Theorem 1

Theorem 1: $\mathrm{Z}_p^{|P_{p,q}|}= \mathrm{Z}_q^{|P_{p,q}|}$ and $t_p^{|P_{p,q}|+1}$=$t_q^{|P_{p,q}|+1}$

Proof: It is obvious that if $|P_{p,q}| = 0$, which means p and q have no common levels, then $Z_p^0 = Z_q^0$ and $t_p^1 = t_q^1$ holds.

For $|P_{p,q}| \geq 1$, which means p and q have $|P_{p,q}|$ common levels. In this case, for $1 \leq i \leq |P_{p,q}|$, as $x_p^i = x_q^i$, $y_p^i = y_q^i$ and $t_p^i = t_q^i$, we always have $z_p^{2i-1} z_p^{2i} = z_q^{2i-1} z_q^{2i}$, $Z_p^i = Z_q^i$ and $t_p^{i+1} = t_q^{i+1}$.

Thus, Theorem 1 holds.

Preposition 1

Preposition 1: The AEL of NA-HE is less than 2 when encoding a k-level window.

Proof: We first compute the total encoding levels (TEL) for a k-level window. To do this, we divide all the coordinates in the window into three subsets: A (the first coordinate in the first line), B(the first encoded coordinates of all lines along S-order except the first line), C(all coordinates not in A and B). For ease of presentation, we denote E_S as the TEL for a subset S of the three subsets.

For A, since it contains the first coordinate only, all the k levels need to be encoded, we have E_A = k.

For B, notice that the TELs for all coordinates in B form a \textbf{recursive centrosymmetric sequence} with a center k, where the left part and right part of this sequence are all recursive centrosymmetric sequences with a center $k-1$. There are exactly one k, two $(k-1)$s,..., 2^{k-1} 1s in this sequence. Therefore, E_B= $\sum_{j=1}^{k} j \times 2^{k-j}$ =$2^{k+1}-k-2$.

For C, the TELs for all coordinates in C_i is the same recursive centrosymmetric sequence as B, where C_i denotes the set of coordinates in C and in the i-th line of the window. Therefore, for all the 2^k lines, we have $E_C = 2^k \times (2^{k+1}-k-2)$.

Thus, the TEL is E_A + E_B + E_C = $2^{2k+1} - k \times 2^k - 2$ and the AEL is $\frac{2^{2k+1}-k\times 2^k-2}{2^{2k}} < 2$ for a k-level window. Preposition 1 holds.