Causal Inference with Treatment Measurement Error

A Non-parametric Instrumental Variable Approach

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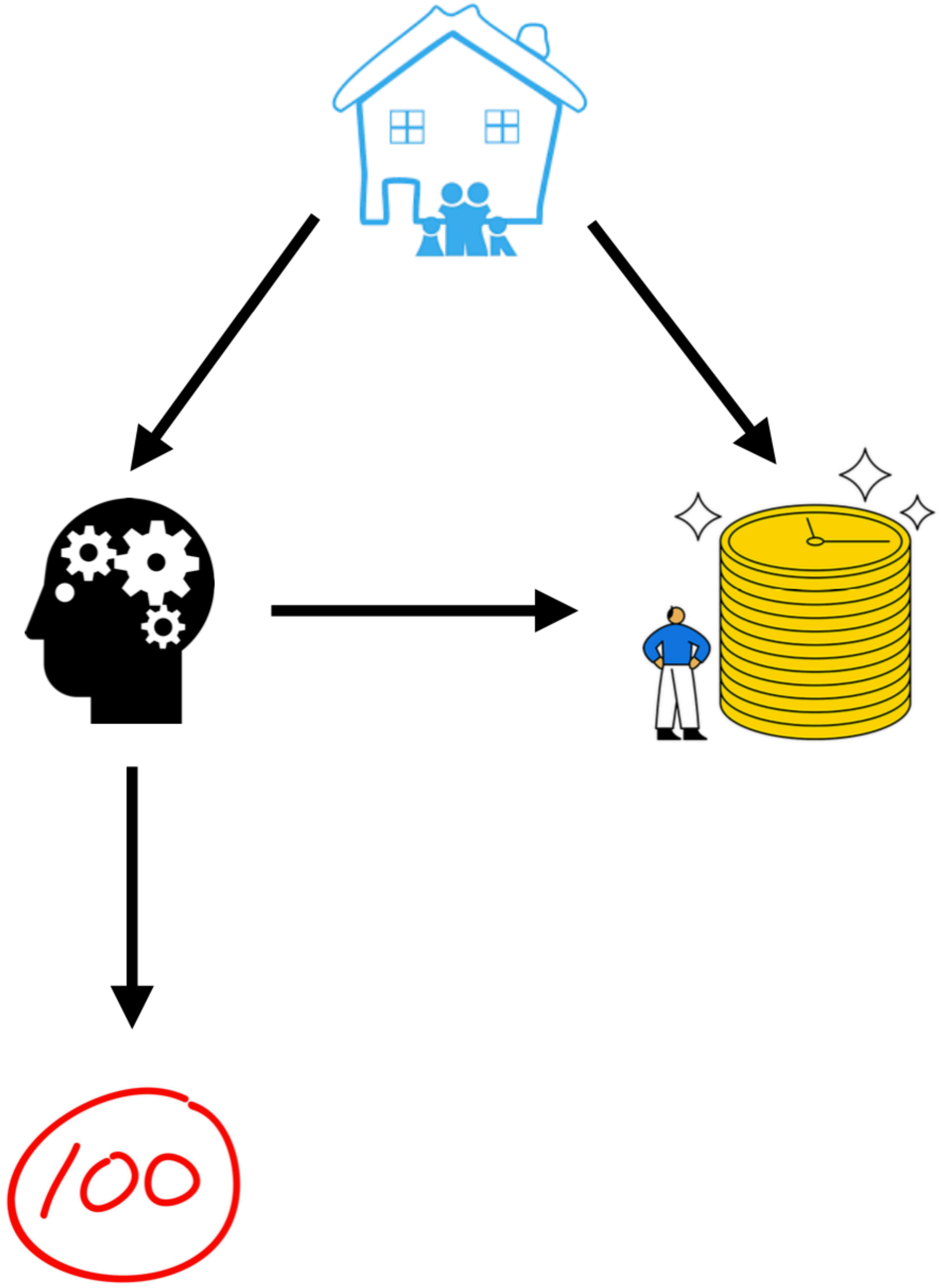

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