Ann. Rev. Astron. Astrophys. 1980. 18 : 363–98

WHITE DWARF STARS

✖2170

James Liebert
Steward Observatory, University of Arizona, Tucson, Arizona 85721

1 INTRODUCTION

Strange objects, which persist in showing a type of spectrum out of keeping with their luminosity, may ultimately teach us more than a host which radiate according to rule.

From the President's Address of Arthur S. Eddington (1922), on the occasion of the Centenary of the Royal Astronomical Society

The violated rule to which Professor Eddington referred was the mass-luminosity relation for dwarf stars. The "strange objects" had the exceedingly low luminosities of the faint red dwarfs, yet their colors were quite bluish. Thus, they came to be called *white dwarfs*. The high surface emissivities required very small radii—of order 10^{-2} $R_{\odot}$—compared with known stars. Yet, at least the companion star to Sirius was known from the orbital solution to have a mass of ~ 1 $M_{\odot}$. This implied interior densities and surface gravities orders of magnitude higher than for dwarf stars. The physical state of such superdense matter could not be understood until the quantum-statistical theory of the electron gas was worked out by E. Fermi and P. Dirac in the mid-1920's. R. H. Fowler then showed that the enhanced pressure of a degenerate electron gas could support an object of stellar mass against its own self-gravitation at precisely the radii of white dwarfs. Chandrasekhar (1939) modeled the basic interior structure and determined the basic mass-radius-density-composition relationships. Most importantly, Chandrasekhar established a critical mass (~ 1.44 $M_{\odot}$) above which stable degenerate configurations cannot exist.

In the wake of their successes, theoreticians called the configurations *degenerate stars* or *degenerate dwarfs*, though observers still used *white dwarfs*. The stars are certainly not main-sequence *dwarfs*, and most are too cool to properly be called *white*. However, the term *degenerate stars* encompasses neutron stars as well. Since all of the above terms are in widespread use, it is futile to argue which of them is the biggest misnomer; we shall use them interchangeably in this discussion.

0066-4146/80/0915-0363$01.00

To this writer's knowledge, there has not been an extensive review of the general properties of single degenerate dwarfs written in the last several years. For exploration of problems in greater depth than is possible here, however, the reader is referred to various specialized reviews and contributions found in the *Proceedings of IAU Colloquium No. 53* on *White Dwarfs and Variable Degenerate Stars*. Earlier comprehensive reviews include those of Weidemann (1968, 1975) and Ostriker (1971, 1972); Bessell (1978) and Van Horn (1979) have provided shorter summaries. Degenerate stars in cataclysmic binary systems are reviewed by Robinson (1976) and Gallagher & Starrfield (1978). For an assessment of the probable role of accreting white dwarfs in higher luminosity X-ray sources, Kylafis et al. (1980) is a useful starting point.

2 OBSERVATIONAL CONSTRAINTS

2.1 *Methods of Discovery and Selection Effects*

White dwarfs have generally been identified in three ways: (*a*) as faint stars in proper motion catalogues, especially the bluer objects; (*b*) in surveys for faint blue stars, generally at high galactic latitudes; (*c*) as faint companions to brighter proper motion or parallax stars. Slit spectroscopy and/or photometric color measurements have generally been required to demonstrate that a given candidate is a degenerate star. Until recently, the majority of established degenerates were culled from the lists of proper motion surveys of W. J. Luyten and the Lowell Observatory (H. Giclas, N. Thomas, and associates). Luyten (1970, 1977) has published extensive lists of likely white dwarfs from the Luyten Palomar and his earlier proper motion surveys. Now the completion of Green et al.'s (1980) sky survey of blue stellar objects, all with spectroscopic classifications, has more than doubled the sample of spectroscopically confirmed degenerate stars. A catalogue of spectroscopically identified white dwarfs has been published by McCook & Sion (1977).

The discovery techniques have inevitably resulted in biasing the known sample in favor of (*a*) hotter stars, (*b*) those with above-average space motions, (*c*) stars with larger than average radii, and presumably lower than average masses and gravities (Shipman 1979a), and (*d*) stars with brighter, common-proper-motion (but well-separated) companions. The search for cool degenerates has also been frustrated by the relatively lower luminosities and effective search volumes for these stars and the prevalence of late-type subdwarfs among the redder stars of high proper motion. Little velocity information is yet available for the stars in Green's sample. However, most do not appear in the proper motion lists, including the Lowell "GD" lists of bluish objects showing only slight motion. Green's discoveries thus should include the low-velocity tail of the distribution of hot white dwarfs, a sample that is missing for the cooler stars.

2.2 *White Dwarfs in Binaries and Clusters*

While the presence of a distant, bright motion companion has resulted in the discovery of many degenerate stars in the known sample, the striking examples of Sirius B and Procyon B next to us in the sky alert us to the fact that most close degenerate companions could go undetected. Since the hottest degenerates stand the best chance of being found in the glare of brighter but redder companions, most of the known close-binary degenerates are hot stars. Of necessity, the studies of the luminosity function and stellar properties of white dwarfs must address the non-close binary. stars only. This is desirable also because close binary evolution will lead to different stellar parameters than for single stars.

However, the desired decoupling of the close binaries from the isolated stars is difficult to achieve. For example, G5-28 was first classified as degenerate M (DM), in part because it had an ultraviolet excess relative to other dwarf M stars. Yet, it is probably a composite dM plus cool white dwarf (Harrington et al. 1975, Liebert 1975). Likewise G107-70 seemed to be a very cool DC degenerate, but the accurate trigonometric parallax placed it 0.75 magnitude brighter than the mean for several other stars with similar photometric color. Then Mrs. B. Riepe noticed that the image was elongated on many Naval Observatory parallax plates; the star was shown to be two barely separated DC white dwarfs (Strand, Dahn & Liebert 1976).

One wonders how many peculiar spectra or atypical parameters found for white dwarfs might be explainable by hidden duplicity. In Section 3.4 we must consider seriously the possibility that some helium atmosphere white dwarfs have very low masses and are products of binary cannibalism. Greenstein (1979a) noted an ubiquitous pairing of very hot white dwarfs with companion stars. The unexplained X-ray emissions from Sirius B (Section 3.1) and the ultraviolet abundance anomalies in V471 Tauri (Guinan & Sion 1979) may yet owe their existence to some kind of particle flow in detached systems.

White dwarfs have been found in several open clusters, including some with turnoff masses $\gtrsim 5\ M_{\odot}$. The implications of these for pre–white dwarf mass loss and progenitor masses are discussed in Section 4.2. Richer (1978) has claimed the probable discovery of white dwarfs in the southern globular cluster NGC 6752. Richer (1979) has obtained low dispersion spectra showing that at least one of his candidates has a photometric parallax and Balmer line strength consistent with its identification as a DA white dwarf at the cluster distance. Clearly the resolution and limiting magnitudes offered by the space telescope may permit the discovery of large numbers of globular cluster white dwarfs.

2.3 *Colors, Spectral Types, and Kinematic Properties*

White dwarfs have been discovered ranging from the colors of O stars through K stars; one or two with early M colors have recently been found. These divide spectroscopically into 1. the "DA" sequence with hydrogen lines, covering nearly the full range in color, and 2. a group of non-DA or generally helium-rich types, also covering the full temperature range of white dwarfs. In Figure 1, we group these by temperature and approximate helium-to-hydrogen atmospheric composition, according to the analyses discussed in Section 3.

The spectra of white dwarfs generally appear monoelemental. The definitions of the basic types appear in Greenstein (1960). However, the recent discovery of some showing both helium and hydrogen lines has complicated the already unwieldy historical system of spectral types. Traditionally, the DA,F designation has been used for stars showing strong Balmer lines and weak Ca II. Analogously, DBA has recently been used for DB stars showing a trace of hydrogen. We now must also introduce other combinations where the first letter after "D" designates the primary spectroscopic type, and the next letter a secondary type. Thus DAO describes a hot star showing primarily Balmer lines but with weak He II λ4686. For convenience we omit the comma between letters.

Notwithstanding the biases in the known sample, it is clear that most local white dwarfs are members of the old disk population. There are small admixtures of young disk and halo stars (Eggen & Greenstein 1967). Attempts to find kinematic differences among the various spectral groupings have been frustrating and inconclusive (cf Sion & Liebert 1977). One reason is the difficulties with obtaining and interpreting radial velocities

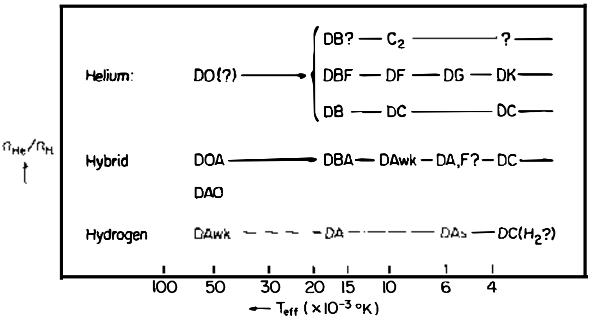

Figure 1 White dwarf spectroscopic T_{eff}—composition sequences. Stars with an actual n_{He}/n_H determinations are plotted in a similar diagram in Liebert et al. (1979c). The basic spectral types were defined in Greenstein (1960); hybrid types used here have the lead symbol (after D) representing the dominant atmospheric constituent (He or H), and the last symbol the secondary constituent appearing in the spectrum.

(see Section 3.2), which has largely limited the exercise to examining distributions of tangential velocities. Trimble & Greenstein (1972) and Wegner (1974) have studied the total space motions of the mostly-DA objects for which they measured radial velocities. A second limitation is that the total stellar ages of white dwarfs may blur any kinematic distinctions of their progenitors. Finally, the mean space motions of white dwarfs may increase with their cooling ages—aside from the selection effects—so that comparison must be made using samples having similar temperature distributions. There is currently no convincing evidence for 1. any significant difference between the kinematic properties of hot DA and non-DA stars, or that 2, the relative numbers of DA and non-DA depend on kinematic properties.

Most authors have assumed that the subgroup of non-DA's showing C_2 bands do come predominantly from a halo population, and hence could be the exceptions to the above statements. However, the previous use of *UBV* colors to estimate absolute magnitudes apparently resulted in the distances and velocities of several C_2 stars being greatly overestimated. The nearby C_2 stars having reasonable trigonometric parallaxes show a mean tangential velocity not significantly different from other groupings of cool white dwarfs (Humphreys et al. 1979; see also Sion, Fragola & O'Donnell 1979).

3 STELLAR AND ATMOSPHERIC PARAMETERS

3.1 *Astrometric Mass Determinations and Radii*

Mass estimates obtained through atmospheric analyses of colors, energy distributions, and line profiles (i.e. through gravity and temperature determinations) are only of statistical value; the reasonably accurate mass determinations for individual white dwarfs are those obtained from binary orbit solutions. Two of these (Sirius B and Procyon B) are in close binary systems with massive companions; the others are in more distant, lower mass systems, and hence are likely to be more useful for comparison with field white dwarfs.

SIRIUS B The most recent mass determination is $1.053 \pm 0.028\ M_{\odot}$ (Gatewood & Gatewood 1978). Its proximity to the brightest star in the sky has complicated the determination of its temperature and radius. The Balmer line profiles and detected soft X-ray flux are consistent with the Greenstein, Oke & Shipman (1971) values of 32,000 K and $\sim 0.0078\ R_{\odot}$. However, the recent far ultraviolet flux measurements and limits on extreme ultraviolet (EUV) radiation suggest $T_{\mathrm{eff}} \simeq 27{,}000$ K and $0.009 \pm 0.002\ R_{\odot}$ (Savedoff et al. 1976; see also Cash et al. 1978, Koester 1979a, Böhm-Vitense et al. 1979). The soft X-ray flux, localized primarily to the "B" component by HEAO-2 imagery (Giacconi 1979), must apparently be attributed to nonthermal processes such as a corona, for which a convincing theoretical justification is lacking (Section 4.3).

PROCYON B The mass of 0.63 $M_\odot$ may be well determined, but the temperature, radius, and even the spectral type are uncertain due to the proximity of the companion.

40 ERI B Heintz' (1974) astrometric solution for the BC pair yielded 0.43 $\pm 0.02\ M_\odot$ for the white dwarf. This value seems preferable to the 0.372 $M_\odot$ advocated by Moffett, Barnes & Evans (1978), who forced the "C" component to fit a mass-luminosity relationship for late M dwarfs. Shipman (1979a) fits $T_{\rm eff} = 16{,}900$ and $0.0124 \pm 0.0005\ R_\odot$ for 40 Eri B, which is in good agreement with the analysis of Koester, Schulz & Weidemann (1979); the new radius determination is nearly 20% lower than the original Matsushima & Terashita (1969) value.

