Nonlinear Tidal Dissipation in Binary White Dwarfs

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1 Introduction and Goals of Proposed Research

1.1 White Dwarf Binaries

Compact white dwarf (WD) binary systems, with orbital periods in the range of minutes to hours, are important for a range of astrophysical problems. They are the most important sources of gravitational waves (GWs) for the Laser Interferometer Space Antenna (LISA)[1]. They are also thought to produce interesting optical transients such as underluminous supernovae[2], Ca-rich fast transients[3], and tidal novae[4]. Most importantly, they have been proposed as the likely progenitors of type Ia supernovae (e.g. [5, 6] or more recently[7, 8]). While presently only a few tens of compact WD binaries are known[9], *Gaia* (currently gathering data) is expected to expand the catalog to a few hundreds[9] (results based on *Gaia*'s second data release have already begun to appear[10, 11]), and the Large Synoptic Survey Telescope (LSST, first light scheduled for 2020) will likely detect a few thousand more[9]. These observations will significantly advance the understanding of WD binaries and their evolution. My proposed theoretical and computational research is well-timed to take advantage of these new advances.

In spite of the broad importance of WD binaries, the evolution of these systems prior to their final mergers is not well understood. Much of this uncertainty comes from our imprecise understanding of tidal interactions, which play an important role during a compact WD binary's inspiral[12]. Previous studies have shown that these interactions manifest as tidal excitation of internal gravity waves (IGW), waves in the WD fluid restored by the buoyancy force due to density stratification[13]. As these waves propagate outwards towards the WD surface, they grow in amplitude until they break, as do ocean waves on a shore, and transfer both energy and angular momentum from the binary orbit to the outer envelope of the WD[12, 13].

Previous works have found that the dissipation of IGW can generate significantly more energy than thermal radiation from the isolated WD surface and is thus a major contributor to the WD energy budget[12, 14]. However, these works parameterized the wave breaking process in an ad hoc manner. The details of dissipation, namely the location and spatial extent of the wave breaking, affect the observable outcome: dissipation near the surface of the WD can be efficiently radiated away and brightens the WD, while dissipation deep in the WD envelope causes an energy buildup that results in energetic flares[4]. Works in other fields based on numerical simulations show that strongly nonlinear wave breaking behaves differently than predictions based on linear and weakly nonlinear theory[15, 16]. Such fully nonlinear numerical simulations have not been performed for WDs.

1.2 Goals of Proposed Research

Characterizing the location and spatial extent of tidal dissipation in WD binaries will require numerical simulation to capture the turbulent cascade to small scales that causes wave breaking. **I propose to study the dynamical effects of tidal dissipation via nonlinear internal gravity wave (IGW) breaking in binary WDs.** There are three specific objectives of my proposed research:

- Objective A: Characterize the location, spatial extent, and other properties of wave breaking in realistic WD models via direct numerical simulation. The location and spatial extent of wave breaking will furnish a simple yet effective parameterization of tidal dissipation in a range of WD models. Section 2 describes the steps required to acheive this goal.
- Objective B: Predict signatures of tidal dissipation over a wide range of possible WD systems. In particular, I will study the impact of tidal heating on the luminosity of WDs in binaries and explore the possibility of producing observable flares. Section 3 details my plan to predict observational manifestations of tidal heating.
- Objective C: Compute modified GW templates for LISA that account for changes in the phase evolution of the orbit due to tidal dissipation. Section 4 elaborates on how I will perform this computation.

2 Objective A: Nonlinear Tidal Dissipation

2.1 Background and Preliminary Work

The current understanding of tidal synchronization in WD binaries is laid out in [12]: Tidal forces from the companion excites IGWs in the deep envelope of the WDs. These IGW propagate outwards and undergo wave breaking in the outer envelope of the WD, locally depositing angular momentum and synchronizing the WD spin to the binary orbit. A similar process also operates in binaries consisting of early type stars[17, 18], the only major difference being in the specifics of wave excitation[1]. Nevertheless, direct numerical simulation of the wave breaking process has not been performed in either of these systems. Since wave breaking is a strongly nonlinear phenomenon, where a larger wave breaks down into many smaller-scale waves, numerical simulation is paramount to an accurate understanding of the tidal dissipation process.

