Student names

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Date

01/12/2021

PHY 103 Discovery Labs

Ecole Polytechnique

Resonances

Oscillations can be observed in electrical, optical, fluid, chemical, biological and ecological, economical, and many other domains. In mechanical systems across all scales, from micro-devices present in your cell phone to the trees and buildings. These objects can all vibrate and are characterized by a *natural* or *resonant* frequency, as shown in Fig. 1. The resonance is sometimes desired, such as the vibration of a musical instrument that creates the sound. Other times they must be avoided, such as by tuning a building's resonant frequency to be different from typical frequencies of earthquakes.

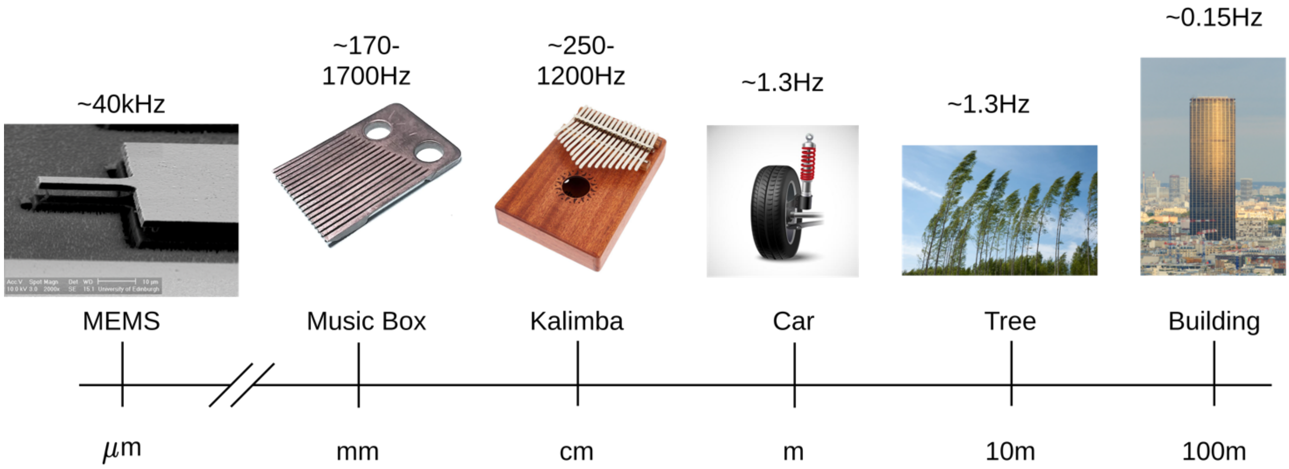


Figure 1: Objects at different scales and their natural frequencies.

At the micro-scale (~µm), some micro-electromechanical systems (MEMS) are vibrated at their natural frequency to keep time or to measure the mass of nanoparticles. Musical instruments such as the music box and kalimba vibrate through a range of frequencies with their multiple prongs to produce different notes. On an order of 1 meter, car suspensions are explicitly designed to handle vibrations caused by driving over bumpy roads. At a slightly larger scale (~10m), trees vibrate as well in response to the wind, for example. At the largest scale of man-made objects (~100m), buildings are designed to handle forced vibrations, such as those from earthquakes.

A common feature of vibrating systems is that they require the presence of both an inertial element, e.g. a mass, and a restoring element, e.g. a spring. Often the mass and the spring are lumped together into a single physical, such as in the vibration of a cantilever beam, which has a mass and an elasticity. This makes understanding the influence of each effect complex.

Fortunately, we can also separate the inertial and restoring elements, by connecting a solid body to external springs: the simple harmonic oscillator. In this lab, we will study this system to identify the system's natural frequency and how it changes with the physical parameters. We will then understand how an oscillator responds when it is forced at low or high frequencies, or near its resonant frequency.

I - Free Mass-spring oscillator

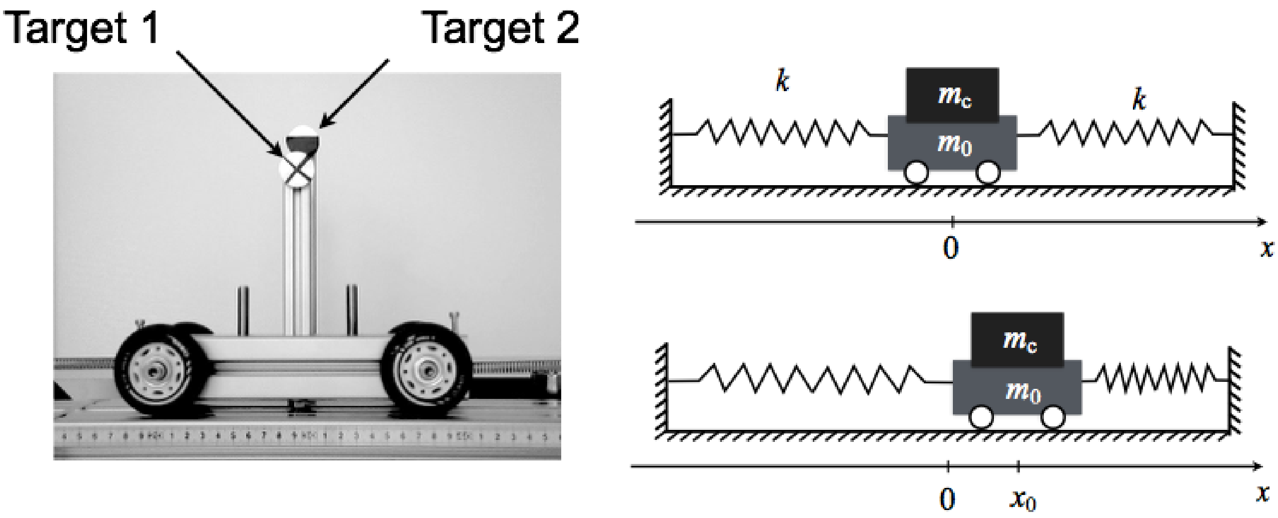


Figure 2 : Mass spring system and its experimental realization.
NOTE that the mass of each cart is written on it

We will begin by considering the spring-mass system, shown in Figure 1. The apparatus consists of a cart, several springs, and several calibrated masses.

