1 Mass Ratio Distribution

I retried using 19 values of fixed q, running each 1000 times, instead of sampling. I only finished two $a_{\rm out,eff}$ values. As a reminder:

$$m_{12} = 50M_\odot, \qquad m_3 = 30M_\odot, \qquad a_0 = 100\,\mathrm{AU},$$
$$e_0 = 10^{-3}, \qquad e_{\rm out,0} \in [0, 0.9], \qquad \cos I_0 \in [\cos 50^\circ, \cos 130^\circ].$$

The plots are shown in Fig. 1. Recall that $e_{\rm os}$ is defined such that

$$\left\langle \frac{\mathrm{d}\ln a}{\mathrm{d}t} \right\rangle_{\rm LK} \sim \frac{1}{t_{\rm GW,0}\, j^6(e_{\max})}, \tag{1}$$

$$j^6(e_{\rm os}) \equiv j_{\rm os} = \frac{t_{\rm LK}}{t_{\rm GW,0}}, \tag{2}$$

$$= \frac{256}{5} \frac{G^3 \mu m_{12}^3}{m_3 c^5 a^4 n} \left(\frac{a_{\rm out,eff}}{a}\right)^3. \tag{3}$$

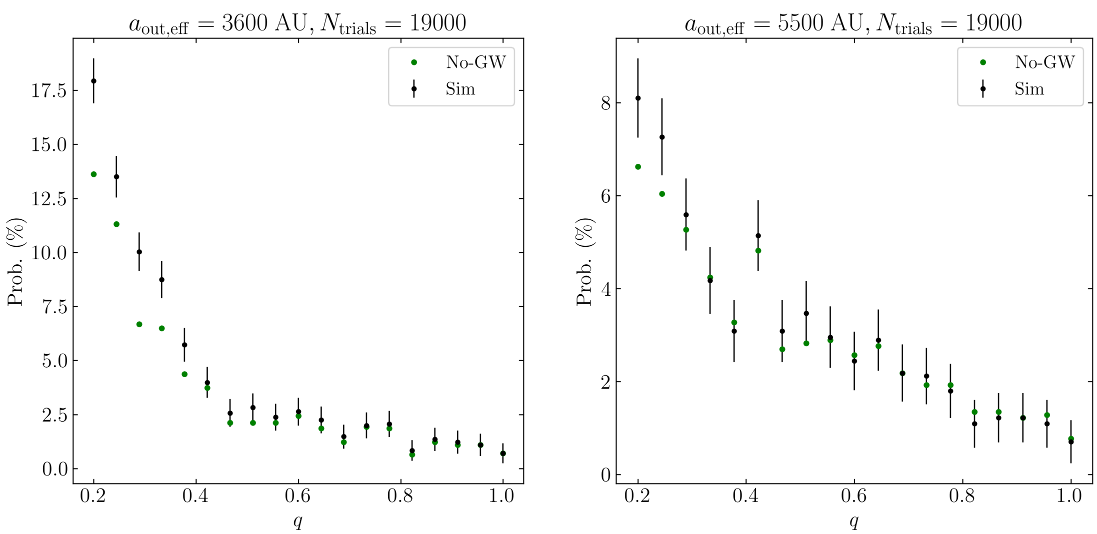

Figure 1: Merger fraction distribution for $a_{\rm out,eff} = 3600, 5500$ AU respectively, where each q has 1000 trials. Error bars are just $\sqrt{N}$ for N counts. Green dots denote predicted merger fractions if $j_{\min}$ ever dips below $3j(e_{\rm os})$ (I added a small fudge factor, it yields a small but not significant improvement) the one-shot merger criterion. Underprediction of merger rates is expected: once large eccentricities are reached, future coalescence is accelerated.

2 Example of Octupole-Enhanced Mergers

For Bin's example, where $a_0 = 10$ AU and $a_{\rm out,eff} = 300$ AU, I ran the $e_{\rm out} = 0.4$ case a long time ago. It shows the octupole-enhanced case, see Fig. 2.

3 SRF-free $e_{\max}$ Plot

We discussed the possible interest of this, turning off apsidal precession and plotting $e_{\max}$. Very little changes, as seen in Fig. 3.

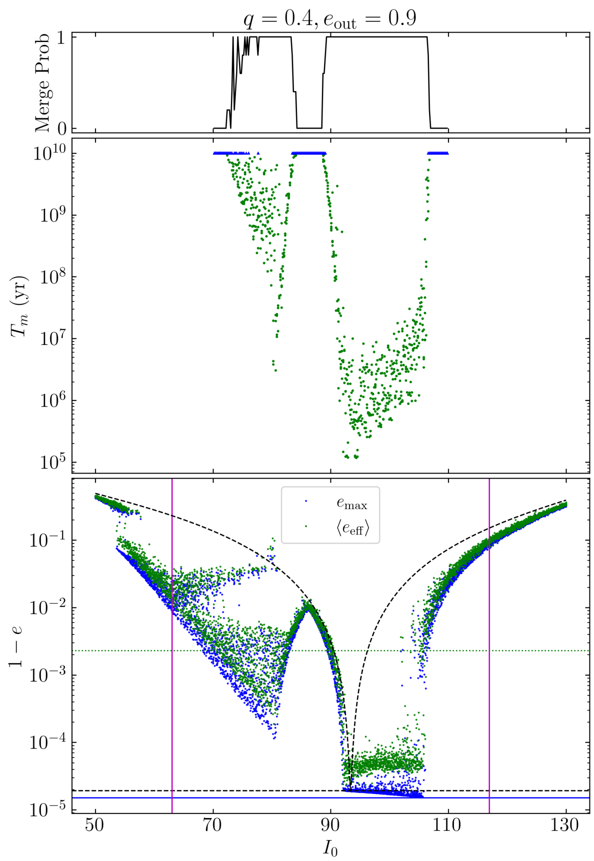

Figure 2: Example of octupole-enhanced mergers. Prograde orientations do not reach $e_{\rm os}$ (i.e. blue dots remain above horizontal blue line) but are still able to merge, as their $e_{\rm eff}$ (green dots go below green line).

