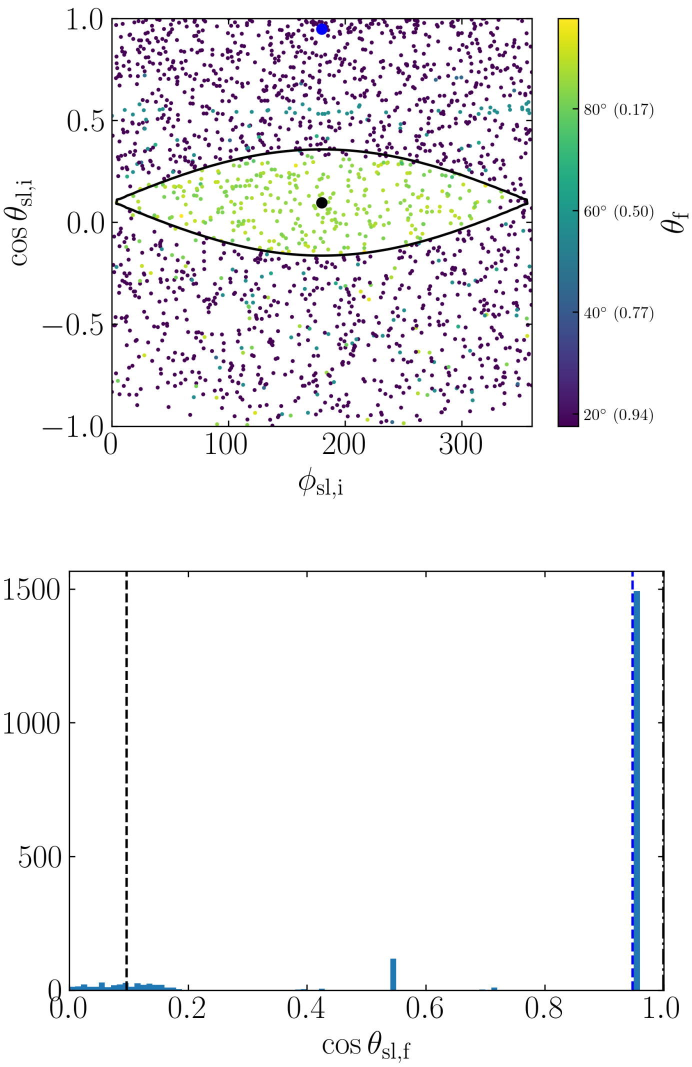

Figure 1: Outcomes and histogram.

1 3-planet CS

I have a few new plots since the group meeting presentation Friday. I primarily just added a few other choices of resonant angles and changed the coordinate system to have $\hat{\mathbf{l}} \propto \hat{\mathbf{z}}$. I focused on the case where $\alpha = 10g_1$, $g_1 = 10g_2$, $I_1 = 10^\circ$, and $I_2 = 1^\circ$, for which the outcomes and histogram are given in Fig. 1. This results in Fig. 2. I also have a few sample plots like this for different g_2 values and different I_2 values.