Recall that the theoretical relation between the (natural) oscillation frequency f_0, the spring stiffness k_0 and the mass M_0 of a spring-mass system is:

$$f_0 = \frac{1}{2\pi}\omega_0 = \frac{1}{2\pi}\sqrt{\frac{k_0}{M_0}}$$ (Equation 1)

Preliminary remarks

- What are the units of f, ω, k and M?

f-Hz=s^-1 ω- Hz=s^-1 k- N*m^-1 M- kg

I.1 Set up the oscillation experiment

Now place the cart on the rail, making sure that the target pattern is facing the camera. Choose the pair of springs with the lowest stiffness (k) (i.e. for which the oscillation frequency is the lowest to easily track the cart position) and attach the cart to the springs and then the springs to the platform. If the two springs are of equal stiffness, the cart should be well centered on the platform.

You will be taking movies of the oscillating mass, using a video camera. In order to set up the experiment, make sure that you can take a movie of the mass during its movement and that you can save the movie on the computer.

- Start by aligning the camera so that the target is visible and in focus for the whole duration of the film.
- Make sure that the webcam is connected, through the USB wire, then launch the software "Logitech Webcam Software" on the desktop. Choose "Quick Capture" + Video mode. Click on the central button to start and then to stop the movie acquisition. Movies are saved in C:\Users\trex-bachelor10\Videos\Logitech Webcam, or another folder for which there will be a shortcut on the computer desktop.
- Try to take a movie of the cart oscillating.

- We will use **Tracker software** to study the motion of the cart. Make sure that the target is visible and in focus on all of the frames.
  - Open the video by clicking on the desired file and dragging it in the Tracker window.
  - Click on the clip settings button located on the main tool bar. This will give access to the camera frame rate and allows you to select only the relevant frames.
  - Calibrate the video: set the scale in mm using the scale attached to the set-up.
  - Set the origin and coordinate axis
  - Create a Point mass and click on the Autotracker menu. Select the image sequence, Shift-control-click it on the target to track and adjust the size and location of the zone to track. *You will get best results by selecting a zone that contains the "X" mark but that does not exceed the area of the target.*
  - Click on the button Search on the Autotracker window.
  - After the tracking is finished, you can analyze the data. For this right-click on the plot and choose "Analyse". A window named "Data Tool" opens, showing the position of the tracked point mass as a function of time. To measure the oscillation frequency, you can either measure the time required for the cart to perform n oscillations, or click on Analyse + Curve Fits.

I.2 Measure the frequency

a. Uncertainty of the measurement

We want to measure the frequency of the oscillator on several identical experiments to be able to evaluate the measurement uncertainty. Do not hesitate to add masses to the cart to decrease the oscillation frequency.

Experimental protocol:

- For a starting position x_0 = 3 cm, make five successive movies of the cart oscillations. (Make separate movies, as they will generate smaller files that are easier to manipulate).

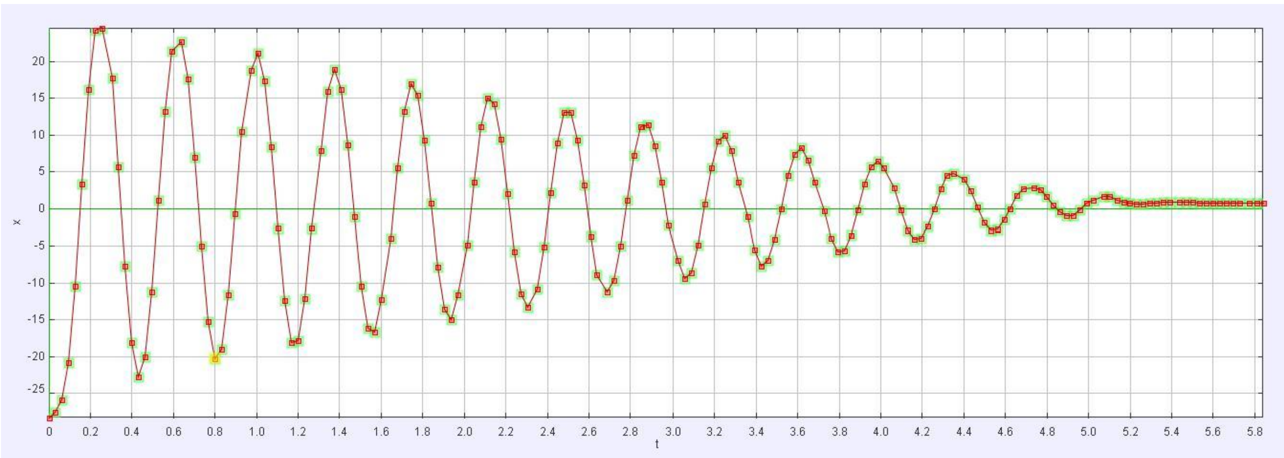
Data analysis:

- Import the first movie into the **Tracker** software. Following the procedure explained in section 1.2 and in the tracker documentation, track and locate the position of the cart as a function of time.
- Using the pointer tool of Tracker, measure the period of oscillations directly on the plot. Deduce the frequency from this measurement.
- Repeat the operation for each of the five movies.

Presentation of the results:

- Show below one of the five graphs of the position as a function of time. Do not forget to label your axes, add a title. Represent on this plot the oscillation period.

Position(x in millimeters) vs Time(t in seconds)



One period is approximately 0.37 seconds.

- Fill the table below

Trial	1	2	3	4	5
f (Hz)	*2.71 Hz*	*2.60 Hz*	*2.40 Hz*	*2.71 Hz*	*2.60Hz*

Conclusion:

- Now conclude on the cart oscillation frequency for x_0 = 3 cm, for a loading mass m_0.
- Assess the uncertainty of your measurement and present it as follow:

$$X = (\underline{X} \pm \Delta X)\ \mathrm{SI}$$

Where X is the variable you measure, $\{X_1\ X_2,\ \ldots, X_n\}$ the measured values, $\underline{X}$ the average over the n measurements (here n = 5), and ΔX the standard deviation:

$$\Delta X = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}}.$$

f≈2.60±0.13 Hz

According to you, what can be the error sources in your experiment?

The execution of the experiment could have caused some error sources in this experiment, that is in each trial since the cart was moved to the left by 3 cm by a human pulling it when letting go of it some initial speed to the left could have been given to it with varying magnitude in each trial due to the inability of the human eye and hands to "measure" or rather sense this small but important initial movement to the left. This should, in theory, have no effect on the period since the period is only dependent on the stiffness of the spring and the mass of the system. It should, in fact, only affect the phase. In reality, since the system is subject to external non-conservative forces and hence some energy is dissipated, the time period is affected by the initial velocity of the cart. Regardless, the percentage uncertainty in the value obtained is only around 4.86% which could also be attributed to random errors in the experiment.

b. Influence of initial conditions

We want to see the dependence of the oscillation frequency on the initial position.

Experimental protocol:

- Take some movies of the cart oscillation for five different starting positions x_0 (x_0 = 1, 2, 4, 5 cm).

