

Mean Flow Steepening in Internal Gravity Wave Breaking

Group Meeting Presentation

Yubo Su

Nov 30, 2018

Background

Problem Setup

- 2D plane parallel atmosphere, continuous train IGW from below.
- Equations to filter sound waves (actually surprisingly contentious):

$$\vec{\nabla} \cdot \vec{u} = 0, \quad (1)$$

$$\begin{aligned} \frac{\partial \rho_1}{\partial t} - \frac{u_{1z} \rho_0}{H} + \vec{u} \cdot \vec{\nabla} \rho_1 \\ = [-\Gamma(z) + \mathcal{D}] \rho_1 + \left\{ F e^{-\frac{(z-z_0)^2}{2\sigma^2}} \cos(k_x x - \omega t) \right\}, \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} + \frac{\vec{\nabla} P_1}{\rho_0} + \frac{\rho_1 \vec{g}}{\rho_0} - \frac{\vec{\nabla} P_1}{\rho_0} \rho_1 \\ = [-\Gamma(z) + \mathcal{D}] \vec{u} \end{aligned} \quad (3)$$

- $\Gamma(z) \propto 2 + \tanh\left(\frac{z-z_H}{w}\right) + \tanh\left(\frac{z_b-z}{w}\right)$, $\mathcal{D} \sim \nabla^k$. (\square numerical, $\{\}$ forcing).
- If $\frac{d \ln \rho_0}{dz} = H$ scale height \gg domain of simulation, can use $\rho_0(z) = \rho_0$, *Boussinesq approximation*. Else, \sim *anelastic*.

Background

Digression: Brunt-Väisälä Frequency and IGW

- The above system is a strange mix of the incompressible/anelastic approximations!
- In the stars, $N^2 = g^2 \left(\frac{d\rho_0}{dP_0} - \frac{1}{c_s^2} \right)$ where $c_s^2 \rightarrow \infty$ is the sound speed.
- In atmospheric sciences, $\frac{d\rho_0}{dP_0} = \frac{1}{gH} = \frac{1}{(282 \text{ m/s})^2} \sim \frac{1}{c_s^2}!$
- Introduce entropy: $S = c_v \ln P - c_p \ln \rho$. If $c_p \gg c_v$, equivalent to $c_s^2 \rightarrow \infty, \Delta\rho = 0$.
If $c_p \sim c_v \gg 1$ however, $\Delta S = 0$ instead for atmospheric *anelastic* approximation.
- In our system:
 - $gH \ll c_s^2$ and so indeed we may use the equations on the previous slide rather than the atmospheric anelastic equations.
 - Our equations do not conserve horizontal momentum ρu_x exactly, but do so up to second order in perturbation quantities, and mean flow interactions are a second order effect.

- Perturbation (\vec{u}, ρ_1, P_1) carries average horizontal momentum flux $\langle F_{p,x} \rangle_x = \langle \rho u_x u_z \rangle$.
- Induces mean flow

$$\langle u_x \rangle_x \equiv \bar{U}_x(z) \neq 0 = \frac{\langle u_x u_z \rangle_x}{c_{g,z}}. \quad (4)$$

- Critical layer (equivalent to corotation resonance in other systems): where Doppler-shifted frequency (in fluid rest frame) $\omega \Rightarrow \omega - k_x \bar{U}_x = 0$.
- Since $\vec{u} \propto e^{z/2H}$, so $\bar{U}_x \propto e^{z/H}$, $\exists z_c : \omega - k_x \bar{U}_x(z_c) = 0$.