STEIN 2051B This white dwarf is of great potential interest because it is a cooler, helium-atmosphere star. However, the astrometric mass is uncertain and remains "double-valued." Strand (1977) now argues that the "A" component is itself double. On the basis of the mass ratio determined between the red dwarf "A" system and the white dwarf "B", the derived mass for the latter is either 0.50 $M_\odot$ or 0.72 $M_\odot$, depending on whether "A" includes a low mass (0.02 $M_\odot$) companion or two components of nearly equal masses. The temperature has been estimated by Liebert (1976) as 7050 ± 400 K, but the radius determination is much less certain (see Section 3.4) than for the DA stars.

G107-70AB This barely resolved cool white dwarf pair (Section 2.2) now has a preliminary orbit determination by Harrington et al. (1980); this yields a total mass of $0.92 \pm 0.22\ M_\odot$ for both stars, implying a low average mass.

MASS-RADIUS RELATION The current "best" estimates for 40 Eri B and Sirius B are not in serious disagreement with the predictions of Hamada-Salpeter (1961) models for interior compositions lighter than iron. For Sirius B, the match to theory requires a radius ($\sim 0.007\ R_\odot$) at the low end of the range allowed by the ultraviolet data. If the mass were significantly below 0.4 $M_\odot$ for 40 Eri B, the recent Shipman radius estimate becomes a better match to the curve for iron than for lighter elements.

3.2 *Temperatures, Radii, and Gravities of DA Stars*

Several major observational and theoretical projects have recently improved our understanding of the distributions of radii, gravities, and masses among hydrogen-atmosphere degenerates. Larger problems remain with the interpretation of DA stars near the extremes in temperature.

The temperature of a DA star can be rather precisely estimated from careful measurements of the Balmer jump and the Paschen continuum. A basic temperture scale for DA stars is fairly well established for the range 30,000 K $\gtrsim T_{eff} \gtrsim$ 7000 K. Greenstein & Oke's (1979) IUE ultraviolet scans for DA stars are generally consistent with the optically determined scale. Multichannel energy distributions by Greenstein and Strömgren colors have been widely used in recent analyses by Greenstein (1976, 1979c), Shipman (1979a), Koester, Schulz & Weidemann (1979 = KSW), Bessell & Wickramasinghe (1978), and Wegner (1979b). The Strömgren colors offer great photometric accuracy with telescopes smaller than the Hale 5-m, but systematic differences among observers' filter sets and techniques have led to some divergence in the Strömgren results. Since there is some contamination of the Strömgren b bandpass by the Hβ wing in strong-lined stars, Tapia's (1978) color system—originally proposed by Wickramasinghe and Strittmatter—offers some advantages in measuring the Paschen continuum slope. However, these colors have not yet been compared with recent model atmospheres (e.g. by Shipman or the Kiel group).

Radii may be directly estimated for the several dozen DA stars with reasonable distance estimates. While this may be done after determining T_{eff} and a bolometric correction, Shipman (1972), KSW, and Shipman (1979a), for example, compute the radius directly from the surface brightness method (Gray 1967), an approach that permits more explicit error analysis. The uncertainty in the radius determination for any object is usually dominated by the uncertainty in its parallax; for the small sample with distance determinations, it is possible to assess the mean radius for the sample, and the dispersion of radii independent of the surface gravity determination. KSW found a mean radius of 0.0120 $R_\odot$ ($\pm$0.002) for 64 parallax DA stars; Shipman reported 0.01237 $R_\odot$ for his parallax sample. However, Shipman argued that the available stars may be enough like an apparent-magnitude-limited sample that we are biased towards discovering those with larger than average luminosities and radii. He suggests that the true mean is closer to 0.0111 $R_\odot$, implying also a larger mean mass.

Surface gravities affect the strengths and profiles of the Stark-broadened Balmer lines, especially above $\sim$12,000 K. However, the wings are so extensive in hot DA stars that an entire theoretical synthetic spectrum should be fit to the energy distribution to avoid errors in defining the true continuum. Accurate, high resolution spectrophotometry is now available for a large sample of DA stars (Greenstein & Vauclair 1979, Greenstein 1979c, Weidemann & Koester 1980). Previously most efforts to derive surface gravities from atmospheric analyses used colors only. The Balmer jump has maximum gravity sensitivity in the range 10,000–15,000 K, but uncertainties in the model energy distributions due to treatment of convection are important in most of this region. Photometric and multichannel flux calibration errors are also a major problem and lead to systematic uncertainties probably exceeding 0.2 in the mean $\overline{\log g}$ for DA stars determined in this way. Observational errors, and discrepancies among different Strömgren observers, are a major uncertainty in the determination of the dispersion of surface gravities as well. The corresponding multichannel results from KSW, Greenstein (1979c), and Shipman & Sass (1980) agree tolerably well on $\overline{\log g} \sim 8.0 \pm 0.2$. However, KSW favor a smaller intrinsic dispersion than Shipman, leading to a disagreement on the implied mass dispersion of DA stars.

Surface gravities for DA stars have been independently determined from measurements of the redshifts of the Balmer lines (Greenstein & Trimble 1967, Trimble & Greenstein 1972, Wegner 1974); the agreement in $\overline{\log g}$ between the two methods has been less than satisfying. The problem of extracting the gravitational effect from the space velocity and pressure shifts is a nontrivial one and the line centers are ill-defined on early photographic spectra. Only a dozen or so DA stars have independent radial velocities from companions. However, higher resolution coudé spectra (Greenstein & Peterson 1973, Greenstein et al. 1977) revealed that DA stars have remarkably sharp non-LTE cores. This permitted line centers for the brighter stars to be more accurately measured. Red-sensitive image tube photographic plates covering Hα were later compared with Boksenberg spectrophotometry of Hβ and Hγ lines. These results and those of Wegner (1974) led Greenstein et al. to a mean gravitational redshift for DA stars of 45 ± 6 km s^{-1}. This implies $\overline{\log g} \simeq 8.25$, appreciably higher than the result obtained from the energy distributions. With the uncertainties discussed previously in the atmospheric $\overline{\log g}$ value, the difference should perhaps be regarded as only marginally significant. The discrepancy propagates into the derived mean masses for DA stars as well.

Masses and mass dispersions may then be indirectly determined from either the surface gravity or radius results, using the theoretical mass-radius or equivalent mass-gravity relation. As discussed in Section 4.1, it is generally assumed that the interior mean molecular weight in this relation is that for carbon, oxygen, or elements of similar weight; the composition dependence is small unless the core were composed of iron. KSW found mean masses of 0.46 $M_{\odot}$ and 0.63 $M_{\odot}$ computed from the theoretical mass-radius and mass-gravity relations of Hamada & Salpeter (1961), respectively. However, the mass required to fit the mean gravitational redshift estimate ($+45$ km s^{-1}) is significantly higher still at $\simeq 0.75\ M_{\odot}$. Moffett et al. (1978) also recently emphasized the discrepancy between the gravitational and atmospheric results. Shipman's (1979a) selection effect could mean that the true mean mass is higher than that for the observed samples, but the selection should apply to the gravitational redshift sample as well. The dispersion in the mass distribution is also controversial. KSW conclude that a standard deviation of $\pm 0.10\ M_\odot$ is appropriate, relying mostly on their "weighted" sample of surface gravity determinations, though values higher than 0.15 $M_\odot$ were found for some of their data subsets. KSW placed most emphasis on the narrow "log g" dispersion for the modest sample of stars observed by Graham (1972) and Green (1977) in the most gravity-sensitive 10,000–15,000 K region. Accurate observations of more DA stars in this region are highly desirable, since other methods suggest a 1 σ mass dispersion nearer to ± 0.2 (Shipman & Sass 1980, Wegner 1979b, Bessell & Wickramasinghe 1978). Shipman relies heavily on his large dispersion of radii, $\sigma(R)/R \sim 0.23$, which he argues is three times the expected observational uncertainty. This transforms to $\sigma(M)/M \gtrsim 0.20$. In any case, none of the authors find convincing evidence for DA stars with masses well above 1.0 $M_\odot$ or below 0.4 $M_\odot$.

THE HOTTEST DA STARS Temperature determinations for the hottest DA stars (> 40,000 K) require flux measurements nearer to the intensity maxima in the far ultraviolet ($\sim$ 1000 Å) or even the EUV (100–1000 Å). HZ43 and Feige 24 were among the first extrasolar EUV objects detected (Lampton et al. 1976, Margon et al. 1976); the former was identified as a soft X-ray source (Hearn et al. 1976). Recent model atmosphere analyses by Auer & Shipman (1977) and Wesselius & Koester (1978) indicate temperatures of 50,000–60,000 K for these objects. Because of the uncertainties, there is little useful information concerning the radii or masses of the hottest hydrogen stars. Shipman (1979b) outlines the particular pitfalls of calculating models for degenerates at high effective temperatures.

THE COOLEST DA STARS Hydrogen lines have been seen in white dwarfs at temperatures as low as $\sim$5500 K—see the analysis of BPM 4729 by Wickramasinghe & Bessell (1979). The onset of convection and other problems greatly increase the uncertainties in the T_{eff} and radii determinations below $\sim$10,000 K. Furthermore, stars below 8,000 K are too cool for either the Balmer jump or Balmer profiles to serve as useful surface gravity indicators, independent of abundance effects (e.g. Shipman 1972, Wehrse 1977). This is most unfortunate, given that the mean parameters might be expected to change if some DA atmospheres are mixed with helium-rich envelopes during the cooling process (Section 4.3). There is the suggestion in the analysis of G128-7 ($T_{eff} \sim$ 5800 K) by Wehrse & Liebert

(1980a) that this cool object may have a higher than normal surface gravity. This tentative result was possible because the Hα profile fits a $T(\tau)$ atmospheric run indicative of the onset of H_2 molecule formation, which *is* gravity sensitive. Furthermore, there is the possibility of identifying cooler degenerates with hydrogen in their atmospheres through the detection of the pressure-induced H_2 dipole bands in the infrared, as attempted by Mould & Liebert (1978). The onset of this opacity source is gravity sensitive, but the available opacity calculations are too crude to be of much use.

3.3 *Abundances of DA Stars*

HELIUM ABUNDANCES Most hydrogen-atmosphere degenerates are both helium-poor and metal-poor; where detailed limits have been possible, the atmospheres are shown to be composed of nearly pure hydrogen. The high energy fluxes measured for the hottest stars HZ43 and Feige 24 were used to determine $n(\mathrm{He})/n(\mathrm{H}) \gtrsim 10^{-6}$ for the former and perhaps as high as 10^{-3} for the latter (Auer & Shipman 1977, Wesselius & Koester 1978, Malina 1979). However, there are hot "DAO" stars like HZ34 with detectable He II λ4686; Koester, Liebert & Hege (1979) found $n(\mathrm{He})/n(\mathrm{H}) \sim 10^{-1.7}$ for this star. These analyses assumed homogeneous atmospheric compositions, but one must also consider the possibility that the hot atmospheres are layered (see Section 4.3), as Heise & Huizenga (1980) first proposed for HZ43.

HZ34 has no known spectroscopic counterparts among cooler stars with hydrogen-dominated atmospheres. He I lines have not yet been found in DA stars above $\sim$ 12,000 K; this permits limits of $< 10^{-2}$ for the best cases. Below the He I line detection limit, it is difficult to infer $n(\mathrm{He})/n(\mathrm{H})$ limits because the effect of enhanced helium on Balmer profiles and energy distributions is small and difficult to distinguish from surface gravity effects.

METAL ABUNDANCES Wehrse's (1972) synthetic spectrum for a white dwarf with solar composition and temperature dramatized how easy it should be to see neutral metal lines in a cool DA atmosphere. While the limits are not as stringent as for lower-opacity helium atmospheres, the absence of Ca II H & K at 5500 K yields a calcium upper limit of only $\sim 10^{-4}$ of solar (Wickramasinghe & Bessell 1979). Since Stark broadening becomes unimportant below $\sim$ 7000 K, the hydrogen lines become considerably narrower than for hotter stars and one expects any metallic lines to be narrow as well. Hintzen & Strittmatter (1975) argued that the gravity insensitivity of these profiles might result in some alleged metal-rich DA degenerates being misclassified as subdwarfs. However, the detailed model analysis of Wickramasinghe et al. (1977) separates the surface gravities successfully using narrow-band colors and the profiles of high Balmer lines. Indeed, no cool hydrogen-rich degenerate showing metal lines other than

"K" of Ca II has ever been established. Now Bessell & Wickramasinghe (1979) maintain that most of the stars photographically classified DA, F are really either 1. cool subdwarfs, 2. DA degenerates, the calcium lines coming from the night sky spectrum, or 3. helium-atmosphere degenerates with some hydrogen and calcium. It is important to demonstrate whether such a well-observed degenerate DA,F as Ross 627 truly belongs to the last category. However, very few if any new DA,F classifications have been claimed since the advent of sky subtraction spectrophotometry.

