

# Mean Flow Steepening in Internal Gravity Wave Breaking

Group Meeting Presentation

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# Background

## Problem Setup

- 2D plane parallel atmosphere, continuous train IGW from below.
- Equations to filter sound waves (actually surprisingly contentious):

$$\vec{\nabla} \cdot \vec{u} = 0, \quad (1)$$

$$\begin{aligned} \frac{\partial \rho_1}{\partial t} - \frac{u_{1z} \rho_0}{H} + \vec{u} \cdot \vec{\nabla} \rho_1 \\ = [-\Gamma(z) + \mathcal{D}] \rho_1 + \left\{ F e^{-\frac{(z-z_0)^2}{2\sigma^2}} \cos(k_x x - \omega t) \right\}, \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} + \frac{\vec{\nabla} P_1}{\rho_0} + \frac{\rho_1 \vec{g}}{\rho_0} - \frac{\vec{\nabla} P_1}{\rho_0} \rho_1 \\ = [-\Gamma(z) + \mathcal{D}] \vec{u} \end{aligned} \quad (3)$$

- $\Gamma(z) \propto 2 + \tanh\left(\frac{z-z_H}{w}\right) + \tanh\left(\frac{z_b-z}{w}\right)$ ,  $\mathcal{D} \sim \nabla^k$ . ( $[\ ]$  numerical,  $\{ \}$  forcing).
- If  $\frac{d \ln \rho_0}{dz} = H$  scale height  $\gg$  domain of simulation, can use  $\rho_0(z) = \rho_0$ , *Boussinesq approximation*. Else,  $\sim$  *anelastic*.

- Perturbation  $(\vec{u}, \rho_1, P_1)$  carries average horizontal momentum flux  $\langle F_{p,x} \rangle_x = \langle \rho u_x u_z \rangle$ .
- Induces mean flow

$$\langle u_x \rangle_x \equiv \bar{U}_x(z) \neq 0 = \frac{\langle u_x u_z \rangle_x}{c_{g,z}}. \quad (4)$$

- Critical layer (equivalent to corotation resonance in other systems): where Doppler-shifted frequency (in fluid rest frame)  $\omega \Rightarrow \omega - k_x \bar{U}_x = 0$ .
- Since  $\vec{u} \propto e^{z/2H}$ , so  $\bar{U}_x \propto e^{z/H}$ ,  $\exists z_c : \omega - k_x \bar{U}_x(z_c) = 0$ .

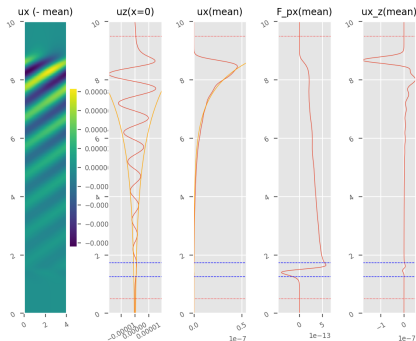
- Since critical layers almost always induce full absorption (recall,  $\propto \exp\left[-\pi\sqrt{\text{Ri}^2 - \frac{1}{4}}\right]$ ,  $\text{Ri} = \frac{N}{\bar{U}'_x}$ ), hypothesis:
  - $\bar{u}_1$  is excited, induces  $\bar{U}_x$  mean flow.
  - Once  $\bar{U}_x$  satisfies critical layer criterion,  $F_{p,x}$  is fully absorbed.
  - Horizontal momentum goes into spinning up more fluid up to  $\bar{U}_{x,crit} = \frac{\omega}{k_x}$ .
  - Thus, critical layer should propagate down.
- Exactly *quasi-biennial oscillation* theory (Lindzen 1980, 1982), assume critical layer breaks down eventually.
- Already different from naive “goes nonlinear and deposits locally” theory!

# Simulations

## Large Anelastic, Low-Amplitude

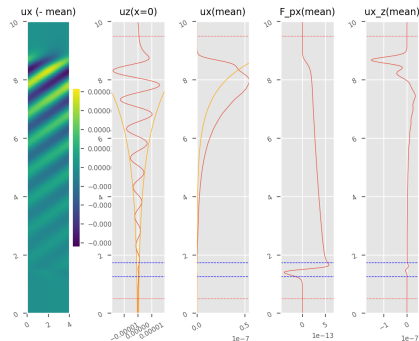
- Permit  $z \in [0H, 10H]$ , full  $\rho_0 \propto e^{-z/H}$ , allow  $\vec{u} \propto e^{z/2H}$  to source growing mean flow.
- Low-Amplitude ( $k_z \xi_z \ll 1$  everywhere). Orange = analytical solution.

