

**Figure 1:** SNe mass transfer function

We want to answer what the primordial BH  $q$  distribution is in a few simplified cases if:

- The ZAMS masses are randomly drawn Salpeter IMF  $P(M) \propto M^{-2.35}$ , then go supernova following <https://ui.adsabs.harvard.edu/abs/2017MNRAS.470.4739S/abstract> (bounded by large/small Z)
- The ZAMS mass ratio is uniform.
- The ZAMS mass ratio is uniform in  $\log q$ .

For reference, the supernova mass transfer function is shown in Fig. 1

## 1 Corrections to Appendix A

I found Appendix A is wrong:  $P(q) \propto q^{-p}$  using the convention  $q \geq 1$ , but not in our convention! See Fig. 2. To draw the distributions, I use either

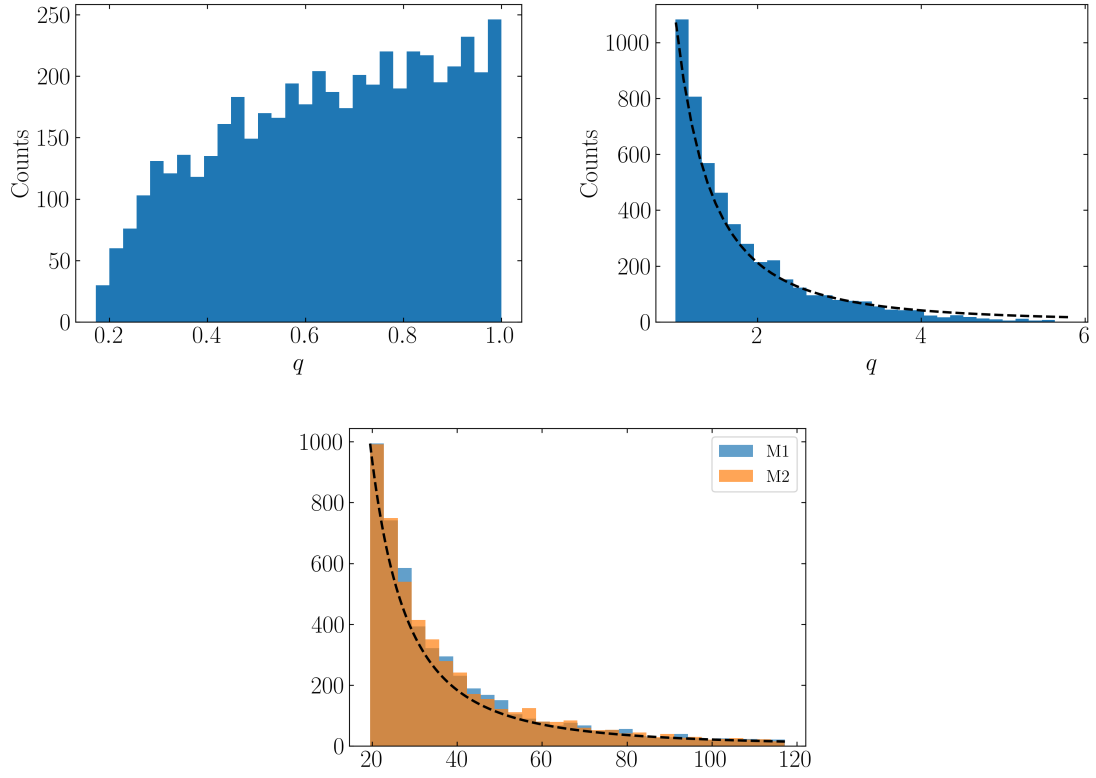
$$q = \min\left(\frac{m_2}{m_1}, \frac{m_1}{m_2}\right) \leq 1, \quad (1)$$

or max and  $\geq 1$ , where  $m_{1,2}$  are drawn from  $P(m) \propto m^{-2.35}$ . I double checked the Moe & di Stefano paper, and under their (2) they really assume that  $P(q \leq 1) \propto q^{-p}$  as well, so I think this might be a misconception?

