Checking the Hamiltonian

Equation numbers refer to `evection.pdf` ,

In [1]:

```
import sympy as sp
import numpy as np

# phi = 2(\varpi - \lambda_out)
G, m1, m2, c, a, m3, aout, I, phi, nout = sp.symbols(r'G m_1 m_2 c a m_3 a_{out} I
e = sp.symbols('e', real=True, positive=True)
m12 = m1 + m2
mu12 = m1 * m2 / m12

H_gr0 = 3 * G**2 * m1 * m2 * m12 / (c**2 * a**2)
# Eq 32 / H_gr0
H = (
    -1 / sp.sqrt(1 - e**2)
    - (G * m3 * mu12 * a**2) / (aout**3 * H_gr0) * (
        sp.Rational(1, 16) * ((6 + 9 * e**2) * sp.cos(I)**2 - (2 + 3 * e**2))
        + sp.Rational(15, 32) * (1 + sp.cos(I))**2 * e**2 * sp.cos(2 * phi)
    )
)
# display(H)
```

In [2]:

```
eps = m3 * a**4 * c**2 / (3 * G * m12**2 * aout**3)
# (33)
H2 = -(
    1 / sp.sqrt(1 - e**2)
    + eps * (
        sp.Rational(1, 16) * ((6 + 9 * e**2) * sp.cos(I)**2 - 3 * e**2)
        + sp.Rational(15, 32) * (1 + sp.cos(I))**2 * e**2 * sp.cos(2 * phi)
    )
)
display(sp.simplify(H - H2))
```

$$\frac{a^4 c^2 m_3}{24 G a_{out}^3 (m_1 + m_2)^2}$$

In [3]:

```
# (38)
J2 = 1 - sp.sqrt(1 - e**2)
J3 = sp.sqrt(1 - e**2) * (1 - sp.cos(I))
A3 = (
    sp.Rational(3, 16)
        * (2 * J2 - J2**2)
        * (2 - 6 * (J3 / (1 - J2)) + 3 * ((J3 / (1 - J2)))**2)
    + sp.Rational(3, 8) * sp.cos(I)**2
)
B3 = sp.Rational(15, 32) * (2 * J2 - J2**2) * (
    4 - 4 * (J3 / (1 - J2)) + (J3 / (1 - J2))**2
)
H3 = -1 / (1 - J2) - eps * (A3 + B3 * sp.cos(2 * phi))
display(sp.simplify(H3 - H2))
```

0

In [4]:

```
# (42), without nout
Gamma = -J2 / 2
A4 = (
    sp.Rational(3, 16)
        * (-4 * Gamma - 4 * Gamma**2)
        * (2 - 6 * (J3 / (1 + 2 * Gamma))+ 3 * ((J3 / (1 + 2 * Gamma)))**2)
    + sp.Rational(3, 8) * (1 - J3 / (1 + 2 * Gamma))**2
)
B4 = sp.Rational(15, 32) * (-4 * Gamma - 4 * Gamma**2) * (
    4 - 4 * J3 / (1 + 2 * Gamma) + (J3 / (1 + 2 * Gamma))**2
)
# display(sp.simplify(A4 - A3))
# display(sp.simplify(B4 - B3))
H4 = -1 / (1 + 2 * Gamma) - eps * (A4 + B4 * sp.cos(2 * phi)) # - 2 * Gamma * nout
display(sp.simplify(H4 - H3))
```

0

In [5]:

```
# checking (49-51)

Gsymb, J3s = sp.symbols('\Gamma J_3')
# use Gamma, J3 as symbols, (1 + 2Gamma)^{-1} => (1 - 2Gamma + 4Gamma^2)
A4symb_small = (
    sp.Rational(3, 16)
        * (-4 * Gsymb - 4 * Gsymb**2)
        * (2 - 6 * (J3s * (1 - 2 * Gsymb + 4 * Gsymb**2))
           + 3 * ((J3s * (1 - 2 * Gsymb + 4 * Gsymb**2)))**2)
    + sp.Rational(3, 8) * (1 - J3s * (1 - 2 * Gsymb + 4 * Gsymb**2))**2
)
A4_zeroorder = A4symb_small.subs(Gsymb, 0)
A4_firstorder = sp.expand((A4symb_small - A4_zeroorder) / Gsymb).subs(Gsymb, 0)
A4_secondorder = sp.expand((A4symb_small - A4_zeroorder - A4_firstorder * Gsymb) /
display(sp.simplify(A4_zeroorder))
display(sp.simplify(A4_firstorder))
display(sp.simplify(A4_secondorder))

B4symb_small = sp.Rational(15, 32) * (-4 * Gsymb - 4 * Gsymb**2) * (
    4 - 4 * J3s * (1 - 2 * Gsymb + 4 * Gsymb**2) + (J3s * (1 - 2 * Gsymb +Gsymb**2)
)
B4_zeroorder = B4symb_small.subs(Gsymb, 0)
B4_firstorder = sp.expand((B4symb_small - B4_zeroorder) / Gsymb).subs(Gsymb, 0)
B4_secondorder = sp.expand((B4symb_small - B4_zeroorder - B4_firstorder * Gsymb) /
display(sp.simplify(B4_zeroorder))
display(sp.simplify(B4_firstorder))
display(sp.simplify(B4_secondorder))
```

$$\frac{3(J_3-1)^2}{8}$$

$$-\frac{15J_3^2}{4}+6J_3-\frac{3}{2}$$

$$\frac{45J_3^2}{4}-\frac{15J_3}{2}-\frac{3}{2}$$

$$0$$

$$-\frac{15J_3^2}{8}+\frac{15J_3}{2}-\frac{15}{2}$$

$$\frac{45J_3^2}{8}-\frac{15J_3}{2}-\frac{15}{2}$$

In [6]:

```
# compare to constant-subtracted case
eps_symb = sp.Symbol('\epsilon')
#A4_small = (A4_firstorder * Gsymb + A4_secondorder * Gsymb**2)
A4_small = ((-sp.Rational(3, 2) + 6 * J3s) * Gsymb + (-sp.Rational(3, 2)) * Gsymb**
#B4_small = (B4_zeroorder + B4_firstorder * Gsymb)
B4_small = -sp.Rational(15, 2) * Gsymb

H4rot = H4 = -(- 2 * Gsymb + 4 * Gsymb**2) - eps_symb * (A4_small + B4_small * sp.c

P = 2 * (1 - nout - eps_symb * (12 * J3s - 3) / 4)
Q = 4 - 3 * eps_symb / 2
R = sp.Rational(15, 2) * eps_symb
# (55)
H5 = Gsymb * P - Gsymb**2 * Q + R * Gsymb * sp.cos(2 * phi)

#display(sp.simplify(H4rot - H5).subs(Gsymb, 0))
#display(sp.simplify((H4rot - H5) / Gsymb).subs(Gsymb, 0))
#display(sp.simplify((H4rot - H5) / Gsymb**2).subs(Gsymb, 0))
display(sp.simplify(H4rot - H5))
```

0

In [7]:

```
display(Gamma)
```

$$\frac{\sqrt{1-e^2}}{2} - \frac{1}{2}$$