PHY101 FINAL PROJECT

Physics in the Kitchen

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CONTENTS

1
MEASURE OF AN OIL VISCOSITY

1.1 Preliminary Measurements

- Ball mass: $m_{\text{tol}} = 0.2$ g
- Ball radius: $r_{\text{bat}} = 1.6$ cm
- Density of the oil: $\rho_{\text{oil}} = 941 \text{Kg/m}^3$

1.2 Equation of the problem

1.2.1 • Forces acting on the ball

From the description of the problem, we derive that weight and the viscous frictional force are acting on the ball. Weight is given by $\vec{W} = m\vec{g}$ and drag is obtained by Stoke's formula $\left(\vec{F}_d = -6\pi n\vec{r}\right)$). We obtain the following free body diagram from said equations:

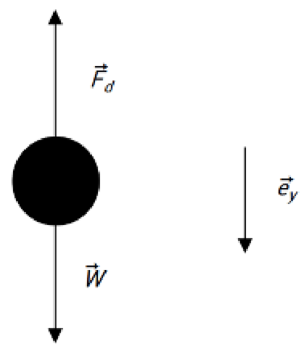


Figure 1: Free Body Diagram of the ball

We will also choose the origin to be on the starting position, with the positive y axis pointing downwards.

Principle used to find the equation of the movement of the ball: Newton's Second Law

1.2.2 • Equation of the Movement

By applying Newton's Second law we get:

$$ma\vec{e}_z = \vec{W} + \vec{F}_d = mg\vec{e}_z - -6\pi\eta rv\vec{e}_z$$

and $m\frac{dr}{dx} = mg - 6\pi nrv$ if we project it along the z axis

1.2.3 • Expression of the speed

By solving said differential equation with the integrating factor method we obtain:

$$\frac{dv}{dt} + \frac{V}{T} = \frac{v_\infty}{T}$$
$$\leftrightarrow e^{\frac{t}{\pi}}\frac{dv}{dt} + e^{\frac{t}{T}}\frac{V}{T} = \frac{d}{dt}\left(e^{\frac{t}{\tau}}v\right) = e^{\frac{t}{\pi}}\frac{V_\infty}{T}$$
$$\rightarrow v(t) = V_\infty + ce^{\frac{t}{T}}$$

Where c is a constant which depends on the initial conditions. Since we drop the ball with no initial speed, $v(O) = 0$, and:

$$v_\infty + c = 0 \rightarrow c = -v_\infty$$

Hence, speed is given by:

$$v(t) = v_\infty\left(1 - e^{-\frac{t}{\tau}}\right)$$

1.2.4 • Physical meaning of V_∞ and r

From the equation of movement we can derive the expressions of v_∞ and T :

$$\frac{dv}{dx} + \frac{6\pi\eta rv}{m} = g$$
$$\rightarrow v_\infty = \frac{mg}{6\pi\eta r},\ \mathrm{T} = \frac{m}{6\pi\eta r}$$

From the equation of speed, we derive that $\lim_{t\to+\infty} v(t) = v_\infty$. Therefore, v_∞ is the terminal velocity of the ball, i.e. the maximum velocity attainable by an object as it falls through a fluid. At this speed, the ball's net force is zero as the external forces cancel out. We can observe this by inputting $v = v_\infty = \frac{mg}{6any}$ into the equation of movement:

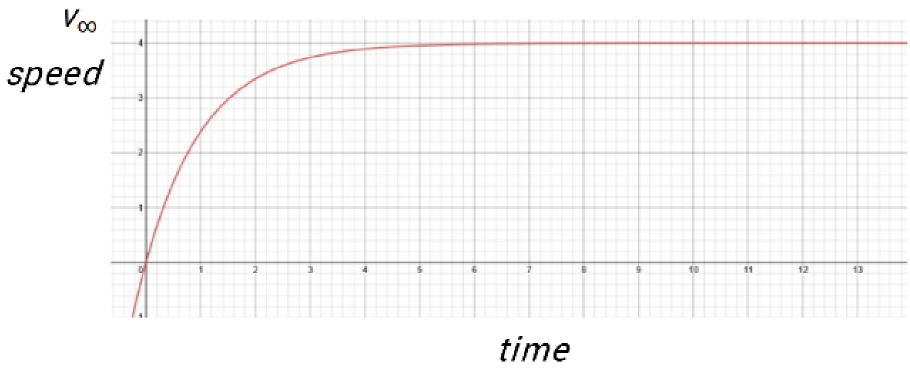
$$mg - 6\pi\eta rv_\infty = mg - 6\pi\eta r\frac{mg}{6\pi\eta r} = 0$$

On the other hand, T determines how much time it takes for the ball's speed to be very close to terminal velocity. From the equation of speed, we derive that v_∞ is reached faster for low values of T and it is reached slower for higher values of T. Moreover, from the equation for T we know that its dimension is time.

1.2.5 • Description of the movement of the ball

By inputting the equation of speed on Desmos, we obtain the following graph

(where speed is on the y-axis and time is on the x-axis):
As seen on the graph, speed will start being zero, only to increase rapidly until it approaches



