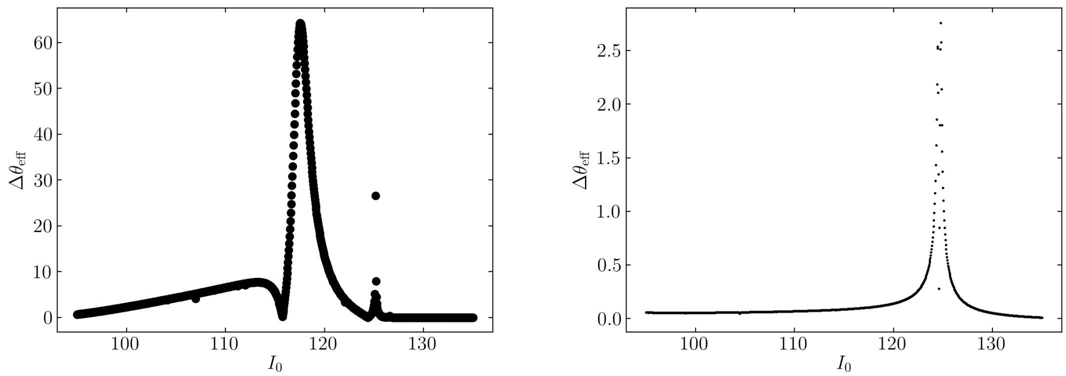

Figure 1: Last week plot. $\Delta\theta_{\rm eff}$ (in degrees) using numerical LK solutions to evolve an initial spin $\hat{\mathbf{S}} = \hat{\mathbf{L}}$ by 500 LK cycles, for $e_0 = 0.003$ in the Paper I and Paper II regimes respectively. Inclinations are sampled $I_0 \in [95°, 135°]$.

Very brief update since last week.

1 Recap

Recall we found that, when setting $e_0 = 3 \times 10^{-3}$ (the eccentricity at the start of the LK cycle), we can numerically obtain the amplitude of oscillations of $\theta_{\rm eff}$ as a function of I_0 the inclination at the start of the LK cycle, where

$$\Delta\theta_{\rm eff} \equiv \theta_{\rm eff,max} - \theta_{\rm eff,min}, \tag{1}$$

and we obtained as shown in Fig. 1. We found evidence consistent with the hypothesis that the toy spin model, coupled with numerical integrations of LK + pericenter advance, generates these oscillations in $\theta_{\rm eff}$.

We then asked: for a real LK + GW system, e_0, I_0 will vary over the course of the inspiral. What is the behavior of $\Delta\theta_{\rm eff}$ for this realistic sequence of orbital parameters $e_0(t)$ and $I_0(t)$[1]? For clarity, I call these parameters $e_{\min}(t)$ and $I_{\min}(t)$, so that e_0, I_0 can refer to the initial value at the start of the inspiral.

2 This Week

I first sampled $\Delta\theta_{\rm eff}$ over a grid of $(e_{\min}, I_{\min})$ (uniform in $I \in [90.5, 135]$ and log-uniform in $e \in [10^{-3}, 0.9]$). I computed this for both the Paper I and Paper II regimes. This is given in Fig. 2.

Focusing on the right plot (paper II regime), it appears $\Delta\theta_{\rm eff}$ never exceeds a few degrees in the Paper II regime, which is of primary interest. To better quantify this, we can make a plot of $\Delta\theta_{\rm eff}(e_{\min}(t), I_{\min}(t))$ using the $e_{\min}(t)$ and $I_{\min}(t)$ from the LK + GW inspiral simulation using the fiducial paper II parameters (and $e_0 = 10^{-3}$, $I_0 = 90.5°$). This is given in Fig. 3.

It bears noting that, in our actual LK + GW inspiral simulations at $I_0 = 90.5°$, the actual deviation of $\theta_{\rm eff}$ from the beginning to end of the simulation is $\ll 1$ degree, much smaller than the $\Delta\theta_{\rm eff}$ given in these plots. However, indeed, plots show that $\theta_{\rm eff}$ varies on a scale of ~ 5–$10°$ right before merger, as shown in Fig. 4. I haven't had time to check the relevant timescales, but I suspect this implies the system adiabatically enters and exits the resonance, thus $\theta_{\rm eff}$ returns to its initial value despite an intermediate period of oscillation. So this seems to suggest that $\Delta\theta_{\rm eff}$ being small is not sufficient

[1]This is formally a discrete sequence of eccentricities and inclinations, but it's easier to think about time-indexed.

