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Week 1 Report

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Title: Apply Reachability Analysis to our first Example System

1 Useful informations

1. **Reachability package**: [link]
2. **Carleman linearization**: [link]
3. **Reachability of weakly nonlinear systems using Carleman linearization**: [link]

2 Introduction to Reachability Analysis of Carleman Linearization for quadratized system

Here we use the [Reachability package] which is in julia and the system code [link]. Here we bring our first example system to the reachability analysis. The system is:

$$x' = -x + ax^3$$

From quadratization, we know that we can introduce a new variable $w_0 = y = x^2$ to quadratize the system. Then for the original equation, we have:

$$\begin{cases} x' = -x + axy \\ y' = -2y + 2ay^2 \end{cases}$$

Then we use the Carleman linearization to linearize the system. Follow the algorithm in the paper [Reachability of weakly nonlinear systems using Carleman linearization], we have the $F_1 \in \mathbb{R}^{n\times n}$ matrix for linear part:

$$F_1 = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}.$$

And for the nonlinear part we have $F_2 \in \mathbb{R}^{n\times n^2}$ matrix:

$$F_2 = \begin{bmatrix} 0 & 2a & 0 & 0 \\ 0 & 0 & 0 & a \end{bmatrix}.$$

where we have the basis for the nonlinear part is $[x^2, xy, yx, y^2]$. Since we use quadratization algorithms to quadratize the system, we should at most have this 4 basis.

Therefore, we can use the algorithms from the paper **Epidemic model(SEIR)** to analyze this sample system. The code is [here].

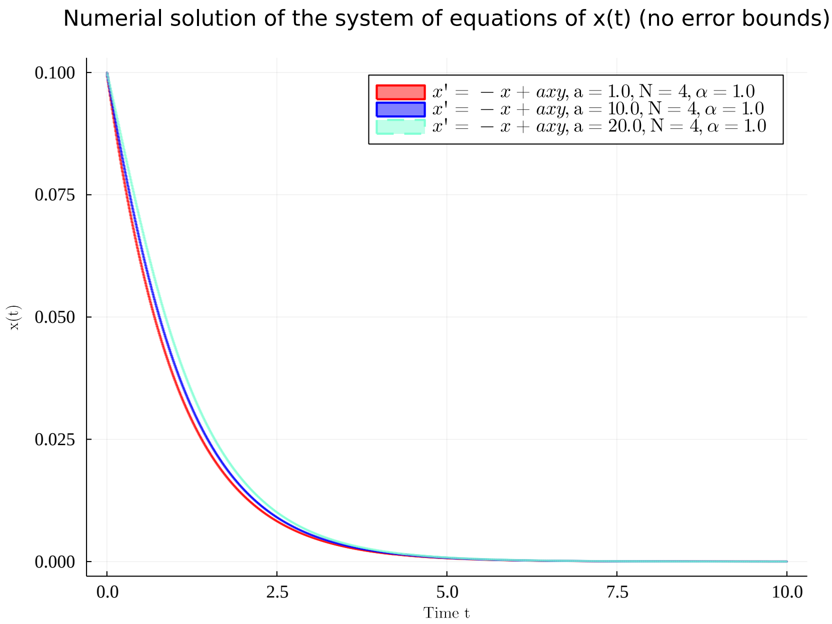
3 Results of the plot without Error Bound