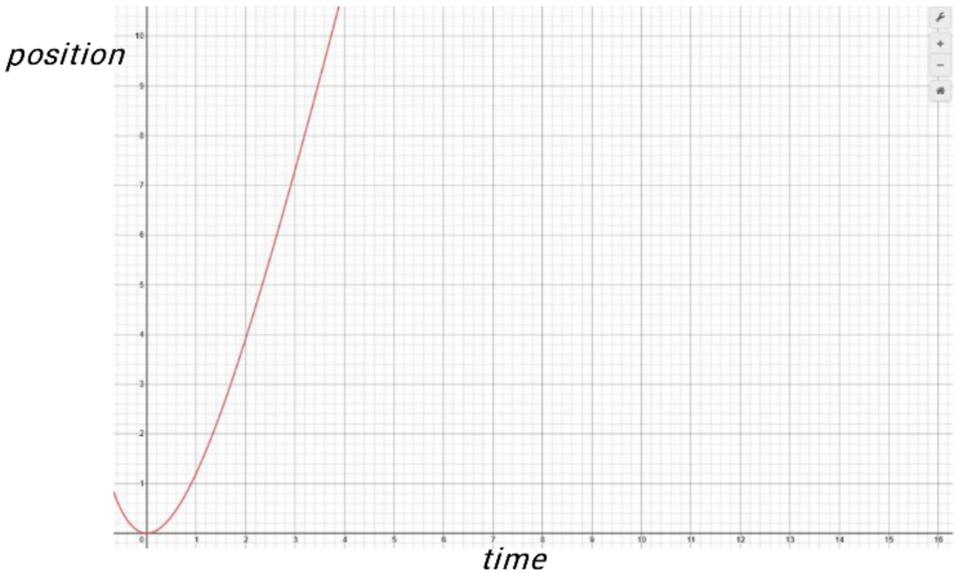
terminal velocity. At this point, speed will almost remail constant. Since position is given by the integral of velocity with respect to time, we can obtain an equation for its position:

$$x(t) = \int vdt = v_\infty \left(t + \mathrm{T}e^{-\frac{t}{\mathrm{T}}}\right) + c$$

Where c is a constant to be determined from the initial conditions. Due to $x(O) = 0$,

$$c = -v_\infty \mathrm{T}$$
$$x(t) = v_\infty \left(t + \mathrm{T}\left(e^{\frac{t}{T}} - l\right)\right)$$

By inputting this new equation into Desmos, we can observe the trajectory of the ball with respect to time.

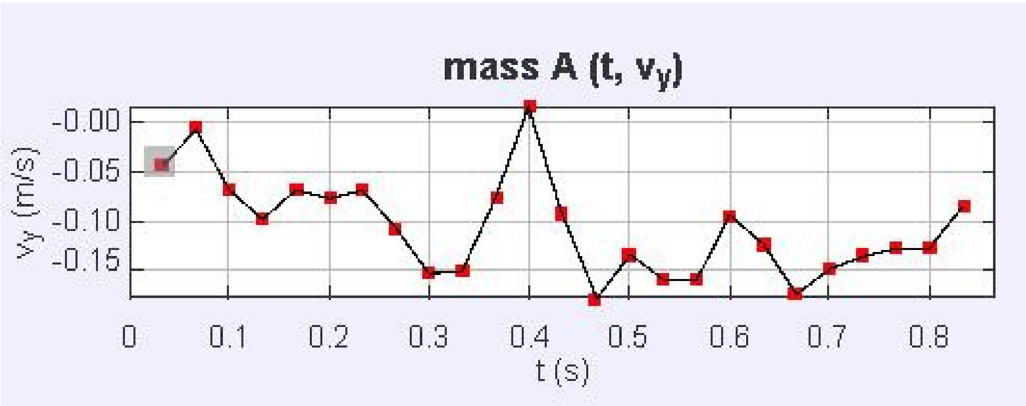


1.3 Measurement of the oil viscosity

1.3.1 • Terminal Speed

To measure the terminal velocity, we took a video of a pill (unfortunately we could not find a better spherical object) being dropped into a glass of sunflower oil and we uploaded it into Tracker. In order to facilitate the detection of the pill by Tracker, we placed a lamp opposite to the camera. We proceeded to use the auto tracker software to obtain a graph of the pills velocity with respect to its speed.

Unfortunately, the y axis could not be properly defined, which resulted in a negative velocity on the graph.



We did not take into account the peak in which velocity reaches zero as this was due to the pill not being tracked by the program. From said graph, we chose the last points in which speed appeared to be constant and took the average, ending up with:

$$v_\infty = 0.147 \text{ m/s}$$

1.3.2 • Viscosity

We can obtain the viscosity from the previous data and the equation for terminal velocity:

$$v_\infty = \frac{mg}{6\pi\eta r} \to \eta = \frac{mg}{6\pi v_\infty r}$$
$$\eta = \frac{0.2 * 10^{-3} * 9.81 \text{ m/s}^2}{6\pi * \frac{0.147 \text{ m}}{s} * 1.6 * 10^{-2}} = 4.6 * 10^{-2}\text{Kg/ms}$$

By searching for the dynamic viscosity of sunflower oil, we obtained that it should be around 4.538×10^{-2} Kg/ms (source: link).

1.3.3 • Characteristic Time

We use the previous graph to find a point in which the velocity is closest to 95% of the terminal velocity and divide the time taken to reach said point to calculate the characteristic time. Hence obtaining:

$$\mathrm{T} = \frac{2.45\ \mathrm{s}}{3} = 0.82\ \mathrm{s}$$

Moreover, we can predict t thanks to our initial equation:

$$\mathrm{T} = \frac{m}{6\pi\eta r} = \frac{0.2 * 10^{-3}\mathrm{Kg}}{6\pi * 4.6 * \frac{10^{-2}\mathrm{Kg}}{ms} * 1.6 * 10^{-2}\ \mathrm{m}} = 0.0144\ \mathrm{s}$$

As observed above, the results do not match which could be due to some errors caused by Tracker as we were unsure at what exact time the pill reached a steady velocity due it fluctuating heavily.

2
HOW HIGH DOES A POPPING CHAMPAGNE CORK FLY

Clumsy ejection of champagne corks causes many inconveniences, including the emergency landing of an airliner due to the oxygen masks coming out when a cork hits the ceiling of the plane, or a Victorian painting being damaged during an auction.

