

1 Tertiary-Induced BBH Merger Fraction Simulations

I completed $a_{\text{out,eff}} = [2800, 3600, 4500, 5500, 7000]$, and also am starting calculations for $a_{\text{out,eff}} = [1200, 2000]$ (not yet completed). The other parameters are:

$$\begin{aligned} [h]m_{12} &= 50M_{\odot}, & m_3 &= 30M_{\odot}, & a_0 &= 100 \text{ AU}, & e_0 &= 10^{-3}, \\ q &\in [0.2, 1], & e_{\text{out},0} &\in [0, 0.9], & \cos I_0 &\in [\cos 50^\circ, \cos 130^\circ]. \end{aligned}$$

All distributions are uniform. The restricted range in I_0 is because the other regions are never octupole-active (at least not at $a_{\text{out,eff}} = 3600$). The resulting merger probabilities as a function of q , defined as

$$f_{\text{merge}} = 100 \times \frac{\cos 50^\circ - \cos 130^\circ}{2} \times \frac{\# \text{ merged}}{\# \text{ run}}, \quad (1)$$

are shown in Fig. 1

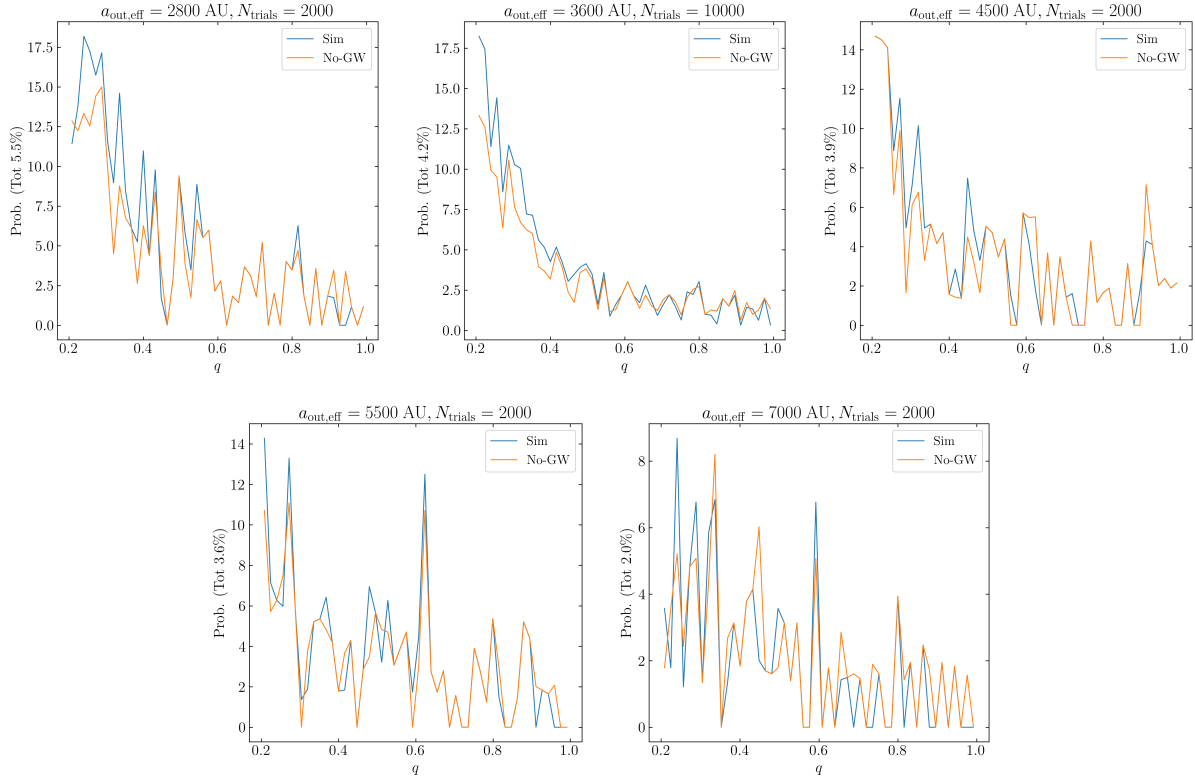


Figure 1: Merger probabilities as a function of q .

The cumulative merger probabilities as a function of $a_{\text{out,eff}}$ are shown in Fig. 2

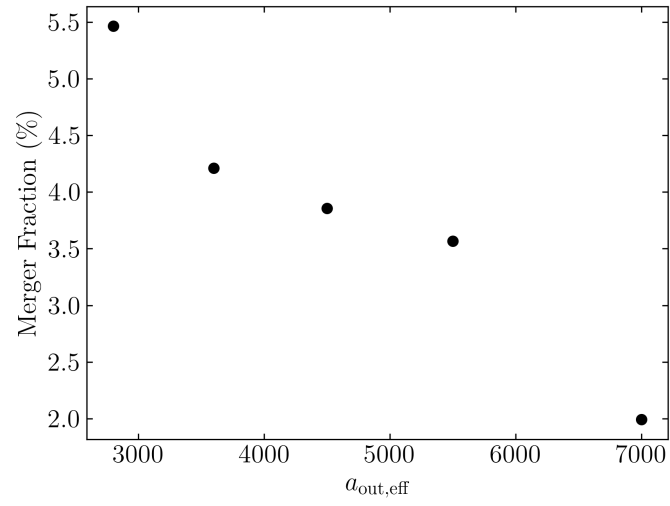


Figure 2: Total merger probabilities as a function of $a_{\text{out,eff}}$.