The higher continuum opacities of hotter DA stars preclude such stringent limits on composition from the absence of heavy element lines in these stars. However, at the highest temperatures, trace abundances of metal could be very influential on the EUV and far ultraviolet energy distributions. More accurate high energy flux distributions may permit a test of how rapid the onset of gravitational and thermal diffusion is in a hydrogen-atmosphere degenerate beginning its cooling evolution.

3.4 *Temperatures, Radii, and Gravities for Helium-Atmosphere Stars*

The atmospheric and stellar parameters for helium-atmosphere degenerates are not nearly so well determined as for the DA stars, and for a number of reasons. There are greater uncertainties in the continuous helium opacities and in the line broadening for the various He I transitions. Atmospheric convection is generally important. For the cooler stars in particular, the absence of electrons in the metal-poor atmospheres leads to very low He^- continuous opacities, very high atmospheric transparencies and correspondingly high atmospheric pressures and densities; this in turn leads to greater uncertainties for cool helium stars due to the onset of non-ideal gas effects, and a dependence on unknown trace abundances of unseen metallic electron donors. Thus, the model atmosphere calculations for non-DA stars have not been as comprehensive as for DA stars. However, new grids relevant to DB and DO temperatures are being completed by Wickramasinghe (1979), Koester (1979b), and Wesemael & Van Horn (1979). These models include both line blanketing and convection. For the cooler stars in which the physical uncertainties are greater, available calculations have generally been aimed at fitting individual objects of types DF, DG, DK, C_2, or DC.

Pending the conclusions from the new series of DB models, the best available determinations of temperature, radii, and gravities for DB and somewhat cooler helium stars are published in Shipman (1979a). Though the uncertainties in the model fluxes are considerable, Shipman finds a similar mean radius for 28 helium stars (0.0111 $R_\odot$) as for the DA sample. He finds a remarkably similar relationship between the monochromatic flux and the (g-r) color of Greenstein's (1976) data for non-DA and DA stars. This is similar to the completely empirical claim of Barnes, Evans & Parsons (1976) and Moffett, Barnes and Evans (1978) that a similar relationship between surface brightness and (V-R) broad-band color exists for white dwarfs and a variety of lower gravity stars. However, the uncertainties in Shipman's flux-color relation may correspond to a 10% uncertainty in the mean radius. Furthermore, Koester's (1979b) preliminary conclusions from fitting colors and equivalent widths of DB stars to his new models are that $\overline{\log g} \sim 7.42$ (vice ~ 8.0 for DA stars), corresponding to only $M \sim 0.27\ M_{\odot}$ for a theoretical interior composed of carbon. For this $\overline{\log g}$ to be compatible with Shipman's radii, the interior of DB stars would have to be composed of much lighter elements (e.g. helium). There is currently no theoretically sensible way to produce core helium degenerates with $M \lesssim 0.4\ M_{\odot}$ from single star evolution (see Section 4.1). It is of course possible that the discrepantly low mean mass requirement will disappear when more accurate temperatures, radii, and gravities are obtained. However, Alcock (1979) and other envelope theorists argue that all DB masses need to be below $0.4\ M_{\odot}$ anyway in order to allow the small amount of matter accreted from the interstellar medium to be mixed in a sufficiently deep convective envelope so that the purity of observed DB atmospheres may be maintained.

Nather, Robinson & Stover (1979) have recently suggested a way in which close binary evolution might produce a single degenerate star with a helium core. They argue that the ultrashort period variable stars showing only helium lines—AM CVn and G61-29—are binaries consisting of helium degenerates with the low-mass secondary likely to be completely dissolved before the process finishes. The total masses of these systems are highly uncertain. However, Conti, Dearborn & Massey (1980) find that the short-period sdO system LB3459 clearly has a total mass $< 0.4\ M_{\odot}$ and may be a detached evolutionary precursor to the AM CVn stage. Thus, the completion of the evolution could result in a single helium white dwarf or subdwarf of very low mass, devoid of hydrogen, and having probably a high enough temperature to be at or above the "DB" range. Since the luminosities and lifetimes of the AM CVn variables are uncertain, it is difficult to assess what fraction of existing DB stars could be accounted for by such precursors.

Model atmosphere calculations for cooler helium-atmosphere stars have generally been aimed at fitting specific stars with abundance anomalies. The typical cool star is DC, with no spectral features to use. The electron supply, which controls the amount of He^- continuum opacity, is dominated by elements heavier than helium, many of which do not have strong transitions in the optical spectrum from which useful abundance limits may be inferred. The temperature uncertainties for cool DC stars may exceed

10%. Likewise, temperature and surface gravity inferences from fitting line profiles in DF-DG-DK stars are of dubious value.

For helium-atmosphere stars cooler than about 6000 K, greater uncertainties occur in the opacities, equations of state, and line broadening physics due to the proximity of third bodies on what are treated as two-body interactions. Simple model energy distributions currently predict such low temperatures for the reddest known DC-DK stars with good parallaxes that many of these stars would be required to have abnormally large radii or low masses, if they are assumed to have helium atmospheres (Mould & Liebert 1978, Shipman 1979a). Hence, Shipman argues that these stars too cool to show Balmer lines generally have hydrogen atmospheres; it is of course also possible that the helium-atmosphere temperature fits are simply ~ 20% lower than they should be.

Böhm, Carson, Fontaine & Van Horn (1977) have published the first exploratory calculations of the nature of pure helium envelopes at temperatures below 5000 K; they used a more detailed Thomas-Fermi equation of state and other refinements, but they allowed no metallic electron donors to the opacity. At temperatures below 4000 K, it appears that the atmospheres of such stars (if they exist) could be characterized by such "strange" effects as coulomb interactions, efficient convection, pressure ionization, degeneracy, and electron conduction; they could have extremely flat temperature $T(\tau)$ profiles, making it difficult for absorption features to be seen—see also Böhm (1979).

3.5 *Abundances in Helium-Atmosphere Stars*

HELIUM-TO-HYDROGEN RATIOS Because of the reduced continuum opacities at most temperatures, more stringent upper limits on hydrogen and heavy elements may be established for the helium-atmosphere white dwarfs. Virtually all well-studied helium-atmosphere cases have nearly pure helium atmospheres, despite the appearance of strong hydrogen and neutral metal lines in the spectra of some.

A few of the hottest known degenerates have helium atmospheres and "DO" spectra dominated by He II lines. The first abundance analysis of the prototype HZ21, assuming a homogeneous composition, indicated significant hydrogen abundance with log $n_{He}/n_H \sim +1$ (Koester, Liebert & Hege 1979). The hydrogen abundance in such a star is difficult to derive given the coexistence of He II Brackett lines at essentially the same wavelengths as the Balmers. However, the large ratio of He I/He II λ4686 strengths required a temperature below 50,000 K; then, the He II transitions alone could not account for the strength of the absorption at Brackett/Balmer wavelengths. Wray, Parsons & Henize (1979) argue that HD149499B, discovered from the *Skylab* UV experiment, may also have a significant hydrogen abundance but at a higher T_{eff} than HZ21. Wegner (1979a) found its optical spectrum to be DO.

Figure 5 in Liebert et al. (1979c) summarizes recently published data on degenerates having helium-to-hydrogen abundance determinations. It is curious that the analyzed stars hotter than 45,000 K like HZ21 cover an extensive range in n_{He}/n_H, while the known cooler stars showing hydrogen lines are still very helium-rich objects near the DB-DC lower limit $\sim 10^4$ (for no hydrogen lines). However, as discussed in the reference, selection might have worked against the discovery of any cooler stars with intermediate n_{He}/n_H abundances.

HEAVIER ELEMENTS IN HELIUM ATMOSPHERES The great majority of helium-atmosphere degenerates show no optical absorption features due to elements heavier than helium; the exceptions belong to either the metallic-line (DF-DG-DK) or C_2 (λ4670) spectral groups. Most of the former show *only* lines of Ca II H&K (DF stars) over a wide range in temperatures. A handful have three or more identified elements from good quality optical or ultraviolet (IUE) spectra and have had detailed atmospheric analyses; these have inspired a lively debate as to the origins of the surface heavy elements (Section 4.3). They range in temperature from the DBF star GD40 at 14,000 K to the heavily blanketed LP701-29 at $\sim$ 4000 K. The dominant atmospheric constituent of LP701-29 is controversial (Dahn et al. 1978, Cottrell et al. 1977). Calcium, magnesium, iron, silicon, sodium, and hydrogen are among the trace elements that have been detected in various objects. Unfortunately, no CNO elements have been directly measured —except in the λ4670 stars—though these may control the electron supply for the He^- continuum opacity above $\sim$ 7000 K.

Table 1 Some interesting abundance ratios for helium-rich white dwarfs

Name (WD #)	Ca/H	Mg/Ca	Si/Ca	Fe/Ca	Ref.[a]
GD40 (0300−013)	$\gtrsim 3 \times 10^{-3}$			—	SGB
GD401 (2215+388)	$\gtrsim 10^{-4}$	$\gtrsim 0.4$	---		CG
R640 (1626+369)	$\sim 2.5 \times 10^{-6}$	89	42	2.6	CG, L
G165-7 (1328+308)	$\gtrsim 3 \times 10^{-4}$	13:	37:	44:	WL
VMa2 (0046+052)	$\gtrsim 10^{-7}$	20:	$\gtrsim 100$:	20:	G
Solar	2.0×10^{-6}	13	16	20	A

[a] References:
A—Allen (1973)
CG—Cottrell & Greenstein (1980)
G—Grenfell (1974)
L—Liebert (1977)
SGB—Shipman, Greenstein & Boksenberg (1977)
WL—Wehrse & Liebert (1980b)

More detailed discussions of abundance determinations in specific metallic line stars may be found in Vauclair et al. (1979), Greenstein (1979a), Vauclair (1979), and Böhm (1979). Some of the abundance ratios most relevant to the issues discussed in Section 4.3 are summarized in Table 1.

3.6 *Rotation*

Much theoretical attention has been devoted to the implications of rapid rotation in degenerate dwarfs (cf Ostriker 1971); after all, such configurations could greatly exceed the Chandrasekhar mass limit and be eventual supernova candidates. However, the simple reality is that—save for those in close binaries—most white dwarfs rotate slowly, if at all. Various investigators have worried that rapidly rotating degenerates might be disguised as simple "DC" stars. Wickramasinghe & Strittmatter's (1970) and Kuzma's (1979) calculations showed that hydrogen lines should still be visible, except perhaps under rather extreme circumstances; Milton (1974) still argued that some differentially rotating hydrogen-atmosphere stars might appear with DAwk or even DC spectra. In any case, it is clear that DA stars with hydrogen lines weak for their color are in reality quite rare: Weidemann & Koester (1980) have disproved most of the prior claims. Any rotating "DC" group should include some at blue colors, where their lack of lines would quickly be noticed by the spectroscopic surveyor. Furthermore, the narrow DA distributions in the two-color diagrams, and the sharp line cores in DA and probably DB stars do not indicate any of the effects of moderate rotation rates calculated by Milton.

Slow rotation periods have been determined or inferred for some of the ZZ Ceti variable DA stars and for the polarization/spectrum-variable magnetic stars—see references given in Sections 3.7 and 3.8. These periods imply rotational velocities of only 0.4–8 km s^{-1}; the nonvariable strongly magnetic stars may not be rotating at all!

The best observational limits on nonvariable white dwarfs are those obtained from coudé-resolution line profiles of the brighter DA stars (Greenstein & Peterson 1973, Bessell & Wickramasinghe 1975, Greenstein et al. 1977). These Hα and Hβ profiles have revealed surprisingly sharp line cores, probably formed under non-LTE conditions high in the atmospheres. Greenstein et al. (1977) report a possible emission reversal in the Hα line of one cool DA star. Projected rotation velocities for eight well-observed stars do not exceed 40 km s^{-1}; several cases with somewhat broader cores could have $v \sin i \simeq$ 50–60 km s^{-1}—see also Kuzma (1979). Comparable observations for DB stars are not yet available. However, several cooler DB stars observed at ~ 2 Å Cassegrain resolution (at Steward Observatory and elsewhere) also appear to have sharp line centers.