In this part, you will estimate the height reached by a champagne cork after expulsion, by studying energy transfers from one form to the other during the process. In the first part, you will consider the motion of the cork right after being expelled from the bottle (projectile motion). In the second part you will consider the complete ejection process.

2.1 Preliminary measurements

Before getting started, you need to measure some of characteristics of the cork and the cylindrical bottle neck, which will be involved in the problem.

Measure of the parameters of the problem:

- length of the cork initially in contact with the bottle neck: $I_{neck} = 0.0013m$
- radius of the bottle neck: $r_{neck} = 0.008m$
- mass of the cork: $m_{cork} = 8.9g$

2.2 Projectile motion of the cork

NOTE: the experiment of this part can be a little more challenging to implement, so if you don't manage to do it, don't waste too many bottles of champagne, you can use the video of popping champagne bottle available on Moodle. For a cheaper experiment, you can also use cider bottles.

This first part considers only the vertical projectile motion of the cork right after being expelled. You will measure the ejection speed v0 of the cork and determine the maximal height that is reached.

2.2.1 • Equation of the problem

First of all, let's determine theoretically the motion of the ball based on energy considerations.

Question: Is the mechanical energy of the cork conserved post-ejection? Determine the maximum height reached H as a function of its initial ejection speed V_0.

We know that the cork will be flying through air. This tells us that there will be some air drag acting on the cork. With drag being a non-conservative force, the total mechanical energy won't be saved.

As for the height, we know that the cork will stop at the maximum height. Thus, we use that fact to get:

$$H_{\max} = \frac{v_O^2}{2g} - \frac{w_{drag}}{mg}$$

or, if we neglect air resistance:

$$H_{\max} = \frac{v_0^2}{2g}$$

2.2.2 • Evaluation of the initial speed v_0

NOTE: Be very cautious during the experiment and make sure not to stand in the way of the popping cork. We also recommend to conduct the experiment outdoor to avoid damaging your ceiling.

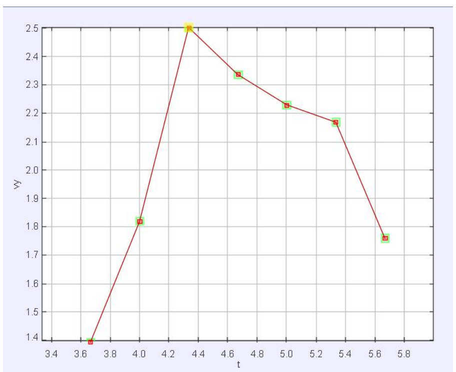
Carefully remove the muselet (wire cage that fits over the cork) from the bottle and take a movie of the cork as it slides out of the bottle and get ejected. If you are filming with a recent cell phone, adjust the parameter of the image acquisition to have the highest possible frame rate (100 frames per second is a good frame rate). Extract the initial speed v_0 from your video.

If you home set-up allows for it, film the entire trajectory of the cork to find out its maximum height.

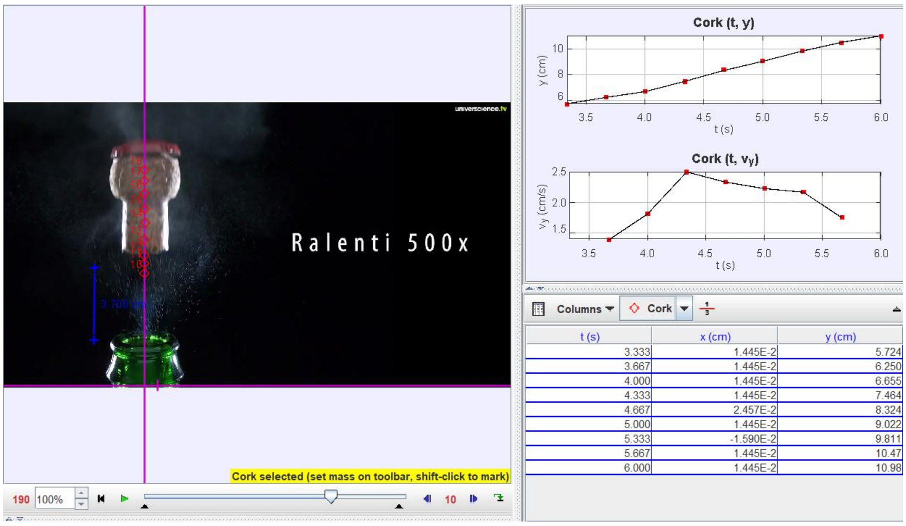
Question: Detail your measurement procedure to get v_0: (you can use images and schematics)

Using tracker, we made three different measurements:

Thus, we got initial velocities: 15 m/s, 10.5 m/s and 15 m/s, with the mean average being 12.625 m/s. Velocity unit: cm/s, time unit: s



We had to locate the point manually in Tracker, so the data is not entirely accurate. However, the velocity does match universal standards that we found



Question: Experimental value of the initial speed V_0 and the final height H_{exp} (if you managed to measure it)
$V_0 = 12.625m/s$
$H_{exp} = N/A$

Question: Based on the value of V_0 measured experimentally, what should be the maximum height reached by the cork? How does it compare with the actual height H_{exp} (if you managed to film the full cork motion)?
Based on our measurement, the height should be: 8.12m

2.3 Complete ejection process

The carbon dioxide dissolved in the champagne is in equilibrium (HENRY equilibrium) with the carbon dioxide gas in the air so that the pressure under the cork remains constant at about 7 bars at 9°C. When the muselet is removed, the cork slowly moves up along the bottle neck before getting expelled. We will now analyze energy transfer during the whole expulsion process to estimate the height reached by the popping cork. In particular, we will estimate the work of pressure forces and friction forces exerted by bottle neck.

