1 Summary of J0045-7319 with MESA

1.1 Parameters & Background

PSR J0045-7319 is in the SMC. The following properties are measured:

- $L = 1.2 \times 10^4 L_\odot$.
- $T_{\rm eff} = (2.4 \pm 0.1) \times 10^4$ K, giving $R = 6.4 R_\odot$.
- $P_{\rm orb} = 51.17$ day.
- $q \equiv M/M_{\rm NS} = 6.3$ the mass ratio.
- $\dot{P} = -3.03 \times 10^{-7}$.
- $v \sin i = 110$ km/s surface rotation rate of the star. Breakup $v = \sqrt{GM/R} \approx 500$ km/s.

Kaspi+1996 (doi:10.1038/381584a0) assume that the NS has mass $1.4 M_\odot$[1], giving a stellar mass of $8.8 M_\odot$ and an inferred separation $a = 126 R_\odot$. Kumar & Quatart (doi:10.1086/305091) use a "Yale model" to infer a convective core of extent $r_{\rm c} = 0.23R = 1.38 R_\odot$ and mass $M_{\rm c} = 3 M_\odot$.

Later, a study by Thorsett & Chakrabarty (doi:10.1086/306742) compare to stellar models to infer a stellar mass of $10 M_\odot$ and NS mass of $1.58 M_\odot$ instead.

Separately, the model of dynamical tides includes a density dependence as:

$$T \propto \frac{\rho_{\rm c}}{\bar{\rho}_{\rm c}} \left(1 - \frac{\rho_{\rm c}}{\bar{\rho}_{\rm c}}\right)^2. \tag{1}$$

This factor is not considered in previous works. It is maximized at 4/27 when $\rho_{\rm c}/\bar{\rho}_{\rm c} = 1/3$. Assuming this density ratio, the convective core needs to be at least $0.93 R_\odot$ in extent to generate the observed $\dot{P}$ without rotating above *core* breakup, i.e. satisfying $\Omega_{\rm c} \leq \sqrt{GM_{\rm c}/R_{\rm c}^3}$.

Puzzle: Even for a very large core $R_{\rm c} \sim 1.5 R_\odot$, the core still needs to be rotating substantially faster than $\sqrt{GM/R^3}$, the entire star's breakup rotation rate, see Figure 8 of the draft. This requires substantial differential rotation. Is this believable?

NB: This is in line with the Kumar & Quataert result, which estimates a star rotating quite near surface critical rotation but without the density correct factor.

1.2 My MESA Models

The current thinking is not to include these results unless there's a very good chance of finding a useful, updated stellar structure.

I'm not too confident that I've implemented the spin prescription accurately/correctly, but the sorts of results I'm obtaining are shown in Fig. 1. The three columns show: (i) finding the correct mass that evolves through the observed (T, R), (ii) showing the evolution of the stellar radius (black), core radius (red), and core H_1 fraction (black) as the best-fitting model evolves, with the vertical blue line showing the time that it matches the observed stellar properties, and (iii) propagation diagram at the time of best fit.

The problem with these simulations is that: (i) the convective core is never more than $\sim 0.7 R_\odot$, because some stellar evolution off the MS is required to match the (L, R), and (ii) the density ratio

[1] At the time, all NS mass measurements were consistent with $1.4 M_\odot$ so they took it as a fixed parameter.

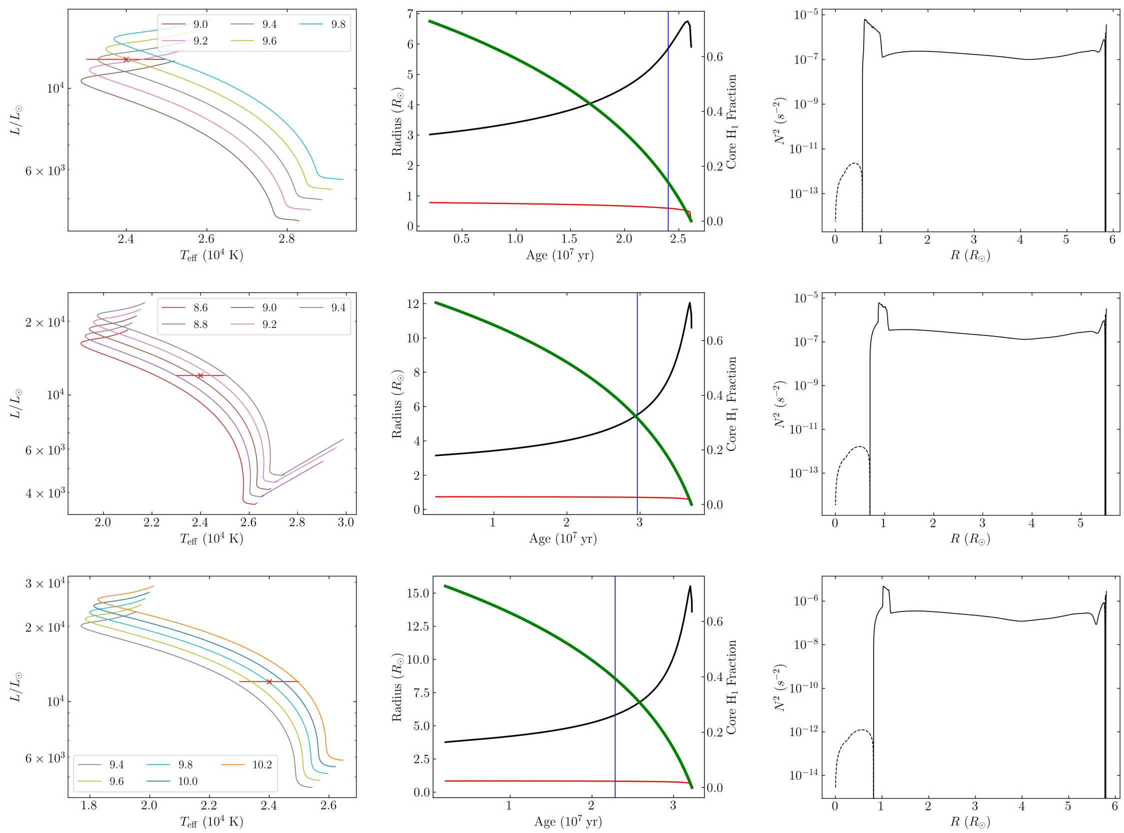

Figure 1: The three sets of simulations I present here are shown in each row: (i) low metallicity ($Z = 0.0012$), non-rotating, (ii) rotating with large convective overshoot, (iii) same as (ii) but with metallicity $Z = 0.004$. The first column shows the L-T diagram, with which we try to match the observed parameters, the second column shows the evolution of some parameters of the best-fitting star as a function of age, and the third column shows the propagation diagrams at the time of best fit (dashed lines denote negative values). The best fitting masses are (i) $M = 9.6M_\odot$, (ii) $M = 9.0M_\odot$, and (iii) $M = 9.8M_\odot$.

above is actually closer to 0.75, requiring a stellar core $\gtrsim R_\odot$ to produce the required stellar torque. Since $T \propto R^5$, this bound is actually somewhat tight.

Puzzle: It seems that the cores generated by MESA outright cannot produce the expected tidal torque even if rotating at core breakup. Even if they could, they would still require substantial differential rotation.