Data analysis:

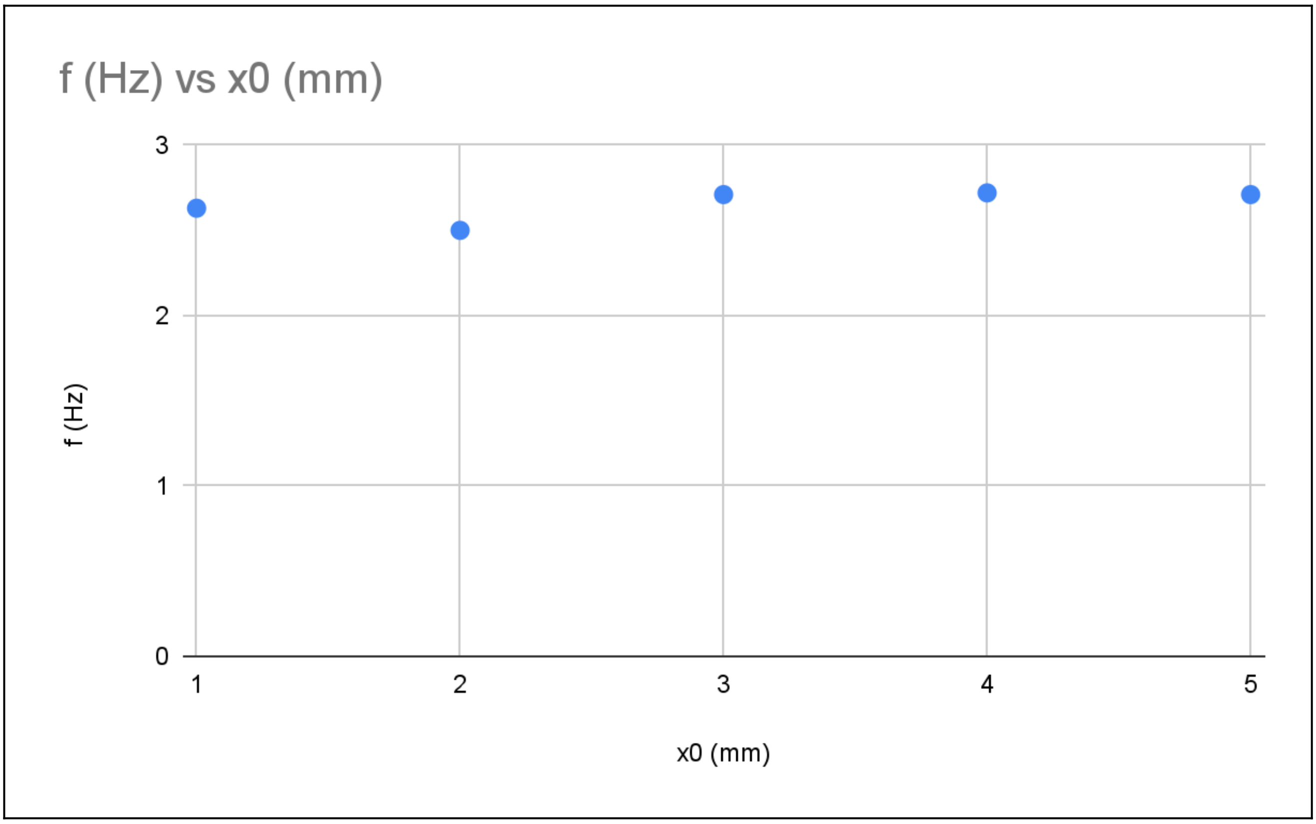
- Following the protocol explained above, measure the frequencies with **Tracker** and report them in Table 2.

Table 2: Oscillating frequency for various initial positions

x_0 (mm)	1	2	3	4	5
f (Hz)	*2.63 Hz*	*2.50Hz*	*2.71 Hz*	*2.72 Hz*	*2.71Hz*

Presentation of the results:

- Plot the measured frequencies as a function of starting position. Show the graph below. Do not forget to label your axes, add a title.



Conclusion:

- According to the precision on the measurement, what can you say about the influence of initial condition on the oscillating frequency?

Using the data obtained in part I, since the standard deviation (or uncertainty) in the data collected is lower than the uncertainty obtained in part I, the difference in the values can be attributed to the errors in the experiment making all of them approximately equal. Thus the initial position of the cart with respect to its equilibrium position (or in other words its displacement) doesn't affect the oscillating frequency. This is to be expected as, in theory, the oscillating frequency is only dependent on the stiffness of the spring and the mass of the system.

- Deduce from the oscillating frequency the equivalent stiffness k_{eq}.

We know that in theory:

$$f_0 = \frac{1}{2\pi}\omega_0 \quad = \quad \frac{1}{2\pi}\sqrt{\frac{k_0}{M_0}}$$

So:

$$k_0 = 4\pi^2 \times f_0^2 \times M_0$$

Using the data obtained in part II:

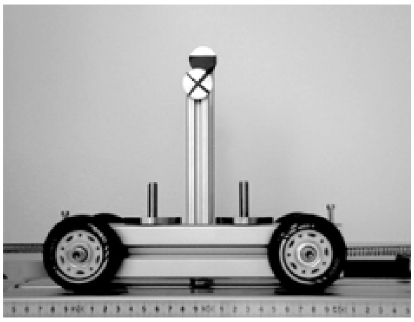
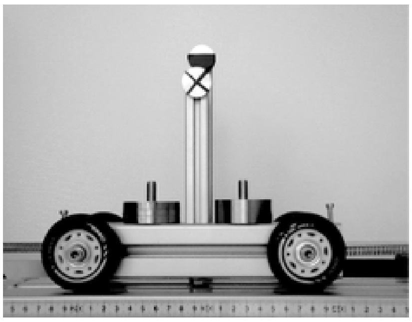
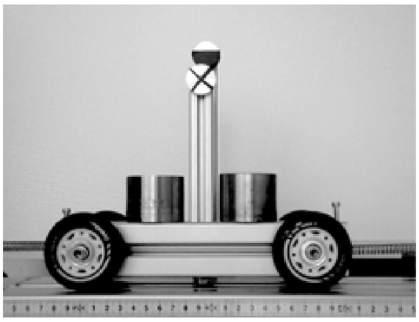
$$k_{eq} = M_0 \times (278 \pm 19)*10\text{^-3} \;\; \text{N/mm}$$

c. Influence of the mass

We now want to see the influence of the total mass on the oscillating frequency.

