1. General boundary-value problem

$$\nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + au = f, \tag{1}$$

$$-\boldsymbol{n} \cdot (-c\nabla u - \alpha u + \gamma) = g - qu \; on \; S_2, \tag{2}$$

$$u = s \; on \; S_1. \tag{3}$$

2. Problem in cartesian coordinates

Equation (1) in cartesian coordinates is written as:

$$-\left[\frac{\partial}{\partial x}\left(c_x \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(c_x \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(c_z \frac{\partial u}{\partial z}\right)\right] - \left[\frac{\partial}{\partial x}(\alpha_x u - \gamma_x) + \frac{\partial}{\partial y}(\alpha_y u - \gamma_y) + \frac{\partial}{\partial z}(\alpha_z u - \gamma_z)\right] + \left[\beta_x \left(\frac{\partial u}{\partial x}\right) + \beta_y \left(\frac{\partial u}{\partial y}\right) + \beta_z \left(\frac{\partial u}{\partial z}\right)\right] + au = f, \tag{4}$$

3. Equation in electrolyte

$$\nabla \cdot \left(-\begin{bmatrix} D & 0 & -\mu C_-^\infty \, exp\left(\frac{e}{k_B T} U_a^{(0)}\right) \\ 0 & D & \mu C_+^\infty \, exp\left(-\frac{e}{k_B T} U_a^{(0)}\right) \\ 0 & 0 & \varepsilon_0 \varepsilon_a \end{bmatrix} \nabla \boldsymbol{u}_2 - \begin{bmatrix} -\mu \nabla U_a^{(0)} & 0 & 0 \\ 0 & \mu \nabla U_a^{(0)} & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{u}_2 \right) + \begin{bmatrix} i\omega & 0 & 0 \\ 0 & i\omega & 0 \\ F & -F & 0 \end{bmatrix} \boldsymbol{u}_1 = 0, \tag{5}$$

$$-\boldsymbol{n} \cdot (-c\nabla \boldsymbol{u}_2 - \alpha \boldsymbol{u}_2 + \gamma) = \begin{bmatrix} 0 \\ 0 \\ \delta \Sigma_S(\boldsymbol{r}_S) \end{bmatrix} \; on \; S_2 \tag{6}$$

$$\boldsymbol{u}_2 = \begin{bmatrix} 0 \\ 0 \\ -\boldsymbol{E}_{ext} \cdot \boldsymbol{r} \end{bmatrix} \; on \; S_1 \tag{7}$$

where

$$\boldsymbol{u}_2 = \begin{bmatrix} \delta C_-(\boldsymbol{r},\omega) \\ \delta C_+(\boldsymbol{r},\omega) \\ \delta U_a(\boldsymbol{r},\omega) \end{bmatrix}, \tag{8}$$

$$c_2 = \begin{bmatrix} D & 0 & -\mu C_-^{\infty} \, exp\left(\frac{e}{k_B T} U_a^{(0)}\right) \\ 0 & D & \mu C_+^{\infty} \, exp\left(-\frac{e}{k_B T} U_a^{(0)}\right) \\ 0 & 0 & \varepsilon_0 \varepsilon_a \end{bmatrix}, \alpha_2 = \begin{bmatrix} -\mu \nabla U_a^{(0)} & 0 & 0 \\ 0 & \mu \nabla U_a^{(0)} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \gamma_2 = 0, \tag{9}$$

$$\beta_2 = 0, a_2 = \begin{bmatrix} i\omega & 0 & 0 \\ 0 & i\omega & 0 \\ F & -F & 0 \end{bmatrix}, f_2 = 0, \tag{10}$$

$$g_2 = \begin{bmatrix} 0 \\ 0 \\ -\boldsymbol{n} \cdot [\varepsilon_0 \varepsilon_i \nabla \delta U_i(\boldsymbol{r}_S,\omega)] + \delta \Sigma_S(\boldsymbol{r}_S) \end{bmatrix}, q_2 = 0, s_2 = \begin{bmatrix} 0 \\ 0 \\ -\boldsymbol{E}_{ext} \cdot \boldsymbol{r} \end{bmatrix}. \tag{11}$$

4. Equation in Stern layer

$$i\omega \delta \Sigma_S(\boldsymbol{r}_S,\omega) = \nabla \cdot \left[D_S \nabla \delta \Sigma_S(\boldsymbol{r}_S,\omega) + \mu_S \Sigma_S^{(0)} \nabla \delta U_S(\boldsymbol{r}_S,\omega) \right]. \tag{12}$$

where

$$\boldsymbol{u}_3 = \delta \Sigma_S(\boldsymbol{r},\omega), \tag{13}$$

$$c_3 = D_S, \alpha_3 = 0, \gamma_3 = -\mu_S \Sigma_S^{(0)} \nabla \delta U_S(\boldsymbol{r}_S,\omega)\,, \tag{14}$$

$$\beta_3 = 0, a_3 = i\omega, f_3 = 0, \tag{15}$$

$$g_3 = 0, q_3 = 0, s_3 = 0, \tag{16}$$

Equation (5) and (12) are coupled and should be solved simultaneously.