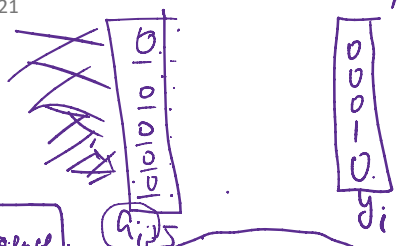
交叉熵 计算Loss

2020年2月16日 9:21

五分类 . OHE. one hot encoding.



KL. divergence

$$\sum_{i=1} y_i \ln a_i \rightarrow -y_k \ln a_k \quad -\ln a_k$$

P.

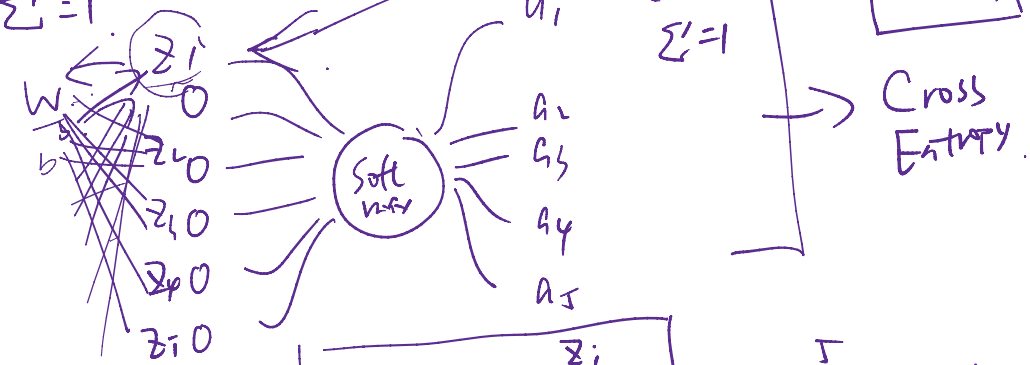
a_i [0.1, 0.1, 0.1, 0.2, 0.5] → 第五类

[0, 0, 0, 1, 0]

$-y_4 \ln a_4 = -\ln 0.2$

(0,1) $\sum = 1$

Softmax



$$a_i = \frac{e^{z_i}}{\sum_{k=1}^{5} e^{z_k}} \qquad \sum_{k=1}^{5} a_k = 1$$

1. 求导

2. 写为复合

Softmax: $S_i = \frac{e^{z_i}}{\sum_j e^{z_j}}$

real. 真实标签

真实标签 [0 0 0 ... 1 0 0]

交叉熵 $Loss = -\sum y_k \cdot \ln a_k = -\ln a_j$

$$\frac{\partial Loss}{\partial z_i} = \frac{\partial Loss}{\partial a_i} \cdot \frac{\partial a_i}{\partial z_i}$$

若 $j = i$

$$\frac{\partial Loss}{\partial a_i} = (-\ln a_i)' = -\frac{1}{a_j}$$

$$\frac{\partial a_i}{\partial} = \frac{\partial}{\partial}\left(\frac{e^{z_i}}{\sum e^{z_k}}\right) = \frac{e^{z_i}\sum e^{z_k} - e^{z_i} \cdot e^{z_i}}{(\sum e^{z_k})^2}$$

$$\frac{\partial a_i}{\partial z_i} = \frac{\partial}{\partial z_i}\left(\frac{e^{z_i}}{\sum_k e^{z_k}}\right) = \frac{e^{z_i}\sum_k e^{z_k} - e^{z_i}\cdot e^{z_i}}{\left(\sum_k e^{z_k}\right)^2}$$

$$= \frac{e^{z_i}}{\sum e^{z_k}} - \left(\frac{e^{z_i}}{\sum e^{z_k}}\right)^2$$

$$= a_i - a_i^2 = a_i(1-a_i)$$

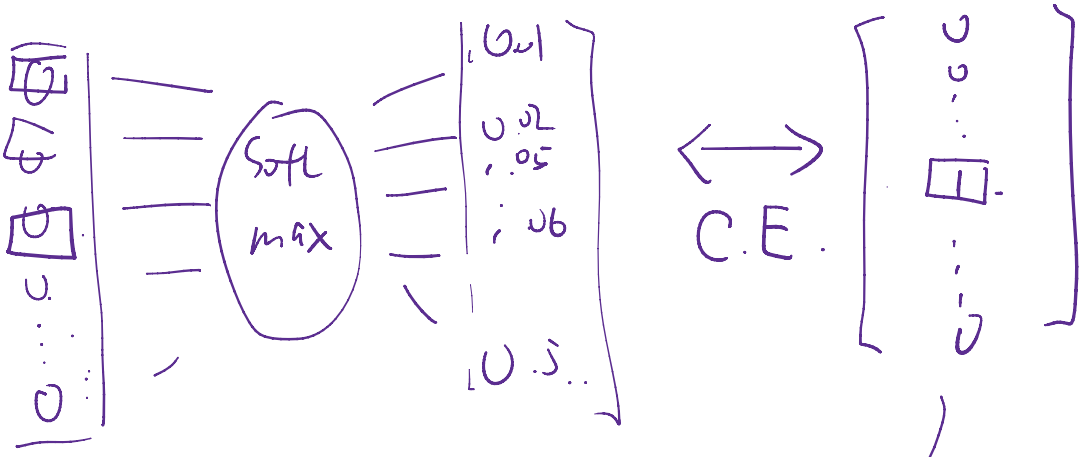
$$\frac{\partial Loss}{\partial z_i} = -\frac{1}{a_j}\cdot a_j(1-a_j) = a_j - 1$$

若 $j \neq i$.