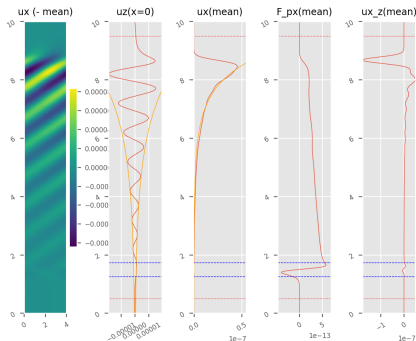
- Since critical layers almost always induce full absorption (recall, $\propto \exp\left[-\pi\sqrt{\text{Ri}^2 - \frac{1}{4}}\right]$, $\text{Ri} = \frac{N}{\bar{U}'_x}$), hypothesis:
 - \bar{u}_1 is excited, induces \bar{U}_x mean flow.
 - Once \bar{U}_x satisfies critical layer criterion, $F_{p,x}$ is fully absorbed.
 - Horizontal momentum goes into spinning up more fluid up to $\bar{U}_{x,crit} = \frac{\omega}{k_x}$.
 - Thus, critical layer should propagate down.
- Exactly *quasi-biennial oscillation* theory (Lindzen 1980, 1982), assume critical layer breaks down eventually.
- Already different from naive “goes nonlinear and deposits locally” theory!

Simulations

Large Anelastic, Low-Amplitude

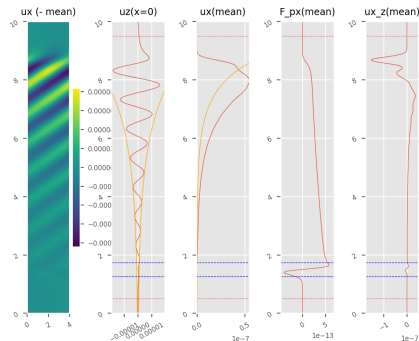
- Permit $z \in [0H, 10H]$, full $\rho_0 \propto e^{-z/H}$, allow $\vec{u} \propto e^{z/2H}$ to source growing mean flow.
- Low-Amplitude ($k_z \xi_z \ll 1$ everywhere). Orange = analytical solution.

Config: linear1 (t=300.00, kx(1.57, -6.28), w=0.24, v_gz=-0.04)



(a) Early Low-A

Config: linear1 (t=816.50, kx(1.57, -6.28), w=0.24, v_gz=-0.04)



(b) Later Low-A

Simulations

Large Anelastic, Low-Amplitude

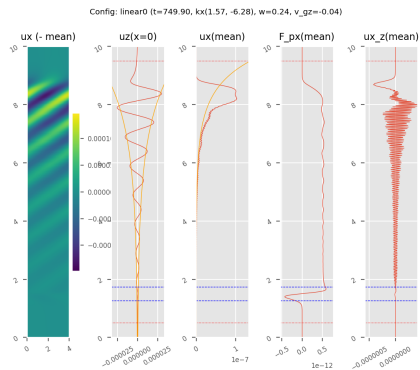


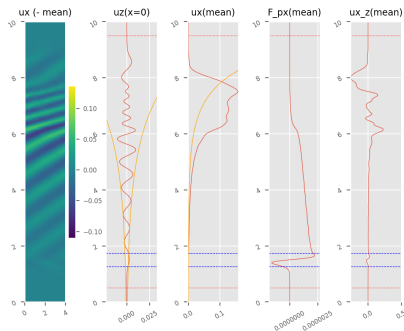
Figure: Low-Amplitude, nearly zero viscosity.

Simulations

Large Anelastic, High-Amplitude

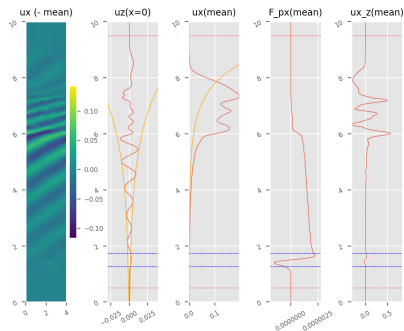
- Note steepening region ($N = 1$, so $\frac{\partial \bar{U}_x}{\partial z} = \text{Ri}^{-1}$).

Config: nl1 (t=483.32, kx(1.57, -6.28), w=0.24, v_gz=-0.04)



(a) Lower-res.