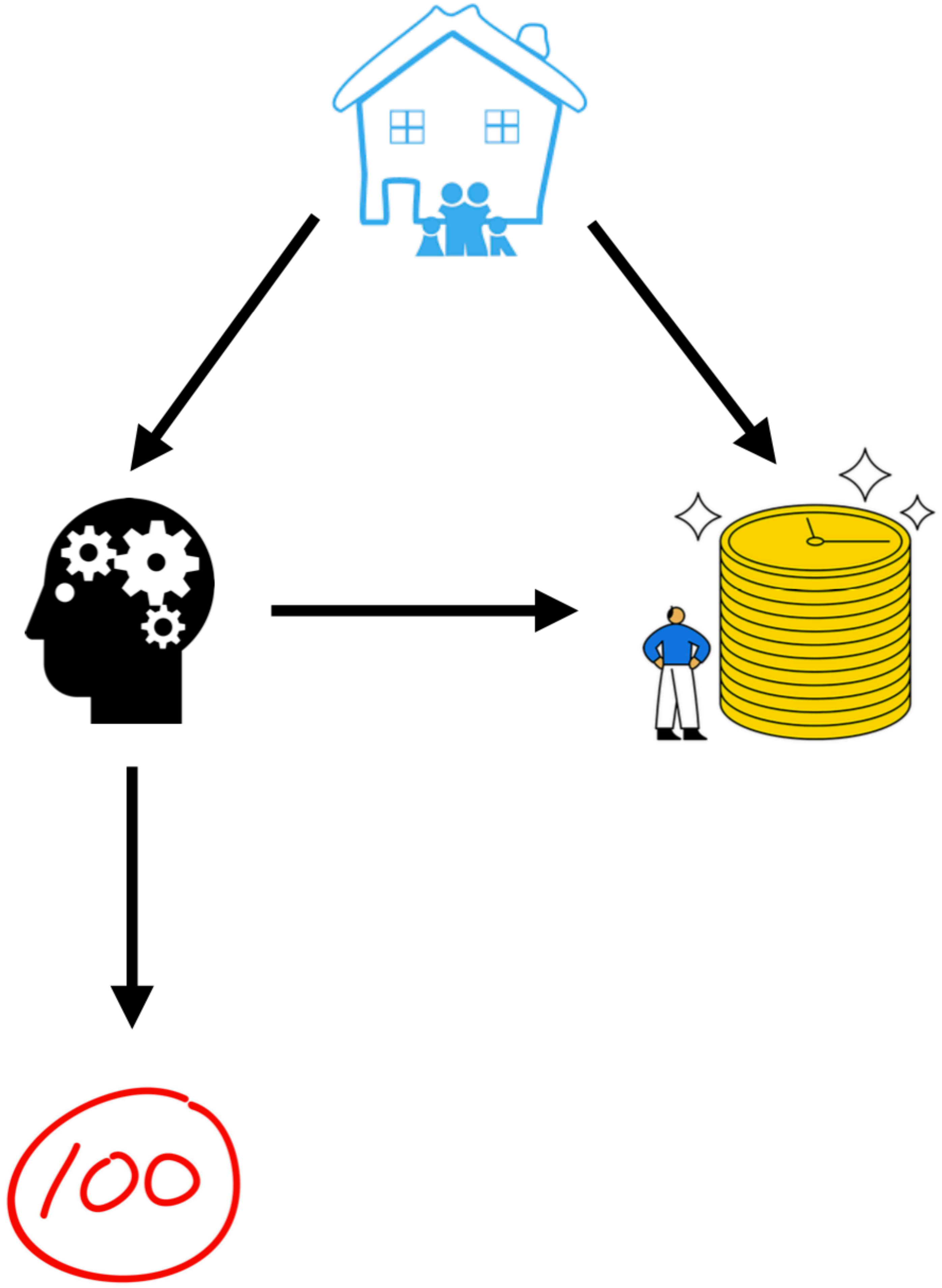

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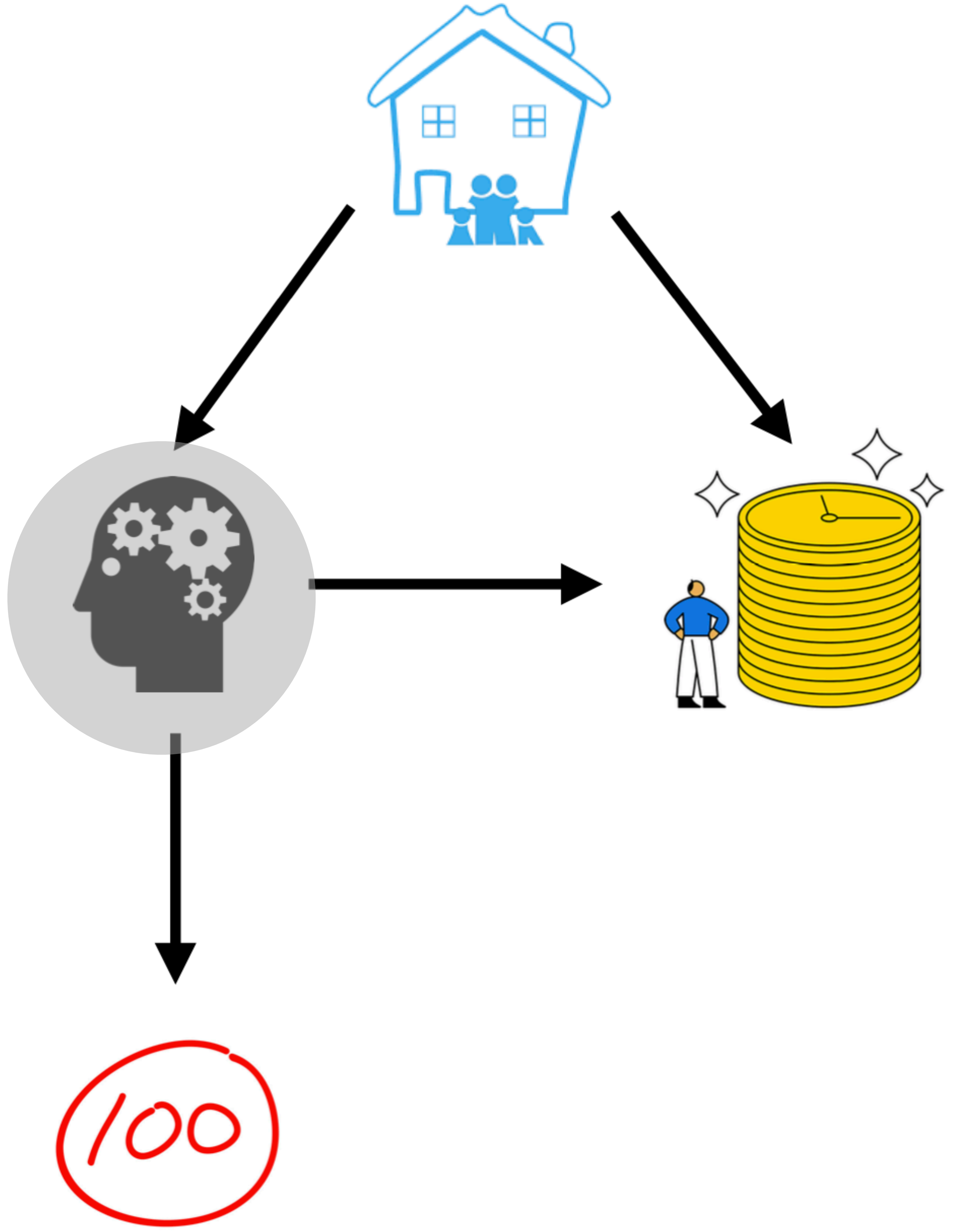

