Cassini State Separatrix Hopping

Group Meeting Presentation

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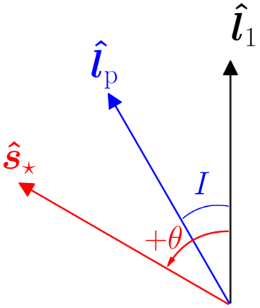
Apr 19, 2019



- In corotating ($\hat{l}_p$ fixed) frame,

$$\frac{\mathrm{d}\hat{s}}{\mathrm{d}t} = \left(\hat{s}\cdot\hat{l}_1\right)\left(\hat{s}\times\hat{l}_1\right) - \eta\left(\hat{s}\times\hat{l}_p\right).$$

  - $\eta = \frac{|g|}{\alpha}$: g is $\hat{l}_1$ precession around total angular momentum axis, α spin precession.

- Hamiltonian

$$\mathscr{H} = \frac{\left(\hat{s}\times\hat{l}_1\right)^2}{2} - \eta\left(\hat{s}\cdot\hat{l}_p\right).$$

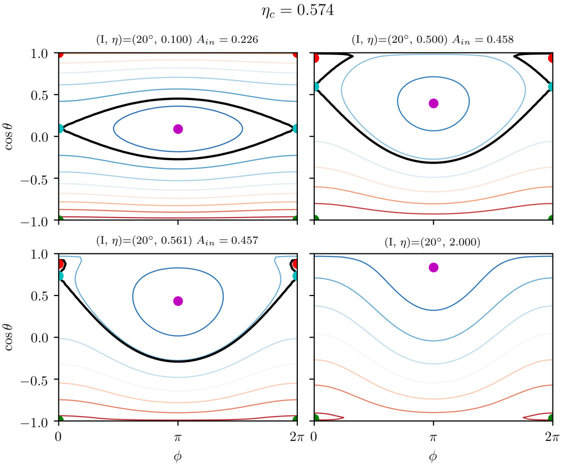


Figure: Black line corresponds to *separatrix*. Equipotential surface joining two saddle points, *flows cannot cross*. Area can be numerically estimated.

Cassini States

Separatrix

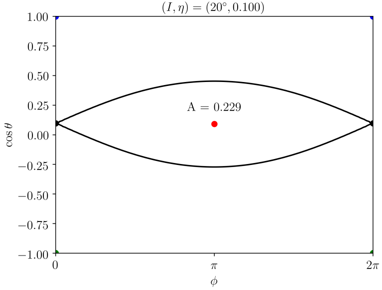


Figure: Red, purple Cassini states are stable, *attracting* with dissipation.

- Zoom in on $\eta = 0.1$ case (left).
- **If weak dissipation & random IC, what is fate of system?**
- *Hypothesis:*

Outside	Alignment (77.1%)
Inside	High obliquity (22.9%)

Introduce tidal force $\left(\frac{\mathrm{d}\hat{s}}{\mathrm{d}t}\right)_{tide} = \epsilon\hat{s} \times \left(\hat{l}_1 \times \hat{s}\right) = -\epsilon \sin\theta\hat{\theta}$. Fate?

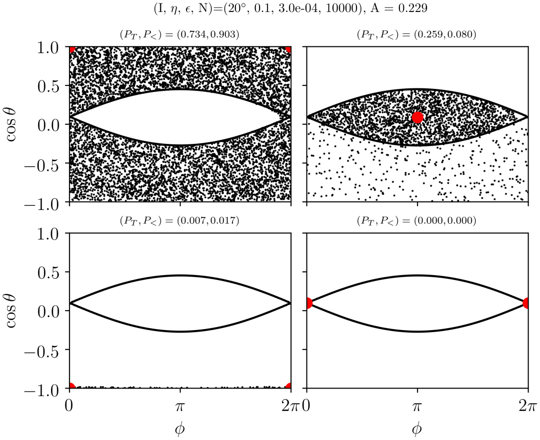


Figure: In other sims $\epsilon \to 0$, $P_{hop} \to 0.08$, *nonzero*!

- *Revised hypothesis*: **74%** align, **26%** high obliquity,

Above	Goes to alignment (38.55%)
Inside	High obliquity (22.9%)
Below	High-obliquity 8% (3.1%), rest align (35.55).

- Data: **73.4%** align, **25.9%** high obliquity!

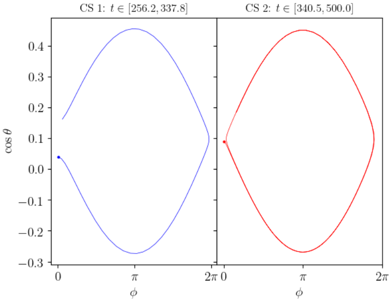
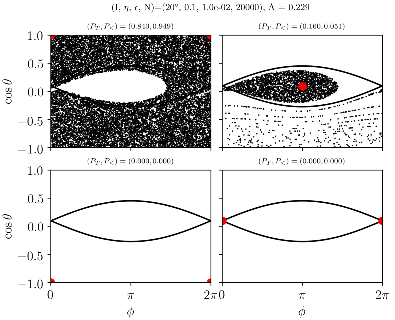


Figure: Trajectories at moment of crossing θ_4, converging to two attracting CSs.

Separatrix Hopping
Heteroclinic Orbits



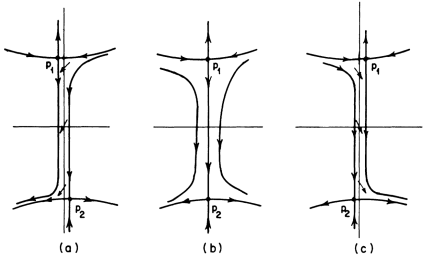


Figure: Heteroclinic orbit (*separatrix*) under perturbation.

Separatrix Hopping
Hypothesis

- Modified Cassini state under tides, $\delta\phi_4 = +\frac{\epsilon}{\eta \sin I}$.
- Opens gap $\propto \epsilon$, but alignment strength also $\propto \epsilon$ (draw).
- Thus, hopping probability $\propto \mathcal{O}(\epsilon^2)$.

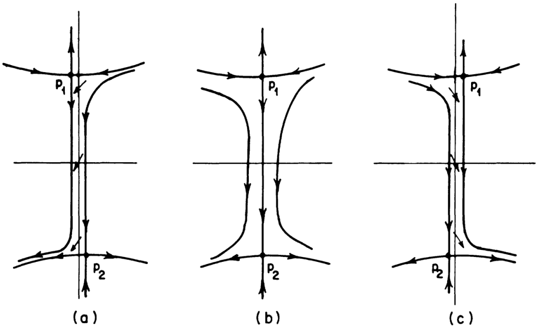


Figure: Heteroclinic orbit (*separatrix*) under perturbation.

Separatrix Hopping
Varying η

- Tried varying η, fixed ϵ. Maybe $P_{hop} \propto \eta A$?:

η	0.025	0.05	0.1	0.2
P_{hop}	0.01	0.028	0.08	0.25
A	0.115	0.163	0.229	0.320