3.7 *Variability*

ZZ CETI STARS Aside from those known to be in binary systems, the only white dwarfs that are established photometric variables are the ZZ Ceti class of DA stars in the 10,000–14,000 K temperature range. Several recent reviews (e.g. McGraw 1979, Robinson 1979, Dziembowski 1979) describe well the properties and puzzles posed by these complicated pulsators. The light curves show variation amplitudes of only 0.01–0.30 magnitudes and periods ranging over $\sim$ 200–1200 seconds; no variation has yet been found at or near the fundamental (1–10^s) pulsation periods of degenerate stars. Multiple pulsation modes occur in all known stars, and the period structure and amplitudes may vary. The light curves are generally thought to be explicable as nonradial g-mode pulsations, as developed from the suggestion by Warner & Robinson (1972), though high radial overtones may also be excited (Starrfield, Cox & Hodson 1979). The curves may also show an effect on the derived modes attributable to rotation of the star; the inferred rotation periods range from a few hours to a few days, similar to those found for magnetic stars which are spectrum variables, and not inconsistent with the DA spectroscopic limits. The ZZ Ceti stars appear spectrophotometrically normal at optical and ultraviolet wavelengths (Greenstein 1979b).

While the nonradial g-mode interpretation has enjoyed some qualitative success in matching ZZ Ceti period spectra, the physical basis for the pulsations has yet to be developed. Many authors have noted that the ZZ Ceti position in the H-R diagram is not inconsistent with an extrapolation of the Cepheid instability strip to log g $\sim$ 8. Hence, most investigators have explored the possibilities that the pulsational driving is due to an underlying He II ionization layer at $\sim$ 50,000 K—the primary source of instability in classical Cepheids—or the Stellingwerf (1978) He II opacity edge near 150,000 K—which may power pulsations in higher gravity Beta Cephei and Delta Scuti stars. Dziembowski (1979) has now demonstrated that theoretical models may be unstable to g-mode pulsations, assuming that there is some helium at the required depths. However, Starrfield et al. (1979) have also found radial instabilities for nearly pure hydrogen compositions. Whatever the mechanism is, the theory must also explain why only specific high overtones appear in certain stars while other overtones, particularly lower ones, are not excited. It is clear that more realistic envelope structures must be incorporated into these calculations, models which attempt to account for the prior evolution in surface abundances of the cooling white dwarf. Hot DA white dwarfs are extremely helium deficient at their surfaces; the theoretical diffusion calculations (Section 4.3) have demonstrated that most surface helium should have sunk quite deep in the envelope, leaving an outer layer mass (M_H) of almost pure hydrogen. One may speculate that the M_H parameter holds the key to what fraction of DA stars cooling to 12,000 K becomes variables; the slow rotations inferred for some stars suggest that rotation is not a key property.

More detailed study of the pulsation properties should enhance our knowledge of the physical structure and evolution of DA degenerates. The detection of period changes would even afford a direct measurement of the cooling time (McGraw 1977), since Osaki & Hansen (1973) predict a linear period-luminosity relation. The slope of the period increase should depend inversely on the mean molecular weight of the interior. However, the stars have proven to be among the universe's better clocks: Stover, Robinson & Nather (1978) report a limit of $|\dot{P}|^{-1} > 10^{11}$ for Ross 548.

OTHER DEGENERATE VARIABLES Variability has been reported for a few non-DA degenerates, including the DC star G44-32 and the helium-rich suspect AM CVn = HZ 29. The latter is almost certainly a low-mass binary (Section 3.4), while the variability of the former is not well established (McGraw 1977). An updated list of degenerate suspects searched for variability is given by Hesser, Lasker & Neupert (1979). These authors emphasize that very little is known about variability in most white dwarfs on time scales $\gtrsim$ 30 minutes.

PG1159-035 The first stellar observations with the Arizona/Smithsonian 4.5-m multiple mirror telescope resulted in the discovery of a new kind of rapid variable star (McGraw et al. 1979, 1980). The object was selected from the Palomar Green faint blue stars because of its very blue color and unique spectrum (Green & Liebert 1979). Despite the drastically higher temperature, the object showed a power spectrum somewhat similar to some ZZ Ceti stars, with strong periodicities near 8.9^m and 7.6^m. Thus, pulsation is the most likely cause of the variability. Further study of the spectrum resulted in the identification of sharp lines due to such ions as He II and C IV, often blended together. Hence, the object is likely to be of lower surface gravity than the DO degenerates, though it may be a helium-rich precursor.

3.8 *Magnetism*

Excellent recent reviews by Angel (1977, 1978) and Landstreet (1979) preclude the need for much attention to this subject here. Thirteen isolated magnetic degenerates are now known, with surface field strengths ranging from 3 to over 100 megagauss. At least three have hydrogen-dominated atmospheres, and at least two have helium-rich atmosphere with some hydrogen. The temperature range (5600–22,000 K) appears representative. It is not clear that there are any significant differences in the space velocities from the sample of nonmagnetic white dwarfs. Stringent limits on surface magnetic field strengths as low as a few kilogauss are available for some DA stars whose line centers have been observed at coudé dispersion. However, Angel, Landstreet & Borra (1980) argue that the distribution of magnetic field strengths and limits for the available sample are not inconsistent with the hypothesis that the stars occur with an equal distribution ($\simeq$ 0.5–1%) of field strengths per octave to $\sim$ 300 megagauss. These authors and Angel (1979) argue that magnetic degenerates may be descended from main-sequence magnetic A stars, whose fossilized core fields must presumably survive subsequent evolutionary stages; this idea might imply higher than average progenitor masses for the magnetic degenerates and lower than average space motions for a sufficient sample of hot magnetics.

4 THE ORIGIN AND EVOLUTION OF DEGENERATE DWARFS

4.1 *The Birthrate of Hot White Dwarfs*

The rate at which degenerate stars are currently being produced is best estimated from the empirical density and luminosity function of hot white dwarfs. The result is insensitive to uncertainties in cooling theory, which are more important at lower temperatures, and to the time dependence of the stellar birthrate function, since hot white dwarfs have not cooled very long. Early attempts to derive the birthrate were reviewed in Weidemann (1968). By far the best determination is due to the comprehensive survey of faint blue stars by Green (1977, 1980). His 18-inch Schmidt plate survey of the sky resulted in the discovery of some 3000 blue stellar objects with $\delta \geq -10°$ and galactic $|b| \geq 30°$; a subset of some 89 objects was used in the initial estimate of the luminosity function. The sample was argued to be complete for B (Magnitude) < 15.7 and for photographic colors corresponding to white dwarf with $M_v \gtrsim 12.75$. Green's (1977) luminosity function corresponds to a white dwarf birthrate of 1.4×10^{-12} pc^{-3} yr^{-1}, which compares with earlier estimates in the range of $1\text{–}5 \times 10^{-12}$. Green also discussed the rather modest constraints the hot white dwarf birthrate imposes on galactic star formation models. His modeling seems consistent with a scale height of $\sim$ 250 pc for the young white dwarf distribution. Chiu's (1978) counts to much larger distances ought to provide a more accurate scale height determination for blue degenerates, though his value of 400–500 pc seems surprisingly large for what is argued to be an old disk population.

4.2 *Links to Prior Evolution*

The birthrate and other information determined for white dwarfs holds implications for identifying progenitor stars and tracing their prior evolution. The kinematical properties (Section 2.3) indicate that most progenitors were originally 1–2 $M_\odot$ Population I stars, but the existence of white dwarfs in young clusters may mean that some stars were initially much more massive. The low remnant masses of most DA's (Section 3.2) indicate that extensive envelope mass loss is a widespread phenomenon of post–main-sequence evolution; their slow rotation rates imply that most of the angular momentum of main-sequence stars is lost in subsequent evolution. It is desirable to match the birthrates with the formation rates of evolved giants, Miras, hot subdwarfs, planetary nebulae, and any other suspected progenitor stages, and to determine whether the atmospheric composition groupings can be identified with specific progenitor candidates. Forging the link with what has generally been learned about post–main-sequence stellar evolution can also improve our understanding of white dwarf parameters.

EVOLUTIONARY CONSTRAINTS ON INTERIOR PARAMETERS The advances in stellar evolution theory since the advent of Henyey-method calculations constrain the core compositions of white dwarfs more accurately than is currently possible from the mass-radius relation (Section 3.1). The calculations have established that any star developing a helium core with $\gtrsim 0.45$ $M_\odot$ will ignite helium producing a core of carbon and/or oxygen. Single star evolution cannot produce a helium remnant with <0.45 $M_\odot$ within the lifetime of the universe unless mass loss were so efficient as to strip away the entire hydrogen envelope before the helium core accumulates 0.45 $M_\odot$; only mass exchange in close binaries may produce such a low mass helium remnant (Section 3.4). Likewise, the expected cooling of the carbon-oxygen core due to neutrinos implies that the core mass should approach or exceed the Chandrasekhar limit before carbon burning—either explosive or nonexplosive—would commence (Paczynski 1970, Mazurek & Wheeler 1980). Below this mass limit, remnant degenerates should result from gravitational contraction following the end of the double-shell-source burning stage; they should generally have cores composed of carbon and oxygen, with a thin outer layer of helium ($\sim 10^{-2}$ $M_\odot$) on top of which an even thinner hydrogen layer might survive a final envelope ejection.

ACCOUNTING FOR THE MASS LOSS The relatively low masses and narrow mass dispersions derived for isolated DA degenerates (Section 3.2) greatly limit the buildup of mass in the carbon-oxygen core during late nuclear-burning stages. This is especially true if the calculations must account for some progenitor masses of $\gtrsim 5$ $M_\odot$. Extensive mass loss prior to any planetary nebula ejection must occur, presumably by a "stellar wind" on the giant and asymptotic giant branches. The Reimer's (1975) expression

$$\dot{M} = 4 \times 10^{-13} \eta \frac{(L/L_\odot)(R/R_\odot)}{(M/M_\odot)} M_\odot \mathrm{yr}^{-1} \tag{1}$$

has the right dimensions for work done by the stellar luminosity in pushing mass away from the surface, and has a crude empirical calibration for the efficiency η. Other quasi-empirical expressions, e.g. Fusi-Pecci & Renzini (1975), make rather similar predictions. Still, it also appears that some stars have significant winds even on the subgiant branch (Dearborn et al. 1976) at luminosities too low for the Reimer's formula to predict any significant effect.

Several investigators have combined assumed formulae for mass loss with normal stellar evolution theory in order to derive theoretical relations between the progenitor and remnant masses. Following the stellar wind losses, Wood & Cahn (1977) assumed that any remaining hydrogen envelope may be ejected as a planetary nebula when the more massive stars reach the high luminosity side of the Mira instability region. The incorporation of an assumed stellar birthrate and initial mass function then permitted the mass distribution of remnants to be predicted. In general, these theoretical distributions (Weidemann 1977a, Wood & Cahn 1977) 1. have a sharp cutoff at the lower mass limit, generally near 0.5–0.6 $M_{\odot}$, and 2. predict that only $\sim 50\%$ of all remnants should be within 0.1 $M_{\odot}$ of this lower mass edge, while the remainder should form a skewed distribution of higher masses.

The theoretical distributions thus predict too many high mass remnants with $\not\lt 1$ $M_{\odot}$ compared to the empirical results. In fact, the known DA distribution offers little evidence for a high mass non-gaussian "tail" (cf Wegner 1979b). Even the well-studied degenerates in the Hyades and Pleiades clusters, with apparent progenitor masses of $\not\lt 2$ $M_{\odot}$ and $\not\lt 5$ $M_{\odot}$ respectively, appear to fit only marginally higher surface gravities and masses than the mean values (Weidemann 1975, Sweeney 1976, Shipman 1979a), and are spectroscopically normal. While the upper mass limit for white dwarf progenitors remains controversial (see below), the problem with accounting for the high mass progenitors underscores our lack of understanding of mass loss processes in late stages of evolution.

