PHY101 - MECHANICS AND HEAT

Homework 4

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CONTENTS

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1.1 A

We take system $= \{M + m\}$
Since no external torque around the center of the planet and from angular momentum theorem we know the angular momentum is conservative. Therefore we have the equation

$$5R \cdot mv_0 sin(\alpha) = m\dot{\theta}r^2$$

$$\dot{\theta} = \frac{5R \cdot v_0 sin(\alpha)}{r^2}$$

1.2 B

We take system $= \{M + m\}$. Since there is no external force therefore the mechanical energy E_m is the sum of potential and kinetic energy. Since the mechanical energy is conservative. we have

$$\frac{1}{2}m(v_L^2 - v_0^2) = E_{G1} - E_{G2} \quad (1)$$

$$\frac{1}{2}m(v_L^2 - v_0^2) = \frac{GMm}{R} - \frac{GMm}{5R} \quad (2)$$

$$v_L^2 - v_0^2 = \frac{8GMm}{5R} \quad (3)$$

$$v_L = \sqrt{v_0^2 + \frac{8GMm}{5R}} \quad (4)$$

1.3 C

Since the mechanical energy is conservative, therefore we have the equation

$$\frac{1}{2}mv_0^2 - \frac{GMm}{5R} = \frac{1}{2}mv_L^2 - \frac{GMm}{R}$$

$$\frac{25sin^2\theta - 1}{2}v_0^2 = \frac{4GM}{5R}$$

$$sin(\theta) = \frac{1}{5}\sqrt{1 + \frac{8GM}{5v_0^2 R}}$$

$$\theta = sin^{-1}(\frac{1}{5}\sqrt{1 + \frac{8GM}{5v_0^2 R}})$$

2

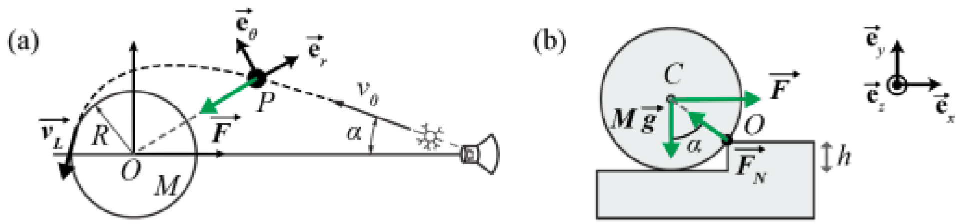

Figure 1: (a) Grazing instrument package. (b) Rolling wheel

2.1 A

When the wheel lifts off there are only three forces apply on the wheel, the gravity, the tension and the normal force from the corner. Therefore we take system= *wheel* and draw the free body graph.

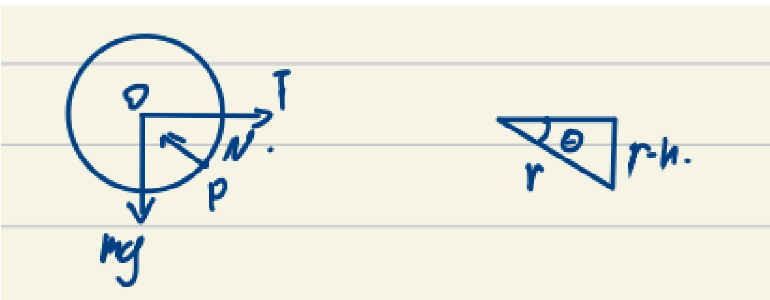

Figure 2: Body diagram of the ball and corner

We choose point p as the original point for the rotation. When the moment the wheel is going to lift off, the net torque is 0.

$$\sum \overrightarrow{\tau} = rFsin(\theta) - mg \cdot rsin(\frac{\pi}{2} - \theta) + N \cdot sin(0) = 0$$

Then we have

$$rFsin(\theta) = mg \cdot rcos(\theta)$$

$$F = mg \cdot \frac{cos(\theta)}{sin(\theta)} \tag{5}$$
$$= mg \cdot \frac{\frac{\sqrt{r^2-(r-h)^2}}{r}}{\frac{r-h}{r}} \tag{6}$$
$$= mg\frac{\sqrt{r^2-(r-h)^2}}{r-h} \tag{7}$$
$$= \frac{mg\sqrt{h(2r-h)}}{r-h} \tag{8}$$

2.2 B

Since $F = \frac{mg\sqrt{h(2r-h)}}{r-h}$ in order to analyze when τ tend to infinite, we denote $u = r - h$

$$F = mg\sqrt{\frac{h(2r-h)}{(r-h)^2}} = mg\sqrt{\frac{(r-u)(r+u)}{u^2}} \tag{9}$$
$$= mg\sqrt{\frac{r^2}{u^2} - 1} \tag{10}$$

In order to discuss this question, we assume h is the variable which is changable since $u = r - h$. When u tends to 0 which means h tends to r. Therefore τ tends to infinite. So when h tends to r, the force F become infinite. If $h \geq r$ then the wheel is not able to lift off.

When the height of the step is close to the redius of the wheel, F become infinite

3

We denote H the height of the barge underwater when the cow is not on it. When the cow steps on it, it sinks an additional height h.

- We consider the system {barge} before the cow steps on it:

  External forces: the weight $M\vec{g}$ (with M the mass of the barge), and the buoyancy force of the water that is equal to the weight of displaced water $\overrightarrow{F_b} = -\rho lwH\vec{g}$.

  The system is at rest, so according to Newton's second (or first) law : $M\vec{g} + \overrightarrow{F_b} = \overrightarrow{0} \Rightarrow Mg = \rho lwHg$

- We consider the system { barge + cow } after the cow steps on the barge:

  External forces : the weight $(M+m)\vec{g}$ (with m the mass of the cow), and the buoyancy force of the water $\overrightarrow{F_b} = -\rho lw(H+h)\vec{g}$.

  The system is at rest, so according to New ton's laws : $(M+m)\vec{g}+\overrightarrow{F_b} = \overrightarrow{0} \Rightarrow (M+m)g = \rho lw(H+h)g$

Using the previous equations, we obtain: $mg = \rho lwhg \Rightarrow m = \rho lwh = 900$ kg