1 Canonical Parameters

We use the canonical values: $M_2 = 1.4M_\odot$, $q = 6.3$, $e = 0.808$, orbital period $P = 51.17$ days, and $\dot{P} = -3.03 \times 10^{-7}$, and an inferred $8.8M_\odot$ for the massive star (giving $a = 126R_\odot$). The massive star is assumed to have core mass $3M_\odot$ and radius $1.38R_\odot$. The form of energy dissipation can be written

$$\dot{E}_{\rm in} = \hat{\tau}\Omega g(e, \Omega_{\rm s}/\Omega), \tag{1}$$

where $\hat{\tau}$ is the torque assuming a circular orbit, $\Omega_{\rm s}$ is the spin of the star, Ω is the orbital angular frequency of the orbit, and e is the eccentricity.

Then, since

$$\dot{E}_{\rm g} = \frac{GqM_2^2}{3a}\frac{\dot{P}}{P}, \tag{2}$$

we can calculate $\dot{P}$ as a function of $\Omega_{\rm s}$. Figure 1 shows the result of this calculation and compares it to the observed $\dot{P}$.

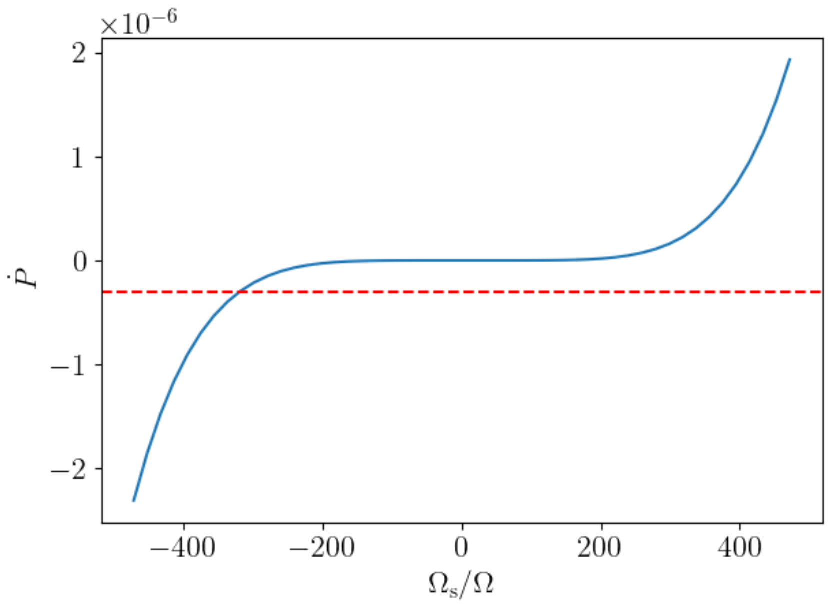

Figure 1: $\dot{P}$ as a function of $\Omega_{\rm s}$ using the canonical parameters for PSR J0045-7319.

2 MESA

I present the results of my MESA simulations here. The observations for the star that we must match are $L = 1.2 \times 10^4 L_\odot$ and $T = (2.4 \pm 0.1) \times 10^4$ K. The three sets of simulations I present here are: (i) low metallicity, non-rotating, (ii) rotating with large convective overshoot, (iii) same as (ii) but with metallicity $Z = 0.004$. These are shown in Fig. 2

It is also worth noting that this implies Zahn's formula, given in the circular case by

$$\tau = \frac{3}{2}\frac{GM_2^2R^5}{a^6}E_2\left(\frac{2\,|\Omega - \Omega_{\rm s}|}{\Omega_{\rm s,c}}\right)^{8/3}, \tag{3}$$