$$\frac{\partial a_j}{\partial z_i} = \frac{\partial}{\partial z_i}\left(\frac{e^{z_j}}{\sum_k' e^{z_k}}\right) = \frac{0 - e^{z_i}\cdot e^{z_j}}{\left(\sum_k' e^{z_k}\right)^2}$$

$$= -a_i \cdot a_j$$

$$\frac{\partial Loss}{\partial z_i} = -\frac{1}{a_j}\cdot -a_i a_j = a_i$$



Soft max

$$\frac{e^2}{e^2+e^3+e^4} \rightarrow \begin{pmatrix} 0.09 \\ 0.24 \\ 0.67 \end{pmatrix}$$

$$\frac{e^{2-k}}{e^{2-k}+e^{3-k}+e^{4-k}}$$

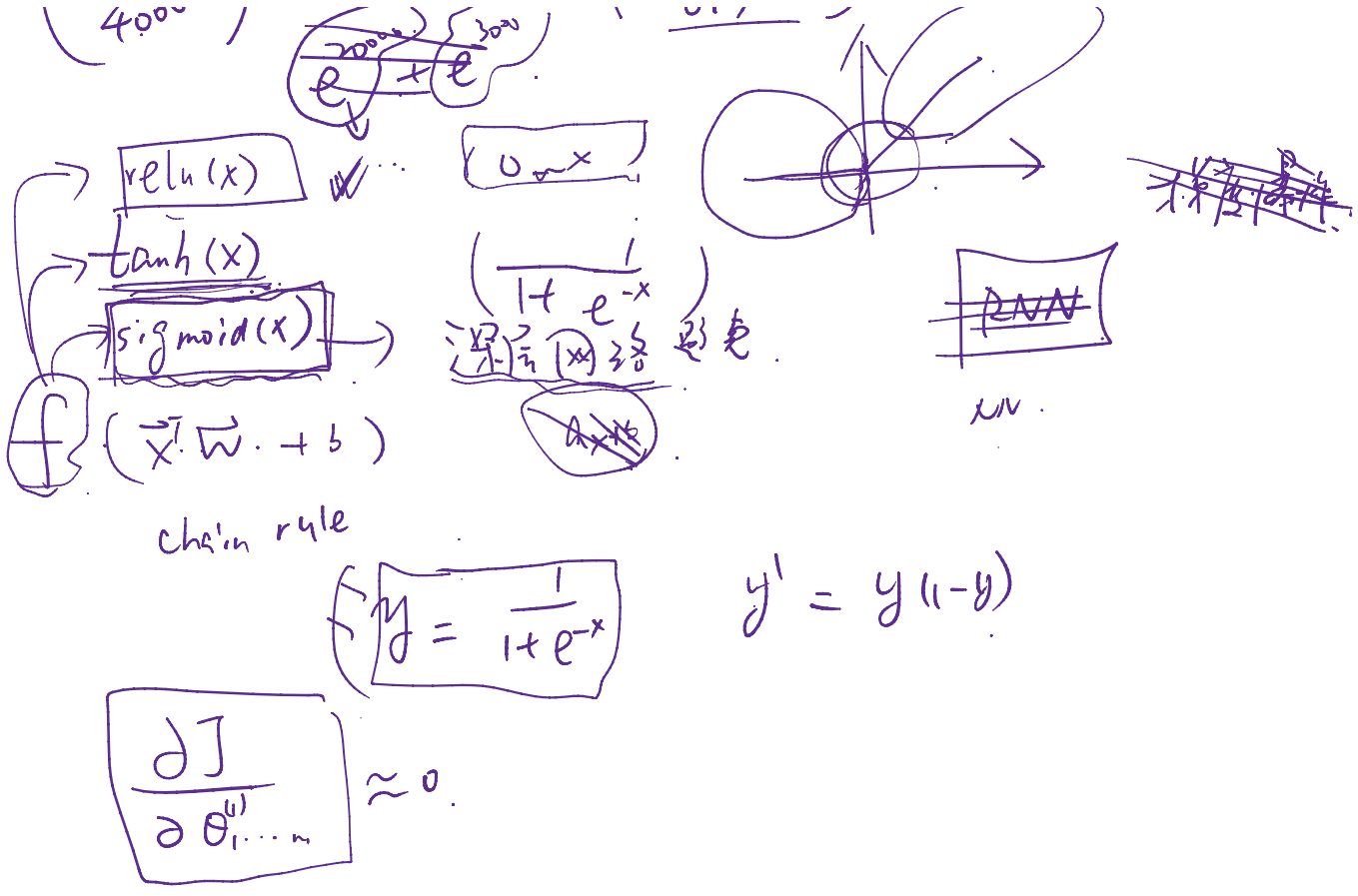
$$\frac{e^2/X}{(e^2+e^3+e^4)/X} \quad e^{\max(x)} \quad X=(2,3,4)$$

$$\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \rightarrow \text{Softmax} \rightarrow \begin{pmatrix} 0.09 \\ 0.24 \\ 0.67 \end{pmatrix}$$

大的更大.
小的更小.

$$\begin{pmatrix} 20000 \\ 30000 \\ 40000 \end{pmatrix} \rightarrow \text{Softmax} \rightarrow \begin{pmatrix} 0.0 \\ 0.0 \\ 0.9 \end{pmatrix}$$

$$\frac{e^{20000}}{e^{20000}+e^{30000}}$$


400°
relu(x)
tanh(x)
sigmoid(x)
f
$(\vec{x}\cdot\vec{w}+b)$
chain rule
$y = \frac{1}{1+e^{-x}}$
$y' = y(1-y)$
$\frac{\partial J}{\partial \theta_{1\cdots n}^{(l)}} \approx 0$