PHY101 - MECHANICS AND HEAT

Homework 2

June 18, 2022

Yubo Cai

ÉCOLE POLYTECHNIQUE
IP PARIS

CONTENTS

1

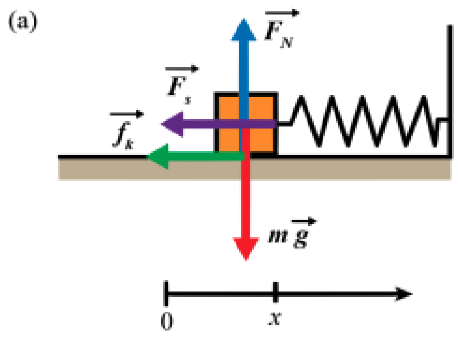

Figure 1: The instruction graph of exercise 1

- We first take the system of {M}
- Frame of the reference as the ground

Since the friction, therefore the block is not in the conservative system, which means part of the kinetic energy loss during the motion. At the rest point, the only mechanic energy is the potential of the spring, and the net force at that point is 0. Since it's in the equilibrium, we assume the x_0 is the final possition where the block rest.

$$\begin{cases} K_i = \frac{1}{2}Mv_0^2 \\ U_{g,i} = 0 \\ U_{s,i} = 0 \end{cases} \quad \begin{cases} K_f = 0 \\ U_{g,f} = 0 \\ U_{s,f} = \frac{1}{2}kl^2 \end{cases}$$

The normal force is not doing work as it is perpendicular to the displacement. The work of the nonconservative frictional force is:

$$W_{f_k}^{\text{ac}} = \int_i^f \overrightarrow{f_k} \cdot d\overrightarrow{OM} = \int_0^l (-bxMg\overrightarrow{e_x}) \cdot (dx\overrightarrow{e_x}) = -bMg\int_0^l xdx = -bMg\left[\frac{1}{2}x^2\right]_0^l = -\frac{1}{2}bMgl^2$$

The Work-Kinetic energy theorem $\Delta E_m = \Delta(K + U_g + U_s) = W_{f_k}^{\text{nc}}$ then rewrites:

$$\frac{1}{2}kl^2 - \frac{1}{2}Mv_0^2 = -\frac{1}{2}bMgl^2 \Rightarrow l = \sqrt{\frac{M}{k + bMg}}v_0$$

2

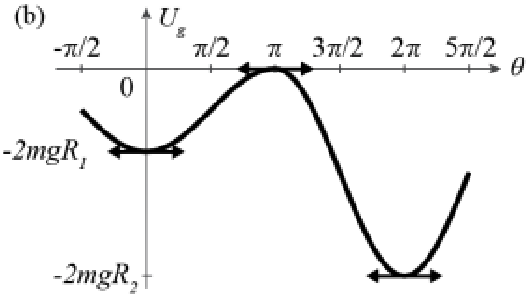

Figure 2: Graph of the gravitational potential energy U_g as a function of angle θ.

2.1 A

We choose an upward vertical z-axis with origin at C_1. $U_g(z) = mgz + C$ with C a constant determined by $U_g(B) = mgz_B + C = 0 \Rightarrow C = -mgz_B$. So $U_y = mg\,(z - z_B)$.

In part (1), $z = -R_1 \cos\theta$ and $z_B = R_1$, so that $U_{g,1}(\theta) = -mgR_1(1 + \cos\theta)$

In part (2), $z = -\left(R_2 - R_1\right) - R_2\cos\theta$ and $z_B = R_{1,}$ so that $U_{g,2} = mg\left(R_1 - R_2 - R_2\cos\theta - R_1\right) \Rightarrow U_{g,2}(\theta) = -mgR_2(1 + \cos\theta)$

2.2 B

As the graph shown

2.3 C

Equilibrium angular positions are extremum of the function $U_g(\theta)$, meaning $\frac{dU}{d\theta} = mgR_{1,2}\sin\theta = 0$. Angles satisfying this condition are $\theta = 0, \theta = \pi$, and $\theta = 2\pi$.

Stable equilibrium are minimum of $U_g(\theta)$, meaning $\frac{d^2U}{d\theta^2} = mgR_{1,2}\cos\theta > 0.\theta = 0$ and $\theta = 2\pi$ are stable equilibrium.

Unstable equilibrium are maximum of $U_g(\theta)$, meaning $\frac{d^2U}{d\theta^2} < 0.\theta = \pi$ is an unstable equilibrium.

2.4 D

(a) Forces acting on the particle are either conservative (gravitational force) or not doing work (normal force exerted by the track, perpendicular to the displacement), so the system is conserB with non zero speed (so $K(B) > 0$), meaning $E_m = K(B) + U_g(B) > U_g(B)$. In other words, it has to overcome the potential energy barrier at B.

$$E_m = E_m(A) > U_g(B) = 0 \Rightarrow \frac{1}{2}mV_0^2 - mgR_1 \geq 0 \Rightarrow V_0 > \sqrt{2gR_1}$$

Note: How do we know the constraining force exerted by the track is not doing work (and is perpendicular to the path)? This force $\vec{F}$ ensures that the motion is constrained along the pre-determined path: in other words, it ensures that the direction of velocity is always tangential to the path. Hence, $\vec{F}$ changes only the direction of the velocity $\vec{v}$ (and not its magnitude), and thus it is normal to $\vec{v}$ (since it induces an acceleration normal to $\vec{v}$). On the other hand, the displacement $\overrightarrow{dr}$ is parallel to $\vec{v}$. Consequently $\vec{F} \cdot \overrightarrow{dr} = 0$ and the constraint force does no work.

(b) $E_m(A) = E_m(F) \Rightarrow \frac{1}{2}mV_0^2 - mgR_1 = \frac{1}{2}mV_f^2 - 2mgR_2$

$$\begin{aligned} &\Rightarrow V_f^2 = V_0^2 + 2g\left(2R_2 - R_1\right) \\ &\Rightarrow V_f = \sqrt{V_0^2 + 2g\left(2R_2 - R_1\right)} \end{aligned}$$

(c) To exit the track at S, the ring needs to have a mechanical energy higher than $U_g(S)$, which is the case if the particle is able to overcome the potential energy barrier at B. So the condition is the same as the one for reaching F.