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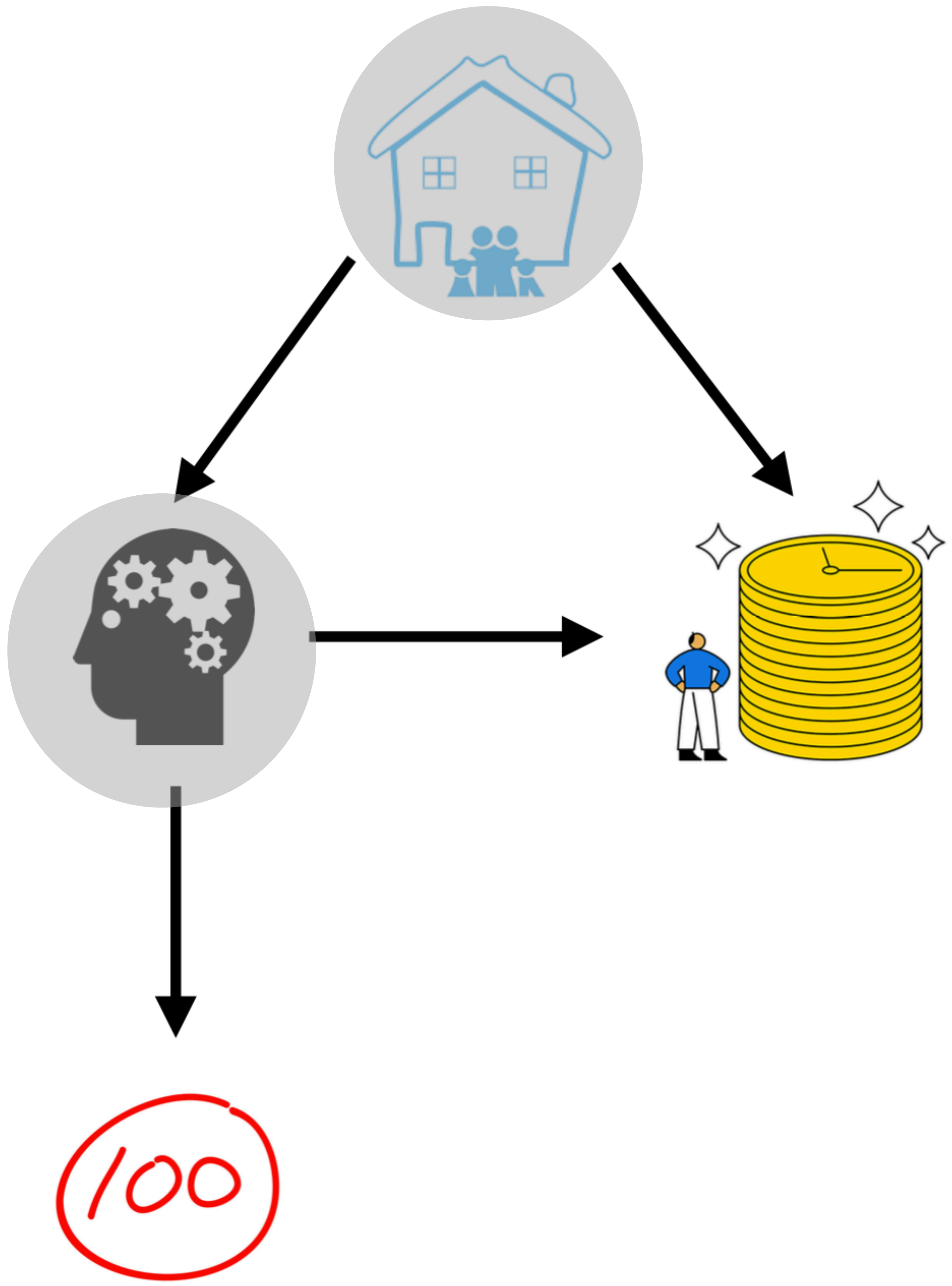

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