2.3.1 • Data of the problem

- Gravity acceleration: g = 9.8 ms^{-2}
- Pressure inside the bottle: P = 7 bar

- Friction force: $\vec{f}_{\text{friction}} = -kS_{lat}\vec{a}_z$ with k the friction coefficient (to be determined), S_{lat} the lateral contact surface between the cork and the neck and $\vec{u}_z$ the vertical axis oriented upwards. (NOTE: This expression may seem contradictory with the notion introduced in class that friction does not depend on the surface of contact. Here, it reflects the fact that, as the slides out of the bottle neck, the force per unit area pressing it against the bottle remains the same while the area of contact decreases.)
- The friction force of the air is neglected once the cork is expelled.

2.3.2 • Formalizing the problem

We consider that the ascension of the cork along the neck is a very slow process and that the pressure inside the champagne bottle is maintained at the saturating vapour pressure P . Figure 1 represents the schematics of the ejection process.

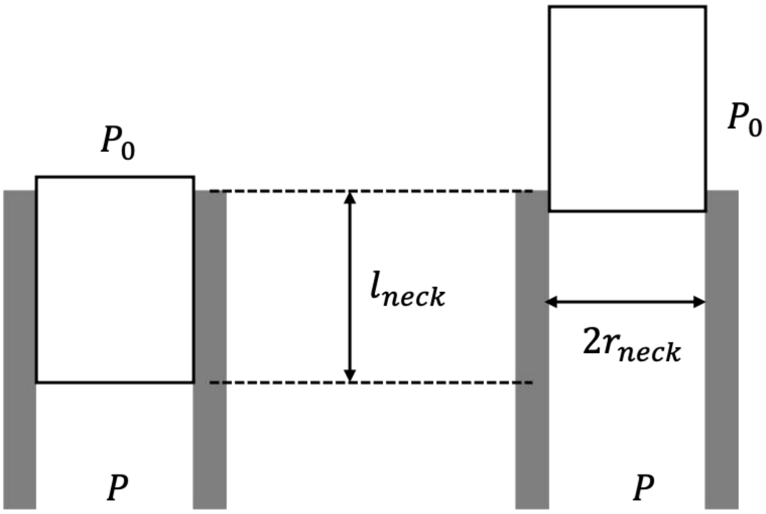
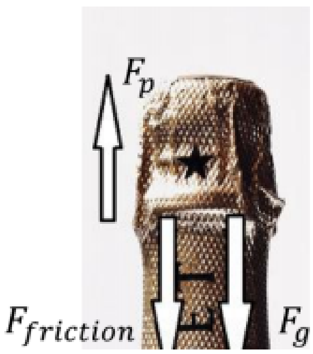
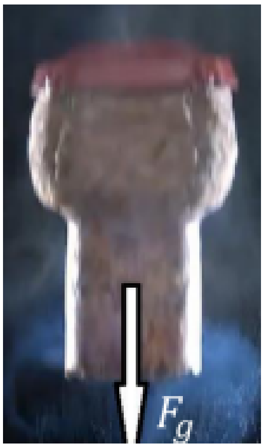


Figure 2: Schematics of the complete ejection process

Question: Forces acting on the cork before ejection: (draw the free body diagram for the cork)



Question: Forces acting on the cork after ejection: (draw the free body diagram for the cork)



2.3.3 • Work of pressure forces

Determine the work done by pressure forces during the ascend of the cork in the bottle neck.

Since the process is quick, and the volume change isn't large, we will assume the pressure stays constant, and thus, the pressure force work stays constant.

$$W_p = (P - p_0)\pi r^2 l$$

2.3.4 • Work of friction forces and determination of the constant k

Determine the work done by friction forces during the ascend of the cork in the bottle neck.

Since the lateral are changes, we will have to integrate our work formula:

$$W_f = \int_i^f FdOM = \int_i^f -k2\pi rldOM = k2\pi r \left(\frac{1_1^2}{2} - \frac{1_f^2}{2}\right) = k2\pi r \frac{1^2}{2}$$

To determine k, we will use the equilibrium condition before the cork starts moving, when friction force and pressure forces balance out (we neglect weight here).

Equilibrium on the cork before movement:

At the equilibrium the friction force of the full lateral area is equal to the pressure force:

$$-k2\pi l = (P - p_0)\pi r^2$$

Expression of the friction coefficient k:

$$k = (P - p_0)\frac{r}{2l}$$

2.3.5 • Estimation of H

After moving up the bottle neck, the cork is expelled and has a vertical projectile motion. To determine the height H reached by the cork, let's consider energy transfers between the initial time (the cork starts to move) and the final one when it reaches its maximal height.

Question: Using the Work-Kinetic energy theorem between those two instants determine the height reached by the cork.

We know that there is work done by the pressure force and friction force, and it will give us the initial kinetic energy of the cork. That energy will be converted into gravitational potential energy, leaving us with the equation:

$$W_p + W_f = mgH$$

$$H = (P - p_0)\pi r^2 \frac{l}{2}$$

Question: Compute the final height. Compare it to the height found with the first method. (Highlight the assumptions made in both cases and the validity of these assumptions).

$$H = 8.97m$$

3
HARD-BOILED OR FRESH EGGS

Question: Explain your experimental procedure and give an approximate value of the angular speed for which the egg stands upright: (you can put schematics or images here)

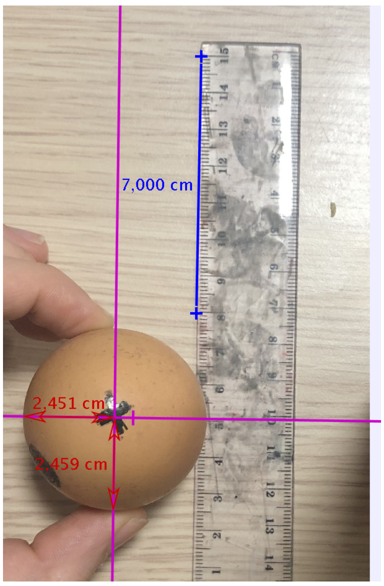
In order to determine the angular speed for which the egg stands up, we will follow the following experimental procedure.