Config: linear1 (t=300.00, kx(1.57, -6.28), w=0.24, v\_gz=-0.04)



(a) Early Low-A

Config: linear1 (t=816.50, kx(1.57, -6.28), w=0.24, v\_gz=-0.04)



(b) Later Low-A

# Simulations

## Large Anelastic, Low-Amplitude

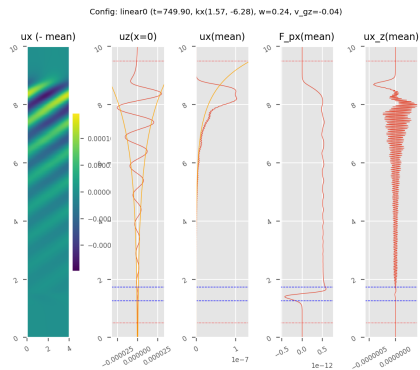


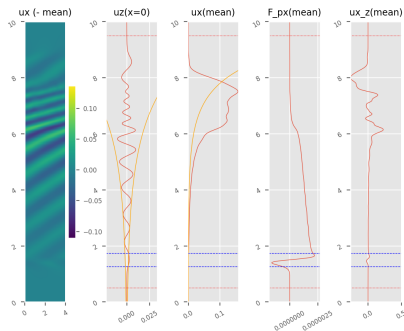
Figure: Low-Amplitude, nearly zero viscosity.

# Simulations

## Large Anelastic, High-Amplitude

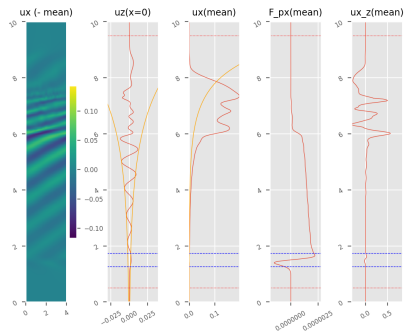
- Note steepening region ( $N = 1$ , so  $\frac{\partial \bar{U}_x}{\partial z} = \text{Ri}^{-1}$ ).

Config: nl1 (t=483.32, kx(1.57, -6.28), w=0.24, v\_gz=-0.04)



(a) Lower-res.

Config: nl\_full (t=483.25, kx(1.57, -6.28), w=0.24, v\_gz=-0.04)

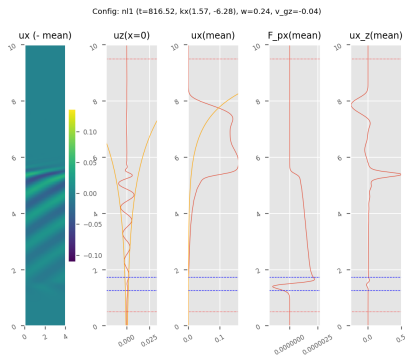


(b) Double  $N_z$ .

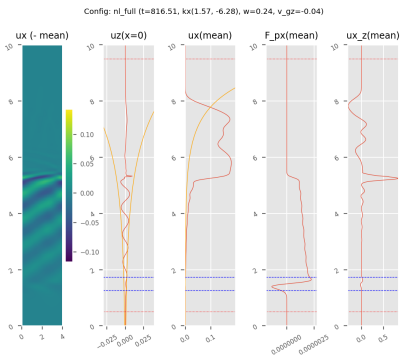
# Simulations

## Large Anelastic, High-Amplitude

- Later



(a) Lower-res.



(b) Double  $N_z$ .



# Simulations

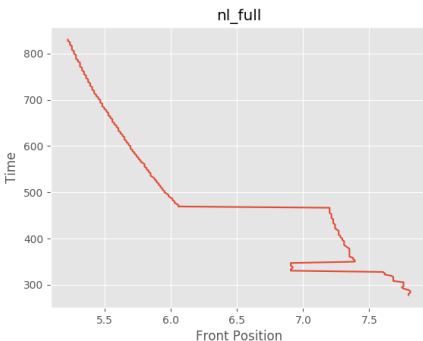
## Large Anelastic, Comments

- While damping layers pump up  $\bar{U}_x$ , ran on higher amplitude, damping layer is not chief cause of  $\bar{U}_x$  critical layer behavior.
- Would expect reflections, but *viscously limited*?
  - Indeed,  $\text{Ri}^{-1} \sim |\vec{k}|d$  where  $d$  is the width of the spinup layer, then
$$\nu\rho_0\frac{\bar{U}_{x,c}}{d^2} \sim \frac{F_{p,x}}{d}.$$
- Matching  $F_{p,x} = \langle \rho_0 u_x u_z \rangle$  with spinning up mass  $F_{p,x} = u_{front}\rho_0\bar{U}_{x,crit}$  lets us predict  $u_{front}$ , next page.

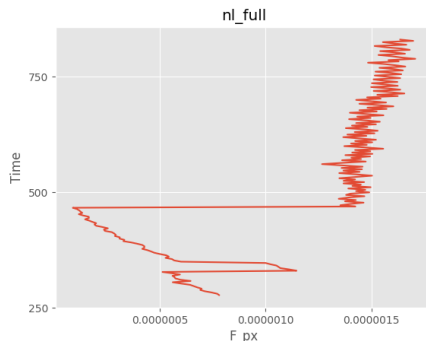
# Simulations

Large Anelastic, Predicting  $u_{front}$

- Front position is  $z_f = \operatorname{argmax}_z \frac{d\bar{U}_x}{dz}$ , while  $F_{p,x} = 2F_{p,x}(z_f)$  ( $\sim$  halfway in front).
- Predicts  $u_{front} = \frac{F_{p,x}}{\rho_0(z)\bar{U}_{x,c}} \approx 2.2 \times 10^{-3} NH$  (using  $z \approx 5.5H$ ), or  $2H/3$  in  $300N$ . Pretty close, seems to imply perfect absorption.