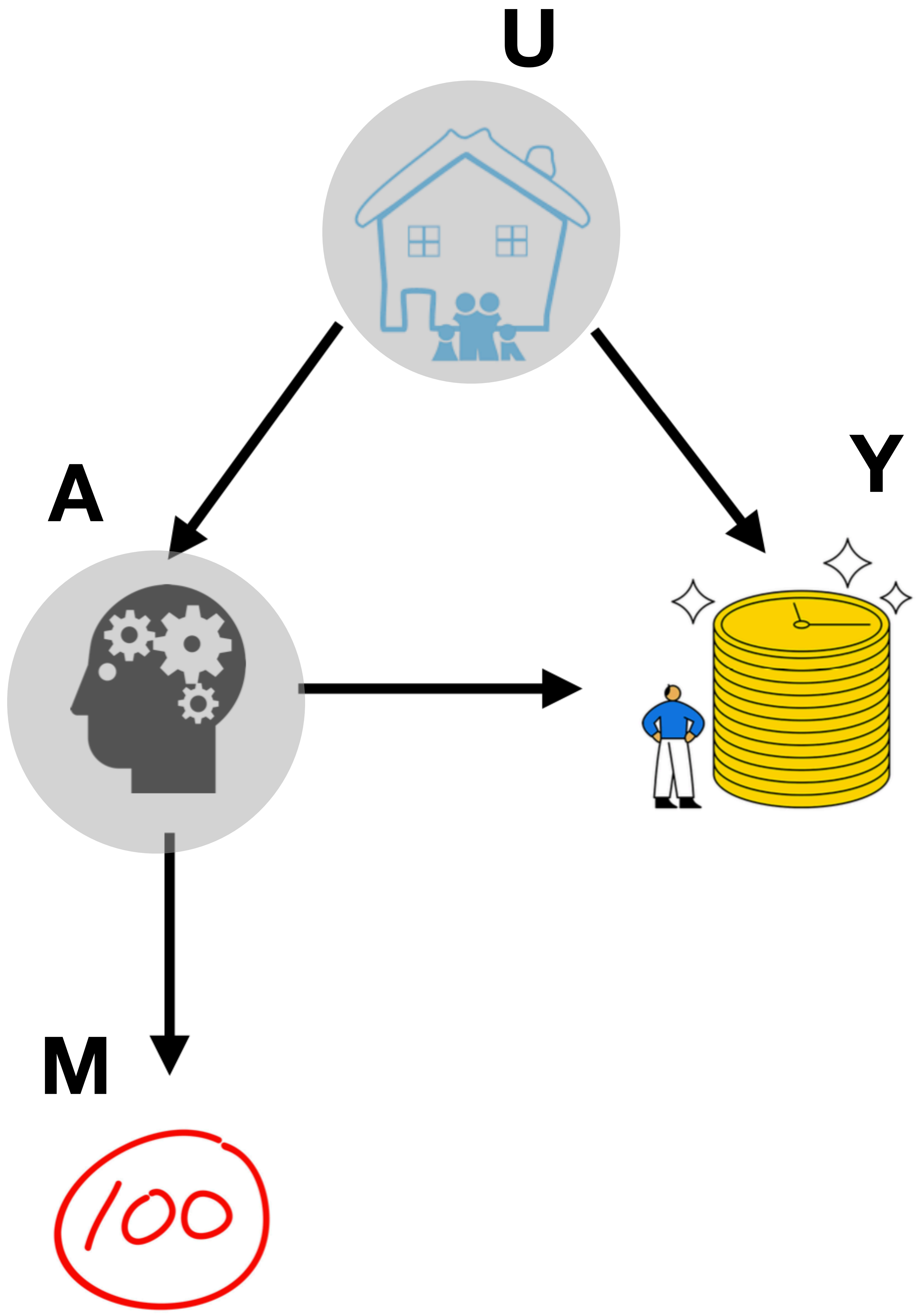

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