Firstly, we will determine the key characteristics of the hard-boiled egg (cooked in boiling water for approximately 15 minutes): its dimension and mass.

We measured the egg's mass by using a digital scale: 69g. We measured the egg's width (a) and height (b = c) using the Tracker software. To do so, we proceeded to take an image of the egg from above lying both on its horizontal and vertical axes (see Figure 1), using a millimeter-measured ruler as a sizing reference. However, we estimate there to be uncertainty due to human error when taking the images or distortion of the camera lens.

We obtained the following measurements:

- m = 69 g
- a = 3.246 cm
- b = c = 2.456 cm (average of the three obtained values)



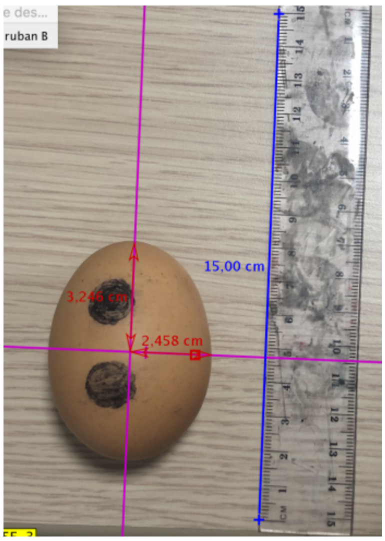


Figure 3: Image and measurements of the egg's key characteristics

With this data in hand, we proceeded to evaluate the angular speed for which the egg stands up.

In order to do so, we made several videos of the egg being released at various speeds, using a smartphone camera. We chose to film these experiments from two different angles:

- horizontally, placing the smartphone on the ground, against a square, to be assured that the videos were being taken arallel to the central axis of the egg when released (see figure 4)

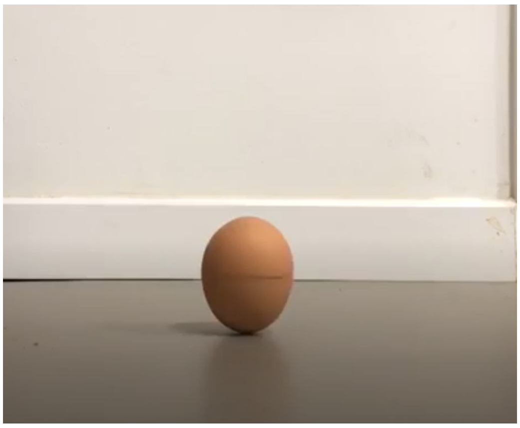


Figure 4: Screenshot of a video taken of the egg rotating on its vertical axis.

- And from above, following a similar procedure

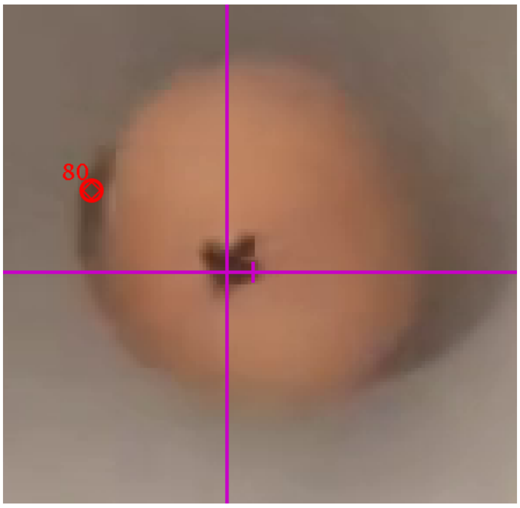


Figure 5: Figure 4: Screenshot of a video taken of the egg rotating on its vertical axis

We then proceeded to film a video in which the egg successfully rose to its vertical axis at, and imported in on Tracker, to determine the releasing speed which allowed this to be possible.

We encountered other problems when analyzing the egg's rotation. Notably, the mark drawn on the egg was seemingly too thin and the frame rate of the smartphone camera we used was quite low, so it was difficult to mark the specific point in each frame on the tracking software. In order to minimize this imprecision, we filmed the experiment with a slow motion camera, at 240 fps, which was the highest frame rate available on our smartphone.

In order to track the egg's angular velocity at the critical "tilting" moment (we chose to track this velocity as the egg was falling down from its vertical axis to its horizontal axis), we drew a mark in the center of the egg's axis of rotation (extremity of the "a" axis) and set it as the center of our frame of reference. (See figure 4). Given that the egg hovered or moved around when rising onto its vertical axis, we redefined this reference point in each frame, so that it followed the egg's movement.

Using Tracker, we tracked the egg's angular frequency by marking the cross drawn on the egg's shell (see red mark in Figure 4) as a "point mass" and then implementing the "self tracker method" (which tracks this point at every frame during the critical tilting) and found the various moments (times) at which the cross appeared in the front of the egg.

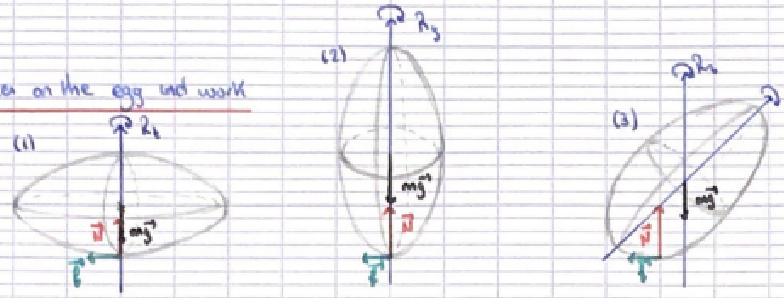
We considered other possible tracking methods such as calculating the difference between neighboring apparitions of the point mass(T1 - T0, T2 - T1, T3 - T2, each equal to ΔT) at a same point and determining the angular frequency of the egg by calculating the average of these time differences. Then, multiplying by 2pi, determining the egg's critical angular velocity. However, we considered that this method, given its manual nature, would be too imprecise.

