Exciting Internal Gravity Waves, Toy Problem

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- 2D hydrodynamics with driving boundary condition, produce gravity waves.
- Begin with linear incompressible case, equations:

$$\frac{\partial \rho_1}{\partial t} = 0, \tag{1a}$$

$$\vec{\nabla} \cdot \vec{u}_1 = 0, \tag{1b}$$

$$\frac{\partial \vec{u}_1}{\partial t} = -\frac{\vec{\nabla} P_1}{\rho_0} + \vec{g}. \tag{1c}$$

- Consider stratified atmosphere $\rho_0(x,z,t) = \rho_0 e^{-z/H}$.

- Four variables $(\rho_1, P_1, u_{1x}, u_{1z})$, all first order in equations of motion, four boundary conditions.
- Periodic BCs in x (2).
- Wavelike solutions $u_{1z} \propto e^{z/2H} e^{i(k_x x + k_z z - \omega t)}$, u_{1x}, P_1, ρ_1 similar up to phase and terms $\sim \mathcal{O}(H^{-1})$.
- Driven bottom BC: $u_{1z}(x, 0, t) = A\cos(k_x x + \omega t)$ (1).

- Top BC can be chosen Dirichlet $u_{1z}(x, L_z, t) = 0$, analytically tractable.
- Driving term will pump energy into the system, without dissipation becomes like driven undamped SHO. In full nonlinear problem not an issue...
- More realistically, two solutions: radiative boundary conditions and damping zone.
- We use damping zone, add term $\frac{\mathrm{d}}{\mathrm{d}t}\vec{q} = L\vec{q} - f(z)(\vec{q} - \vec{q}_0)$ where

$$f(z) = \begin{cases} \Gamma\left[1 - \frac{(z-L_z)^2}{(z_{damp}-L_z)^2}\right] & z > z_{damp} \\ 0 & z < z_{damp} \end{cases}. \tag{2}$$

- Linear incompressible hydrodynamics with periodic BCs in x, driving term at $z = 0$ and either Dirichlet or reflection-free BCs at $z = L_z$.
- Spectral code Dedalus, built-in Runge-Kutta timestepper (simplest to characterize).

Results

Waveform Agreement

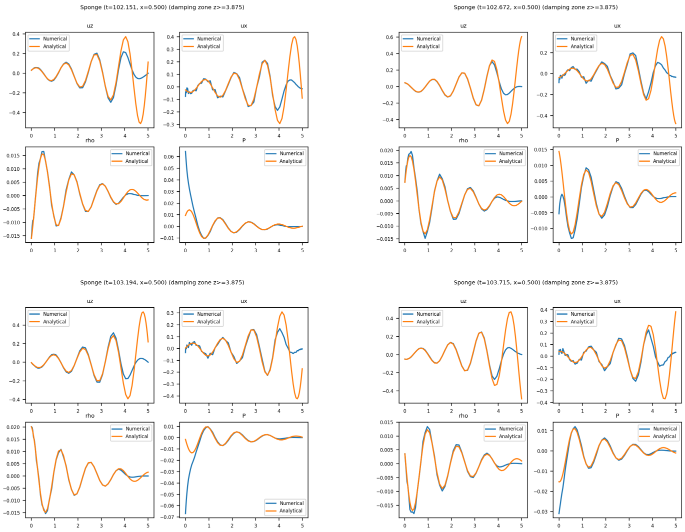

Figure: Agreement of numerical and analytical solution at four adjacent snapshots

- $$RMS = \frac{\sqrt{\sum\limits_{x,z,t}\left(u_{1z}^{(N)}(x,z,t) - u_{1z}^{(2N)}(x,z,t)\right)^2}}{\sqrt{\sum\limits_{x,z,t}\left(u_{1z}^{(N)}(x,z,t)\right)^2}}$$

../sims/2d_strat_conv/t_conv.png ../sims/2d_strat_conv/x_conv.png

../sims/2d_strat_conv/z_conv.png