THE PROGENITOR MASS LIMIT The faint blue stars found by Romanishin & Angel (1980) in several open clusters with 3–6 $M_{\odot}$ turnoff masses indicate that the Pleiades case is not an isolated example and that some stars initially with $\gtrsim 5$ $M_{\odot}$ evolve into white dwarfs. Nevertheless, there is some evidence favoring an upper mass limit for progenitors (M_u) less than $5 M_{\odot}$. The possibility of non-coeval star formation in young clusters introduces a potentially serious uncertainty. Landolt (1979) and Stauffer (1980) have shown that some low mass stars in the Pleiades fit pre–main-sequence contraction ages of $\gtrsim 10^8$ years, or more than twice the turnoff age. While there may prove to be some explanation other than differing ages for these objects, a high mass counterpart of ~ 4 $M_{\odot}$ could become a white dwarf in

$\sim 10^8$ years. Additionally, the white dwarf counts in the Hyades apparently support a lower M_u value (van den Heuvel 1975), though this estimate is subject to uncertainties in the cluster initial mass function at high masses, the small numbers of stars, the difficulties with assessing membership, and the problem of escaped members. A statistically lower M_u is also favored by Taylor & Manchester (1977) in accounting for the pulsar/neutron star birthrate and scale-height evidence. However, the pulsar birthrate they wish to satisfy may be uncertain by as much as 600 (Arnett & Lerche 1980). Furthermore, Shipman & Green (1980) find that recent revisions in the stellar birthrate function permit them to accommodate the pulsar progenitors with a much higher M_u value than Taylor & Manchester used.

Finally, various authors have argued that differing angular momenta or magnetic field configurations for evolved stars should result in differential mass loss rates anyway—that is, M_u should vary with these properties (Weidemann 1977a). These possibilities are difficult to evaluate given the lack of a physical understanding of the mass loss mechanisms for the simpler assumptions of zero angular momenta and zero magnetic fields! The mean value of M_u thus remains uncertain despite a great deal of recent research, but is likely to be $\gtrsim 5\ M_{\odot}$.

PLANETARY NEBULAE AND HOT SUBDWARFS The white dwarf birthrate ($\dot{N}_{WD}$) may be compared with those for earlier phases of stellar evolution in order to determine which stars may generally be precursors. Planetary nebulae stars have long been assumed to be immediate progenitors, given that they are thought to be in the final stage of gravitational contraction and that the hottest white dwarfs approach the region of the Harman-Seaton sequence in the H-R diagram. In fact, Liebert, Greenstein & Green (1980) have now found that the central star of the low-surface-brightness planetary Abell 7 actually has the spectrum of a DAO white dwarf. With the older distance scale (Seaton 1968), the estimated birthrate for planetaries was substantially higher (Cahn & Kaler 1971) than Green's determination of $\dot{N}_{WD}$. However, there are persuasive arguments in favor of increasing the distance scale of planetaries by 30–45%, which would result in a nebula birthrate $\gtrless \dot{N}_{WD}$ (Cudworth 1974, and especially Weidemann 1977b). The birthrates are therefore consistent with the assumption that most white dwarfs pass through the planetary nebula stage.

Other groups of stars suspected of being white dwarf progenitors are the subdwarf O and B stars, particularly those found to have relatively high surface gravities. Since most of these stars have been found at high galactic latitudes (e.g. Greenstein & Sargent 1974), they are known to be evolved stars from an old stellar population, though many may be disk rather than halo population objects. They are generally too hot and/or too low in luminosity to be plausibly associated with horizontal branch (helium burning) evolution, yet some exist at temperatures much too low to be fit by theoretical models for nonbinary gravitationally contracting stars of reasonable masses ($\gtrsim 0.4\ M_{\odot}$). Indeed, planetary nebulae stars are generally much hotter. Though Green's (1977) preliminary estimate implies that the space densities of both sdB and sdO stars were too low for them to be precursors for most white dwarfs, the uncertainties in the luminosities may be great enough for this to remain an open question.

Since stars classified sdO are generally believed to have helium-rich atmospheres, it is tempting to suppose that these may be progenitors for some helium-atmosphere white dwarfs. Schönberner (1977) has evolved a series of asymptotic branch giant models stripped of outer hydrogen envelopes, following the evolution to the white dwarf stage. He finds that configurations with $\sim 0.7\ M_{\odot}$ may cross through the regions of R CrB and other helium-rich stars (cf Hunger 1975), though the evolutionary tracks again do not account for the lower temperature sdO stars having high surface gravities. Schönberner finds that the number densities and time scales for evolved helium-rich stars suggest a lower limit death rate of $\gtrsim 4 \times 10^{-14}\ \mathrm{pc}^{-3}\ \mathrm{yr}^{-1}$, within an order of magnitude of the birthrate inferred for helium-atmosphere white dwarfs. It may be possible that the cooler helium sdO's require consideration of much lower masses and binary evolution (see Section 3.4).

4.4 *Envelope Evolution of White Dwarfs*

A variety of physical processes may change the surface abundances over the cooling time of a white dwarf. There is currently very extensive theoretical work on these problems, and the imminent availability of ultraviolet and X-ray spectra and fluxes for some hot stars promises to test some of the ideas concerning diffusion, mixing, stellar winds, and coronae. The author recently compared the empirical numbers and abundance distributions for cool DA, metallic-line and carbon-band white dwarfs with those that might be expected if convective mixing or interstellar accretion were taking place (Liebert 1979). Likewise, Vauclair (1979) has reviewed various envelope processes in detail—see also Böhm (1979), Vauclair, Vauclair & Greenstein (1979) and individual papers by Alcock, Michaud, Fontaine, and others referenced below. Here we attempt only a brief resumé of the potentially important physical processes.

DIFFUSION AND ACCELERATION PROCESSES Since the work of Schatzman (1958), it has been recognized that diffusion processes can operate very quickly to separate elements in quiescent white dwarf envelopes. It was apparent that helium and the heavier elements could sink completely out of hot white dwarf atmospheres—leaving an essentially pure hydrogen upper layer—on time scales of < 100 years. Recent comprehensive calculations by Fontaine & Michaud (1979), Michaud & Fontaine (1979), Vauclair, Vauclair & Greenstein (1979), and by Alcock & Illarionov (1980) generally confirm and greatly extend this conclusion. The downward diffusion is caused by both pressure and temperature gradients, but radiative acceleration tries to drive material outwards. The net downward diffusion velocity may be expressed (adapting from the Vauclair et al. paper):

$$v = \frac{\bar{D}m}{kT}(g_{\mathrm{GT}} - g_{\mathrm{rad}}) \tag{2}$$

where $\bar{D}$ is a diffusion coefficient and m the mass for a given ion; whether the ion sinks or rises thus depends on the relative sizes of the gravitational plus thermal acceleration (g_{GT}) and the radiative acceleration (g_{rad}). In the case of elements whose line transitions are not saturated, the latter may be approximated (Michaud et al. 1976) by

$$g_{\mathrm{rad}} = 1.7 \times 10^{-4}\,\frac{T_{\mathrm{eff}}^4 R^2}{A\,T\,r^2}\,\mathrm{cm\ s^{-2}} \tag{3}$$

where T_{eff} is the effective temperature, T is the temperature at radius r, R is the stellar radius, and A is the atomic mass number of the element being considered. The ions must have transitions with large f-values near the peak wavelength of the flux distribution. Thus g_{rad} may become competitive for light CNO ions with ultraviolet resonance transitions near the surfaces of very hot stars. Vauclair et al. expect that some ions having $g_{\mathrm{rad}} \gtrsim g_{\mathrm{GT}}$ could be suspended in equilibrium or even expelled from very hot photospheres. However, when $g_{\mathrm{rad}} < g_{\mathrm{GT}}$ the element will sink rapidly and permanently downwards. Whether ions are suspended or ejected from the atmosphere depends on details of the outer boundary conditions of the envelope problem (see the subsection on accretion).

Saturation effects can reduce g_{rad} far below the estimates provided by Equation (3). The Vauclair et al. calculations suggest that saturation could effectively prevent any enhancements of CNO ions for hydrogen envelopes. Indeed, no elements other than hydrogen have yet been detected in early IUE ultraviolet spectra of hot DA stars (Greenstein & Oke 1979). However, in helium-rich envelopes, higher pressures produce relatively larger line widths thus reducing the effects of saturation. The calculations suggest that significant CNO abundances may be supported for a while at the tops of hot helium stars. Still, the onset of convection below about 60,000 K introduces a different deterrent. Due to the T^{-1} dependence in Equation (3), diffusion of heavy elements at the lower boundary of the homogeneous convective region could restrict the operation of the radiative acceleration process to very hot helium stars (Fontaine & Michaud 1979, Alcock & Illarionov

1980). Hence, radiation effects without accretion should not be relevant to the observed Ca II in a DB star such as GD40 at $\sim 15{,}000$ K (from Shipman, Greenstein & Boksenberg 1977). Again, the best chances for detecting heavy ions rest with ultraviolet spectrophotometry of hot helium-rich degenerates. Green & Liebert (1980) found C IV $\lambda 1550$ in at least two Palomar Green (PG) stars classified DO.

THE QUESTION OF CORONAE Numerous investigators have suggested that hot white dwarfs may have coronae which 1. inhibit accretion of hydrogen onto DB stars (Strittmatter & Wickramasinghe 1971), 2. contribute to the galactic soft X-ray background (Strittmatter, Brecher & Burbridge 1972), or 3. account for the soft X-ray flux from Sirius B (see Section 3.1). The first authors suggested that a hot corona-stellar wind might be generated for stars having a convective outer envelope—helium stars above $\lesssim 20{,}000$ K and hydrogen stars below $\gtrsim 12{,}000$ K. This idea was independently suggested and developed by Böhm & Cassinelli (1971), who calculated large acoustic fluxes for helium white dwarf envelopes.

Now Muchmore & Böhm (1978) have attempted preliminary modeling of the temperatures, pressures, and energy distributions for hot non-DA coronae, based on the main assumptions 1. that a corona is powered by turbulence (acoustic noise) generated in the envelope, 2. that acoustic fluxes may be estimated by the Lighthill-Proudman theory, and 3. that the corona structure adjusts itself to minimize the total flux emitted, i.e. the Hearn (1975) minimum flux theory. In contrast to the solar corona, this model produces coronae that have temperatures of $\sim 5 \times 10^6$ K and extend only tens of kilometers above the photosphere. The Muchmore-Böhm coronae lose energy by direct radiation, electron thermal conduction downwards, and by material outflow. However, the input energy estimate in assumption 2. is very uncertain because of the dependence on the eighth power of the maximum convective velocity; furthermore, the accuracy of the simple Hearn formalism 3. has been widely questioned.

Thus, the Böhm modeling offers only a qualitative physical foundation, but its predicted high energy fluxes may soon be tested from space observations, and it carries implications for other envelope problems. The total stellar EUV and soft X-ray radiation would be significantly enhanced by coronal radiation, particularly shortwards of atmospheric helium opacity edges (see also Böhm 1979, Böhm & Kapranidis 1980). However, a simple calculation suggests that not enough high energy photons would be created to produce detectable emission lines at He II $\lambda 4686$. Likewise, we note that the predicted corona is not nearly hot enough nor extended enough to selectively accelerate and eject protons from a DB atmosphere according to the mechanism of Michaud & Fontaine (1979). Still, it is implicit that there would be some outward mass flow, and its properties in inhibiting interstellar accretion have not yet been explored.

There remains no strong physical basis for postulating a corona around DA stars as hot as Sirius B. Lampton & Mewe (1979) suggest that Sirius B has a subsurface ionized helium layer at $\tau \gtrsim 4$ that provides the required acoustic power for a minimum flux corona. The outer hydrogen layer must be exceedingly thin yet cannot be mixed by the mechanical energy transport, unless it is resupplied by accretion from the primary. Still, Shipman (1979b) reminds us that the solar corona is neither static nor homogeneous in character. It is difficult to assess such an ad hoc, layered DA atmosphere model, given the difficulties with obtaining accurate line profile observations of Sirius B. The HEAO-1 steep X-ray energy distribution does not seem readily compatible with the thermal bremsstrahlung expected from a corona (Lampton et al. 1978). Hence, as of this writing, the Sirius X-ray question remains unanswered.

Not surprisingly, searches for soft X-ray emission from white dwarfs have been directed towards hot objects (Lampton & Kahn 1978). Yet it is in cooler DA stars at about the T_{eff} of the ZZ Ceti variables that deep convective envelopes develop which might readily provide coronal heating. The Starrfield et al. work (see Section 3.7) even suggests that acoustic modes are involved in the pulsations, which might therefore support a corona, as originally suggested by Strittmatter et al. (1972). However, the Böhm (1979) model suggests that such cool DA coronae would have temperatures well below 10^6 K, so that X-ray emission would be minimal.