<u>Experimental protocol:</u>

- Take some videos of the oscillation of the cart for different mass. The cart mass m_0 is known. You have at your disposal metal plates of mass $\Delta m \approx 326$g that you can fix on the cart to vary the total mass m_0+Δm of the system.

- Run the experiment starting from x_0 = 5 cm for five different masses: without additional mass (only the cart), and adding successively weight. Report the measurements in the table below.

<u>Data analysis:</u>

- Following the protocol explained above, obtain the tracks for the target for all experiments. Keep the data as it will be useful in future sections.
- Measure the frequency as a function of the total mass and report it in Table 3.

Table 3 : Oscillating frequency as a function of the mass of the system

m_{added} (kg)	0	0.326	0.326	0.326	0.326
M_{total} (kg)	m_0	$0.326+m_0$	$0.652+m_0$	$0.978+m_0$	$1.304+m_0$
f (Hz)	2.71 Hz	2.23Hz	2.01Hz	1.89 Hz	1.73 Hz

Presentation of the results:

- Plot the measured frequency as a function of the mass of the system. Attach the graph below. Do not forget to label your axes, add a title.

We need to determine m_0.

From earlier we know that:

$$k_0 = 4\pi^2 \times f_0^2 \times M_0$$

So since k_0 is constant, by using this equation for m_0 and $0.326+m_0$ m_0 can be determined using the following equality

$$4\pi^2 \times f_0^2 \times m_0 = 4\pi^2 \times f_1^2 \times (0.326 + m_0) \quad , where\ f_0\ and\ f_1\ are\ the\ oscillating\ frequencies\ for$$

$$m_0\ and\ m_0 + 0.326\ respectively$$

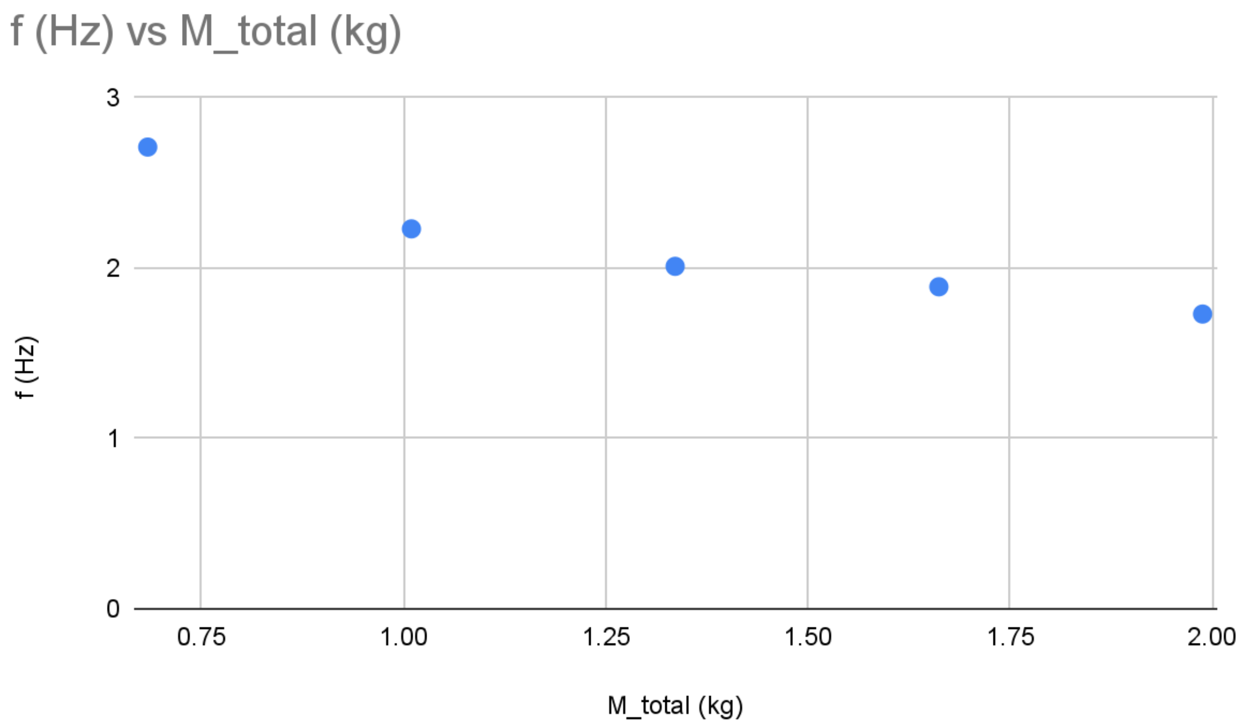
which gives:

$$f_0^2 \times m_0 = f_1^2 \times (0.326 + m_0)$$

simplifying gives:

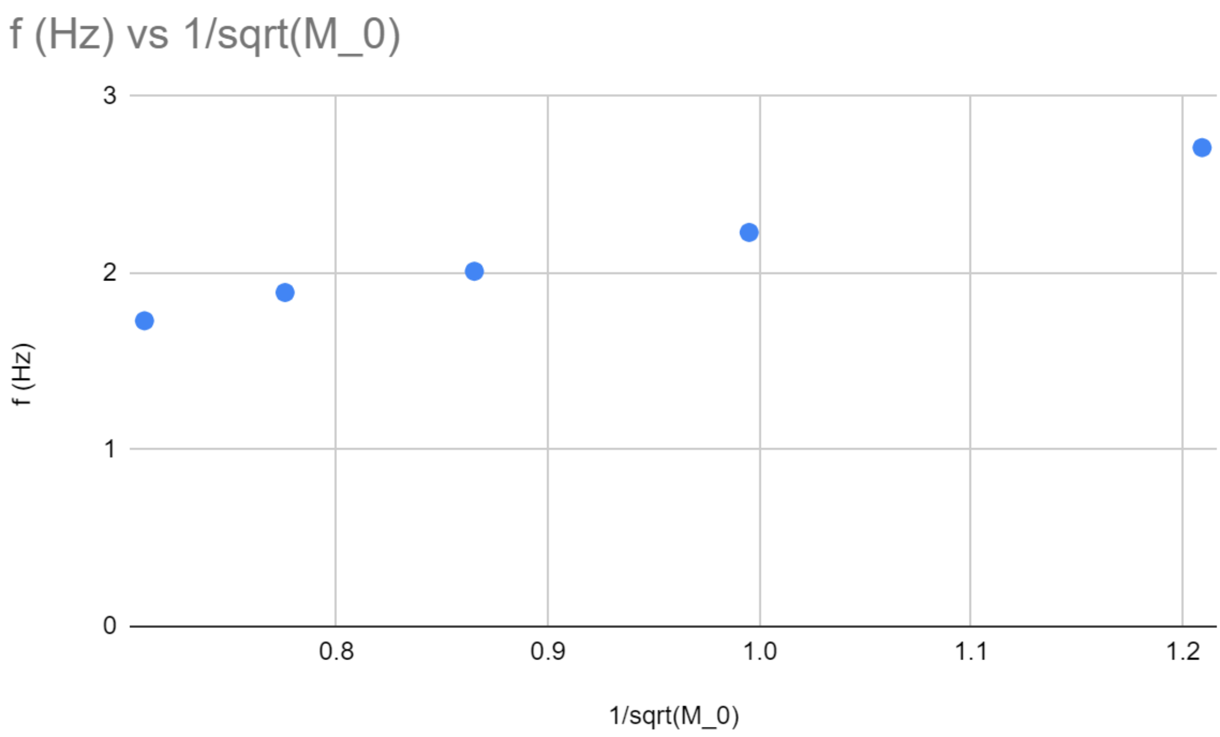
$$m_0 = \frac{0.326}{(\frac{f_0}{f_1})^2 - 1} \approx 0.684\ kg$$

