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90° Attractor

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ABSTRACT
Abstract

Key words: keywords

1 INTRODUCTION

This problem is important as it will be important for self-driving cars, curing cancer, and the search for extraterrestrial intelligence.

2 ANALYTICAL RESULTS

2.1 Equations of Motion

We study Lidov-Kozai oscillations due to an external perturber to quadrupole order and include precession of pericenter and gravitational wave radiation due to general relativity. Consider an inner BH binary with masses m_1 and m_2 having total mass m_{12} and reduced mass μ orbited by a third BH with mass m_3. Call a_3 the orbital semimajor axis of the third BH from the center of mass of the inner binary, and e_3 the eccentricity of its orbit, and define effective semimajor axis

$$\bar{a}_3 \equiv a_3\sqrt{1-e_3^2}. \tag{1}$$

We adopt the test particle approximation such that the orbit of the third mass is fixed in space.

We then consider the motion of the inner binary, described by orbital elements $(a, e, \Omega, I, \omega)$. The equations describing the motion of these orbital elements is then (Storch & Lai 2015; Liu & Lai 2018; Peters 1964)

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{a}{t_{GW}}, \tag{2}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{15}{8t_{LK}}e\sqrt{1-e^2}\sin 2\omega \sin^2 I, \tag{3}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{3}{4t_{LK}}\frac{\cos I\left(5e^2\cos^2\omega - 4e^2 - 1\right)}{\sqrt{1-e^2}} + \Omega_{GR}, \tag{4}$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{15}{16}\frac{e^2\sin 2\omega \sin 2I}{\sqrt{1-e^2}}, \tag{5}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{3}{4t_{LK}}\frac{2\left(1-e^2\right) + 5\sin^2\omega(e^2 - \sin^2 I)}{\sqrt{1-e^2}}. \tag{6}$$

Here, we have defined

$$t_{LK}^{-1} = n\left(\frac{m_3}{m_{12}}\right)\left(\frac{a}{\bar{a}_3}\right)^3, \tag{7}$$

$$t_{GW}^{-1} = \frac{64}{5}\frac{G^3\mu m_{12}^2}{c^5a^4}\frac{1}{(1-e^2)^{7/2}}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right), \tag{8}$$

$$\Omega_{GR} = \frac{3Gnm_{12}}{c^2a\left(1-e^2\right)}, \tag{9}$$

and $n = \sqrt{Gm_{12}/a^3}$ is the mean motion of the inner binary.

We are ultimately interested in the evolution of the spin vector of the inner BHs. Neglecting spin-spin interactions, they evolve independently as

$$\frac{\mathrm{d}\hat{\mathbf{S}}}{\mathrm{d}t} = \Omega_{SL}\hat{\mathbf{L}} \times \hat{\mathbf{S}}, \tag{10}$$

$$\Omega_{SL} = \frac{3Gn\left(m_2 + \mu/3\right)}{2c^2a\left(1-e^2\right)}. \tag{11}$$

APPENDIX A: EQUATIONS OF MOTION

We follow Kinoshita (1993); Storch & Lai (2015) and write down Hamiltonian in the frame where

APPENDIX B: DIFFERENCE FROM CHAOTIC STELLAR SPIN DYNAMICS

- LK is not a perturbation for us (compared to $\hat{\mathbf{S}} \cdot \hat{\mathbf{L}}$ dynamics), it is significantly dominant. This corresponds to the $\mathcal{A} \ll 1$ regime of SL15. They obtain a neat bifurcation due to separatrix crossing, which is not observed in our LK simulations, so this cannot in spirit be a similar mechanism.
- SL15 focuses on adiabatically changing $\mathcal{A}$ and seeing how it encounters resonances. In our problem, nothing nontrivial can occur if $\mathcal{A}$ changes slowly.
- Our Hamiltonian takes on form $H = (*)\,\mathbf{\Omega}_{SL} - \mathbf{R}\cdot\hat{\mathbf{S}}$. This will never have any resonances since it's perfectly linear; anything that looks nonlinear is a pure consequence of coordinates (e.g. multiplication of θ and ϕ terms).

REFERENCES

Kinoshita H., 1993, Celestial Mechanics and Dynamical Astronomy, 57, 359

Liu B., Lai D., 2018, The Astrophysical Journal, 863, 68

Peters P. C., 1964, Physical Review, 136, B1224

Storch N. I., Lai D., 2015, Monthly Notices of the Royal Astronomical Society, 448, 1821

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