Config: nl_full (t=483.25, kx(1.57, -6.28), w=0.24, v_gz=-0.04)

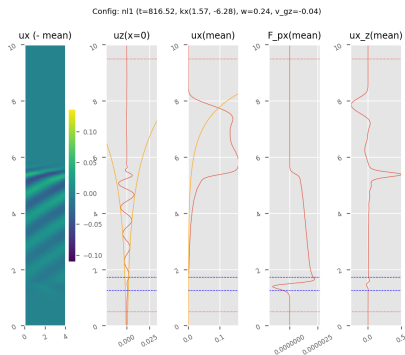


(b) Double N_z .

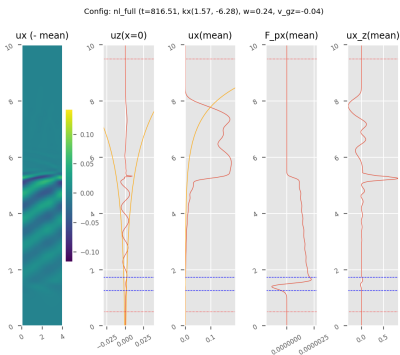
Simulations

Large Anelastic, High-Amplitude

- Later



(a) Lower-res.



(b) Double N_z .

Simulations

Large Anelastic, Comments

- While damping layers pump up \bar{U}_x , ran on higher amplitude, damping layer is not chief cause of \bar{U}_x critical layer behavior.
- Would expect reflections due to $\text{Ri} \gtrsim 1/2$, different from hypothesis!

- No reflections seen, could be *viscously limited*?

- Indeed, $\text{Ri}^{-1} \sim |\vec{k}|d$ where d is the width of the spinup layer, then

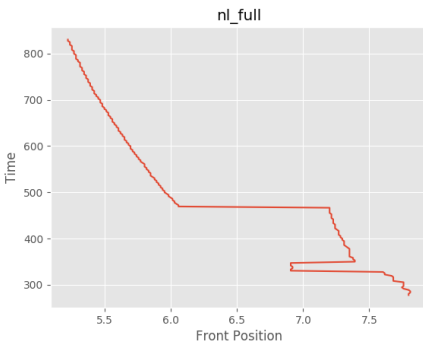
$$\nu \rho_0 \frac{\bar{U}_{x,c}}{d^2} \sim \frac{F_{p,x}}{d}.$$

- Separately, matching $F_{p,x} = \langle \rho_0 u_x u_z \rangle$ with spinning up mass $F_{p,x} = u_{front} \rho_0 \bar{U}_{x,crit}$ lets us predict u_{front} , next page.

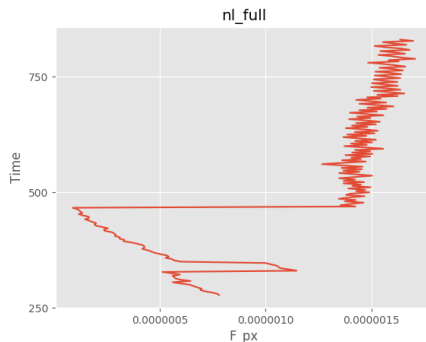
Simulations

Large Anelastic, Predicting u_{front}

- Front position is $z_f = \operatorname{argmax}_z \frac{d\bar{U}_x}{dz}$, while $F_{p,x} = 2F_{p,x}(z_f)$ (\sim halfway in front).
- Predicts $u_{front} = \frac{F_{p,x}}{\rho_0(z)\bar{U}_{x,c}} \approx 2.2 \times 10^{-3} NH$ (using $z \approx 5.5H$), or $2H/3$ in $300N$. Pretty close, seems to imply perfect absorption.



(a) Front Position. Slight exponential.

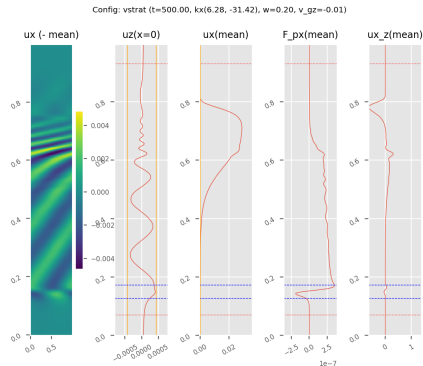
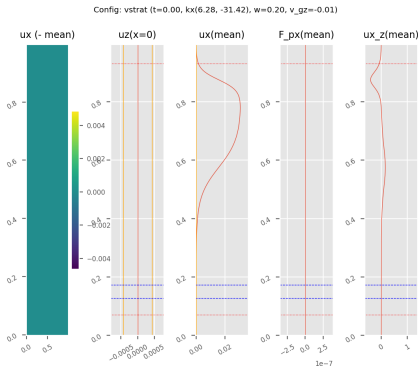


(b) $F_{p,x} = \langle \rho_0 u_x u_z \rangle_x$.