Now we can plot the frequency against the total mass of the system.



- Knowing that the mass-spring system oscillating frequency scales as $f \propto \frac{1}{\sqrt{M}}$, think of a better way to present the data in order to have a straight line of know slope. Plot it and attach it below.

Plotting the oscillating frequency against $\frac{1}{\sqrt{M}}$, M is the total mass of the system should give a linear graph with slope $\frac{\sqrt{k_0}}{2\pi}$(from the equation $f_0 = \frac{1}{2\pi}\sqrt{\frac{k_0}{M_0}}$). Plotting such a graph with the values obtained from the experiment gives:



Trying a proportional fit gives a slope of 2.306 Hz*kg^½.

A linear fit instead of a linear fit is done since the frequency should approach 0 as the 1/sqrt(m_0) approaches 0, in other words, the fit should be of the form f=slope*1/sqrt(m_0).

since the slope is $\frac{\sqrt{k_0}}{2\pi}$, k_0 can be deduced to be 209.932 N/m from this graph which agrees with the value deduced for k_{eq} before since it is between 177.156 and 290.996 N/m with the given uncertainty

Analysis:

- Deduce from the measurements of frequency the stiffness k_{eq} of this system. Comment on the fact that the stiffness of the system is different from the stiffness of each spring?

As deduced before:

$$k_{eq} = M_0 \times (278 \pm 19) * 10\hat{} - 3\ N/mm$$

and $m_0 = \frac{0.326}{(\frac{f_0}{f_1})^2 - 1} \approx 0.684\,kg$

so $k_{eq} = 190 \pm 13 \times 10^{-3}\ N/mm$

The springs seem to be connected in parallel but they are actually connected in parallel as they have the same displacement instead of force applied on them. Thus, $k_{eq} = k_1 + k_2$, where k_1 and k_2 are the stiffness of the first and second spring respectively. But since they are equal in this case $k_{eq} = 2k_1$ so the stiffness of the system is double the stiffness of each spring.

- Predict what would be the equivalent stiffness in the case where the two springs are different, denoting as k_1 and k_2 their respective stiffness.

As mentioned before, $k_{eq} = k_1 + k_2$, where k_1 and k_2 are the stiffness of the first and second spring respectively.

- Springs are commonly used in cars to increase both the driver's control of the car and the comfort of the occupants. Estimate the amount by which the oscillation frequency of a small car (Fiat Panda) changes when carrying one vs. four people. How about for a large car (Mercedes S-class)?

We know that the oscillating frequency is given by:

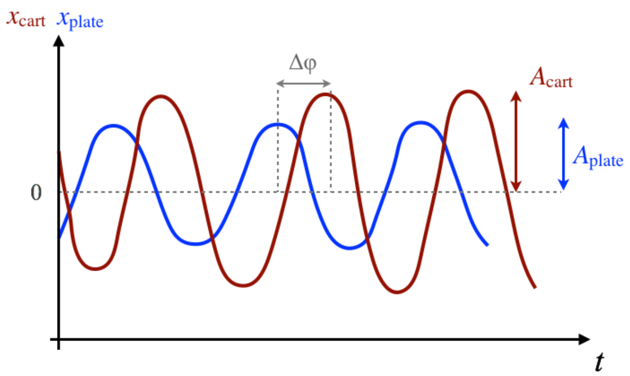
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_0}{M_0}}$$

So if the mass of each person is m and the mass of the car is M, $M_o = M + 4m$

and the frequency is $\frac{1}{2\pi}\sqrt{\frac{k_0}{M+4m}}$. When the three people get out the new frequency is $\frac{1}{2\pi}\sqrt{\frac{k_0}{M+m}}$ which is bigger since the denominator is now smaller and hence the frequency is higher. However, for a large and hence heavier car, the difference is less and hence the frequency increases by a lower amount. So the small car oscillates faster than the large car when the people get out.

I.3 Forced oscillations

We now want to study the response of the cart to a sinusoidal forcing. The cart is placed on a mobile platform that can be oscillated using a DC motor, which is connected to a stabilized power supply. We can therefore vary the oscillation frequency of the platform by modifying the voltage imposed on the motor and see how the cart responds. We will be interested in the variations of amplitude, of frequency, as well as the phase of oscillations. The quantities that we are going to measure are defined on the graph below.



Experimental protocol:

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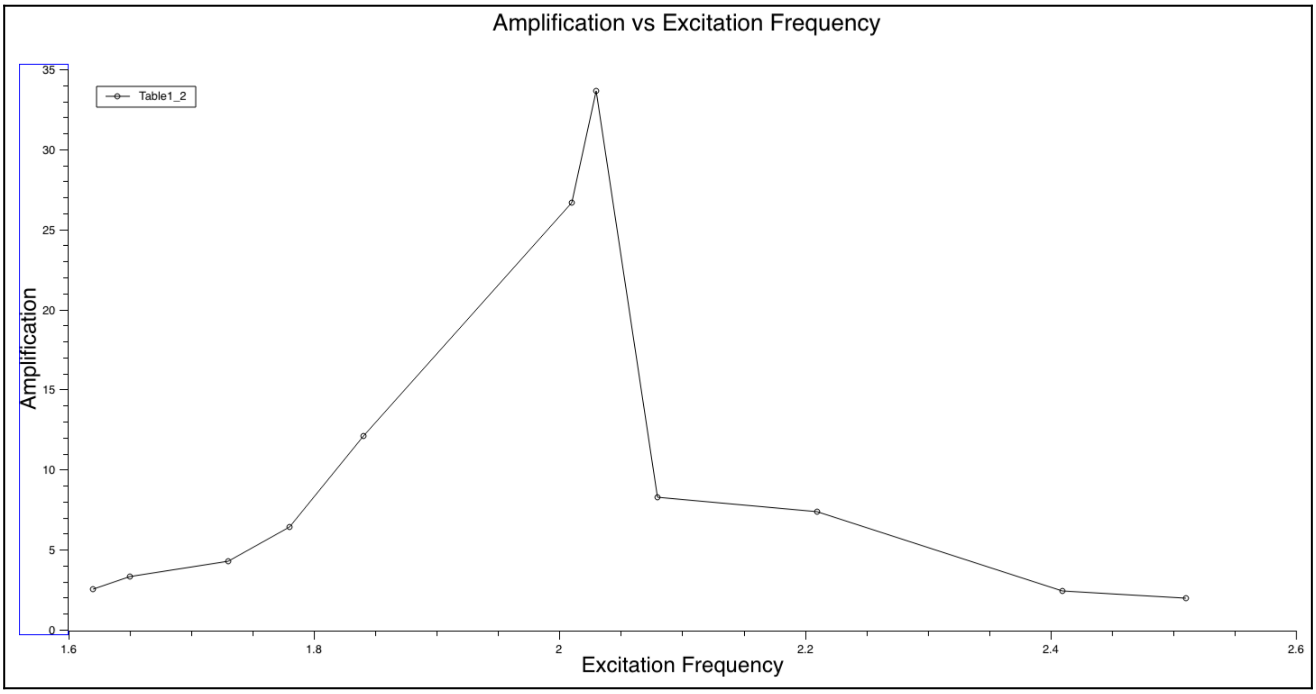
- Switch the motor power on and verify that the vibration table oscillation frequency can be tuned by varying the imposed voltage.
- Make sure that both the table target and the cart target are visible on each frame.
- Load the cart with a mass m = 652g.
- Run the motor and look for the condition that leads to the maximum oscillation amplitude. This gives you an estimate of the resonance frequency. **Note:** when you vary the frequency you should wait for several oscillations for the amplitude to stabilize.
- For 11 forcing voltages, we want to measure both the plate and the cart oscillation amplitudes, frequencies and the phase difference. Make sure to have at least 6 data points close to the estimated resonance.
- Track the position of the two targets, respectively attached to the cart to the vibrating table using **Tracker**. We denote the plate position x_{plate} and the cart position x_{cart}. From the plots of x_{plate} and x_{cart}, determine the frequencies f_{plate} and f_{cart}, the amplitudes A_{plate} and A_{cart} as well as the phase-shift $\Delta\phi$ by comparing the two signals.
  Report them in Table 5 below.

Table 5: Frequency response of the cart to forced oscillations

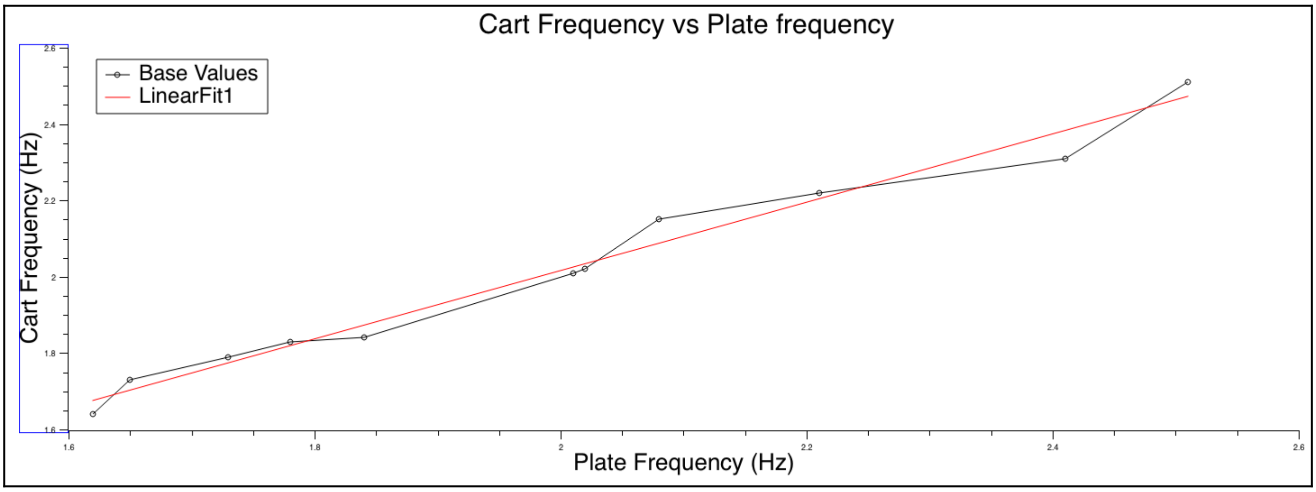
Forcing voltage (V)	6.0	6.3	6.6	6.9	7.2	7.5	7.8	8.1	8.4	8.7	9.0
f_{plate} (Hz)	1.62	1.65	1.73	1.78	1.84	2.01	2.02	2.08	2.21	*2.41*	2.51
f_{cart} (Hz)	*1.64*	*1.73*	*1.79*	*1.83*	*1.84*	*2.01*	*2.02*	*2.15*	*2.22*	*2.31*	*2.51*
A_{plate}(cm)	*0.29*	*0.27*	*0.31*	*0.27*	*0.27*	*0.26*	*0.26*	*0.24*	*0.25*	*0.24*	*0.25*
A_{cart} (cm)	*0.73*	*0.89*	*1.32*	*1.74*	*3.26*	*6.94*	*8.75*	*1.98*	*1.84*	*0.58*	*0.49*
$\Delta\phi$ (rad)	*0.032*	*0.032*	*0.0032*	*0.033*	*0.032*	*0.032*	*0.032*	*0.18*	*0.21*	*0.19*	*0.21*

Presentation of the results

- From the measurements in Table 5, plot the amplification of the movement, defined as $H= A_{plate}/ A_{cart}$, as a function of the excitation frequency. Plot $H(F_{plate})$ and report it below.