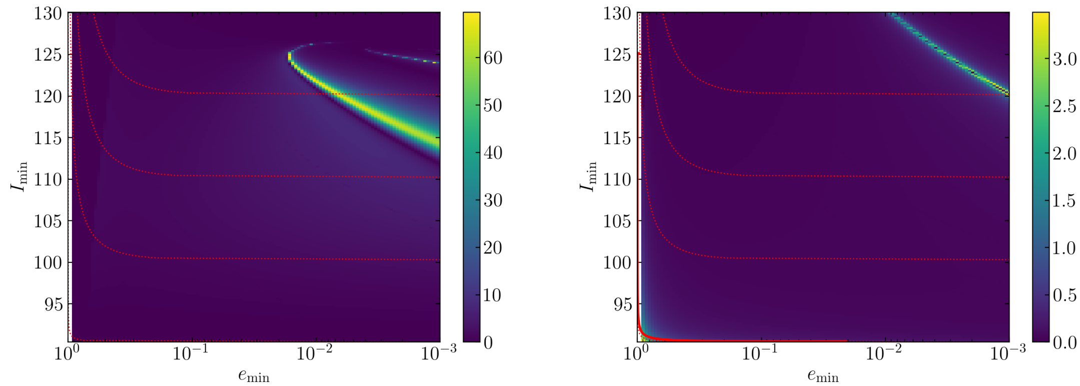

Figure 2: $\Delta\theta_{\rm eff}$ (in degrees, colorbar) plotted over a scan of $e_{\rm min}$ and $I_{\rm min}$, the "osculating" orbital elements relevant for the LK cycle, for Paper I and Paper II parameters respectively. Overplotted are lines of constant Kozai constant (dotted red) and the trajectory swept through using the paper II parameters for $e_0 = 10^{-3}$ and $I_0 = 90.5°$. The real LK + GW simulation shown follows the constant-K curves somewhat, since K is slowly varying under GW dissipation.

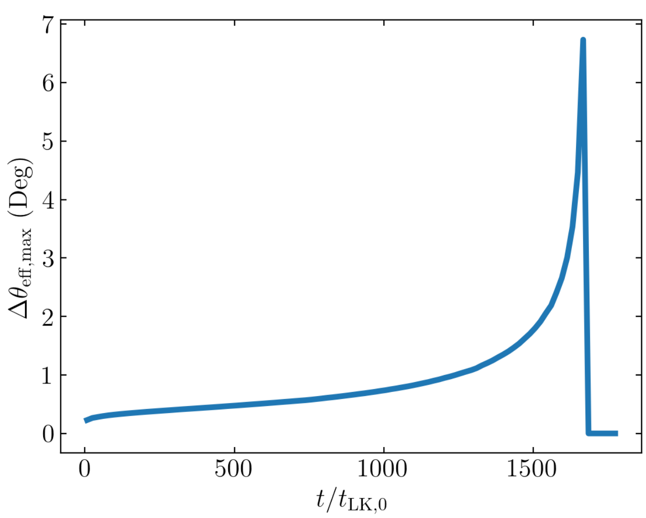

Figure 3: Plot of $\Delta\theta_{\rm eff}\left(e_{\rm min}(t), I_{\rm min}(t)\right)$ (degrees) using the orbital histories from a LK + GW inspiral simulation.

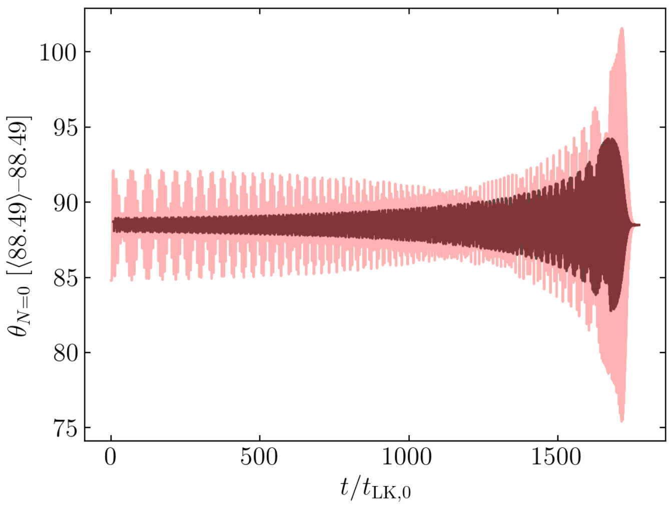

Figure 4: Plot of $\Delta\theta_{\rm eff}$ (black; ignore red; called $\theta_{N=0}$ in this old plot) for $I_0 = 90.5^\circ$ LK + GW inspiral simulation. Important to see is that, near merger, $\theta_{\rm eff}$ has quite large amplitude oscillations, but narrows into its final value.

to guarantee extremely good ($\ll 1$ degree) conservation of $\Delta\theta_{\rm eff}$, only that it won't vary by any more than a few degrees.

The adiabaticity explanation may also be able to explain the Paper I results. In this regime, LK is much milder and so we expect transitions between states to be much more gradual. One competing trade-off is that the significantly milder oscillation gives a much narrower resonance width, so transitions may not necessarily be "more adiabatic" in the paper I regime. This may be necessary to explain why systems in the Paper I regime deviate from exact conservation of $\theta_{\rm eff}$ for $I \gtrsim 100^\circ$.