
Call $P(q; a_{\text{out,eff}})$ the merger probability as a function of q . Questions to consider at this point:

- (1) For what q is the $P(q)$ maximized? ($P(0) = 0$)
- (2) What does $P(q)$ look like with fewer bins (less noise)?
- (3) What does $P(q)$ look like if e_{out} is thermal?
- (4) Does $P(a_{\text{out,eff}})$ vanish at the same place as LL18's analytic expression (in the quadrupole limit)?
- (5) LIGO/LISA band eccentricities?
- (6) What is the primordial q distribution in high-mass binaries?

These are all relatively short, I address them in kind:

1 Maximum $P(q)$

This happens at very small q because e_{os} is a very insensitive function of q , (i.e.. $j(e_{\text{os}})^6 \sim t_{\text{LK}}/t_{\text{GW},0} \propto 1/\mu$). I derived before (probably with some mistakes)

$$j^6(e_{\text{os}}) \equiv j_{\text{os}} = \frac{256}{5} \frac{G^3 \mu m_{12}^2}{c^5 a^4} \frac{1}{n} \frac{m_{12}}{m_3} \left(\frac{a_{\text{out,eff}}}{a} \right)^3, \quad (1)$$

$$= \frac{256}{5} \frac{G^3 \mu m_{12}^3}{m_3 c^5 a^4 n} \left(\frac{a_{\text{out,eff}}}{a} \right)^3. \quad (2)$$

Approximating $j_{\text{lim}} \approx \frac{8\epsilon_{\text{GR}}}{9+3\eta^2/4}$, we obtain the a criterion for one-shot mergers

$$256 \frac{G^{5/2} a_{\text{out,eff}}^3 m_{12}^{5/2} \mu}{5 a^{11/2} c^5 m_3} \gtrsim \left(\frac{8}{9+3\eta^2/4} \frac{3G m_{12}^2 a_{\text{out,eff}}^3}{c^2 a^4 m_3} \right)^6, \quad (3)$$

$$a^{37/2} \gtrsim 5 \cdot 1024 \frac{G^{7/2} a_{\text{out,eff}}^{15} m_{12}^{19/2}}{c^7 m_3^5 \mu (3+\eta^2/4)^6}, \quad (4)$$

$$\left(\frac{a}{a_{\text{out,eff}}} \right) \gtrsim 0.0118 \left(\frac{a_{\text{out,eff}}}{3600 \text{ AU}} \right)^{-7/37} \left(\frac{m_{12}}{50 M_{\odot}} \right)^{17/37} \left(\frac{30 M_{\odot}}{m_3} \right)^{10/37} \left(\frac{q/(1+q)^2}{1/4} \right)^{-2/37}. \quad (5)$$

Note that $\mu/m_{12} = m_1 m_2 / m_{12}^2 = q/(1+q)^2$. Thus, if a is fixed, q must decrease dramatically to cause the system to transition from $e_{\text{lim}} \gtrsim e_{\text{os}}$ to $e_{\text{lim}} \lesssim e_{\text{os}}$. For instance, even going down to $q = 10^{-5}$ produces very little effect as seen in Fig. 1 (very coarse sampling).

2 $P(q)$ with fewer bins & Thermal Distribution & Merger Eccentricity

There was some computer downtime, but I was able to get one $P(q)$ plot with fewer bins, see Fig. 2. I also include the distributions if the distribution of e_2 is thermal in red dots. I also tried using the

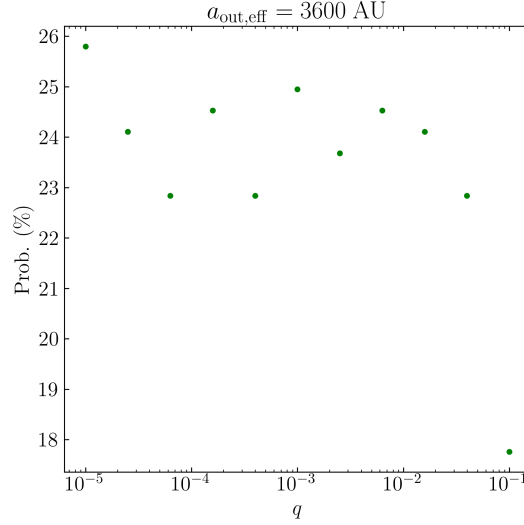


Figure 1: Merger fraction at low q . Indeed, little effect.