IGW breaking has been studied in atmospheric sciences. The wave breaking process proceeds as follows: Initially, as the IGW reaches nonlinear amplitudes, it breaks down via the parametric instability and transfers energy and angular momentum from the wave to the mean flow of the fluid[19]. After the mean flow velocity reaches the horizontal phase velocity of the IGW, a critical layer forms. Analytical calculations show that the IGW is nearly completely absorbed at this critical layer in the linear approximation and endows the atmosphere with a mean horizontal flow[20, 21]. However, when this mean flow absorption was numerically studied including full nonlinear interactions, new phenomena not described by the linear theory (reflection off the critical layer and sharpening of the mean flow) were observed[15, 22]; this nonlinear behavior significantly affected

[1] While IGWs in massive stars are excited at their radiative-convective boundaries, the excitation of IGWs in WDs is more gradual and is associated with sharp composition changes in the stellar envelope[12].

the evolution of the atmosphere over time. *This highlights the importance of numerical simulation in capturing the wave breaking process.*

To gain insight into the tidal dissipation process, I have already begun to adapt the spectral hydrodynamics code Dedalus[23] to study IGW breaking in a 2D isothermal, stratified atmosphere. A spectral code like Dedalus is ideal for simulating complex hydrodynamical phenomena, as spectral methods have no inherent numerical viscosity and so can more accurately resolve the nonlinear cascade to small length scales in wave breaking.

Working with Dr. Daniel Lecoanet (a post-doc at Princeton University, and one of the authors of the Dedalus code) and my advisor, Prof. Dong Lai, I have simulated the nonlinear evolution of an upward-propagating IGW wavetrain excited at the bottom of the atmosphere (see Figure 1 for an example). My simulations show that waves break and deposit horizontal momentum in the fluid, causing the fluid to acquire an average horizontal flow, consistent with previous studies[12]. I have derived simple formulae for the location and spatial extent of the dissipation zone where the IGW is absorbed by the fluid. More interestingly, my simulation reveals a partial reflection of the IGW at the critical layer[24], a phenomenon not considered in the current astrophysical literature but consistent with the aforementioned results[15]. I am preparing these results for publication[24].