(a) Front Position. Slight exponential.

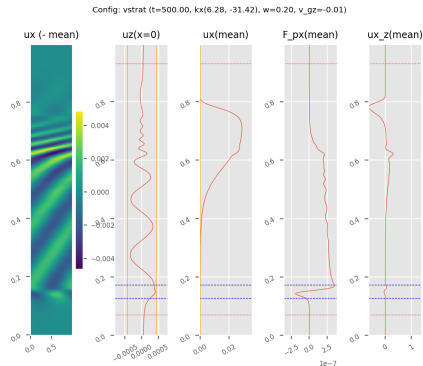
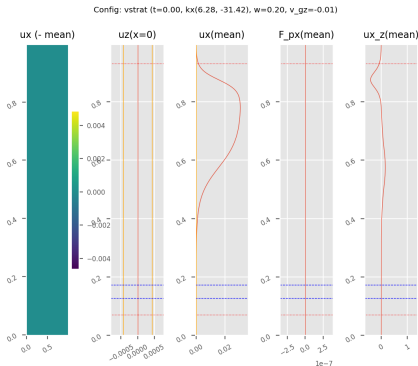


(b)  $F_{p,x} = \langle \rho_0 u_x u_z \rangle_x$ .

# Simulations

## Local Boussinesq

- Go to Boussinesq, “zoom in.” Use  $\mathfrak{D} = \nabla^6$  regularization. Set up initial mean flow  $\bar{U}_x(z)$  such that  $\max_z \bar{U}_x(z) = \bar{U}_{x,c}$ .
- Reflection indeed develops!

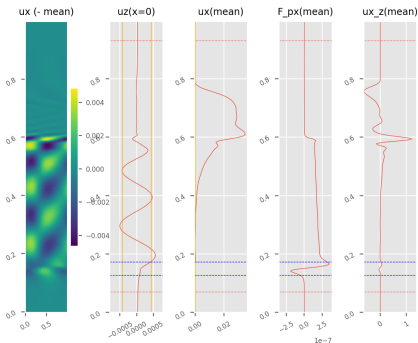


# Simulations

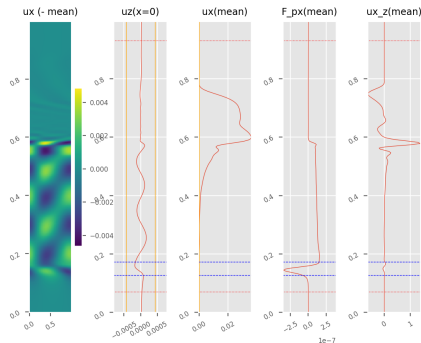
## Local Boussinesq

- Expect Kelvin-Helmholtz instability to develop; resolution-limit? Tried with random noise, no qualitative difference.
- Not viscosity-limited:  $\nu^{(6)} \rho_0 \frac{\bar{U}_{x,c}}{d^6} \ll \frac{F_{p,x}}{d}$ !

Config: vstrat (t=2000.00, kx(6.28, -31.42), w=0.20, v\_gz=-0.01)



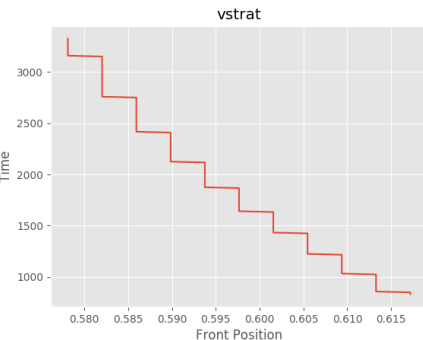
Config: vstrat (t=3249.90, kx(6.28, -31.42), w=0.20, v\_gz=-0.01)



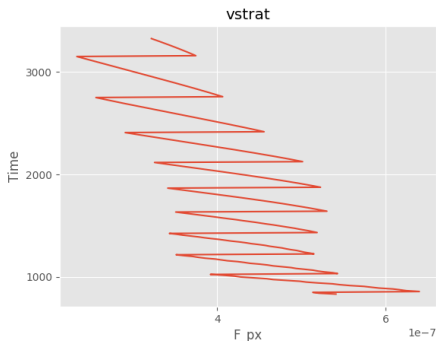
# Simulations

## Comparing $u_{front}$

- Same as for anelastic, but predicts  $u_{front} \approx \frac{4.5 \times 10^{-7}}{\rho_0 \bar{U}_{x,c}} \approx 1.4 \times 10^{-5} HN$  or  $0.014H$  in  $1000N$ .  $t \in [1000, 2000]$  accurate,  $t \in [2000, 3000]$  less so.
- No front slowdown is expected since  $\rho_0$  is constant in space, but is observed; could be explained by increasing reflectivity.



(a) Front Position. Slowing down.



(b)  $F_{p,x}$ .