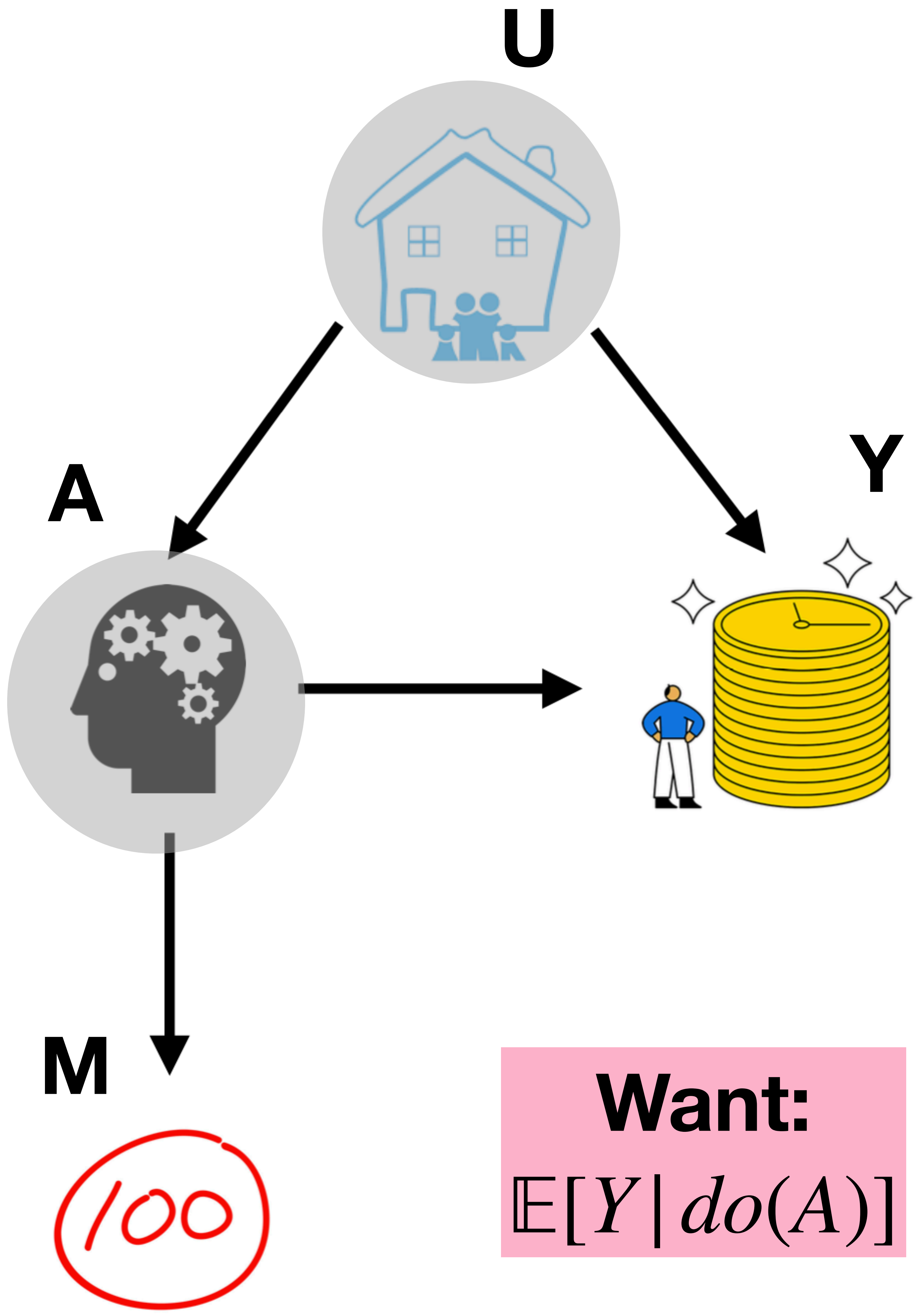

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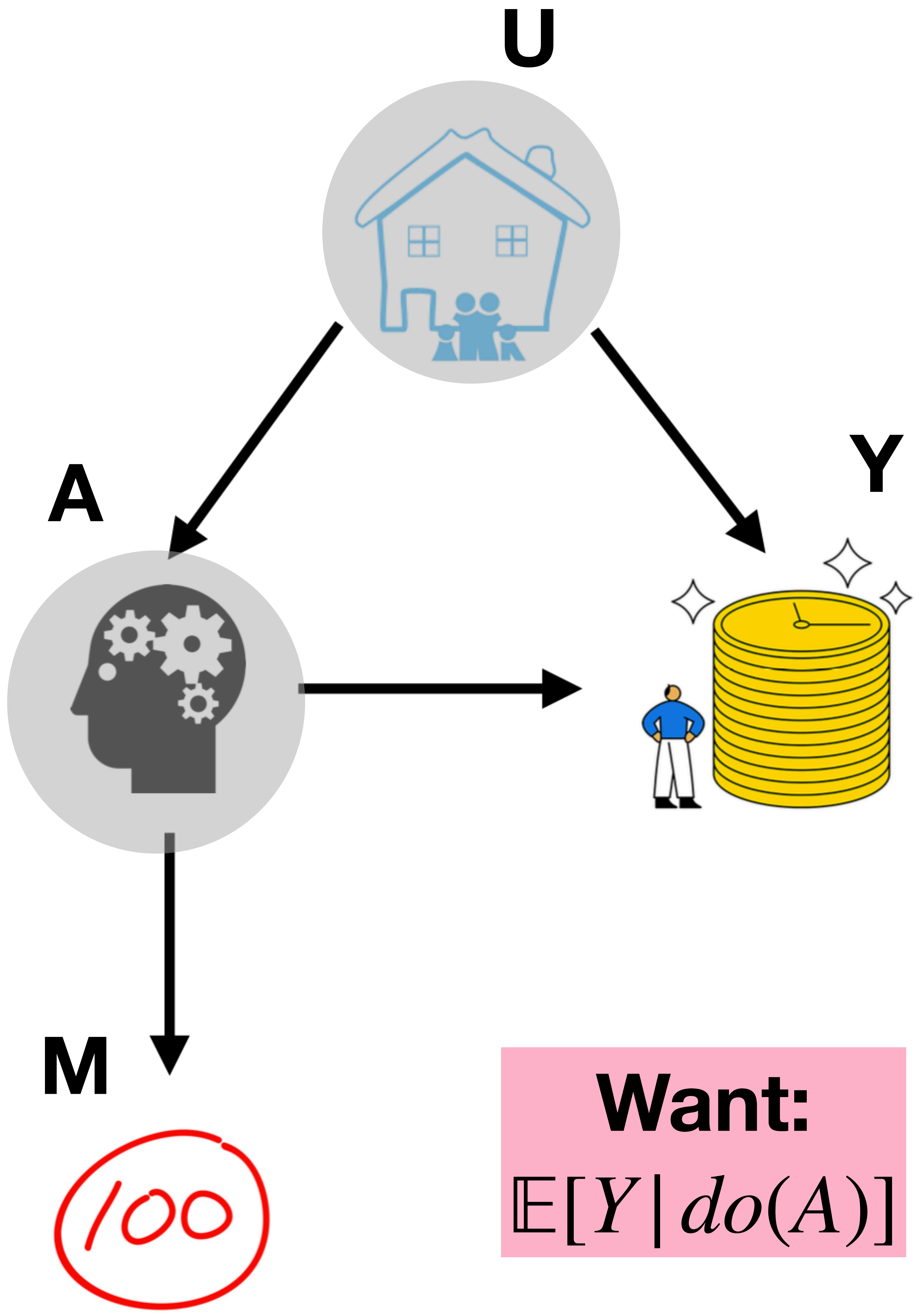
- What if we don't account for them?



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Consequences of confounding/measurement error?

