Dynamical Tides in Eccentric Massive Stellar Binaries

Group Meeting

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- Massive star with eccentric binary companion inducing dynamical tides.
- Primary difficulty: dynamical tides is typically messy, sum over many modes, hard to gain analytical intuition.
- Question: can we obtain a *simple closed form for dynamical tides* in this system?
- Dynamical tide in massive stars due to companion on *circular orbit* (Kushnir et. al. 2017).

$$\tau(\omega; r_c) = \beta_2 \frac{GM_2^2 r_c^5}{a^6} \frac{\rho_c}{\bar{\rho}_c} \left(1 - \frac{\rho_c}{\bar{\rho}_c}\right)^2 \times \left(\frac{\omega}{\sqrt{GM_c/r_c^3}}\right)^{8/3}.$$

- Eccentric forcing is just sum of many circular forcings (Fourier transform, e.g. Vick et. al. 2017)

$$\tau_{\rm tot} = T_0 \sum_{N=-\infty}^{\infty} F_{N2}^2 \,\mathrm{sgn}(\sigma) \tau(\omega = |\sigma|),$$

where $\sigma \equiv N\Omega - 2\Omega_s$ and F_{Nm} are the *Hansen coefficients*

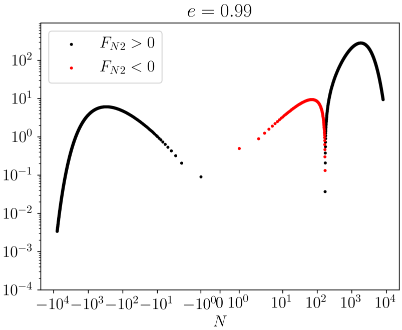
$$F_{Nm} = \frac{1}{\pi} \int_0^{\pi} \frac{\cos[N\mathcal{M}(E) - mf(E)]}{(1 - e\cos E)^2} \,\mathrm{d}E,$$

where f, $\mathcal{M}$, and E are the true, mean, and eccentric anomalies.

- Thus, want to evaluate something of form

$$\tau_{\text{tot}} = \hat{T}(r_c, \Omega) \sum_{N=-\infty}^{\infty} F_{N2}^2(e)\,\text{sgn}\left(N - 2\frac{\Omega_s}{\Omega}\right)\left|N - 2\frac{\Omega_s}{\Omega}\right|^{8/3}.$$

- The F_{N2} look like (note: FT is fastest to compute coefficients, $e = 0.99$ took ~ 2 s)



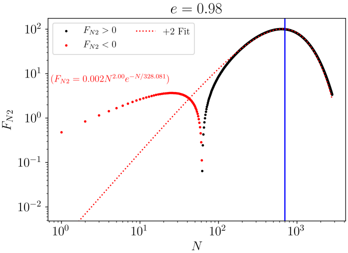
- Key insight: only one important hump ($\sim N_{\text{peri}}$), seek inspired fit.

$$\tau_{\rm tot} = \hat{T}(r_c,\Omega)\sum_{N=-\infty}^{\infty} F_{N2}^2(e)\,\mathrm{sgn}\left(N - 2\frac{\Omega_s}{\Omega}\right)\left|N - 2\frac{\Omega_s}{\Omega}\right|^{8/3}.$$

- Criteria for approximate $F_{N2}(e)$:
  - Should only have one scale, $N_{\rm peri}$
  - Should exponentially fall off for large N (smoothness)
  - $F_{02}(e) \approx 0$.
- Guess: maybe

$$F_{N2}(e) \simeq C(e) N^{p(e)} e^{-N/\eta(e)}. \quad (1)$$

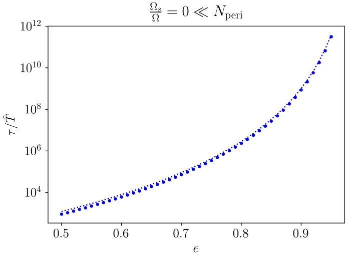
  Turns out $p \approx 2$.
- Furthermore, $\underset{N}{\operatorname{argmax}}\, F_{N2}(e) = p\eta(e)$, so $\eta \simeq N_{\rm peri}/2$.
- C is fixed by normalization (Parseval's).
- Finally, $\int_0^\infty x^p e^{-x}\,\mathrm{d}x \equiv \Gamma(p-1)$.