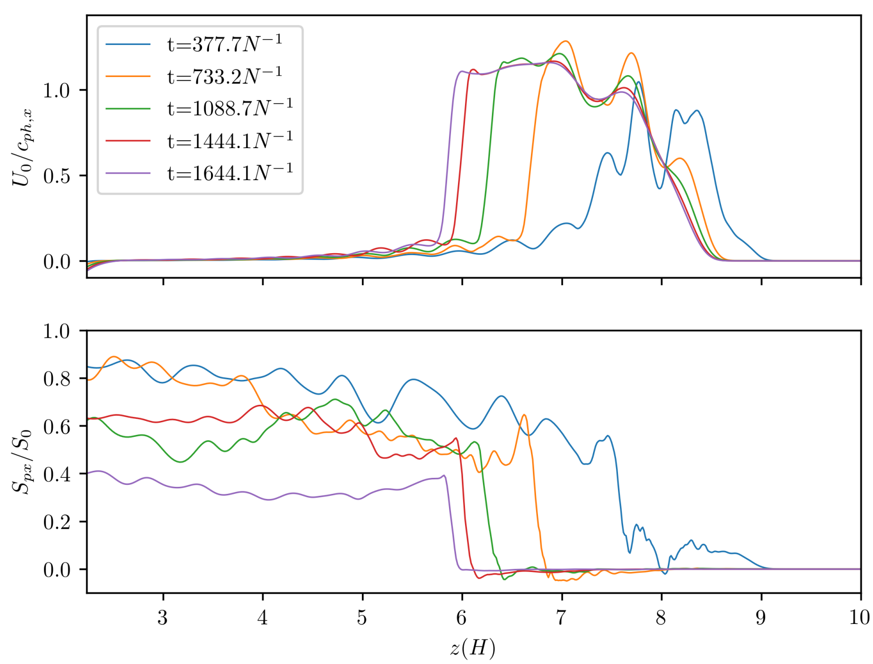

Figure 1: A sample numerical simulation of the evolution of the nonlinear internal gravity wave (IGW) propagating upwards and breaking in a stratified isothermal atmosphere. The upper panel shows the evolution of the average horizontal flow velocity of the fluid U_0 (in units of $c_{ph,x}$ the horizontal phase velocity of the IGW), and the lower panel shows the horizontal momentum flux contained in the IGW $S_{px}(z)$ (in units of the total excited flux S_0). Different lines correspond to different times in the simulation. Times are measured in units of N^{-1}, the inverse of the Brunt-Väisälä frequency (or the buoyancy period), and heights in units of H (the scale height of the density stratification). IGWs are excited at $z = 2H$ and propagate to higher z before undergoing wave breaking. The sharp decrease in S_{px} corresponds to the location of the critical layer where the IGW breaks and drives U_0 towards $c_{ph,x}$; it moves to lower z over time. The apparent decrease in S_{px} at later times indicates wave reflection off the critical layer.

2.2 Proposed Work

It is clear that full numerical modeling of IGW breaking is necessary to fully capture tidal dissipation. The first aim of my proposal is to **extend my preliminary results concerning IGW breaking in stratified atmospheres to characterize the dynamics of nonlinear IGW breaking in realistic WD models.** Via numerical simulation, I will develop models for tidal dissipation that can be used to study long-term WD evolution under tidal heating. I will continue my work in the following stages:

- I will perform simulations examining the validity of my results in spherical geometries to capture tidal effects in WD binaries. I will continue to use Dedalus, which supports spherical coordinates. Although 3D simulations are necessarily more complex than my 2D simulations of stratified atmospheres (see above), the underlying dynamics of IGW breaking are the same.

- I will extend my simulations to realistic WD models and equations of state such as those in [25] as well as those generated by MESA[26] (see Section 3 below), continuing to track the location and spatial extent of the dissipation layer as well as any new phenomena. As WDs vary widely in composition and effective temperature, studying representative WD models is vital to obtaining a robust characterization of tidal dissipation.

2.3 Feasibility and Availability of Resources

In my preliminary 2D simulations, I use 256 horizontal and 1024 vertical spectral modes. These simulations take about 2 days to run on 32 threads using Dedalus[23] using adaptive timestepping. Based on these preliminary results, I intend to perform 3D simulations at resolution $128 \times 128 \times 512$. Including some optimizations I am still developing, I anticipate these simulations will take about 2 weeks each when performed using 64 threads.

The computational resources available to me at Cornell include a cluster with 104 cores/208 threads as well as multiple 32 core/64 thread machines. There are sufficient to perform many such 3D simulations within the timespan of the proposal.

3 Objective B: Tidal Heating and White Dwarf Evolution

3.1 Background

As discussed earlier, compact WD binaries may exhibit a range of transient phenomena: tidal novae[4], underluminous supernovae[2], and Ca-rich fast transients[3] are all hypothesized to arise in WD binary systems. Given that tidal heating can become a significant contributor to a WD's total energy budget, a realistic model of tidal dissipation is important to understanding the thermal evolution of WDs during their binary inspiral.

In [4] (hereafter FL), the authors used MESA[26] to study the production of tidal novae in binary WDs. A simple two-zone parameterization where the tidal heat is deposited throughout the outer zone was used to model tidal dissipation. It was found that cool WDs in sufficiently compact binaries (orbital period $\lesssim 15$ minutes) may incur a thermonuclear detonation of the hydrogen envelope. WD binaries with such short orbital periods have been observed, e.g. SDSS J0651 has a period of 12.75 minutes[27]. FL fitted the observed properties of J0651 to their tidal heating models and finds evidence for tidal heating of the secondary WD in this system.