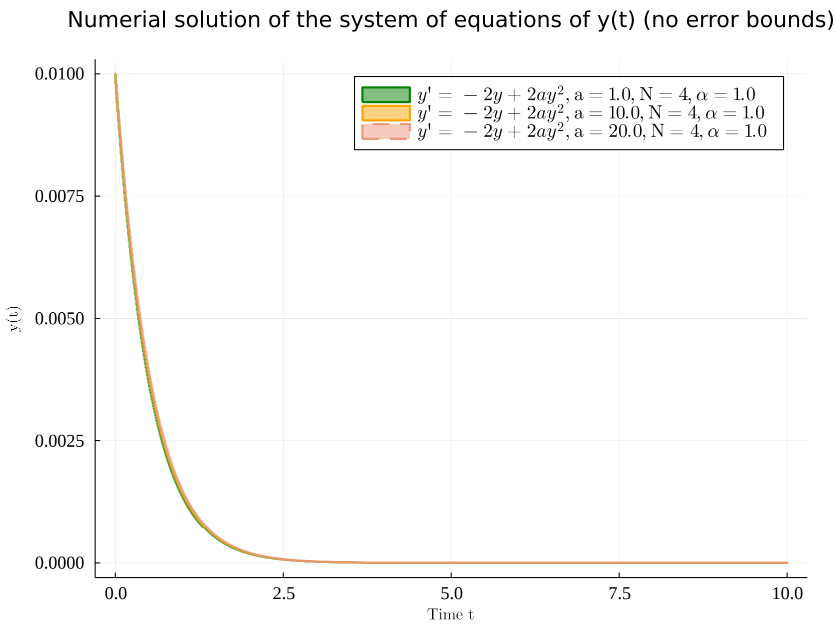
We set the Carlimen Linearization matrix as following

```
function system_carlin(a, alpha)
  F1 = zeros(2, 2)
  F1[1, 1] = -1
  F1[2, 2] = -2 / alpha

  F2 = zeros(2, 4)
```

```
  F2[1, 2] = 2 * a / alpha
  F2[2, 4] = a

  return F1, F2
end
```

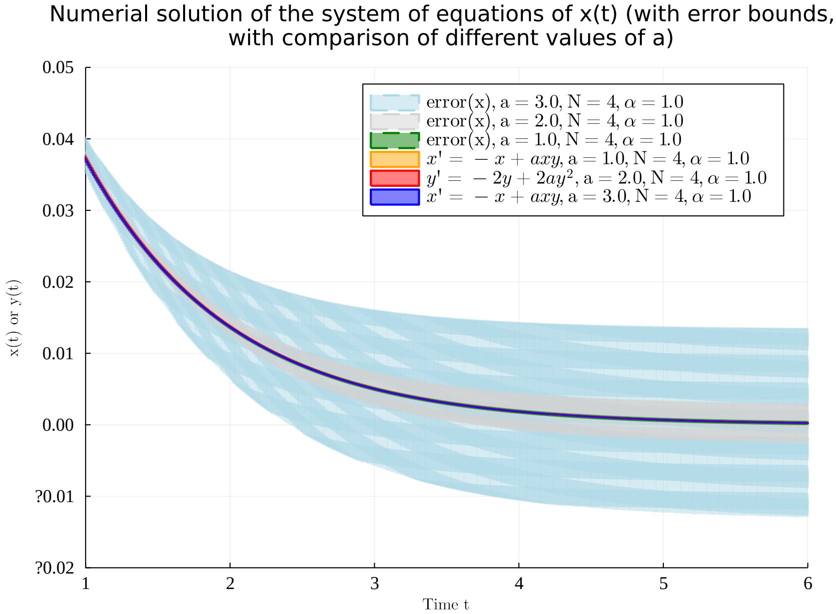
We apply the method **_solve_system_carlin(; N=4, T=30.0, δ=0.1, radius0=0, bloat=false, resets=nothing, alpha, a)** and plot the graph. Here in order to have a better numerical plot, we set $\delta = 0.01$, Tmax = 10 and N=4. Here in order to have better numerical plot, we set $\delta = 0.01$, Tmax = 10, and N=4. Here we only change the parameter a and keep the parameter α as 1. We initialize the value for $X_0 = [x_0, y_0] = [x_0, x_0^2] = [0.1, 0.01]$. In order to have better plot, we have the plot as follows:





4 Results of the plot with Error Bound

Here we apply the error bound method to the system. We have the following result of the plot:



We can find that the error bound works fine, and if we increase the value of a, the error bound get much larger.