep crossing (I).
  - CS2, no sep crossing (II).
  - Cross to CS1 (III).
  - Hop to CS2 (III).
- At $s_c \lesssim 0.3$, $P_{hop} \propto \sqrt{s}$, so closer to sep $\Rightarrow$ higher P_{hop}.

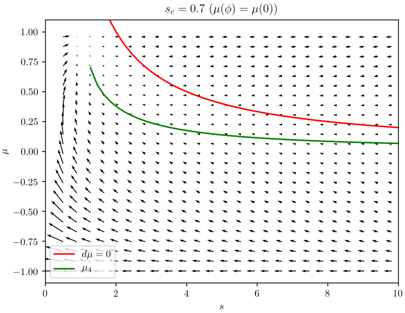


Problem 2

Evolution of Outcome Distribution with s_c



- In the absence of perturber (just the weak tide components), phase portrait takes on shape:
- Green is μ_4 Cassini State 4, above red is $\dot{\mu} < 0$.



Problem 2

Sign of $\left\langle \frac{\mathrm{d}\mu}{\mathrm{d}t} \right\rangle$