3.2 Proposed Work

With the tidal dissipation models I will develop in Objective A (see Section 2), I will be able to perform a binary WD evolution study similar to that in FL but with a realistic tidal dissipation profile (instead of their parameterized model). The second aim of my proposal is to **use my tidal dissipation profiles to simulate a binary WD undergoing tidal heating and make comparisons to observational data.** The location of the tidal heating is a key ingredient in determining whether the deposited tidal energy can be efficiently transfered away or whether the WD experiences sudden detonations. As such, an improved understanding of the properties of tidal dissipation is important to proper forecasting of binary WDs' thermodynamic evolution. Moreover, while only a few sufficiently compact WD binaries were available at the time of writing of FL, *Gaia* data releases 2 and 3 will provide many more compact WD binaries to examine for observational manifestations of tidal heating.

Using MESA, I will evolve various WD models undergoing tidal heating. From these studies, I will extract the increased temperature of the WDs and make comparisons to observational data, in particular to WDs in new *Gaia* data releases. I will also identify the occurrence rate and observational properties of any predicted optical transients such as tidal novae and attempt to identify them among existing detected events. Predictions of the occurence rates of such phenomena are vital to guiding future observations. Finally, comparison of observational data from known WD binaries to my theoretical predictions could yield new constraints on the interior properties of WDs (e.g. crystallization).

This objective is very feasible using the resources available to me at Cornell. Many members of my research group have used MESA in their work on tides in WDs (e.g. [12, 14, 28]) and will be able to advise me. Additionally, as MESA is a 1D stellar evolution code, I will have sufficient computing resources to carry out this investigation.

4 Objective C: Tidal Dissipation and LISA

As discussed in Section 1, WD binaries are an important source of GW radiation for LISA. LISA will attain optimal sensitivity at frequencies 10^{-4}–10^{-1} Hz[29]. Exactly in this frequency range, tidal effects act to synchronize the spin of the WDs to the binary orbit and transfer energy from the orbit into the WDs. While the decay of the binary orbit is still mostly driven by GW radiation, the tidal energy dissipation rate grows to $\sim 10^{-2}$ the GW luminosity[12, 14]. An effect of such a magnitude causes the phase of the emitted GWs to deviate significantly from the point-mass binary prediction; the emitted wave may exhibit "missing cycles" due to tidal effects[12]. GW astronomy uses matched filtering, where a library of template waveforms is matched against instrument data, to identify GW signals. As such, the accuracy and completeness of the template library is of utmost importance.

The final aim of my proposal is **to use my tidal dissipation model to compute WD binary GW waveforms including tidal dissipation for use in the LISA detection pipeline**. This aim is much less computationally expensive than it appears: LISA-band WD binaries can be well described using leading-order post-Newtonian dynamics and including the effect of tidal deformations.[30]. The resultant GW emission can then be accurately computed using the weak gravity quadrupole approximation (see e.g. [9, 31]). Under these two approximations, the GW waveform can be computed analytically without resorting to numerical relativity simulations at all. Thus, I will compute GW waveforms accounting for the additional phase evolution due to tidal dissipation. I will publish my corrected waveforms for use by LISA and the GW community.

5 Project Timeline

During the first year of work and first half of the second year, I will complete calculations of tidal dissipation models. I anticipate that the extension of my 2D plane-parallel work to 3D spherical geometries will be complete within the first half year, while the extension to realistic WD models will take up to a year. I expect that these two results together will produce two peer-reviewed publications in addition to the one currently in preparation.

During the following year, I will use my tidal dissipation model to perform MESA simulations of tidally heated WDs and compare to observational data. I expect my MESA simulations and extracting appropriate observables to take about half a year. I then intend to spend another half year analyzing observational data for potential signatures of tidal heating or tidal novae. I expect both of these phases will produce peer-reviewed publications.

Finally, in the last six months I will compute WD binary GW templates for use by the LISA community. This work will also produce one peer-reviewed publication.

6 Relevance to NASA Objectives

This project is extremely relevant to the NASA Astrophysics Division, both in terms of supporting the division's objectives and enhancing the science output from NASA missions. My work directly relates to NASA objectives: (i) "the interactions of particles under the extreme conditions found in astrophysical situations", (ii) "how complex systems create and shape the structure and composition of the universe on all scales", and (iii) "the development of new techniques that can be applied to future major missions".