Simulations

Local Boussinesq

- Go to Boussinesq, “zoom in.” Use $\mathfrak{D} = \nabla^6$ regularization. Set up initial mean flow $\bar{U}_x(z)$ such that $\max_z \bar{U}_x(z) = \bar{U}_{x,c}$.
- Reflection indeed develops!

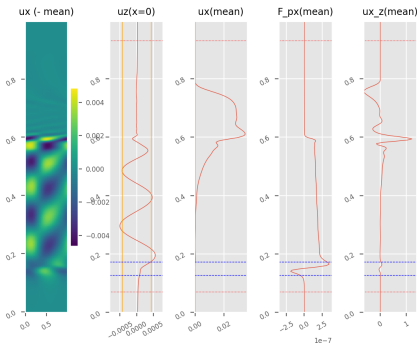


Simulations

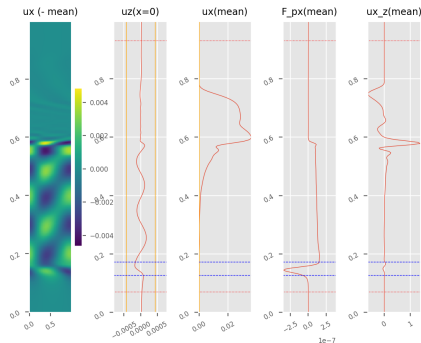
Local Boussinesq

- Expect Kelvin-Helmholtz instability to develop; resolution-limit? Tried with random noise, no qualitative difference.
- Not viscosity-limited: $\nu^{(6)} \rho_0 \frac{\bar{U}_{x,c}}{d^6} \ll \frac{F_{p,x}}{d}$!

Config: vstrat (t=2000.00, kx(6.28, -31.42), w=0.20, v_gz=-0.01)



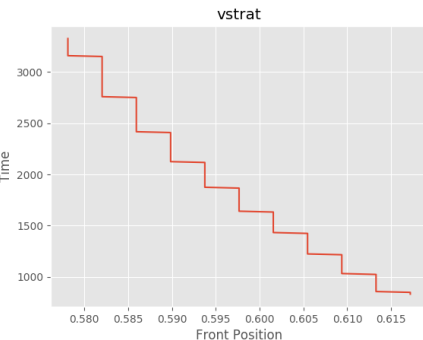
Config: vstrat (t=3249.90, kx(6.28, -31.42), w=0.20, v_gz=-0.01)



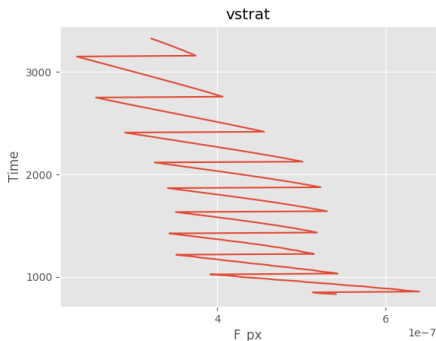
Simulations

Comparing u_{front}

- Same as for anelastic, but predicts $u_{front} \approx \frac{4.5 \times 10^{-7}}{\rho_0 \bar{U}_{x,c}} \approx 1.4 \times 10^{-5} HN$ or $0.014H$ in $1000N$. $t \in [1000, 2000]$ accurate, $t \in [2000, 3000]$ less so.
- No front slowdown is expected since ρ_0 is constant in space, but is observed; could be explained by increasing reflectivity.



(a) Front Position. Slowing down.



(b) $F_{p,x}$.