CONVECTION AND ENVELOPE MIXING The occurrence of significant convective envelopes in cool DA stars and in non-DA stars of most surface temperatures holds several potential implications for envelope evolution:

1. the possible formation of coronae;
2. the dredging of gravitationally diffused heavy elements from deeper in the envelope;
3. penetration and mixing of an underlying carbon-oxygen layer for helium-envelope stars;
4. penetration and mixing of an underlying helium layer into hydrogen atmosphere stars; and
5. the mixing and dilution of accreted surface material.

Coronae (1) were discussed in the previous subsection. The reappearance of diffused heavy elements (2) at the surface of the star during any stage of the cooling process cannot be caused by convective mixing, according to the recent calculations of Fontaine & Michaud (1979) and Vauclair et al. (1979). These authors and Alcock (1979) insist that the appearance of metals in some cool helium degenerates must be caused at least in part by accretion.

Since outer helium convection zones should extend quite deep at cooler T_{eff}, it has been suggested that the sequence of non-DA stars showing C_2 bands covering $6000 \lesssim T_{\mathrm{eff}} \lesssim 10{,}000$ K may result from some dredging of material out of the underlying carbon-oxygen core (3, above). D'Antona & Mazzitelli (1979, 1980) argued that pre–white dwarf evolution should not leave a helium envelope thin enough for the convection to penetrate. Calculations by these authors and by Vauclair & Fontaine (1979) indicate that, if this helium layer *were* small enough, penetration of the μ-discontinuity would result in complete mixing anyway and a nearly pure carbon surface composition for most stars. This would contradict the abundance analyses showing that several $\lambda 4670$ degenerates have helium-dominated atmospheric compositions (Bues 1973, Grenfell 1974) with C/He only $\sim 10^{-3}$. For these stars, a mechanism may exist that inhibits the diffusion of carbon in helium envelopes, regardless of the origin of the surface carbon. On the other hand, the predicted spectra for any stars with carbon-dominated atmospheres are not well established, and might even be DC-like; this leaves open the possibility that real carbon stars exist and have yet to be recognized!

A number of recent investigations (Koester 1976, Vauclair & Reisse 1977, D'Antona & Mazzitelli 1979) have explored the mixing of an outer hydrogen DA layer into an underlying helium envelope (4, above). The treatment of the μ-barrier and the possibilities for convection zones in both layers are serious complications to the evolutionary code. The actual surface T_{eff} at which mixing occurs should vary according to the hydrogen layer mass (M_{H}) and the total stellar mass. Given the theoretical uncertainties and the wide range of possible M_{H} values, it is unclear as to what fraction of stars should undergo mixing. However, Wehrse & Liebert (1980a), Liebert (1979), and Sion (1979) discuss empirical evidence that mixing *has* taken place in some of the stars reaching 6000–7000 K. Whether all DA stars get converted to helium atmospheres at still lower temperatures may be determined from further theoretical work and infrared observations of very cool stars.

THE QUESTION OF ACCRETION The existence of nearly pure helium atmosphere degenerates over a wide range of temperatures has long been a puzzle, given the physical arguments that these stars should accrete gas as they pass through the interstellar medium. Strittmatter & Wickramasinghe (1971) argued that, with the assumption that a tail shock forms to destroy particle momentum perpendicular to the relative motion of the star, the Bondi-Hoyle rate should apply:

$$\dot{M} = 3 \times 10^{21}\, \alpha \frac{M^2 n}{v^3}\ (\mathrm{g\,yr^{-1}}) \tag{4}$$

where $n =$ the interstellar gas density, v is the relative velocity of the star through the gas, α is an efficiency factor, and M the mass in solar units. Taking $n \sim 1\ \mathrm{cm}^{-3}$, $v \sim 30\ \mathrm{km\ s}^{-1}$, they showed that the DB atmosphere should be replenished by interstellar material in $\sim 10^{-1}$ years! If one assumed instead the "minimal" Eddington accretion rate

$$\dot{M} = 10^{14}\, n\bar{R}\bar{M}/v\,(\mathrm{g\,yr}^{-1}) \tag{5}$$

the time scale of $\sim 10^3$ years is still far shorter than the cooling age of a DB star.

A realistic assessment, however, must take into account the inhomogeneous distribution of the interstellar gas. Using $n \gtrsim 10^{-2}\ \mathrm{cm}^{-3}$ for the typical "intercloud medium," the time scales are increased a few orders of magnitude. Koester (1976) argued that the Bondi-Hoyle rate should not apply at such low densities. Truran et al. (1977) suggested that most accretion occurs when a white dwarf encounters an interstellar cloud, as it is likely to do every 10^6–10^7 years. This idea has been developed by Wesemael (1979) and Alcock (1979), who argued that the Bondi-Hoyle mechanism must surely apply to a star moving slowly through a dense gas clump. Hence the more recent analyses still leave us with the same conclusion reached by Strittmatter & Wickramasinghe: either 1. a mechanism to inhibit accretion, especially the accretion of hydrogen, must be identified, or 2. the accreted material must be sufficiently diluted by an envelope mixing process. In the former category, a number of ideas have been suggested. The formation of a hot corona or "mini" H II region might deter accretion of protons (cf Vauclair et al. 1979). However, it is unclear that a low-lying, low temperature Böhm corona would prevent protons from reaching the surface. If electric fields create an ion wind in helium envelopes (Michaud & Fontaine 1979), then protons could be selectively expelled. However, the existence of such a wind is challenged by Alcock (1979). Finally, we note that the combination of a kilogauss magnetic field and a modest rotation rate provides another possibility for "batting away" charged particles from the magnetospheric environment, a possibility that has not been explored. Given the modest likely rotation rates of DB stars (Section 3.6), attention has focussed on convection zones as a means of diluting the accreted hydrogen if it does reach the surface. However, the outer convection layers are relatively thin for the hotter stars; Alcock (1979) argues that the DB stars must have masses $< 0.4\ M_\odot$ in order to have deep enough layers for sufficient dilution to occur. Given that convective envelopes deepen with decreasing temperature, Alcock's argument may imply that the hottest helium white dwarfs generally have the lowest masses (see Section 3.4).

For the cooler helium-rich stars—the most numerous kind of white dwarf—there is less justification for assuming that accretion of interstellar material may be prevented. These stars have relatively less ultraviolet flux to ionize the approaching gas, relatively lower envelope acoustic fluxes to power a corona, and relatively few free electrons and ions for hypothesized acceleration processes (Liebert 1979). They do offer deeper convective envelopes in which to mix away the pollutants, but now a question arises specifically with regard to those cool helium stars showing metallic lines. The consensus view from the recent theoretical work requires these surface metals to be supplied by accretion, rather than by convective dredging from deeper layers. Vauclair et al. (1979) and Cottrell & Greenstein (1980) argue that the larger-than-cosmic abundance ratios of Mg/Ca in some metallic-line stars are consistent with the accretion hypothesis, though GD401 may be a counterexample (Table 1). Likewise, the metals in the accreted material should diffuse downwards (Alcock 1979), while hydrogen should remain in the convective layer. Thus, the predicted metals-to-hydrogen ratios would be *at or below* solar (interstellar) values, yet real DF-DG-DK stars have calcium-to-hydrogen abundance ratios ranging from about solar to well above solar (Table 1). This would imply that some cooler stars must have an effective screening mechanism that blocks hydrogen accretion in favor of heavier elements.

4.4 *Cooling Evolution and the Luminosity Function*

Extensive theoretical work on degenerate stellar interiors in the last fifteen years supersedes the simple Mestel cooling relation ($t \propto L^{-5/7}$) to describe the cooling of degenerate stars. Mestel (1952) assumed only that the luminosity of a white dwarf is due entirely to the thermal energy of ions, that Kramer's law opacity is applicable with radiative diffusion in a non-degenerate envelope, and that the degenerate core is isothermal. These assumptions grossly oversimplify the physics of the stars—Van Horn's (1971) review is still timely. Yet the sophisticated calculations show that the Mestel relation is a surprisingly good approximation for luminosities that are neither very high nor very low.

NEUTRINO COOLING For the highest luminosity stars the principal uncertainty concerns the role of neutrinos in cooling. Koester (1978) estimated the expected luminosity function of very hot stars from modern calculations with and without the universal Fermi reaction. The predictions differ by factors of ~ 3 in the 50–70,000 K range. Gravitational contraction may also be significant for low mass stars at high luminosities. The heat capacity of the electrons is also incorporated into recent calculations. Unfortunately, the completeness problem, paucity of stars, and uncertainties in assigning temperatures and surface gravities have precluded a clear observational test of the function for $M_v \lesssim +10$ ($\gtrsim 40{,}000$ K). Wesemael (1978) argues that the X-ray background evidence supports the lower luminosity function derived from including neutrinos in the calculation. The combination of the PG star sample, space ultraviolet observations, and accurate model atmospheres for hot stars may soon permit a numerical test.

COLOUMB EFFECTS, CRYSTALLIZATION, AND CONVECTION At lower luminosities, numerous complications in calculating the cooling rate are caused by coulomb interactions and the onset of envelope convection. Plasma interactions complicate the interior conductivity, the radiative opacities in the envelope and transition zone, and the particle ionization in the envelope. As the stars cool, the increasing interactions lead first to a liquid state, then to a crystallization of the ions, with the release of latent heat. During this state the extra energy input lengthens the cooling time. However, this is followed by rapid Debye cooling, due to the drop in the specific heat of the latticed ions. Lamb & Van Horn (1975) have provided a clear discussion of the evolutionary effects, while Shaviv (1979) gives a comprehensive treatment of the coulomb physics.

The onset of crystallization is a gradual process, beginning first in the deep interior and spreading slowly outwards as the degenerate dwarf cools. Hence, the latent heat is deposited over a range in luminosity (cf the 1 $M_{\odot}$ curve of Lamb & Van Horn 1975) and no pileup of stars at a single luminosity is expected. It is thus unlikely that crystallization sequences for different interior compositions (should they exist) could be recognized. The recent calculations (Lamb & Van Horn 1975, Shaviv & Kovetz 1976) also indicate that the onset of the rapid cooling phase—that is, the turndown from the peak of the luminosity function—should not occur until the stars have cooled below $\sim 10^{-4}\ L_{\odot}$. The mass and envelope composition dependence of the peak position is not well established, though the above calculations are for reasonable carbon-oxygen core configurations covering 0.6–1.0 $M_{\odot}$.

The onset of extensive envelope convection offers a faster means for interior energy to escape the star, thus speeding up the cooling process at lower temperatures (Böhm 1968, Van Horn 1970). A comprehensive series of envelope calculations by Fontaine & Van Horn (1976) does not suggest that the uncertainties here could upset the basic cooling results discussed in the previous paragraph. However, envelope coulomb effects complicate the physics, and the basic relation between the core temperature and the surface effective temperature may remain uncertain by up to a factor of two.

THE EMPIRICAL LUMINOSITY FUNCTION Green (1977, 1980) has provided the best determination of the observational luminosity function for $+10 \gtrsim M_v \gtrsim +13$. The data are in good agreement with the Mestel cooling relation, assuming that the white dwarf birthrate has remained constant for $\lesssim 10^9$ years.