2 Planet Octupole Simulations

I ran simulations to determine $e_{\max}(I_0)$, where I_0 is the initial mutual inclination between the planets' orbits. The parameters I used were:

$$\begin{aligned} m_1 &= 1M_{\odot}, & m_2 &= m_3 = 1M_J, & a &= 5 \text{ AU}, & a_{\text{out}} &= 50 \text{ AU}, \\ e_0 &= 10^{-3}, & k_2 &= 0.37, & R_2 &= R_J. \end{aligned}$$

I sampled $I_0 \in [40^\circ, 140^\circ]$, though the upper inclination range doesn't seem always to be sufficient. I tried among $e_2 = [0.1, 0.3, 0.5, 0.6, 0.8, 0.9]$. I also retried all my simulations with

$$m_3 = M_{\odot}, \quad a_{\text{out}} = 500 \text{ AU}.$$

This ensures the same quadrupole strength ($\bar{a}_{\text{out,eff}}$) but causes η to decrease by a factor of ≈ 3000 , satisfying the test particle approximation. This is for verification against the MLL16 fitting formula. The results are shown in Fig. 3 and 4. Each inclination is run for 3 random choices of ω_i, Ω_i angles, while a total of up to 2000 inclinations are sampled uniformly. The value of e_{\max} in these simulations is $1 - e_{\max} \approx 10^{-3}$, in line with the analytical estimate, suggesting my ω_{tide} is correct.