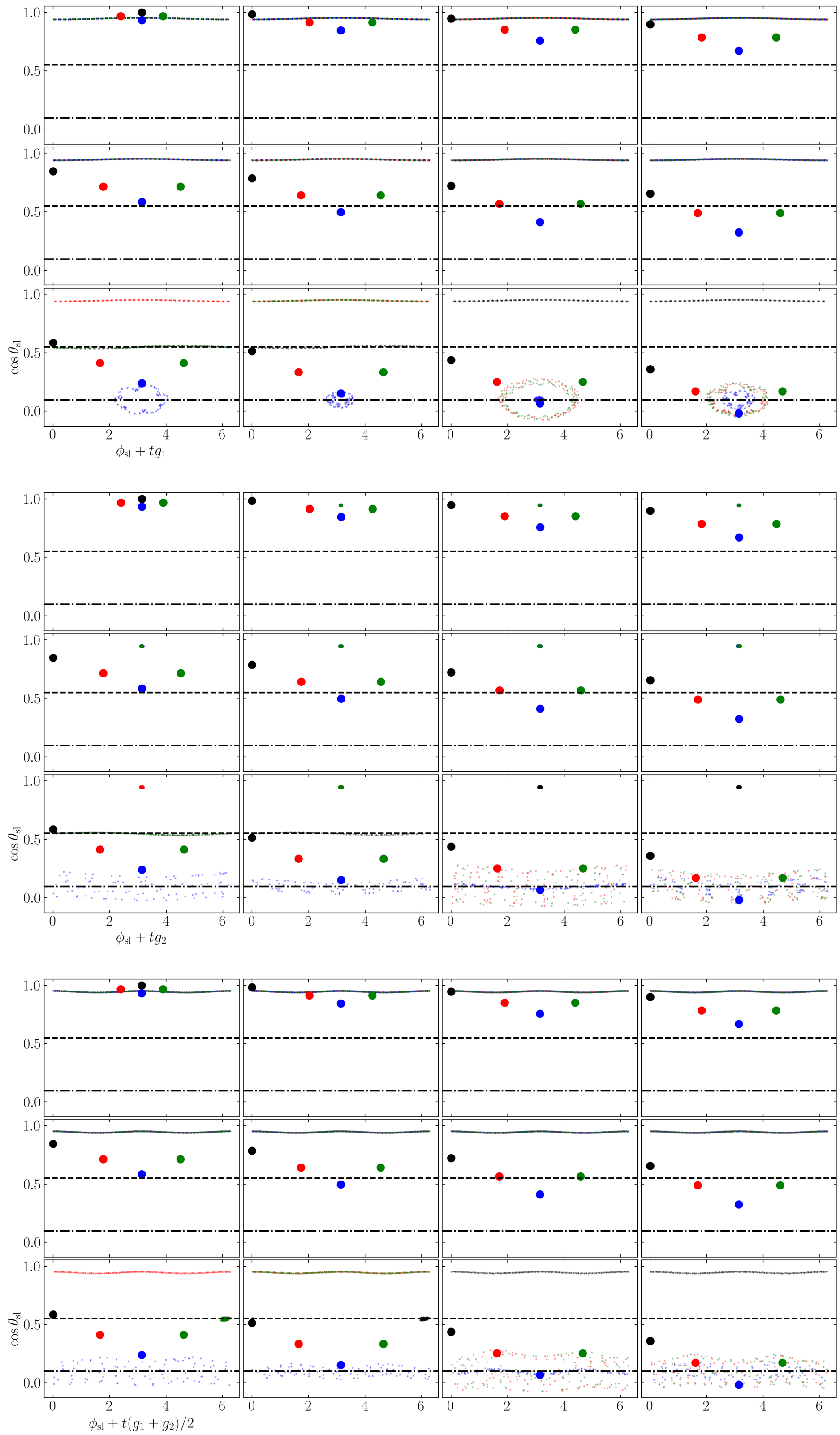

Figure 2: Plot of ICs and final cycles for a variety of initial conditions. The histogram in Fig. 1 suggests that all ICs go to CS2 of mode 2, rather than CS1 of mode 1, and this is confirmed by these trajectories.

Regarding analytical work, I tried to do some work in the frame where $\hat{\mathbf{J}} \propto \hat{\mathbf{z}}$, i.e.

$$\left(\frac{\mathrm{d}\hat{\mathbf{s}}}{\mathrm{d}t}\right)_{\mathrm{rot}} = \alpha\left(\hat{\mathbf{s}}\cdot\hat{\mathbf{l}}\right)\left(\hat{\mathbf{s}}\times\hat{\mathbf{l}}\right) - \bar{g}\left(\hat{\mathbf{s}}\times\hat{\mathbf{z}}\right), \tag{1}$$

$$\hat{\mathbf{l}}(t) = \begin{bmatrix} (I_1 + I_2)\cos\left(\frac{\Delta g t - \phi_0}{2}\right) \\ (I_1 - I_2)\sin\left(\frac{\Delta g t - \phi_0}{2}\right) \\ 1 \end{bmatrix} + \mathcal{O}\left[(I_1 + I_2)^2\right]. \tag{2}$$

but it's quite hard to enforce $\left(\hat{\mathbf{s}}\cdot\hat{\mathbf{l}}\right)$ to be fixed. I will think about what you proposed as well...

2 BH Spin Orbit Attractor

I did some more investigation, and I realized that I wasn't computing the angle correctly: the "attractor" is really defined relative to the changing $\hat{\mathbf{l}}$ vector, and I forgot that $\hat{\mathbf{l}}$ also precesses. When computing the updated angle, using both the old (wrong) and new (Racine 2008) equations, I reproduce the following behavior:

- When $\theta_{s_1 l} = 10°$, $\mathbf{s}_1\cdot\mathbf{s}_2 \to 1$, i.e. the spins align. This causes $\Delta\Phi = 0$. When $\theta_{s_2 l} = 170°$, $\mathbf{s}_1\cdot\mathbf{s}_2 \to -1$, causing the spins to anti-align generally.

- When the masses are equal, $\mathbf{s}_1\cdot\mathbf{s}_2$ is *constant*. So I can't do any equal-mass analysis, unfortunately.

