Problem 0: Temporal Logic Proofs

(i) $\phi_1 = (\Diamond \Box a) \wedge (\bigcirc a)$

$\phi_2 = \Box \Diamond a$

$\phi_1 = (\Diamond \Box a) \wedge (\bigcirc a)$

$= \exists i. (\lambda s. [[\Box a]](s+i) \wedge \lambda s. [[a]](s+1)$

$= \exists i. (\lambda s. (\forall i. \lambda s. [[a]](s+i)))(s+i) \wedge \pi_{s+1} \models a$

$= \exists i. (\lambda s. (\forall i (\pi_{s+i} \models a)))(s+i) \wedge \pi_{s+1} \models a$

$= \exists i\, \pi_{s+i} \models \forall i (\pi_{s+i} \models a) \wedge \pi_{s+1} \models a$

$\phi_2 = \Box \Diamond a$

$= \forall i. \lambda s. [[\Diamond a]](s+i)$

$= \forall i. \lambda s. (\exists i\, \lambda s [[a]](s+i))(s+i)$

$= \forall i. \lambda s. (\exists i\, \pi_{s+i} \models a)\ (s+i)$

$= \forall i. \pi_{s+i} \models (\exists i\, \pi_{s+i} \models a)$

$\phi_1 \neq \phi_2$ since for the following state diagram shows ϕ_2 being satified while ϕ_1 isn't.

ϕ_2 is satisfied as always eventually a is satisfied.

ϕ_1 specifically $(\bigcirc a)$ (next a) would fail when it goes from b to state b.

(ii) $\psi_1 = (\Box\Diamond\neg b) \vee (\Box\Diamond c)$

$\psi_2 = \Box(\Box b \Rightarrow \Diamond c)$

$\psi_1 = (\Box\Diamond\neg b) \vee (\Box\Diamond c)$

$= \Box(\Diamond\neg b \vee \Diamond c)$

$\psi_2 = \Box(\Box b \Rightarrow \Diamond c)$

$= \Box(\neg\Box b \vee \Diamond c)$

$= \Box(\Diamond\neg b \vee \Diamond c)$ (duality rule)

$\psi_1 = \psi_2$

(iii) $S_1 = \Box\Box(d \vee \neg e)$

$S_2 = \neg \Diamond(\neg d \wedge e)$

$S_1 = \Box\Box(d \vee \neg e)$

$= \Box(d \vee \neg e)$ (idempotency law)

$S_2 = \neg \Diamond(\neg d \wedge e)$

$= \Box \neg(\neg d \wedge e)$ (duality law)

$= \Box(d \vee \neg e)$ (De Morgan's law)

Therefore $S_1 = S_2$