Critical Layer Absorption and Tidal Spin-Up

Group Meeting

Yubo Su

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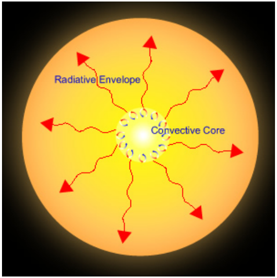
Background

Goldreich & Nicholson 1989

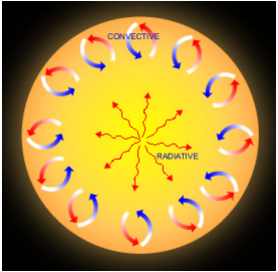
- *Tidal Friction in Early-Type Stars*.
- Tidal torques excite outgoing internal gravity waves (IGW) at boundary between convective core and radiative envelope.
- IGW amplify as they propagate due to density rarefaction.
- Dissipate in the envelope to mean flow,
- Transfers energy from the orbit to the star and synchronizing the spin to the orbit outside-in.



Background

Barker & Ogilvie 2010

- Barker & Ogilvie 2010 present a similar picture for solar-type stars (radiative core and convective envelope).



- IGW propagate inwards instead, amplifying via geometric focusing, and spin up the star inside-out.
  - Perform 2D simulations.
  - Observe two phases of spin-up: mean flow formation, corotation absorption (mean flow corotates with companion).

- Linear, inviscid theory of critical layers predicts almost complete absorption (Booker & Bretherton 1967). Viscosity same (Hazel 1967).
- Weakly nonlinear theory predicts reflection! Predicts reflection appears after $\mathcal{O}(A^{-2/3})$ (Brown & Stewartson 1980).
- 3D plane-parallel simulation shows reflection at nonzero amplitude (Winters & d'Asaro 1994).
- **Question: What happens in other geometries?**

- Begin with incompressible fluid equations in the presence of stratification $\rho_0 \propto e^{-z/H}$ and shear flow $\vec{u}_0 = u_0(z)\hat{x}$:

$$\vec{\nabla}\cdot\vec{u}_1 = 0, \tag{1a}$$

$$\frac{\partial \rho_1}{\partial t} + \left(\vec{u}\cdot\vec{\nabla}\right)\rho_1 - u_{1z}\frac{\rho_0}{H} = 0, \tag{1b}$$

$$\frac{\partial u_{1x}}{\partial t} + \left(\vec{u}\cdot\vec{\nabla}\right)(u_{1x} + u_0) + \frac{1}{\rho_0}\frac{\partial P}{\partial x} = 0, \tag{1c}$$

$$\frac{\partial u_{1z}}{\partial t} + \left(\vec{u}\cdot\vec{\nabla}\right)u_{1z} + \frac{1}{\rho_0}\frac{\partial P}{\partial z} + \frac{\rho_1 g}{\rho_0} = 0. \tag{1d}$$

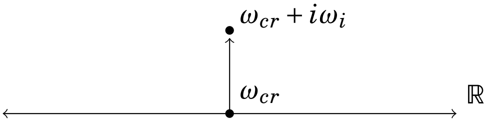
- Linearize and math $(\omega_{cr} = \omega - k_x u_0)$

$$\frac{\partial^2 u_{1z}}{\partial z^2} - \frac{1}{H}\frac{\partial u_{1z}}{\partial z} + \left[\frac{k_x^2 N^2}{\omega_{cr}^2} - k_x^2 - \frac{k_x}{\omega_{cr} H}\frac{\partial u_0}{\partial z} + \frac{k_x}{\omega_{cr}}\frac{\partial^2 u_0}{\partial z^2}\right]u_{1z} = 0. \tag{2}$$

- Near critical layer, $\frac{\partial^2 u_{1z}}{\partial z^2} + \frac{k_x^2 N^2}{\omega_{cr}^2} u_{1z} = 0$, $\omega_{cr} \propto k_x \frac{\partial u_0}{\partial z} \delta z$ where $\delta z = z - z_c$ is distance to critical layer.
- Power series ansatz, $u_{1z} \propto \omega_{cr}^{\alpha} \propto (\delta z)^{\alpha}$ gives

$$\alpha = \frac{1}{2} \pm i \sqrt{\frac{N^2}{\left(\frac{\partial u_0}{\partial z}\right)^2} - \frac{1}{4}} = \frac{1}{2} \pm i \sqrt{\mathrm{Ri} - \frac{1}{4}}. \tag{3}$$

- Move $\omega_{cr} \to \omega_{cr} + i\omega_i$ (growing perturbation), specify branch cuts for u_{1z}!



- Call being above the critical layer $\mathrm{Arg}(\delta z) = 0$, then below the critical layer $\mathrm{Arg}(\delta z) = -\pi$. This gives

$$(\delta z)^{\frac{1}{2} \pm i\mu} = \begin{cases} |\delta z|^{\frac{1}{2} \pm i\mu} & \mathrm{Re}\,\delta z > 0, \\ |\delta z|^{\frac{1}{2} \pm i\mu} e^{-\frac{i\pi}{2}} e^{\mp \mu\pi} & \mathrm{Re}\,\delta z < 0. \end{cases} \tag{4}$$

  - Why specify above? Physically we know only one type of wave above, should have less complexity after branch cut, Papers don't elaborate, I think inspired guess.
- $|\delta z|^{\pm i\mu} = e^{\pm i\mu \ln|\delta z|}$, liberally interpret $\sim e^{\pm i k_z |\delta z|}$.
- If so, $e^{-ik_z\delta z} e^{+\mu\pi}, e^{+ik_z\delta z} e^{-\mu\pi}$. But for IGW, $k_z < 0$ means *upward-propagating*!

Work

2D Plane Parallel Conclusion

$$\begin{tikzcd}
e^{-ik_z\delta z}e^{+\mu\pi} \arrow[dr] & & \\
& {} \arrow[r] \arrow[dl] & 1 \\
e^{+ik_z\delta z}e^{-\mu\pi} & &
\end{tikzcd}$$