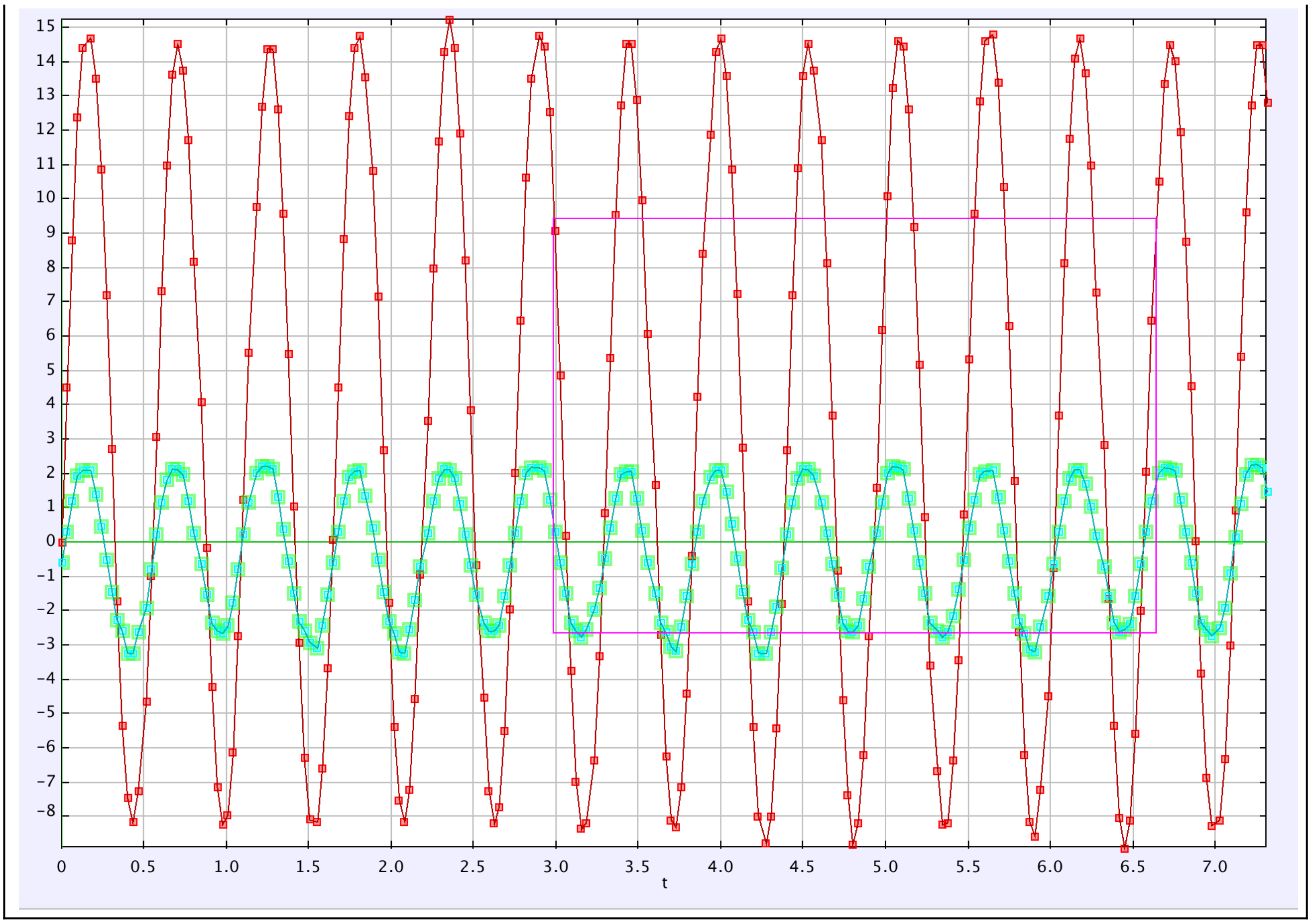


- Plot the cart frequency F_{cart} as a function of F_{plate} and report it below.



Use the "right-click – compare" function on Tracker to plot on the same graph the two oscillation signals (x_{plate} and x_{cart}). Show below the graphs for the lowest forcing frequency, close to the resonance and for the highest forcing frequency.

6.6 V f_{plate} (Hz) = 1.73 Hz



Analysis

- Comment on the evolution of the <u>frequency</u> and <u>amplitude</u> of the oscillations, as the forcing frequency is varied (Distinguish the cases of low frequency, high frequency and intermediate regimes). How is the frequency of maximum amplitude related to the oscillation frequency measured in Section 1.3?

The frequency of the cart and the plate increases and the amplitude of the cart first increases and then decreases. In the relative low and high frequencies, the amplitude is low. However, at a specific frequency, the amplitude reaches the maximum value. And the frequency of maximum amplitude is about the same as oscillation frequency.

- What do you notice about the phase difference $\Delta\phi$ between the cart and the plate?

The phase difference increases when the excitation frequency increases

BONUS : Amplitude evolution for the mass/spring system

You may have noticed that oscillations are damped after a few periods. This is due to unavoidable losses that occur during the motion. In this section we will explore the dependence of the losses on the conditions.

In the real world, free oscillations of engineering systems are damped over time by a resistive force, leading to a decrease in the amplitude oscillations.

In most cases, the resistance force is proportional to the velocity of the motion $\vec{f} = -r\vec{v}$ and is directed in the direction opposite the motion.

According to Newton's second law, one can write

$$ma = -kx - rv$$

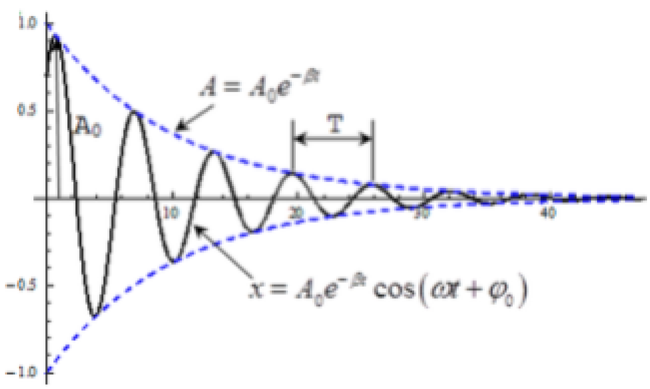
which can be recast into

$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

The harmonic frequency remains the same : $\omega_0^2 = \frac{k}{m}$, but in the presence of a resisting force we now can define the damping coefficient $\beta = \frac{r}{2m}$, which describes how the oscillations will decrease in amplitude, so that the equation writes as :

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2x = 0$$

Denoting A the initial amplitude, this second-order equation admits a solution that can be recast into $x = Ae^{-\beta t}cos(\omega t + \varphi)$ which is represented below:



Oscillations at a time t of amplitude $A(t) = A_0 e^{-\beta t}$ will be reduced so that at a time $t + \tau$, the amplitude is reduced and equal to $A(t + \tau) = A_0 e^{-\beta(t+\tau)}$.

We now introduce the decrement damping δ, which is the ratio of the amplitudes which are separated in time by a period and equal to $\delta = \frac{A(t)}{A(t+T)} = e^{\beta T}$.