At lower luminosities, there is no available sample free of motion selection and other biases. Early difficulties with finding cooler degenerates in proper motion surveys (e.g. Greenstein 1971) led to the suggestion that there might be a deficit of "yellow" degenerates at $M_v \gtrsim +13$ relative to the predictions of the Mestel relation. However, Hintzen & Strittmatter (1974) and especially Greenstein subsequently identified many new cool degenerates. Weidemann (1967), Kovetz & Shaviv (1976), and Sion & Liebert (1977) attempted to correct for the volume-dependence of the surveys at different absolute magnitudes, for successively larger spectroscopic samples. They found that the empirical distribution continues to rise to at least $M_{bol} = +14$. In Figure 2, the method is repeated for Sion's (1979) updated spectroscopic catalogue sample and yields very similar results; based on the result discussed in the next paragraph, stars near $M_{bol} \sim +15$ or $10^{-4} L_{\odot}$ may in fact be the most numerous kind of white dwarf! Green's function with a more secure absolute scale is also shown in Figure 2; that

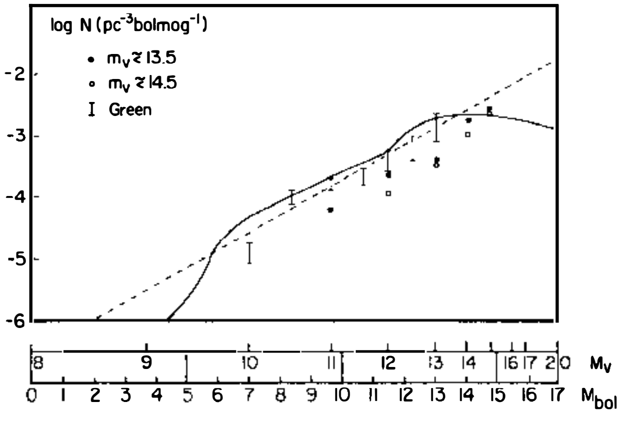

Figure 2 The empirical luminosity function for the known spectroscopic sample of white dwarfs (Sion 1979), updated from Sion & Liebert (1977) employing the technique of Weidemann (1967). Filled circles show values assuming relative completeness to $m_v \simeq 13.5$ (154 stars), open circles to 14.5 (200 stars). Numbers are converted to unit bolometric magnitude intervals using a blackbody bolometric correction. Green's (1977, 1980) empirical luminosity function is converted to the bolometric scale with 1σ error bars. The solid curve is the theoretical function of Lamb & Van Horn (1975) for 1 $M_{\odot}$; the dashed line is the Mestel 2/7 M_{bol} cooling law, which is closer to the shape of the Shaviv & Kovetz (1976) 0.6 $M_{\odot}$ calculation. In the region overlapping with Green, the spectroscopic sample appears to be less than 50% complete, though the slope offers satisfactory agreement. The technique basically hinges on the assumption that the fractional completeness remains essentially independent of absolute magnitude within a constant apparent magnitude limit. While 115 degenerates are now known within the interval $13.5 \lesssim M_v \lesssim 15.5$, too few stars with $M_v \gtrsim 15.5$ have been discovered for this method to be readily applicable. Other arguments indicate that the empirical luminosity function peaks near $M_{bol} \sim +15$.

the Weidemann technique matches tolerably well with Green in the overlap region is reassuring.

Recent observations of very faint Luyten (1976) proper motion stars and comprehensive surveys of brighter stars have failed to find any stars clearly having $M_{\text{bol}} \geq +16$, despite the abundance of cases near $+15$ (Liebert et al. 1979b, Liebert 1979). The reality of the shortfall is strongly supported by the absence of any such stars in proper motion binaries; the apparent magnitude difference would clearly earmark a low luminosity companion to any of the thousands of nearby stars having distance estimates. Yet the theoretical cooling curves predict that the stars spend at least as much time at $+16 \leq M_{\text{bol}} \leq +17$ as in each of the preceding two magnitude intervals. Hence, the cutoff implies 1. serious errors in the cooling calculations at $\lesssim 10^{-4} L_{\odot}$, and/or 2. a decreased white dwarf birthrate $\gtrsim 10^{10}$ years ago.

It is important to establish whether *any* such very low luminosity stars exist—e.g. such as LP131-66 (Liebert et al. 1979a); this will require parallax work, further investigations of the discrepant optical and infrared colors (Liebert 1979), and the calculation of more realistic energy distributions for stars below 5000 K (Böhm et al. 1977, Bessell, Wickramasinghe & Cottrell 1979). A cutoff would be expected if the galactic disk were effectively much younger than the halo, as some recent evidence suggests (e.g. Demarque & McClure 1977).

OTHER LUMINOSITY SOURCES AT LOW LUMINOSITY? Two effects which could help explain the observed paucity of degenerates at $< 10^{-4} L_{\odot}$ are 1. a physical separation of the elements when the core freezes, and 2. accretion of matter from the interstellar medium. Stevenson (1977) argued that the heavy elements in the core should become insoluble at the time the main constituent, i.e. carbon, crystallizes. Stevenson (1979) expects even the separation of oxygen from carbon to be an important energy source. The diffusion could release enough gravitational energy to slow the cooling process greatly, perhaps even suspending the star at the crystallization luminosity for the age of the galaxy!

A similar pileup of stars somewhere near $\simeq 10^{-4} L_{\odot}$ might be expected if interstellar accretion becomes a significant, gravitational energy source. The accretion rate necessary to provide $10^{-4} L_{\odot}$ is $\simeq 10^{-14} M_{\odot}$/year; such a mean rate is precluded in general for non-DA stars above ~ 6000 K, but one may speculate that any barriers to accretion which depend on surface temperature (Section 4.3) might be inoperative at very low T_{eff}. Starrfield & Sparks (1979) argue that such stars could even burn the accreted hydrogen quiescently with little increase in luminosity, though it seems unlikely that very cool stars could have sufficiently high internal temperatures.

Either mechanism discussed above might be expected to produce an excess of stars near the peak of the empirical luminosity function. A substantial excess near $M_{bol} = +14$–15, relative to the cooling predictions, is certainly not ruled out. However, there is still a limitation on the early stellar birthrate function (Liebert et al. 1979b).

THE SPACE DENSITY OF REMNANT DEGENERATE STARS The sum of the observed luminosity function and the numbers of any black degenerates below the detection limit is an important contributor to the local galactic mass. A number of authors have previously pointed out that the numbers of undiscovered cool and invisible degenerates might account for the missing Oort mass (insofar as there remains a "missing mass"). However, the number of single stars accounted for to $M_{bol} = +16$ accounts for only about 0.015 $M_{\odot}$ pc^{-3}, though a sizable increment may be necessary to include the contribution from close binaries. The use of a conventional stellar birthrate function with a disk age of 15×10^9 years would predict a local mass density for non-close binaries as high as 0.06 $M_{\odot}$ pc^{-3} in total remnants (Green 1977, Liebert et al. 1979b); however, the paucity of very low luminosity degenerates argues that the last number is a serious overestimate. The empirical numbers thus offer fair agreement with theoretical expectations based on global galaxy properties, such as those of Hills (1977).

Acknowledgment

I thank Drs. Roger Angel, K. H. Böhm, Dave Dearborn, Richard Green, Jesse Greenstein, Mike Lampton, Roger Malina, John McGraw, Harry Shipman, Ed Sion, Sumner Starrfield, and Francois Wesemael for results in advance of publication and/or useful discussions. The author acknowledges support from National Science Foundation grant No. AST 78-21071.

Literature Cited

Alcock, C. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 202. New York: Univ. Rochester

Alcock, C., Illarionov, A. 1980. *Ap. J.* 235:534

Allen, C. W. 1973. *Astrophysical Quantities*. London: Athlone

Angel, J. R. P. 1977. *Ap. J.* 216:1

Angel, J. R. P. 1978. *Ann. Rev. Astron. Astrophys.* 16:487

Angel, J. R. P. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 313. New York: Univ. Rochester

Angel, J. R. P., Landstreet, J. D., Borra, E. F. 1980. Preprint

Arnett, W. D., Lerche, I. 1980. Preprint

Auer, L., Shipman, H. L. 1977. *Ap. J. Lett.* 211:L103

Barnes, T. G., Evans, D. S., Parsons, S. B. 1976. *MNRAS* 174:503

Bessell, M. S. 1978. *Proc. Astron. Soc. Aust.* 3:220

Bessell, M. S., Wickramasinghe, D. T. 1975. *MNRAS* 171:11

Bessell, M. S., Wickramasinghe, D. T. 1978. *MNRAS* 182:275
Bessell, M. S., Wickramasinghe, D. T. 1979. Preprint
Bessell, M. S., Wickramasinghe, D. T., Cottrell, P. L. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 179. New York: Univ. Rochester
Böhm, K. H. 1968. *Astrophys. Space Sci.* 2:375
Böhm, K. H. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 223. New York: Univ. Rochester
Böhm, K. H., Carson, T. R., Fontaine, G., Van Horn, H. M. 1977. *Ap. J.* 217:521
Böhm, K. H., Cassinelli, J. 1971. *Astron. Astrophys.* 12:21
Böhm, K. H., Kapranidis, S. 1980. Preprint
Böhm-Vitense, E., Dettmann, T., Kapranidis, S. 1979. *Ap. J. Lett.* 232:L189
Bues, I. 1973. *Astron. Astrophys.* 28:181
Cahn, J. H., Kaler, J. B. 1971. *Ap. J. Suppl.* 22:319
Cash, W., Bowyer, S., Lampton, M. 1978. *Ap. J. Lett.* 221:L87
Chandrasekhar, S. 1939. *An Introduction on the Study of Stellar Structure.* Univ. Chicago Press
Chiu, G. 1978. PhD thesis. Univ. Calif., Berkeley
Conti, P. S., Dearborn, D., Massey, P. 1980. Submitted to *MNRAS*
Cottrell, P. L., Bessell, M. S., Wickramasinghe, D. T. 1977. *Ap. J. Lett.* 218:L133
Cottrell, P. L., Greenstein, J. L. 1980 *Ap. J.* In press
Cudworth, K. M. 1974. *Astron. J.* 79:1384
Dahn, C., Hintzen, P., Liebert, J., Stockman, H. S., Spinrad, H. 1978. *Ap. J.* 219:976
D'Antona, F., Mazzitelli, I. 1979. *Astron. Astrophys.* 74:161
D'Antona, F., Mazzitelli, I. 1980. *Ap. J.* 237:In press
Dearborn, D. S. P., Kozlowski, M., Schramm, D. 1976. *Nature* 261:210
Demarque, P., McClure, R. D. 1977. In *The Evolution of Galaxies and Stellar Populations*, ed. B. M. Tinsley, R. B. Larson. New Haven: Yale Univ. Press
Dziembowski, W. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 359. NY: Univ. Rochester
Eddington, A. S. 1922. *MNRAS* 82:436
Eggen, O. J., Greenstein, J. L. 1967. *Ap. J.* 150:927
Fontaine, G., Michaud, G. 1979. *Ap. J.* 231:826
Fontaine, G., Van Horn, H. M. 1976. *Ap. J. Suppl.* 31:467
Fusi-Pecci, F., Renzini, A. 1975. *Astron. Astrophys.* 39:413
Gallagher, J., Starrfield, S. G. 1978. *Ann. Rev. Astron. Astrophys.* 16:171
Gatewood, G. D., Gatewood, C. V. 1978. *Ap. J.* 225:191
Giacconi, R. 1979. Review paper presented at the 154th (Wellesley) A.A.S. Meeting
Graham, J. 1972. *Astron. J.* 77:144
Gray, D. F. 1967. *Ap. J.* 149:317
Green, R. F. 1977. PhD thesis. Calif. Inst. Tech., Pasadena
Green, R. F. 1980. *Ap. J.* 238:In press
Green, R. F., Liebert, J. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 118. New York: Univ. Rochester
Green, R. F., Liebert, J. 1980. In preparation
Green, R. F., Schmidt, M., Liebert, J., Green, J. 1980. *The PG Catalogue.* In preparation
Greenstein, J. L. 1960. In *Stars and Stellar Systems*, Vol. 6, ed. J. L. Greenstein, p. 676. Univ. Chicago Press
Greenstein, J. L. 1971. In *White Dwarfs, Proc. IAU Symp. No. 42*, ed. W. J. Luyten, p. 46. Dordrecht: Reidel
Greenstein, J. L. 1976. *Astron. J.* 81:323
Greenstein, J. L. 1979a. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 1. New York: Univ. Rochester
Greenstein, J. L. 1979b. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 374. New York: Univ. Rochester
Greenstein, J. L. 1979c. *Ap. J.* 233:239
Greenstein, J. L., Boksenberg, A., Carswell, R.; Shortridge, K. 1977. *Ap. J.* 212:186
Greenstein, J. L., Oke, J. B. 1979. *Ap. J. Lett.* 229:L141
Greenstein, J. L., Oke, J. B., Shipman, J. L. 1971. *Ap. J.* 169:563
Greenstein, J. L., Peterson, D. M. 1973. *Astron. Astrophys.* 25:29
Greenstein, J. L., Sargent, A. 1974. *Ap. J. Suppl.* 28:157
Greenstein, J. L., Trimble, V. 1967. *Ap. J.* 149:283
Greenstein, J. L., Vauclair, G. 1979. *Ap. J.* 231:491
Grenfell, T. C. 1974. *Astron. Astrophys.* 31:303
Guinan, E. F., Sion, E. M. 1979. *Bull. Am. Astron. Soc.* 11:683
Hamada, T., Salpeter, E. E. 1961. *Ap. J.* 134:683

