Lidov-Kozai 90° Attractor

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Date

1 Equations

1.1 Bin's Papers

Our major references will be Bin's paper with Diego + Dong in 2015 (LML15) and Bin's later paper with Dong on spin-orbit misalignment (LL18). The target of study is §4.3 of LL18, where a 90° attractor in spin-orbit misalignment seems to appear when the octupole effect is negligible.

When the octupole effect is negligible, we define vectors

$$\mathbf{j} = \sqrt{1-e^2}\hat{n}, \tag{1}$$

$$\mathbf{e} = e\hat{u}. \tag{2}$$

Here, $\mathbf{j}$ is the dimensionless angular momentum vector and $\mathbf{e}$ is the eccentricity vector; see LML15 for precise definitions. Note that $\mathbf{j}\cdot\mathbf{e} = 0$, $j^2 + e^2 = 1$. Then, the EOM for the inner and outer vectors satisfy to quadrupolar order

$$\frac{\mathrm{d}\mathbf{j}_1}{\mathrm{d}t} = \frac{3}{4t_{LK}}\left[\left(\mathbf{j}_1\cdot\hat{n}_2\right)\left(\mathbf{j}_1\times\hat{n}_2\right) - 5(\mathbf{e}_1\cdot\hat{n}_2)(\mathbf{e}_1\times\hat{n}_2)\right], \tag{3}$$

$$\frac{\mathrm{d}\mathbf{e}_1}{\mathrm{d}t} = \frac{3}{4t_{LK}}\left[\left(\mathbf{j}_1\cdot\hat{n}_2\right)(\mathbf{e}_1\times\hat{n}_2) + 2\mathbf{j}_1\times\mathbf{e}_1 - 5(\mathbf{e}_1\cdot\hat{n}_2)\left(\mathbf{j}_1\times\hat{n}_2\right)\right]. \tag{4}$$

Let's assume for the time being that $L_1 \ll L_2$, so the system is sufficiently hierarchical that $\mathbf{j}_2$, $\mathbf{e}_2$ are constants. Note for reference that

$$t_{LK} \equiv \frac{L_1}{\mu_1\Phi_0} = \frac{1}{n_1}\left(\frac{m_0+m_1}{m_2}\right)\left(\frac{a_2}{a_1}\right)^3\left(1-e_2^2\right)^{3/2}. \tag{5}$$

Here, $n_1 \equiv \sqrt{G(m_0+m_1)/a_1^3}$. Finally, the GR effects (Peters 1964) cause decays of $\mathbf{L}$ and $\mathbf{e}$ as

$$\left.\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}\right|_{GW} = -\frac{32}{5}\frac{G^{7/2}}{c^5}\frac{\mu^2 m_{12}^{5/2}}{a^{7/2}}\frac{1+7e^2/8}{\left(1-e^2\right)^2}\hat{L}, \tag{6}$$

$$\left.\frac{\mathrm{d}\mathbf{e}}{\mathrm{d}t}\right|_{GW} = -\frac{304}{15}\frac{G^3}{c^5}\frac{\mu m_{12}^2}{a^4\left(1-e^2\right)^{5/2}}\left(1+\frac{121}{304}e^2\right)\mathbf{e}. \tag{7}$$

Here, $m_{12} \equiv m_1 + m_2$.

Given this system (from LML15), we can then add the spin-orbit coupling term, which is given in LL18 to be

$$\frac{\mathrm{d}\hat{S}_1}{\mathrm{d}t} = \Omega_{SL}\hat{L}_1 \times \hat{S}_1, \tag{8}$$

$$\Omega_{SL} \equiv \frac{3Gn\left(m_2 + \mu/3\right)}{2c^2 a\left(1 - e^2\right)}. \tag{9}$$

Note that μ is the reduced mass of the inner binary. We can drop the back-reaction term since $S_1 \ll L_1$. What is observed is that, as this system is evolved forward in time and GR coalesces the inner binary, $\theta_{sl} \equiv \arccos\left(\hat{S} \cdot \hat{L}_1\right)$ goes to 90° consistently. The relevant figure is Fig. 19 of LL18, which shows that for a close-in, low-eccentricity perturber ($\bar{a}_{\mathrm{out,eff}} \propto a_{out}$), the focusing is significantly stronger. Note that initially, $I \equiv \arccos\left(\hat{L}_1 \cdot \hat{L}_2\right) \approx 90^\circ$ while $\theta_{sl} \approx 0$.

In LL18, an adiabaticity parameter is defined:

$$\mathscr{A} \equiv \left|\frac{\Omega_{SL}}{\Omega_L}\right|, \tag{10}$$

where $\Omega_L \simeq \left\langle \frac{\mathrm{d}\hat{L}_1}{\mathrm{d}t} \right\rangle_{LK}$ to quadrupolar order. As the inner binary coalesces, $\mathscr{A}$ transitions from $\ll 1$ to $\gg 1$ (as Ω_{SL} is a GR effect so ramps up very quickly as orbital separation decreases).

1.2 Intuitive Picture

Looking at LL18, in praticular at Fig 4,