I show this in

This dot product obeys:

$$\frac{\mathrm{d}}{\mathrm{d}\psi}\left(\hat{\mathbf{s}}_1\cdot\hat{\mathbf{s}}_2\right) = \underbrace{\frac{3}{7 - 3\lambda/2}\left[\frac{m_2}{m_1} - \frac{m_1}{m_2} - \lambda\mu\left(\frac{1}{m_1} - \frac{1}{m_2}\right)\right]}_{\beta_-}\hat{\mathbf{J}}\cdot\left(\hat{\mathbf{s}}_1\times\hat{\mathbf{s}}_2\right). \tag{3}$$

Note that this vanishes when the masses are equal, and indeed, numerically I can verify that the $\Delta\Phi$ equilibria disappear! At first glance, this should average out over a precession cycle, so the origin of this attracting behavior is indeed somewhat mysterious. However, the signature and strength of the attractor, the different phenomenology in the retrograde/prograde cases, are both interesting.

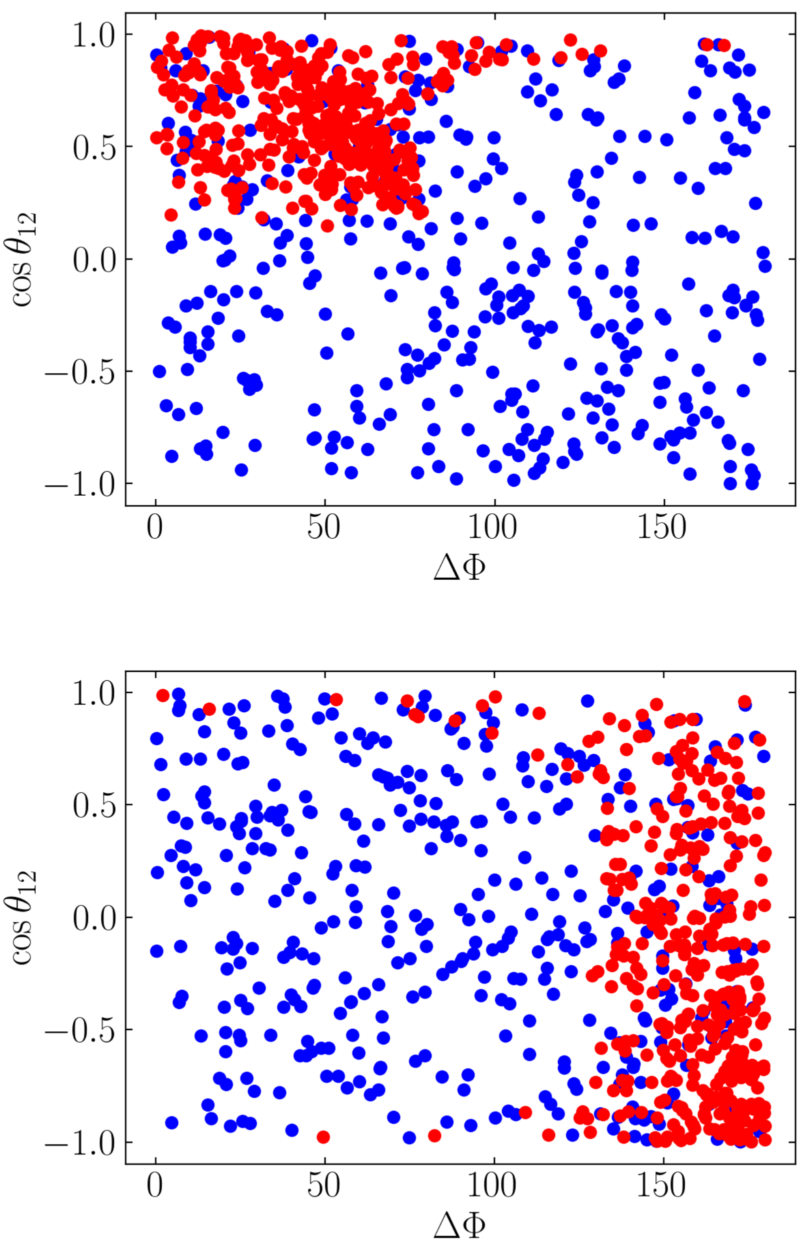

Figure 3: Initial (blue) and final (red) values of $\theta_{12} \equiv \arccos(\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{s}}_2)$ and $\Delta\Phi$ (relative to $\hat{\mathbf{l}}$) for the prograde and retrograde $\hat{\mathbf{s}}_1$ case. Here, $m_1 = 0.55M_\odot$ and $m_2 = 0.45M_\odot$.