Harrington, R., Christy, J., Dahn, C., Strand, K., Liebert, J. 1980. In preparation
Harrington, R., Dahn, C., Behall, A., Priser, J., Christy, J., Riepe, J., Ables, H., Guetter, H., Hewitt, A., Walker, R. 1975. *Publ. US Naval Obs.* 24: Part I
Hearn, D. R. 1975. *Astron. Astrophys.* 40: 355
Hearn, D. R., Richardson, J. A., Bradt, H. V. D., Clark, G. W., Lewin, W. H. G., Mayer, W. F., McClintock, J. E., Primini, F. A., Rappaport, S. A. 1976. *Ap. J. Lett.* 203: L21
Heintz, W. D. 1974. *Astron. J.* 79: 819
Heise, J., Huizenga, H. 1980. *Astron. Astrophys.* In press
Hesser, J. E., Lasker, B. M., Neupert, H. E. 1979. *Ap. J. Suppl.* 40: 577
Hills, J. G. 1977. *Ap. J.* 219: 550
Hintzen, P., Strittmatter, P. A. 1974. *Ap. J. Lett.* 193: L111
Hintzen, P., Strittmatter, P. A. 1975. *Ap. J. Lett.* 201: L37
Humphreys, R. M., Liebert, J., Romanishin, W., Strittmatter, P. A. 1979. *Publ. Astron. Soc. Pac.* 91: 107
Hunger, K. 1975. In *Problems in Stellar Atmospheres and Envelopes*, ed. B. Baschek, W. Kegel, G. Traving, p. 57. Berlin-New York: Springer
Koester, D. 1976. *Astron. Astrophys.* 52: 415
Koester, D. 1978. *Astron. Astrophys.* 65: 449
Koester, D. 1979a. *Astron. Astrophys.* 72: 376
Koester, D. 1979b. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 130. New York: Univ. Rochester
Koester, D., Liebert, J., Hege, E. K. 1979a. *Astron. Astrophys.* 71: 163
Koester, D., Schulz, H., Weidemann, V. 1979b. *Astron. Astrophys.* 76: 232
Kovetz, A., Shaviv, G. 1976. *Astron. Astrophys.* 52: 403
Kuzma, T. J. 1979. *Ap. J.* 227: 548
Kylafis, N. D., Lamb, D. Q., Masters, A. R., Weast, G. J. 1980. Ninth Texas Symposium on Relativistic Astrophysics, Munich, Germany, 1978. *Ann. NY Acad. Sci.* 336: 520
Lamb, D. Q., Van Horn, H. M. 1975. *Ap. J.* 200: 306
Lampton, M., Kahn, S. 1978. *Bull. Am. Astron. Soc.* 10: 511
Lampton, M., Mewe, R. 1979. *Astron. Astrophys.* 78: 104
Lampton, M., Margon, B., Paresce, F., Stern, R., Bowyer, S. 1976. *Ap. J.* 203: L71
Lampton, M., Tuohy, I., Garmire, G., Charles, P. 1978. COSPAR Meeting, Innsbruck, Austria
Landolt, A. 1979. *Ap. J.* 231: 468
Landstreet, J. D. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 297. New York: Univ. Rochester
Liebert, J. 1975. *Ap. J. Lett.* 200: L95
Liebert, J. 1976. *Ap. J.* 210: 715
Liebert, J. 1977. *Astron. Astrophys.* 56: 427
Liebert, J. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 146. New York: Univ. Rochester
Liebert, J., Dahn, C., Gresham, M., Hege, E. K., Moore, R. L., Romanishin, W., Strittmatter, P. A. 1979a. *Ap. J.* 229: 196
Liebert, J., Dahn, C., Gresham, M., Strittmatter, P. A. 1979b. *Ap. J.* 223: 226
Liebert, J., Greenstein, J. L., Green, R. F. 1980. In preparation
Liebert, J., Gresham, M., Hege, E. K., Strittmatter, P. A. 1979c. *Astron. J.* 84: 1612
Luyten, W. J. 1970. *White Dwarfs*. Minneapolis: Univ. Minn. Press
Luyten, W. J. 1976. *LHS Catalogue*. Minneapolis: Univ. Minn. Press
Luyten, W. J. 1977. *White Dwarfs—II*. Minneapolis: Univ. Minn. Press
Malina, R. 1979. PhD thesis. Univ. Calif., Berkeley
Margon, B., Lampton, M., Bowyer, S., Stern, R., Paresce, F. 1976. *Ap. J. Lett.* 210: L79
Matsushima, S., Terashita, Y. 1969. *Ap. J.* 156: 219
Mazurek, T. J., Wheeler, J. C. 1980. *Fundamentals of Cosmic Physics*. In press
McCook, G. P., Sion, E. M. 1977. *Villanova Univ. Obs. Contrib. No. 2*
McGraw, J. T. 1977. PhD thesis. Univ. Texas, Austin
McGraw, J. T. 1979. *Ap. J.* 229: 203
McGraw, J. T., Starrfield, S. G., Liebert, J., Green, R. F. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 377. New York: Univ. Rochester
McGraw, J. T., Starrfield, S. G., Liebert, J., Green, R. F. 1980. In preparation
Mestel, L. 1952. *MNRAS* 112: 583
Michaud, G., Charland, Y., Vauclair, S., Vauclair, G. 1976. *Ap. J.* 210: 447
Michaud, G., Fontaine, G. 1979. *Ap. J.* 229: 694
Milton, R. L. 1974. *Ap. J.* 189: 543
Moffett, T. J., Barnes, T. G., Evans, D. S. 1978. *Astron. J.* 83: 820
Mould, J., Liebert, J. 1978. *Ap. J. Lett.* 226: L29
Muchmore, D. O., Böhm, K. H. 1978.

Astron. Astrophys. 69:113

Nather, R. E., Robinson, E. L., Stover, R. J. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 453. New York: Univ. Rochester

Osaki, Y., Hansen, C. J. 1973. *Ap. J.* 185:277

Ostriker, J. O. 1971. *Ann. Rev. Astron. Astrophys.* 9:353

Ostriker, J. O. 1972. In *Stellar Evolution*, ed. H. Chiu, A. Muriel, p. 211. Cambridge, Mass.: MIT Press

Paczynski, B. 1970. *Acta Astron.* 20:47

Reimers, D. 1975. *Mem. Soc. Roy. Sci. Liège, 6^{te} ser.* 8:369

Richer, H. 1978. *Ap. J. Lett.* 200:L95

Richer, H. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 259. New York: Univ. Rochester

Robinson, E. L. 1976. *Ann. Rev. Astron. Astrophys.* 14:119

Robinson, E. L. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 343. New York: Univ. Rochester

Romanishin, W., Angel, J. R. P. 1980. *Ap. J.* 235:992

Savedoff, M. P., Van Horn, H. M., Wesemael, F., Auer, L. H., Snow, T. P., York, D. G. 1976. *Ap. J. Lett.* 207:L45

Schatzman, E. 1958. *White Dwarfs*. Amsterdam: North Holland

Schönberner, D. 1977. *Astron. Astrophys.* 57:437

Seaton, M. J. 1968. *Astrophys. Lett.* 3:55

Shaviv, G. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 11. New York: Univ. Rochester

Shaviv, G., Kovetz, A. 1976. *Astron. Astrophys.* 51:383

Shipman, H. L. 1972. *Ap. J.* 177:723

Shipman, H. L. 1979a. *Ap. J.* 228:240

Shipman, H. L. 1979b. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 86. New York: Univ. Rochester

Shipman, H. L., Green, R. F. 1980. *Ap. J. Lett.* In press

Shipman, H. L., Greenstein, J. L., Boksenberg, A. 1977. *Astron. J.* 82:480

Shipman, H. L., Sass, C. 1980. *Ap. J.* 235:177

Sion, E. M. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 245. New York: Univ. Rochester

Sion, E. M., Fragola, J. L., O'Donnell, W. C. 1979. *Publ. Astron. Soc. Pac.* 91:460

Sion, E. M., Liebert, J. 1977. *Ap. J.* 213:468

Starrfield, S. G., Cox, A. N., Hodson, S. W. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 382. New York: Univ. Rochester

Starrfield, S. G., Sparks, W. M. 1979. *Bull. Am. Astron. Soc.* 11:663

Stauffer, J. 1980. Submitted to *Ap. J.*

Stellingwerf, R. F. 1978. *Astron. J.* 83:1184

Stevenson, D. J. 1977. *Proc. Astron. Soc. Aust.* 3:167

Stevenson, D. J. 1979. Paper presented at the C.N.R.S. International Colloquium on the Physics of Dense Matter, Paris, Sept. 17–22

Stover, R. S., Robinson, E. L., Nather, R. E. 1978. *Publ. Astron. Soc. Pac.* 89:912

Strand, K. 1977. *Astron. J.* 82:745

Strand, K., Dahn, C., Liebert, J. 1976. *Bull. Am. Astron. Soc.* 8:506

Strittmatter, P. A., Wickramasinghe, D. T. 1971. *MNRAS* 152:47

Strittmatter, P. A., Brecher, K., Burbidge, G. R. 1972. *Ap. J.* 174:L91

Sweeney, M. A. 1976. *Astron. Astrophys.* 49:375

Tapia, S. 1978. PhD thesis. Univ. Ariz., Tucson

Taylor, J. H., Manchester, R. N. 1977. *Ap. J.* 215:885

Trimble, V., Greenstein, J. L. 1972. *Ap. J.* 177:441

Truran, J. W., Starrfield, S. G., Strittmatter, P. A., Wyatt, S. P., Sparks, W. M. 1977. *Ap. J.* 211:539

Van den Heuvel, E. P. J. 1975. *Ap. J. Lett.* 196:L121

Van Horn, H. M. 1970. *Ap. J.* 160:L53

Van Horn, H. M. 1971. In *White Dwarfs, Proc. IAU Symp. No. 42*, ed. W. J. Luyten, p. 97. Dordrecht: Reidel

Van Horn, H. M. 1979. *Physics Today* 32:23

Vauclair, G. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 165. New York: Univ. Rochester

Vauclair, G., Fontaine, G. 1979. *Ap. J.* 230:563

Vauclair, G., Reisse, C. 1977. *Astron. Astrophys.* 61:415

Vauclair, G., Vauclair, S., Greenstein, J. L. 1979. *Astron. Astrophys.* 80:79

Warner, B., Robinson, E. L. 1972. *Nature* 239:2

Wegner, G. 1974. *MNRAS* 166:271

Wegner, G. 1979a. *MNRAS* 187:17

Wegner, G. 1979b. *Astron J.* 84:1384

Wehrse, R. 1972. *Astron. Astrophys.* 19:453

Wehrse, R. 1977. *Mem. Soc. Astron. Italiana* 48:13
Wehrse, R., Liebert, J. 1980a. *Astron. Astrophys.* 83:184
Wehrse, R., Liebert, J. 1980b. *Astron. Astrophys.* In press
Weidemann, V. 1967. *Z. Astrophys.* 67:286
Weidemann, V. 1968. *Ann. Rev. Astron. Astrophys.* 6:351
Weidemann, V. 1975. In *Problems in Stellar Atmospheres and Envelopes*, ed. B. Baschek, W. Kegel, G. Traving, p. 173. New York-Berlin: Springer
Weidemann, V. 1977a. *Astron. Astrophys.* 59:411
Weidemann, V. 1977b. *Astron. Astrophys.* 61:L27
Weidemann, V., Koester, D. 1980. Submitted to *Astron. Astrophys.*
Wesemael, F. 1978. *Astron. Astrophys.* 65:301
Wesemael, F. 1979. *Astron. Astrophys.* 72:104
Wesemael, F., Van Horn, H. M. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 130. New York: Univ. Rochester
Wesselius, P., Koester, D. 1978. *Astron. Astrophys.* 70:745
Wickramasinghe, D. T. 1979. In *White Dwarfs and Variable Degenerate Stars, IAU Colloq. No. 53*, ed. H. M. Van Horn, V. Weidemann, p. 35. New York: Univ. Rochester
Wickramasinghe, D. T., Bessell, M. S., Cottrell, P. L. 1977. *Ap. J. Lett.* 217:L65
Wickramasinghe, D. T., Bessell, M. S. 1979. *MNRAS* 186:399
Wickramasinghe, D. T., Strittmatter, P. A. 1970. *MNRAS* 147:123
Wood, P. R., Cahn, J. H. 1977. *Ap. J.* 211:499
Wray, J. D., Parsons, S. B., Henize, K. G. 1979. *Ap. J. Lett.* 234:L187

Annual Review of Astronomy and Astrophysics
Volume 18, 1980

CONTENTS

INDEXES