Question: Repeat the experiment with a fresh egg. What do you observe? Does the fresh egg stand up? In your opinion, why?

After repeating the experiment with a fresh egg, we notice that the egg does not stand up. We can conjecture that the fresh egg's center of gravity is not constant, given that the fluid within the egg moves around and exerts forces against the inside of the shell as well as within the fluid. Whereas, once it is hard-boiled, the liquid inside the egg becomes firm and the center of gravity of said egg becomes constant, thus allowing friction to help it tilt up to its vertical axis of rotation.

Question: What are the forces applied on the egg? Which of them have a non-zero work during the movement?

· Forces on the egg and work



. One can see that during the whole movement, the forces applied on the egg are:

. The friction force : $\vec{f}$ opposing motion

. The normal force : $\vec{N}$ perpendicular to the support (table) upwards

. The weight : $m\vec{g}$ vertical downwards.

. We recall that for rotational movements, $W = \int \vec{\tau} \, d\theta$.

One can see that for situations (1) and (2), when the egg is rotating solely around its minor or major axis, $\vec{\tau}_N = \vec{0}$ and $\vec{\tau}_w = \vec{0}$ since $\vec{N}$ and $m\vec{g}$ are colinear to the axis of rotation.

Therefore, when the egg is turning either on its minor axis or major axis, weight and normal force have zero work.

Friction will always be opposing motion, so it will provide non zero work during the whole motion.

When the egg is rising, both normal force and weight will provide work as the egg will be rotating on two axes for that period of time.

Question: What is the kinetic energy of the egg rotating at an angular speed ω? How high above the table is its center of mass? Determine its mechanical energy Ez m.

. We consider the egg rotating on its minor axis. (situation 1)

We recall that the kinetic energy was given to be $K = \frac{1}{2} I \omega^2$

Then, $K_z = \frac{1}{2} I_z \omega^2$ and $I_z = \frac{m}{5}(a^2 + b^2)$

Thus, $K_z = \frac{1}{2}\left(\frac{m}{5}(a^2+b^2)\omega^2\right) = \frac{m}{10}(a^2+b^2)\omega^2$.

The egg is at height $h_z = a = c$ above the table.

Therefore, one gets $E_m^z = \frac{m}{10}(a^2+b^2)\omega^2 + mga$

. Similarly, we consider the egg rotating on its major axis (situation 2)

We have $K_y = \frac{1}{2} I_y \omega^2$ and $I_y = \frac{m}{5}(a^2+c^2) = \frac{2m}{5} a^2$

Thus $K_y = \frac{1}{2}\left(\frac{2}{5} m a^2\right)\omega^2$

$= \frac{1}{5} m a^2 \omega^2$

Then, one gets $E_m^y = \frac{1}{5} m a^2 \omega^2 + mgb$ since the egg is at height b above the table

Question: In this second configuration, what is the kinetic energy of the egg rotating at an angular speed ω? How high above the table is its center of mass? Determine its mechanical energy Ey m.

. The egg rises when the mechanical energy associated with the rotation around axis y and z are equal. Then, in order to find the critical angular momentum ω_c one needs to set $E_m^y = E_m^z$

Then, $\frac{1}{5} m a^2 \omega_c^2 + mgb = \frac{m}{10}(a^2+b^2)\omega^2 + mga$. Multiplying by 2 and dividing by m.

$\frac{2}{5} a^2 \omega_c^2 + 2gb = \frac{1}{5}(a^2+b^2)\omega^2 + 2ga$

so $-\frac{1}{5} a^2 \omega_c^2 + \frac{1}{5} b^2 \omega_c^2 + 2g(a-b) = 0$.

$\frac{1}{5}\omega_c^2 (b^2 - a^2) = 2g(b-a)$

$\omega_c^2 = \frac{10g(b-a)}{(b-a)(b+a)} = \frac{10g}{b+a}$

Therefore, one obtains $\omega_c = \pm\sqrt{\frac{10g}{a+b}}$ rad. s^{-1}.

One notes that the $\pm$ notation gives insight about whether the egg is rotating clockwise (negative) or anticlockwise (positive)

Question: What is the expression of critical angular speed c at which the egg stands up? Based on the characteristics of your egg, determine the predicted value of c. Compare it to the angular speed found in the experiments and comment the results.

ω predicted = 41.478 rad/s
ω measured = abs(-2318,290273) degree/ s = abs(-40,46181561) rad/ s with an error of 2.4499358% error

We determined the critical speed by taking the average of the angular velocities found using the tracker software during the critical tilting moment.

We can explain the discrepancy between these values by the following causes of uncertainty:

- **Neglecting friction,** which may have had an impact on the work done on the egg.
- **Human error** when measuring the egg's key characteristics, when releasing the egg and giving it its starting velocity, and when setting the mark on the egg as a point mass manually (by eye).
- **Camera:** Deformation of the camera lens and low frame rate.