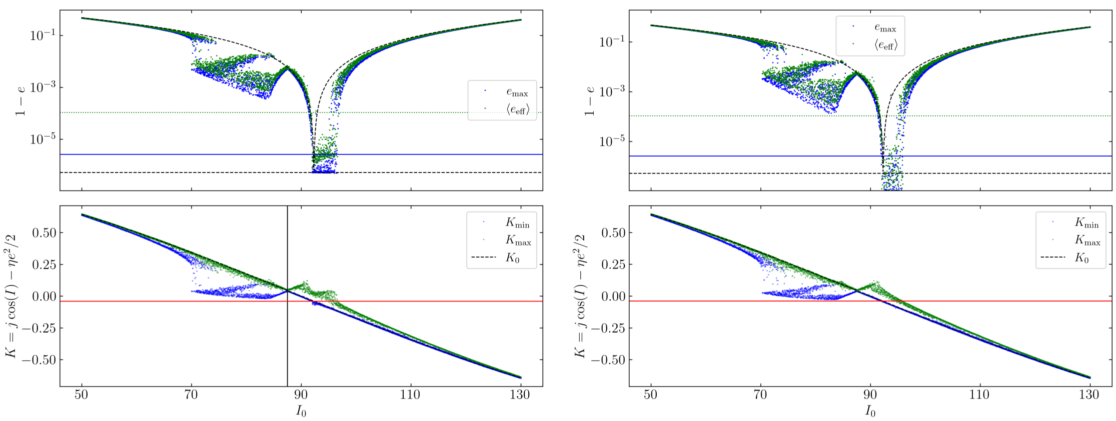

Figure 3: Two $e_{\max}$ distributions for $a_0 = 100\,\mathrm{AU}$, $a_{\mathrm{out,eff}} = 3600\,\mathrm{AU}$, $q = 0.4$ and $e_{\mathrm{out}} = 0.6$. The one on the left has SRF turned off. Very little changes

4 A Signature for the Gap

I have some tentative evidence for why the gap exists. In brief, when $\mathbf{L}_{\mathrm{in}}$ is librating, octupole-induced eccentricity cycles are expected to be heavily suppressed, and only long periods of circulation generate substantial eccentricity cycles. Integrating the octupole equations for just a single period compared to their quadrupole counterparts, I found that when moving to the gapped region, most ICs librate, and outside of the gapped region, most circulate, e.g. see Fig. 4. This is a general feature, though it breaks down somewhat for $q = 0.2$. I may try with a few more ω_1 values at a later date to make this story more robust.

Note that in the test mass, quadrupole limit, $\Delta\Omega = 180^\circ$ only at $I_0 = 90^\circ$, so $\Delta\Omega_{\mathrm{e}} = 180^\circ$ at $I_0 = 90^\circ$ for the librating case. Thus, by integrating the quadrupole, finite-η equations, it may be possible to predict where the gap is (or even obtain a leading order expression).

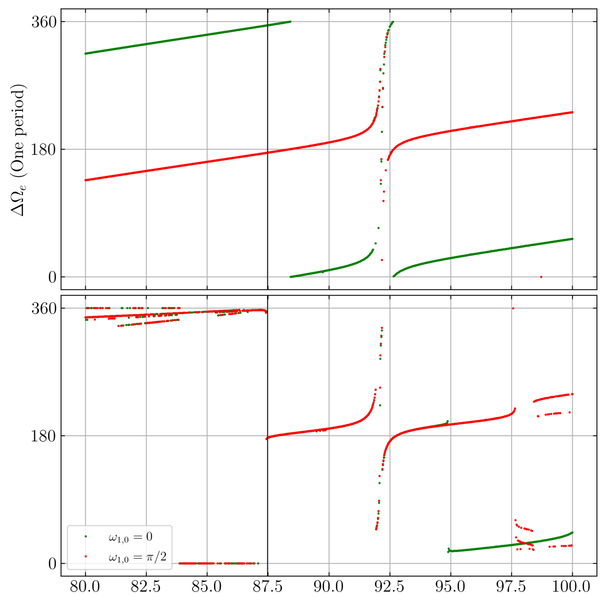

Figure 4: Plot of $\Delta\Omega_\mathrm{e} \sim \Delta\varpi$, where Δ indicates change over a period, so libration means $\Delta\Omega_\mathrm{e} \approx 180°$. Top is quadrupole-only, bottom is with octupole. Vertical black line is the center of the "gap". The detailed parameters used are $a_0 = 100\,\mathrm{AU}$, $a_2 = 3600\,\mathrm{AU}$, $e_2 = 0.6$, and $q = 0.5$.