1 Apr 21, 2020

For the 90° attractor in the LK problem, investigated the $N = 0$ dynamics. Recall EOM

$$\frac{d\hat{\mathbf{S}}}{dt} = \langle -\dot{\Omega}\hat{\mathbf{z}} + \Omega_{SL}\hat{\mathbf{L}}\rangle_{LK} \times \hat{\mathbf{S}}$$
$$+ \left[\sum_{N=1}^{\infty} \hat{\mathbf{\Omega}}_{\text{eff},N} \exp(2\pi i N t/t_{LK})\right] \times \hat{\mathbf{S}}. \quad (1)$$

We ignore $N \geq 1$ for now, assuming resonances are not hit, so we examine

$$\frac{d\hat{\mathbf{S}}}{dt} = \langle -\dot{\Omega}\hat{\mathbf{z}} + \Omega_{SL}\hat{\mathbf{L}}\rangle_{LK} \times \hat{\mathbf{S}}. \quad (2)$$

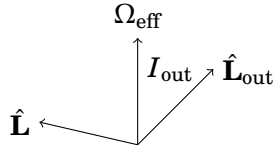Consider rotation by I_{out} given by Figure 1. In equa-

Figure 1: Geometry. I is angle between $\hat{\mathbf{L}}, \hat{\mathbf{L}}_{\text{out}}$ while I_{out} is angle between $\hat{\mathbf{L}}_{\text{out}}, \Omega_{\text{eff}}$.

tions, this requires

$$-\dot{\Omega}\sin I_{\text{out}} + \Omega_{SL}\sin(I + I_{\text{out}}) = 0. \quad (3)$$

We obtain EOM (note that LK-averaged I is almost constant):

$$\frac{d\hat{\mathbf{S}}}{dt} = \left[-\dot{\Omega}\cos I_{\text{out}} + \Omega_{SL}\cos(I + I_{\text{out}})\right]\hat{\mathbf{z}} \times \hat{\mathbf{S}}$$
$$- \dot{I}_{\text{out}}\hat{\mathbf{y}} \times \hat{\mathbf{S}}, \quad (4)$$

$$\Delta\phi(t) = \int_0^t \left[-\dot{\Omega}\cos I_{\text{out}}(\tau) + \Omega_{SL}\cos(I + I_{\text{out}}(\tau))\right] d\tau, \quad (5)$$

$$\Delta\theta = \int_0^{T_m} \dot{I}_{\text{out}}\sin\phi \, dt. \quad (6)$$

Intuition: $\dot{I}_{\text{out}} \sim 1/T_{\text{GW}}$ while $\dot{\phi} \sim \min(\dot{\Omega}, \Omega_{SL})$. Thus, when sufficiently slow, phases cancel, $\Delta\theta \to 0$. **NB:** in full numerical simulations, $\langle\theta\rangle_{LK}$ is the approximately conserved quantity, θ varies within LK cycles.
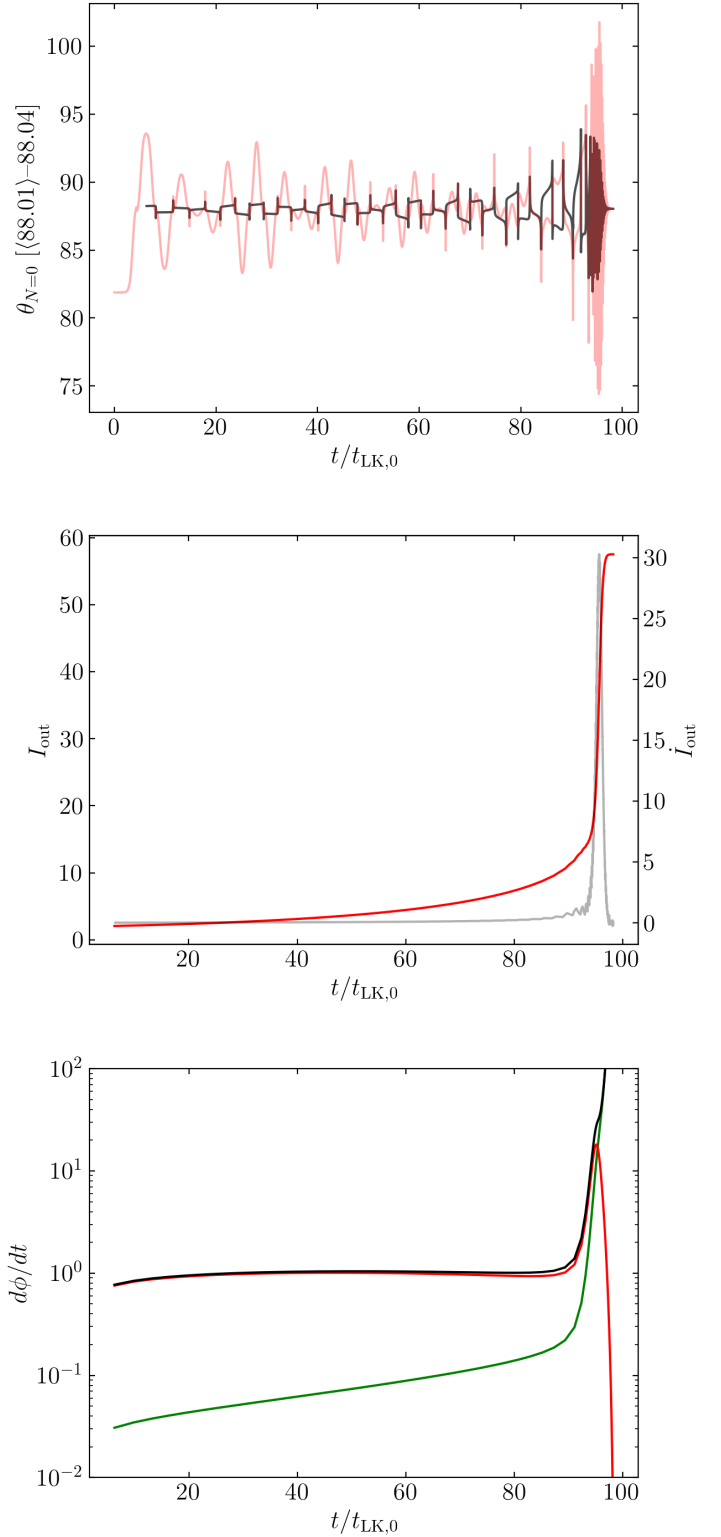
How does this do? Figure 2.

Figure 2: $I = 90.3°$. Top: Comparison of θ (black) vs θ_{eff} from Bin's paper (red). Middle: I_{out} is red, $\dot{I}_{\text{out}}$ is light black. Bottom: Comparison of LK-averaged $\dot{\Omega}$ (red), Ω_{SL} (green), and $\langle\dot{\Omega} + \Omega_{SL}\rangle_{LK}$ (black).