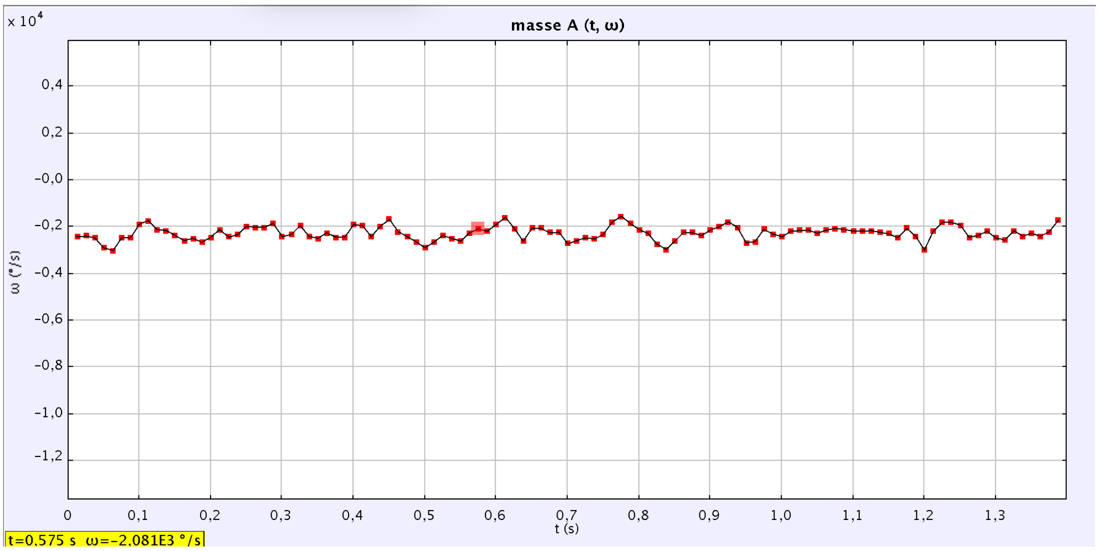


Figure 6: Graph of the values for ω found using Tracker as a function of their corresponding times (in the video).

time in s	w in degree/s
0.0125	-2404.96
0.025	-2384.05
0.0375	-2450.88
0.05	-2890.21
0.0625	-3008.88
0.075	-2457.31
0.0875	-2450.01
0.1	-1875.39
0.1125	-1739.09
0.125	-2107.18
0.1375	-2150.9
0.15	-2356.55
0.1625	-2584.73
0.175	-2487.72
0.1875	-2659.93
0.2	-2434.5
0.2125	-2108.57
0.225	-2426.32
0.2375	-2308.24
0.25	-1968.68
0.2625	-2007.49

Table 1: Table of the values for found using Tracker and their corresponding times (in the video).

4
WILL THE GLASS OVERFLOW?

The last part of this project is an experiment you could do on a sunny summer day. While it's hot outside, you help yourself to a large glass of water and to make it cooler you add one or more ice cubes. But you get a call before you drink your glass and when you come back your ice cube has melted. The question is: can the melting ice in the glass cause it to overflow?

4.1 Experiment of the ice cube in a glass of water

Put an ice cube in a glass and fill it with water, so that the glass is entirely full to the brim. Wait for it to melt (you can put it in hot water to make it melt faster). Take a film of the experiment.

Question: What happens to the level of water as the ice melts? Does the glass overflow?

During the gradual melting of the ice, the water level line in the beaker does

not change, which means the water level line neither rise up or go down.

We now put ice cubes in the beaker (in the second question I will talk about the process of measuring the submerged volume of ice cubes), after which we add warm water at about 50 degrees Celsius until the glass is entirely full to the brim. (like the first picture)

Figure 7: The water level line before the ice melted

By observation, we found that the water level in the beaker did not change until the ice in the beaker was completely melted and the water level remained unchanged. (like the second picture)

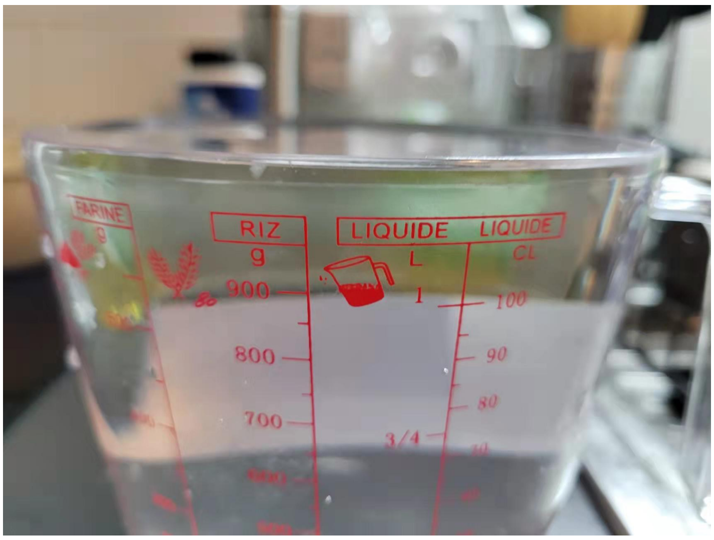


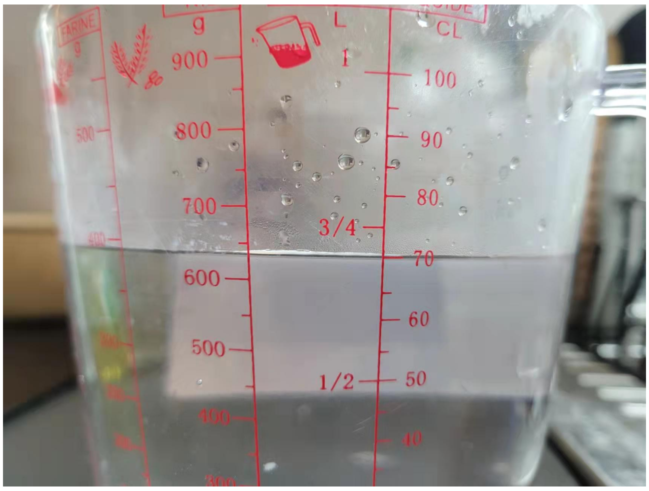
Figure 8: The water level line after the ice melted

Question: On the video, try to measure as well as possible the initial immersed volume of the ice cube.