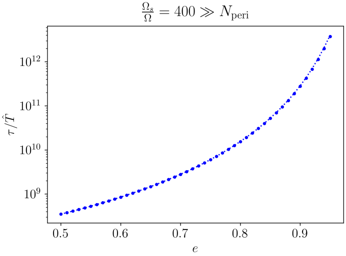


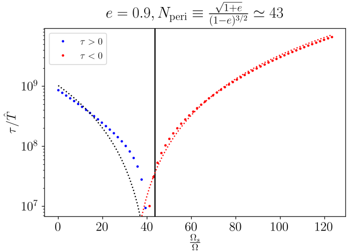
Solution

Tidal Torque

- Use resulting $\tau_{\rm tot}$ in closed form (piecewise for $\Omega_s \gg N_{\rm peri}\Omega/2$ or $\Omega_s \ll N_{\rm peri}\Omega/2$), some small fudge factors, compare with explicit sum:

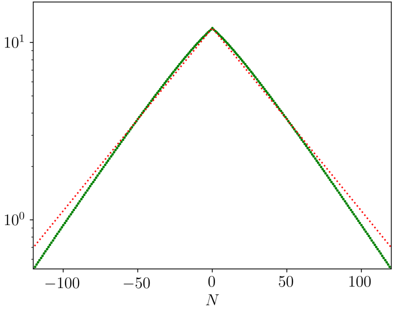






- For heating, same treatment for F_{N2}, need $m = 0$ Hansen coefficients F_{N0} too:

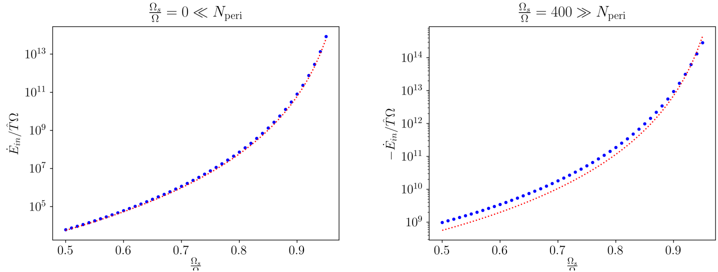
$$\dot{E}_{\rm in} = \frac{1}{2}\hat{T}(r_c, \Omega) \sum_{N=-\infty}^{\infty} \left[N\Omega F_{N2}^2 \,\mathrm{sgn}(\sigma)|\sigma|^{8/3} + \left(\frac{W_{20}}{W_{22}}\right)^2 \Omega F_{N0}^2 |N|^{11/3} \right]. \qquad (2)$$



- Only characteristic scale is still $N_{\rm peri}$.

- Guess $F_{N0} \simeq Ae^{-\frac{|N|}{N_{\rm peri}}}$. Empirically, find

$$F_{N0} \approx Ae^{-\frac{|N|}{N_{\rm peri}/\sqrt{2}}}, \tag{3}$$



Upcoming Work

- Application to J0045+7319?
  - Since e is known, τ_{tot} is very easy to evaluate without approximation.
  - Using earliest works (Lai 1996, Kumar & Quaetert 1997), find $\frac{\Omega_s}{\Omega} \approx -0.37$.
  - Since these works, stellar mass $\sim 10 \pm 1 M_\odot$. Working on recalculating using MESA, numbers seem wrong (K&Q get $M_c = 3M_\odot$, $R_\star = 6R_\odot$, $R_c = 0.23R_\star$, while i get $M_c = 2.75M_\odot, R_\star = 3.5R_\odot, R_c = 0.2R_\star$). Debugging.
- Tidal synchronization timescale as a function of e?