PHY101 - MECHANICS AND HEAT

Homework 1

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IP PARIS

CONTENTS

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1.1 A

We take system {M} and ground as initial reference frame. the M is apply by three forces.

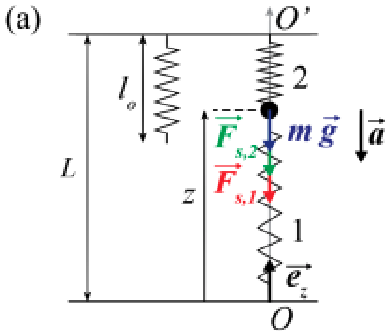

Figure 1: The instruction graph of the string

$\overrightarrow{F_{s,1}}$ is the force from bottom of string.
$\overrightarrow{F_{s,2}}$ is the force from top of string.

Therefore we get the system of equation

$$\begin{cases} \overrightarrow{F_{s,1}} = -k\left(z_e - l_o\right)\overrightarrow{e_z} \\ \overrightarrow{F_{s,2}} = k\left(L - z_e - l_o\right)\overrightarrow{e_z} \end{cases}$$

Since the system is in equilibrium position, therefore

$$\begin{aligned} 0 &= -mg - k\left(z_e - l_o\right) + k\left(L - z_e - l_o\right) \\ \Rightarrow 2kz_e &= kL - mg \\ \Rightarrow z_e &= \frac{L}{2} - \frac{mg}{2k} \end{aligned}$$

1.2 B

From the Newton's first law, we get: $m\vec{a} = m\vec{g} + \overrightarrow{F_{s,1}} + \overrightarrow{F_{s,2}}$

The projection on the z-axis:

$$\begin{aligned}
&m\ddot{z} = -mg - k\left(z - l_o\right) + k\left(L - z - l_o\right)\\
\Rightarrow &m\ddot{z} = -2kz - mg + kL = -2k\left[z - \left(\frac{L}{2} - \frac{mg}{2k}\right)\right]\\
\Rightarrow &\ddot{z} + \frac{2k}{m}\left(z - z_e\right) = 0
\end{aligned}$$

1.3 C

We define $u(t) = z(t) - z_e$. From the previous equation, we get $(\ddot{u} = \ddot{z})$:

$$\ddot{u} + \omega_o^2 u = 0, \text{ with } \omega_o = \sqrt{\frac{2k}{m}}$$

1.4 D

Solutions to the differential equation are of the form $u(t) = A\cos\left(\omega_0 t + \varphi\right)$. At initial time $t = 0 : u(0) = a$ and $\dot{u}(0) = 0 \Rightarrow A\cos\varphi = a$ and $-A\omega_0\sin\varphi = 0 \Rightarrow A = a$ and $\varphi = 0$. We thus have:

$$\begin{aligned}
&u(t) = a\cos\omega_o t\\
\Rightarrow\ &z(t) = u(t) + z_e = a\cos\omega_o t + z_e
\end{aligned}$$

2

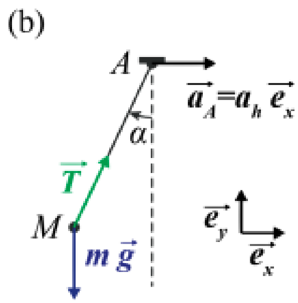

Figure 2: The instruction graph of the tension

2.1 A

We take the system={M} and the frame of reference as ground also considered inertial, the force we have here is weight we have $m\vec{g}$ and tension $\vec{T}$. From the Newton's first law we have:

$$\overrightarrow{0} = m\vec{g} + \vec{T} \Rightarrow \vec{T} = -m\vec{g} \Rightarrow \|\vec{T}\| = mg$$

2.2 B

Newton's second law:

$$m\overrightarrow{a_v} = m\vec{g} + \vec{T} \Rightarrow \vec{T} = m\left(a_v + g\right)\overrightarrow{e_z} \Rightarrow \|\vec{T}\| = m\left(a_v + g\right)$$

The cable is subjected to a higher tension due to the upward acceleration of the load. It feels as if it was lifting a heavier load.

2.3 C

M is stationnary with respect to A, then its acceleration $\overrightarrow{a_M} = \overrightarrow{a_A} = a_h\overrightarrow{e_x}$.

From Newton's second law, we have

$$m\overrightarrow{a_M} = m\vec{g} + \vec{T}$$

Projection on the x - axis: $ma_h = T\sin\alpha$
Projection on the y - axis: $0 = -mg + T\cos\alpha$
We thus get $T\sin\alpha = ma_h$ and $T\cos\alpha = mg$. yielding $\tan\alpha = \frac{a_h}{g}$