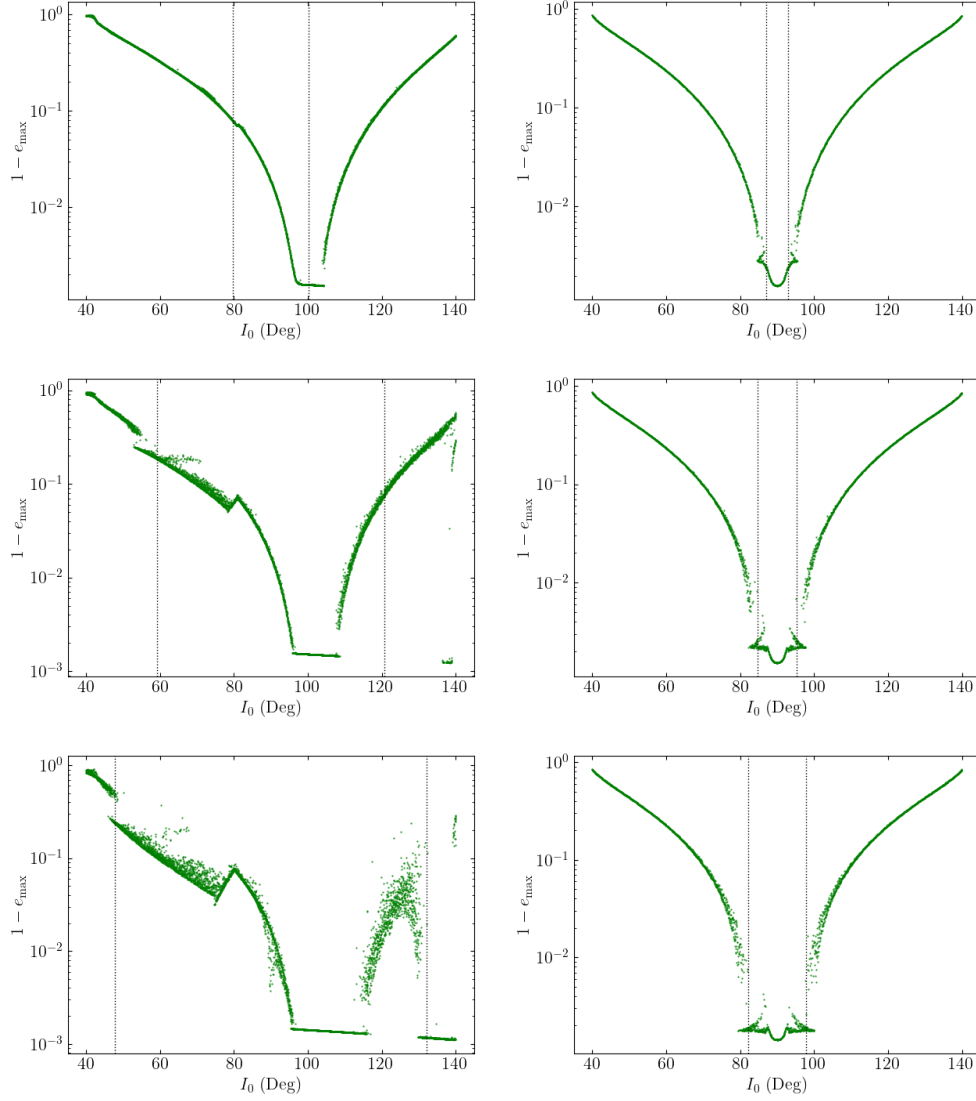


Figure 3: Part 1. Rows are $e_{\text{out}} = [0.1, 0.3, 0.5]$, while columns are for the fiducial parameters and for $m_3 = M_\odot$. Dots indicate e_{max} when run over $500t_{\text{LK}}$ with apsidal precession (due to both GR and tides) but with no dissipation. Vertical lines are fits from MLL16. Note that e_{oct} is larger on the left column by a factor of 10.

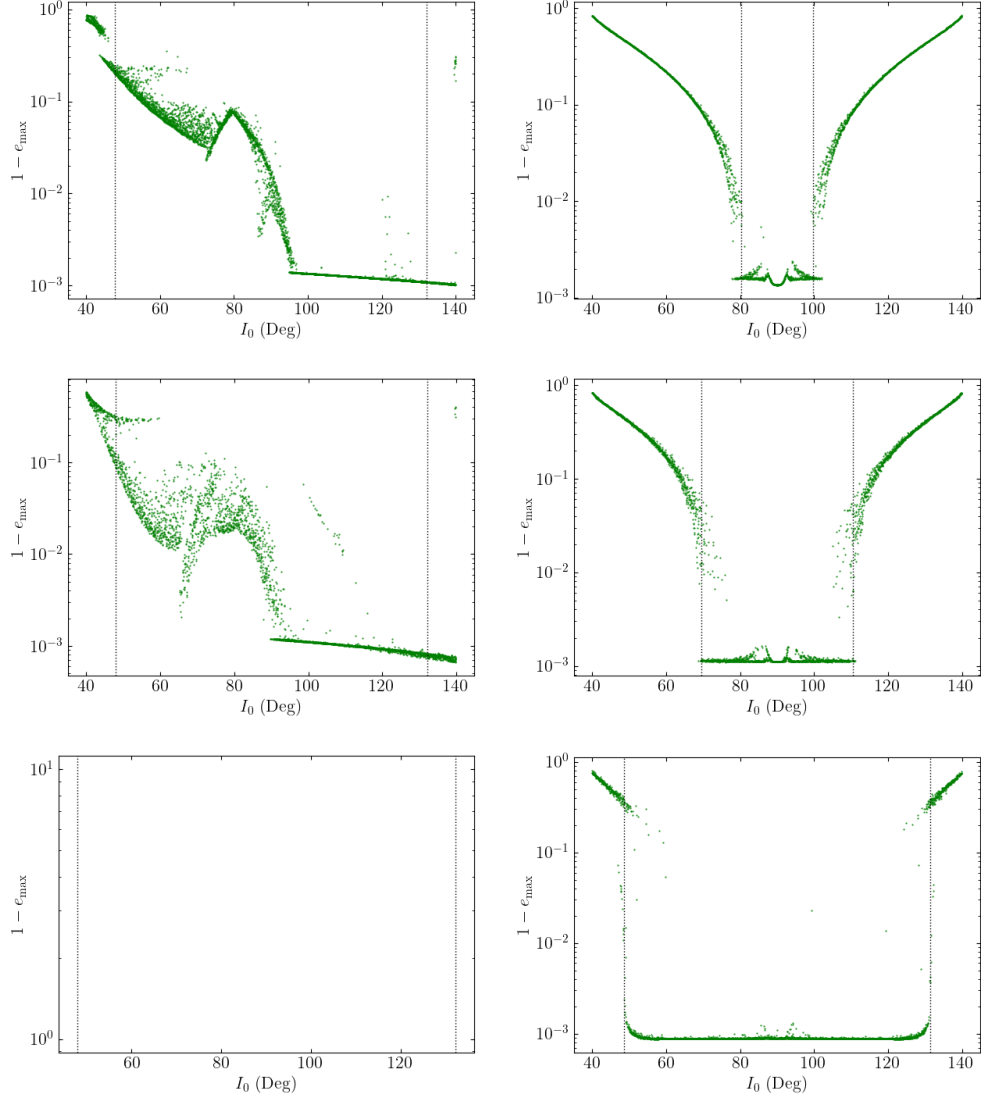


Figure 4: Part 2. Rows are $e_{\text{out}} = [0.6, 0.8, 0.9]$, while columns are for the fiducial parameters and for $m_3 = M_{\odot}$. No simulations are available for $e_{\text{out}} = 0.9$ and $m_3 = M_J$ yet.