$V_{immersed} = 22cl = 0.22L$ (Estimated value)

Measurement Method: Measurement method: We measure the submerged volume of ice by calculating the volume difference between the volume before putting ice and the volume after ice.

We first add 70cl of warm water at 50 degrees Celsius to the beaker.



In the second part, we slowly add ice to the beaker, this step must be slow to prevent water from splashing out of the beaker and thus causing bias in the measurement. After adding the ice into the beaker and wait the system reach the stable status, we found that the water level is at 92cl.

Therefore $V_{immersed} = V_{after} - V_{before} = 92cl - 70cl = 22cl = 0.22L$

Finally we add water into the beaker until the glass is entirely full to the brim.

4.2 Equation of the problem

Let's formalize the problem.

Question: How come that the ice cube is floating in the glass and not sinking to the bottom? From the ice cube equilibrium condition, determine its initial mass as a function of its immersed volume.

From the resources we can find the density of ice and water.

$$\rho_{ice} = 0.92 \text{ g/cm}^3 \text{ and } \rho_{\text{water}} = 1.00 \text{ g/cm}^3$$

Therefore we find the density of ice is smaller than the density of water, from the Archimedes principle we learned from PHY101 we know the ice cube should float in the glass since its smaller density but we will prove this in the following part in mathematical formula.

Computation of immersed volume: From the Archimedes principle we have $F_{\text{float}} = G_{immersed}$. Therefore we have

$$\rho_{ice} \cdot g \cdot V_{ice} = \rho_{\text{water}} \cdot g \cdot V_{immersed}$$

Then

$$V_{\text{immersed}} = V_{ice} \cdot \frac{\rho_{\text{ice}}}{\rho_{\text{water}}}$$

$$V_{\text{immersed}} \approx 0.92 V_{ice}$$

Also we know that

$$M_{ice} = \rho_{\text{water}} \cdot V_{\text{immersed}}$$

In conclusion, the ration of the Volume immersed in water to the Volume of the ice cube is 0.92. **And the initial mass of the ice cube equals to the mass of water in its immersed volume.**

Question: Does the mass of water varies as ice turns into liquid? You can do the experiment by weighting the ice-cube as it melts in an empty glass.

I think there is ambiguity in this question.If the mass of water is the water in the beaker, that is, go out to the ice and the mass of the beaker, the mass of water is certainly increased because the ice melted into water, of course the mass of water increased.

However, If this mass refers to the mass of the entire beaker (including the mass of water and the mass of ice) the overall mass is certainly unchanging, we can treat this whole as a system, and the process of melting ice into water is the internal operation of the system. Also, by measured by the balance we can find the mass doesn't change in the whole process.

Question: How does the amount of extra water (from the melted ice-cube) then compared with that previously displaced by the cube? Do you expect the glass to overflow? Is it coherent with your experiment?

From the previous question we know

$$M_{water} = M_{ice}$$

$$V_{water\ increased} = V_{immersed} = 0.92V_{ice}$$

Therefore we know **the volume of extra water from the melted ice-cube equals to the volume of immersed of the gas.** For this reason, the water in the beaker wouldn't come out which coherent with the observation of our experiment.

Question: Diluted soda is slightly denser than water due to its sugar content. Could a water ice cube melting in a a soda glass cause liquid to spill over the top? Justify your answer.

The water would spill over the top, following is the analysis: We know the density relation of water, soda, ice, diluted soda

$$\rho_{\text{soda}} > \rho_{\text{mixed}} > \rho_{\text{water}} > \rho_{ice}$$

From the **Archimedes principle** we have

$$\rho_{ice} \cdot V_{ice} = \rho_{soda} \cdot V_{immersed} = \rho_{\text{water}} \cdot V_{\text{melted}}$$

Then

$$V_{\text{ice}} > V_{\text{melted}} > V_{\text{immersed}}$$

The volume of beaker

$$V = V_{im\ \text{mersed}} + V_{\text{soda}}$$

In the whole process the **mass of the system is conservative** so we have

$$\rho_{soda} \cdot V_{\text{soda}} + \rho_{ice} \cdot V_{ice} = \rho_{\text{soda}} \cdot V_{\text{soda}} + \rho_{\text{water}} \cdot V_{\text{melted}} = \rho_{\text{mixed}} \cdot V_{\text{mixed}}$$

So we have the relation

$$V = V_{\text{immersed}} + V_{\text{soda}} < V_{\text{mixed}} < V_{\text{soda}} + V_{\text{water}}$$

In conclusion from the relation above we can say that the water will spill out since the volume of the mixed solution is more than the volume of the beaker.

This simple problem can be extended to a much larger scale: with global warming the ice pack and icebergs are melting and we are also seeing a rise in water levels.

Question: In light of the ice cube experiment carried out previously, did the melting of the pack ice and icebergs cause the water to rise? If not, according to you, what would be the cause?

We can analyze this problem by using the physical model of soda. The melting point of seawater is around -1.8 degrees Celsius and a large amount of salt is discharged during the process of seawater freezing. The density of glaciers is still lower than that of pure water. The density of seawater is normally around 1.021.07 g/cm3 which is larger than the density of pure water. Therefore from the soda model from previous question we know the water level **increase**.

The same reason for this case, we can deduce that in the ideal physics model that if the icebergs melted to water the sea level would increase. However, we are analysis this question in a really ideal situation. In reality, the analysis must be integrated with meteorological physics, oceanography, marine geology and other disciplines. This is a multidisciplinary problem, and our analysis by physical modeling is only a very rough analysis.