Towards (i), my work will improve understanding of possible energetic phenomena that can occur in compact object binaries. Towards (ii), my results concerning angular momentum transfer via IGW breaking will be applicable to astrophysical systems beyond WD binaries, for instance in massive stars[32]. Towards (iii), my work has direct relevance to NASA missions by forecasting astrophysical transients related to WD binaries. For instance, tidal novae are theorized to have a similar observational signature to dwarf novae which have been observed with Chandra and the Hubble Space Telescope, among others. Additionally, my GW templates will be important for LISA GW detection efforts.

Research performed by Cornell University is conducted as fundamental research (basic and applied research ordinarily published and shared broadly within the scientific community) and is therefore exempt from the requirements of the International Traffic in Arms Regulations (ITAR) and the Export Administration Regulations (EAR).

References

1. Nelemans, G. *Class. Quantum Grav* **26,** 094030 (2009).
2. Perets, H. B. *et al. Nature* **465,** 322 (2010).
3. García-Berro, E., Badenes, C., Aznar-Siguán, G. & Lorén-Aguilar, P. *MNRAS* **468,** 4815–4821 (2017).
4. Fuller, J. & Lai, D. *ApJL* **756,** L17 (2012).
5. Iben Jr, I. & Tutukov, A. V. *ApJS* **54,** 335–372 (1984).
6. Webbink, R. *ApJ* **277,** 355–360 (1984).
7. Gilfanov, M. & Bogdán, Á. *Nature* **463,** 924 (2010).
8. Maoz, D., Sharon, K. & Gal-Yam, A. *ApJ* **722,** 1879–1894 (2010).
9. Korol, V. *et al. MNRAS* **470,** 1894–1910 (2017).
10. Shen, K. J. *et al. AJ* **865,** 15 (2018).
11. Kilic, M., Hambly, N. C., Bergeron, P, Genest-Beaulieu, C & Rowell, N. *MNRAS* **479,** L113–L117 (2018).
12. Fuller, J. & Lai, D. *MNRAS* **421,** 426–445 (2012).
13. Fuller, J. & Lai, D. *MNRAS* **412,** 1331–1340 (2011).
14. Fuller, J. & Lai, D. *MNRAS* **430,** 274–287 (2013).
15. Winters, K. B. & DAsaro, E. A. *J. Fluid Mech* **272,** 255284 (1994).
16. Barker, A. J. & Ogilvie, G. I. *MNRAS* **404,** 1849–1868 (2010).
17. Zahn, J.-P. *A&A* **41,** 329–344 (1975).
18. Goldreich, P. & Nicholson, P. D. *ApJ* **342,** 1079–1084 (1989).
19. Drazin, P. *Proc. R. Soc. Lond. A* **356,** 411–432 (1977).
20. Booker, J. R. & Bretherton, F. P. *J. Fluid Mech* **27,** 513539 (1967).
21. Hazel, P. *J. Fluid Mech* **30,** 775783 (1967).
22. Jones, W. L. & Houghton, D. D. *J. Atmospheric Sci* **28,** 604–608 (1971).
23. Burns, K. J., Vasil, G. M., Oishi, J. S., Lecoanet, D. & Brown, B. Astrophysics Source Code Library. Mar. 2016. ascl: `1603.015`.
24. Su, Y., Lecoanet, D. & Lai, D. (In preparation).
25. Brassard, P, Fontaine, G, Wesemael, F & Hansen, C. *ApJS* **80,** 369–401 (1992).
26. Paxton, B. *et al. ApJS* **208,** 4 (2013).
27. Brown, W. R. *et al. ApJL* **737,** L23 (2011).
28. Lai, D., Vick, M. & Fuller, J. *Monthly Notices of the Royal Astronomical Society* **468,** 2296–2310. ISSN: 0035-8711 (Mar. 2017).
29. Amaro-Seoane, P. *et al. arXiv* (Feb. 2017).

30. Van den Broek, D., Nelemans, G., Dan, M. & Rosswog, S. *MNRAS:L* **425,** L24–L27 (2012).
31. Peters, P. C. & Mathews, J. *Phys. Rev.* **131,** 435–440 (1 1963).
32. Aerts, C., Mathis, S. & Rogers, T. *ARAAAJ* **57** (2019).