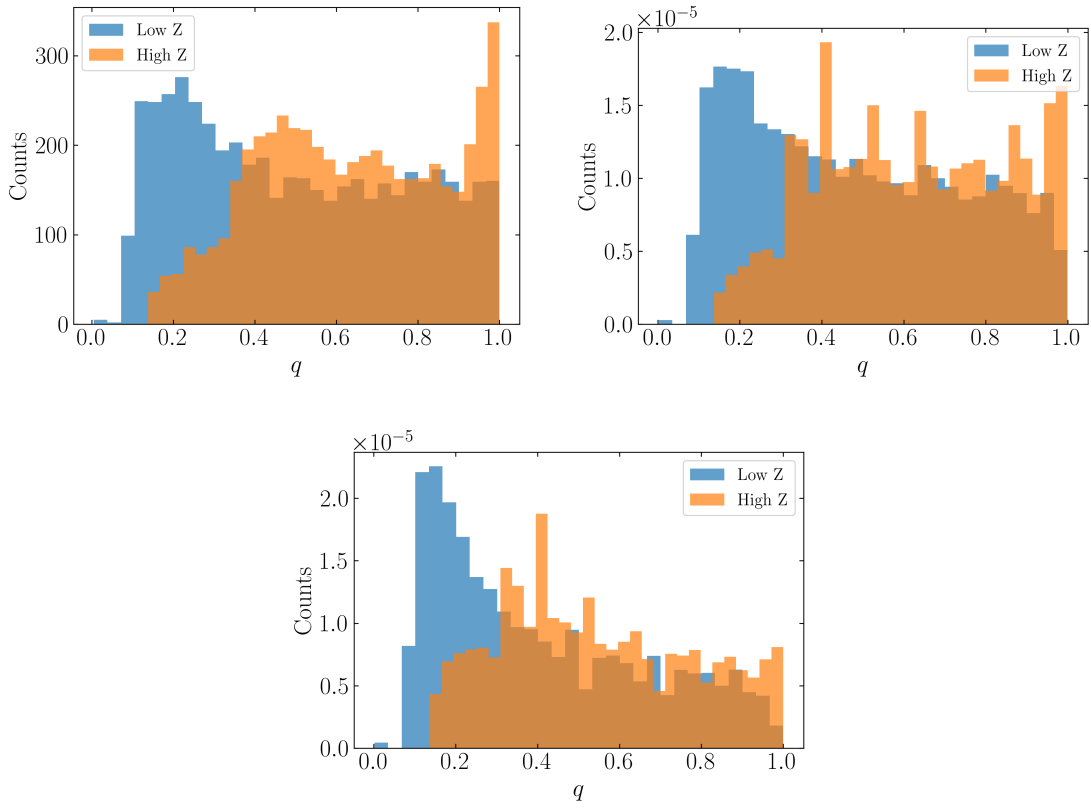
Note that in my Appendix, the calculation doesn't change if we take  $m_2 \geq m_1$ ! But clearly, using  $m_2 \geq m_1$  and the calculation in the text is the correct calculation, and the calculation is incorrect as is. Why?

## 2 Histograms

The three requested plots are shown in Fig. 3. For (i), I just took the masses from the previous section and sent them through the SNe transfer function (Fig. 1). For (ii) and (iii), the procedure is somewhat more complicated; for each value of  $q$  at ZAMS: choose  $m_2 \in [M_{\min}, qM_{\max}]$  and  $m_1 = m_2/q$ . Compute the BH value of  $q_{\text{BH}}$  by sending it through the SNe transfer function, and weight it by  $P(m_1)P(m_2)$ . Repeat for a grid of  $q$  and  $m_2$ , and histogram it all.



**Figure 2:** (i) Histogram of  $q \leq 1$  with random pairings from Salpeter IMF, (ii) histogram of  $q \geq 1$  with random pairings from Salpeter IMF, with  $q^{-2.35}$  overlaid, and (iii) histogram of masses, with  $M^{-2.35}$  power law overlaid, as a sanity check.



**Figure 3:** Distribution of  $q$  after (i) random pairings Salpeter IMF + supernovae, (ii) uniform  $q_{\text{ZAMS}}$ , and (iii) uniform  $\log(q_{\text{ZAMS}})$ .