PHY101 - MECHANICS AND HEAT

Homework 5

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CONTENTS

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1.1 A

The process is isochoric with constant volumn V
the $W_{1\to2} = 0J$ since the volume is constant in the whole process

$$\begin{aligned}\Delta U_{1\to2} &= W_{1\to2} + Q_{1\to2} = Q_{1\to2} \\ &= C_V(T_2 - T_1) = \frac{nR}{r-1}(T_2 - T_1) \\ &\approx 3.78kJ\end{aligned}$$

Therefore $Q_{1\to2} = \Delta U_{1\to2} = 3.78kJ$

1.2 B

The process from $2 \to 3$ is an adiabatic process, which mean there is no heat exchange with external environment, so we have $Q_{2\to3} = 0J$

$$\begin{aligned}\Delta U_{2\to3} &= W_{2\to3} + Q_{2\to3} = W_{2\to3} \\ &= C_V(T_3 - T_2) = \frac{nR}{r-1}(T_3 - T_2) \\ &\approx -1.825kJ\end{aligned}$$

Therefore $Q_{1\to2} = \Delta U_{1\to2} = -1.825kJ$

1.3 C

The process from $3 \to 1$ is an isobaric process, which the pressure P is constant.

$$\begin{aligned}W_{3\to1} &= \int_{V_3}^{V_1} PdV = -P(V_1 - V_3) \\ &= PV_3 - PV_1\end{aligned}$$

Since the ideal gas law we have $PV = nRT$

$$W_{3\to1} = PV_3 - PV_1 = nR(T_3 - T_1) = 1.29kJ$$

$$\Delta U_{3\to1} = C_V(T_3 - T_1) = \frac{nR}{\gamma-1}(T_3 - T_1) = -1.95kJ$$

By the first law of thermodynomics, we get

$$\Delta U_{3\to1} = W_{3\to1} + Q_{3\to1} \Rightarrow Q_{3\to1} = \Delta U_{3\to1} - W_{3\to1} = -3.24kJ$$

1.4 D

$$W = W_{1\to2} + W_{2\to3} + W_{3\to1} = 0J - 1.825kJ + 1.29kJ = -0.535kJ < 0$$

Since the net work is negative, therefore the work is the gas done on the external environment. Therefore the thermodynamic cycle is a heat engine.

1.5 E

The energy we want is the net work down by the gas, the energy we pay is the heat we take from the environment in process $1 \to 2$.

$$\varepsilon = \frac{energy\ we\ want}{energy\ we\ pay} = \frac{|w|}{Q_{1\to2}} = 14.1\%$$

1.6 F

At point 1 we have the ideal gas law $P_1V_1 = nRT_1$ $v_1 = \frac{nR\pi_1}{P_1} \approx 25L$.
Since the process from $1 \to 2$ is isochoric, therefore $V_2 = V_1 = 25L$

$$P_2V_2 = nRT_2 \quad P_2 = \frac{nRT_2}{V_2} = \frac{2nRT_1}{V_1} = 2.00atm$$

at paint 3. since the proces from $3 \to 1$ is isobaric, then $P_3 = P_1 = 1atm$.

$$P_3 \cdot V_3 = nRT_3. \quad V_3 = \frac{nRT_3}{P_3} = 37 \text{ L}$$

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2.1 A

Work done during a quesi-static process $w = -\int_{v_i}^{v_f} Pdv$. Since we know in polytropic process we have $PV^K = P_iV_i^k = P_fV_f^k = constant$. Then we have

$$P = \frac{P_iV_i^k}{V^k} \quad w = -\int_{V_i}^{v_f} \frac{P_iV_i^k}{V^k} dV$$

.

$$\begin{aligned} w = -P_iV_i^k \int_{V_i}^{V_f} \frac{1}{V^k} dV &= -\frac{P_iV_i^k}{-k+1}\left(V_f^{-k+1} - V_i^{-k+1}\right) \\ &= \frac{1}{v-1}\left(P_fV_f - P_iV_i\right) \end{aligned}$$

2.2 B

From the first law of thermodynamic $\Delta U = Q + W$ then $Q = \Delta U - W$.

$$\begin{aligned} \Delta U &= C_V\left(T_f - T_i\right) = \frac{nR}{\gamma - 1}\left(T_f - T_i\right) \\ W &= \frac{1}{k-1}\left(P_fV_f - P_iV_i\right) \\ Q &= \Delta U - W = \frac{nR}{\gamma - 1}\left(T_f - T_i\right) - \frac{1}{k-1}\left(P_fV_f - P_iV_1\right) \end{aligned}$$

Since we have

$$PV = nRT$$

Then we get

$$P_fV_f = nRT_f \; P_iV_i = nRT_i$$

Therefore

$$\begin{aligned} Q &= \frac{nR}{\gamma - 1}(T_f - T_V) - \frac{nR}{k-1}\left(T_f - T_i\right) \\ &= nR\left(\frac{1}{\gamma - 1} - \frac{1}{k-1}\right)\left(T_f - T_i\right) = C(T_f - T_i). \end{aligned}$$

In conclusion we get

$$C = nR\left(\frac{1}{\gamma - 1} - \frac{1}{k-1}\right)$$

2.3 C

Since $k = \gamma$

$$C = nR\left(\frac{1}{\gamma-1} - \frac{1}{\gamma-1}\right) = 0$$

then $Q = C = 0$.

$$W = \frac{1}{\gamma-1}\left(P_f V_f - P_i V_i\right) = \frac{nR}{\gamma-1}\left(T_f - T_i\right) = C_V\left(T_f - T_i\right) = \Delta H.$$

therefore the process is adiabatic since $Q = 0$.

2.4 D

The process is constant-pressure process - isobaric process, if $k = 0 \quad PV^0 = P = constant$ therefore the process is isobaric

$$W = \frac{1}{0-1}\left(P_f V_f - P_i V_i\right) = -P(V_f - V_i)$$

$C_p = \frac{nR\gamma}{\gamma-1}$ since it's constant pressure process, therefore $Q = C_P(V_f - V_i)$

$$\begin{aligned}\Delta U = Q + W &= C_P\left(V_f - V_i\right) - P_i\left(V_\gamma - V_i\right)\\ &= \left(\frac{nR\gamma}{\gamma-1} - P_i\right)\left(V_f - V_i\right)\end{aligned}$$

2.5 E

$k \to +\infty$, then $PV^k = \text{constant}$. $V = \left(\frac{\text{Constant}}{P}\right)^{\frac{1}{k}}$
Since $\frac{1}{k} \to \infty$, then V is constant therefore the process is isochoric then $W = 0$.

$$\Delta U = Q + W = Q = C_V\left(V_f - V_i\right) = \frac{nR}{\gamma-1}\left(V_f - V_i\right)$$