1 Evection Resonance Maximum Growth

If we require the evection resonance condition $\dot{\varpi} \sim \dot{f}_{\rm out}$, or

$$\frac{3Gm_{12}}{c^2 a}\sqrt{\frac{m_{12}a_{\rm out}^3}{m_{123}a^3}} \sim 1, \tag{1}$$

then this can be rewritten as

$$\frac{a^5}{a_{\rm out}^3} \sim \frac{9G^2 m_{12}^3}{c^2 m_{123}}. \tag{2}$$

The ϵ associated with the system can then be rewritten as:

$$\begin{aligned} \epsilon &= \frac{m_3 a^4 c^2}{3Gm_{12}^2 a_{\rm out}^3}, && (3)\\ &= \frac{3m_3}{m_{123}}\left(\frac{v}{c}\right)^2. && (4) \end{aligned}$$

Here, $v = \sqrt{Gm_{12}/a}$ is the orbital velocity of the inner binary. Using the above scalings, we find that $v \propto a^{-1/2} \propto a_{\rm out}^{-3/10}$, and thus $\Delta e \propto a_{\rm out}^{-3/10}$. Is this observed? Well, my $a_{\rm out} = 2.38\,\text{AU}$ has $\Delta e = 0.006$ and my $a_{\rm out} = 238\,\text{AU}$ has $\Delta e \approx 0.0015$, which is in rough agreement.

1.1 With Eccentricity

We showed in our notes that the evection Hamiltonian looks something like

$$\begin{aligned} H(\Gamma,\phi) &= P\Gamma - 4\Gamma^2 + R\cos\phi,\\ P &= 2\left[1 - \Omega_{\rm out}/\Omega_{\rm GR,0} + 3\epsilon/4\right],\\ R &= \frac{15\epsilon}{2}(1 + F_{N2}). \end{aligned} \tag{5}$$

where $\Gamma \approx -e^2/4$, and F_{N2} is the Hansen coefficient. The equilibrium of the Hamiltonian, when it exists is located at

$$\Gamma_{\rm eq} = \frac{P-R}{8} \sim \mathcal{O}(\epsilon). \tag{6}$$

Note that even if $e_{\rm out} = 0.9$, F_{N2} only maximizes at ~ 20, so the evection eccentricity cannot be enhanced by more than a factor of 4–5 realistically except for extremely strong eccentricities: to leading order, F_{N2} is maximized at $N \simeq \left(1 - e_{\rm out}^2\right)^{-3/2}$ at a value of $(1 - e_{\rm out}^2)^{-3/2}$, so we expect an enhancement of the evection resonance eccentricity excitation by $\sim \left(1 - e_{\rm out}^2\right)^{-3/4}$, not a lot.

So, basically, without some sort of exotic $2+1+1$ system, we're probably out of luck.

The problem here is that $\epsilon \sim \Phi_{\rm ZLK}/\Phi_{\rm GR}$ is too small when $\Omega_{\rm out} \sim \dot{\omega}_{\rm GR}$, i.e. the hierarchy of scales between the quadrupole ZLK coupling and the simple Keplerian coupling is too large.