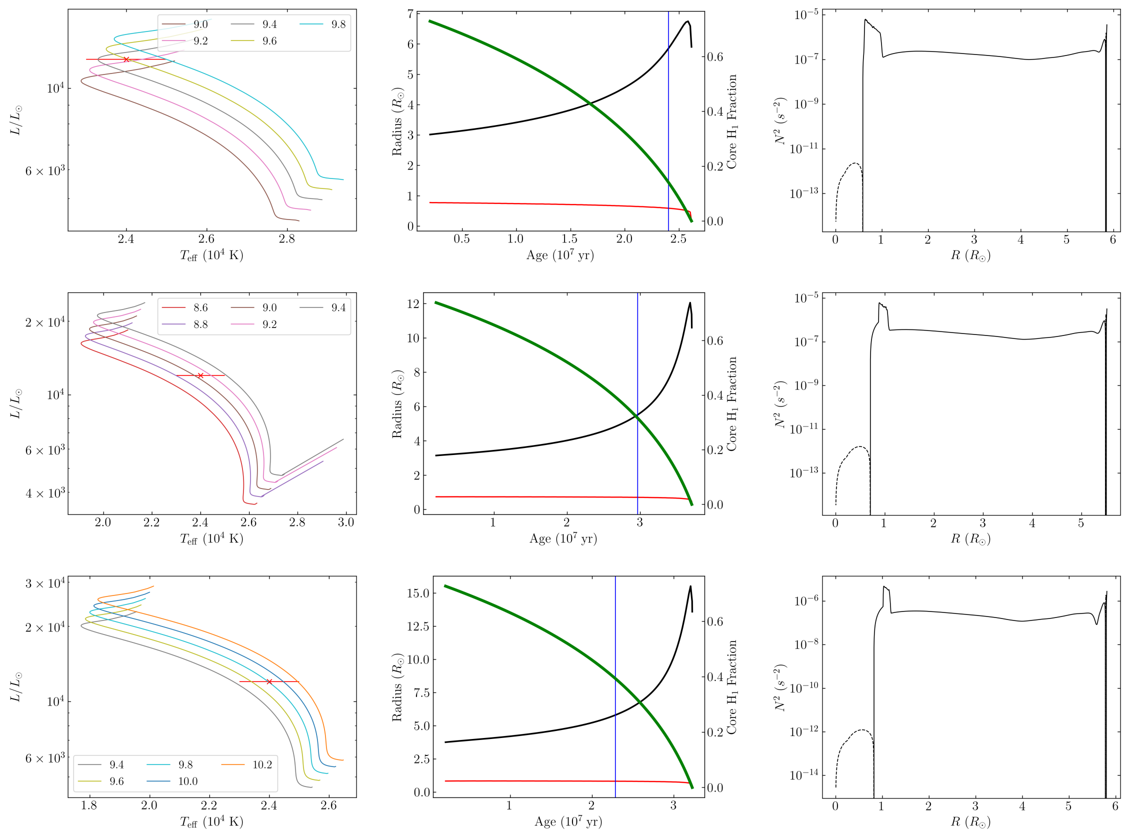

Figure 2: The three sets of simulations I present here are shown in each row: (i) low metallicity, non-rotating, (ii) rotating with large convective overshoot, (iii) same as (ii) but with metallicity $Z = 0.004$. The first column shows the L-T diagram, with which we try to match the observed parameters, the second column shows the evolution of some parameters of the best-fitting star as a function of age, and the third column shows the propagation diagrams at the time of best fit (dashed lines denote negative values). The best fitting masses are (i) $M = 9.6M_\odot$, (ii) $M = 9.0M_\odot$, and (iii) $M = 9.8M_\odot$.

(e.g. Kushnir et. al$\dot{2}$016) depends only on R and not r_c. Since $E_2 = 1.592\times10^{-9}(M/M_\odot)^{2.84}$ (Hurley et. al. 2002; Vigna-Gómez et. al. 2020), we see that this dynamical tide formula predicts that the torque increases as the star expands at constant mass. Our formula predicts that the torque remains roughly constant.

2.1 Constraint

We note that $\Omega_s \le \Omega_{s,c} \equiv \sqrt{GM_c/r_c^3}$ the breakup frequency. This lets us put a constraint on the maximum $\dot{P}$ obtainable from tides:

$$\left|\dot{P}\right| \le -\frac{6\pi}{q}\beta_2\left(\frac{r_c}{a}\right)^5\frac{\rho_c}{\bar{\rho}_c}\left(1-\frac{\rho_c}{\bar{\rho}_c}\right)^2 2^{8/3}\frac{f_2}{(1-e^2)^6}. \tag{4}$$

Here, $\beta_2 \approx 1$, ρ_c is the density at the RCB, $\bar{\rho}_c$ is the average density of the core, and $f_2 = 1+15e^2/2+45e^4/8+5e^6/16$.

If we require -3.03×10^{-7} to be within this range and take $\rho_c/\bar{\rho}_c \approx 0.74$, this requires $r_c \gtrsim 1.19R_\odot$, which is indeed satisfied by the canonical parameters. However, our simulations disagree with these parameters!

It is furthermore worth noting that P is measured, not a; this implies that a is not a constant, but instead $a = \left(GM_2(1+q)/\Omega^2\right)^{1/3}$, so

$$\left|\dot{P}\right| \le -3.03\times10^{-7}\left(\frac{r_c}{1.19R_\odot}\right)^5\left(\frac{qM_2}{8.8M_\odot}\right)^{-5/3}. \tag{5}$$

In other words, if we use larger stellar masses (like our MESA models suggest), the $\dot{P}$ constraint is even more strict!