For practical reasons, we define $\lambda = ln(\delta) = \beta T$.

Preliminary remarks

- What are the units of β, λ and δ?

β: s-1 λ: no unit δ: no unit

Experimental protocol:

- For this section we will re-use the data obtained in the previous section. **No new experiments are required**.

Data analysis:

Following the protocol explained above, measure the position of the cart as a function of time with **Tracker.** Measure the maximum amplitude A_n, n being the number of period. Report the results in Table 6 below.

Table 6: Amplitude evolution

M_{total}(g)					
A_0/A_1					
A_1/A_2					
A_2/A_3					

A_3/A_4					

Analysis

- For each of the masses, plot $\lambda = \ln\ (\delta)$ as a function of n, the number of periods, and show the graph below

Conclusion

- Deduce the damping coefficient β
- Does the rate of decrease of amplitude depend on the added mass?
- Based on the above observation, what do you expect is the physical origin of the damping? How can it be modified?

II - Epilogue:

Now recall that musical notes are the propagation of vibrations from musical instruments making their way through the air to your ears. With what we have learned from this lab and natural frequencies, let’s take a peak at a music box. A music box is a device that spins a drum with knobs that bend and release a comb fork, so that the fork vibrates (see Fig. 97 for what a music box looks like and Fig. 98 for the drum loading and releasing the forks):

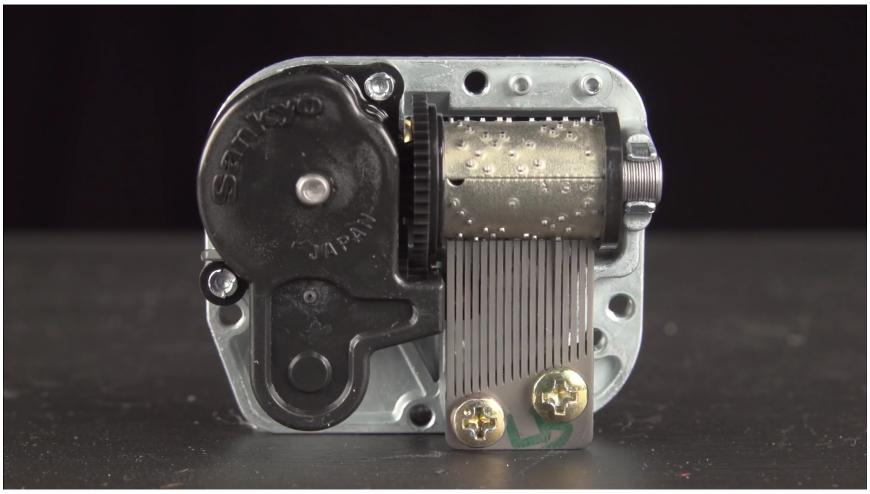


Figure 97: Music box.

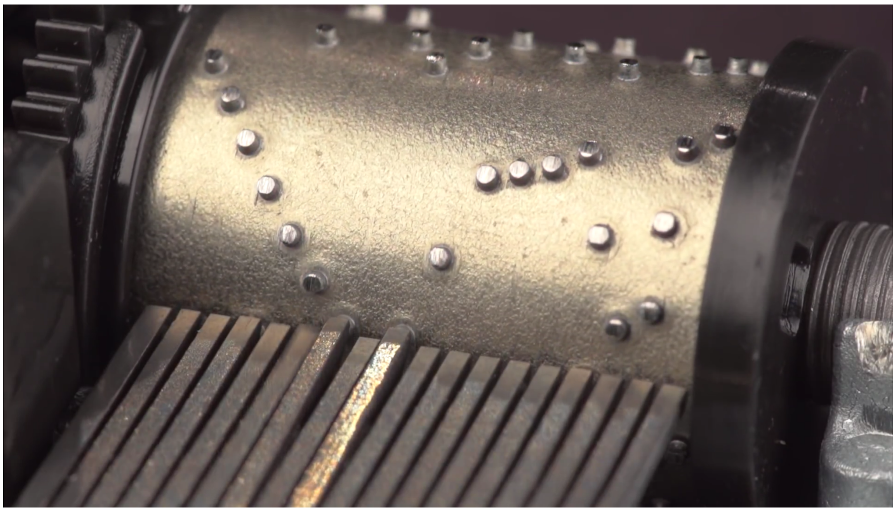


Figure 98: Rotating drum loading and releasing the comb's forks.

The forks vibrate at specific frequencies to produce the notes that you hear. Additionally, these comb forks are typically weighted on the one end, with varying weights.

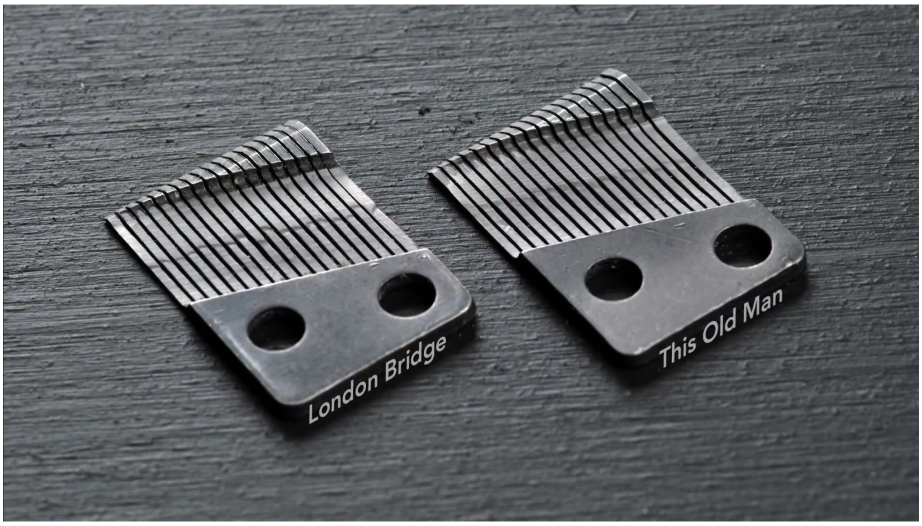


Figure 99: View of the music box combs, showing the weighting of the forks.

Let's consider a few questions about the design of the music box comb:

- Why might the music box comb forks be weighted at the one end given what we have learned in this lab?
- Why are the weights different on the different forks?
- How could you change the weight of the forks to get different notes?
- What would you do if you couldn't change the weighting of the forks but wanted different notes to be played?
- Building off the above questions, what do you think occurs with other musical instruments to produce different notes, such as with a kalimba?