GW-only simulations to uniformly scan a grid of (I_0, e_{out}) for each q , using a much denser grid of q , shown in the blue line. Finally, the eccentricities in the LISA/LIGO band for merging binaries is shown.

For merger eccentricity, I just used the Peters formulas. I defined LISA/LIGO to be $f = 0.1, 10$ Hz respectively, where $f = n/\pi = \sqrt{Gm_{12}/a_{\text{in}}^3}/\pi$.

3 Comparison of Merger Fraction to LL18

Last meeting, we mentioned that a figure like Fig. 8 of LL18 is not useful because it requires assuming distributions on q and e_{out} . However, if we compare the LL18 criterion to our “effective eccentricity” GW-less merger criterion, we notice that their criterion, given by

$$T_{\text{m},0} j^6(e_{\text{m}}) = T_{\text{crit}}, \quad (6)$$

is equivalent to requiring $j(e_{\text{eff}}) \lesssim j_{\text{eff,crit}}$, where $j_{\text{eff,crit}}$ is the effective maximum eccentricity of the LK cycle (equal to the quadrupole e_{max} when octupole effects are small). I haven’t checked this against data.

4 Primordial q distribution in HM binaries

See <https://arxiv.org/pdf/1606.05347.pdf>, in particular Figs. 2, 8, and 11 (NB: Salpeter has $\gamma = -2.35$). Notably, - Abstract - The intrinsic population of binaries with longer orbital periods is even further skewed toward smaller mass ratios. Our result is consistent with the conclusions of Abt et al. (1990), who also find that early B spectroscopic binaries become weighted toward smaller mass ratios with increasing separation

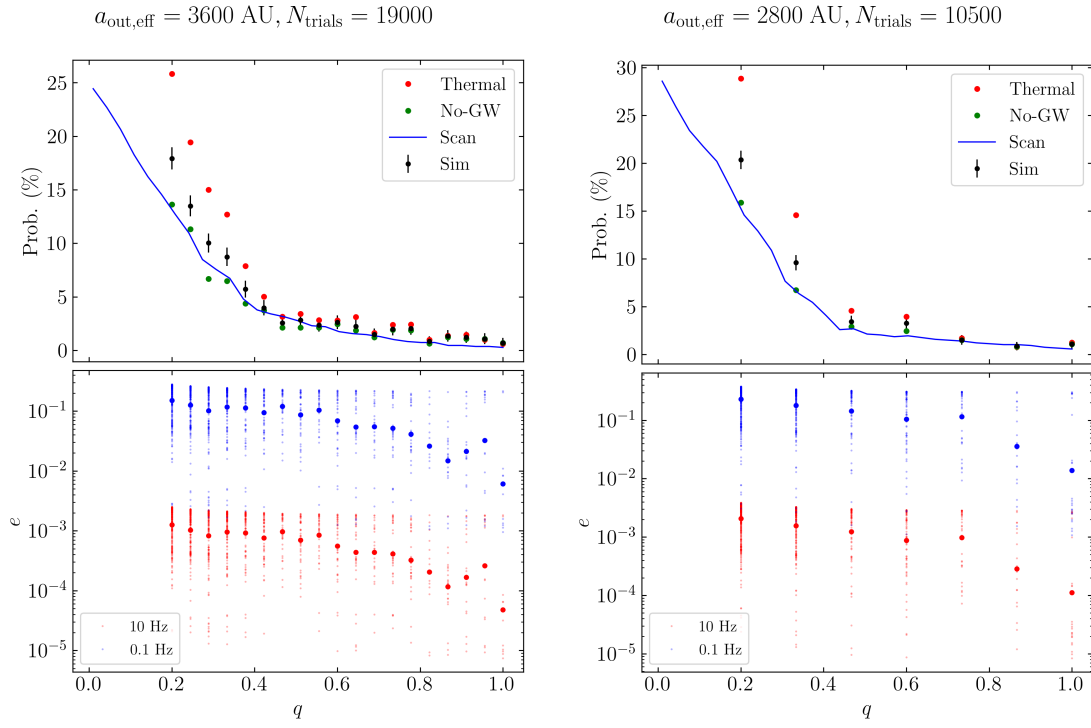


Figure 2: $P(q; a_{\text{out}} = 3600 \text{ AU})$ with 19 bins, and $P(q; a_{\text{out}} = 2800 \text{ AU})$ with 7 bins.