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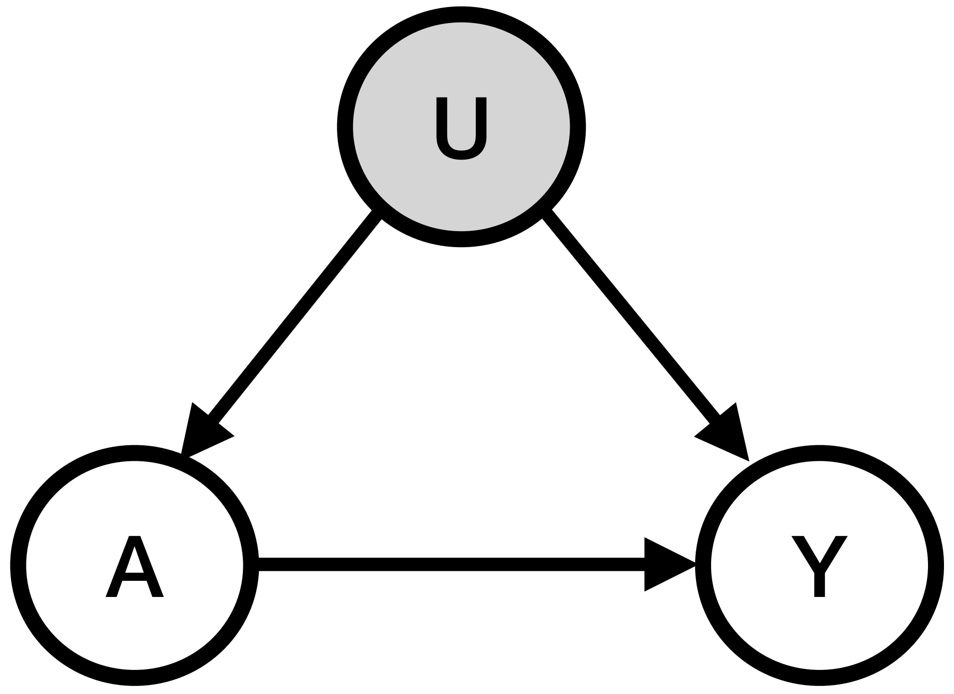

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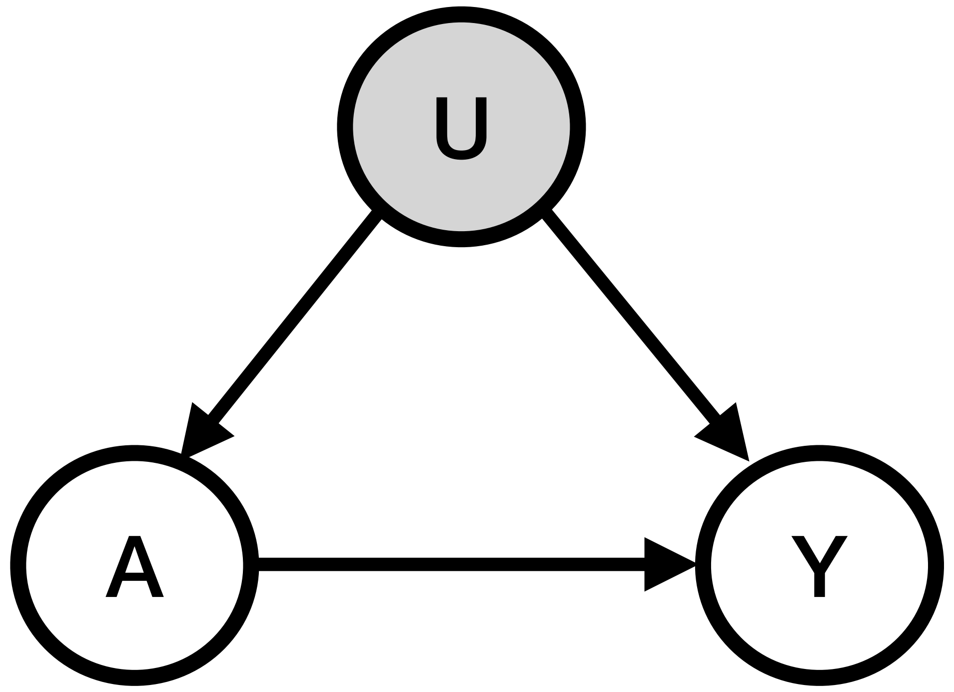

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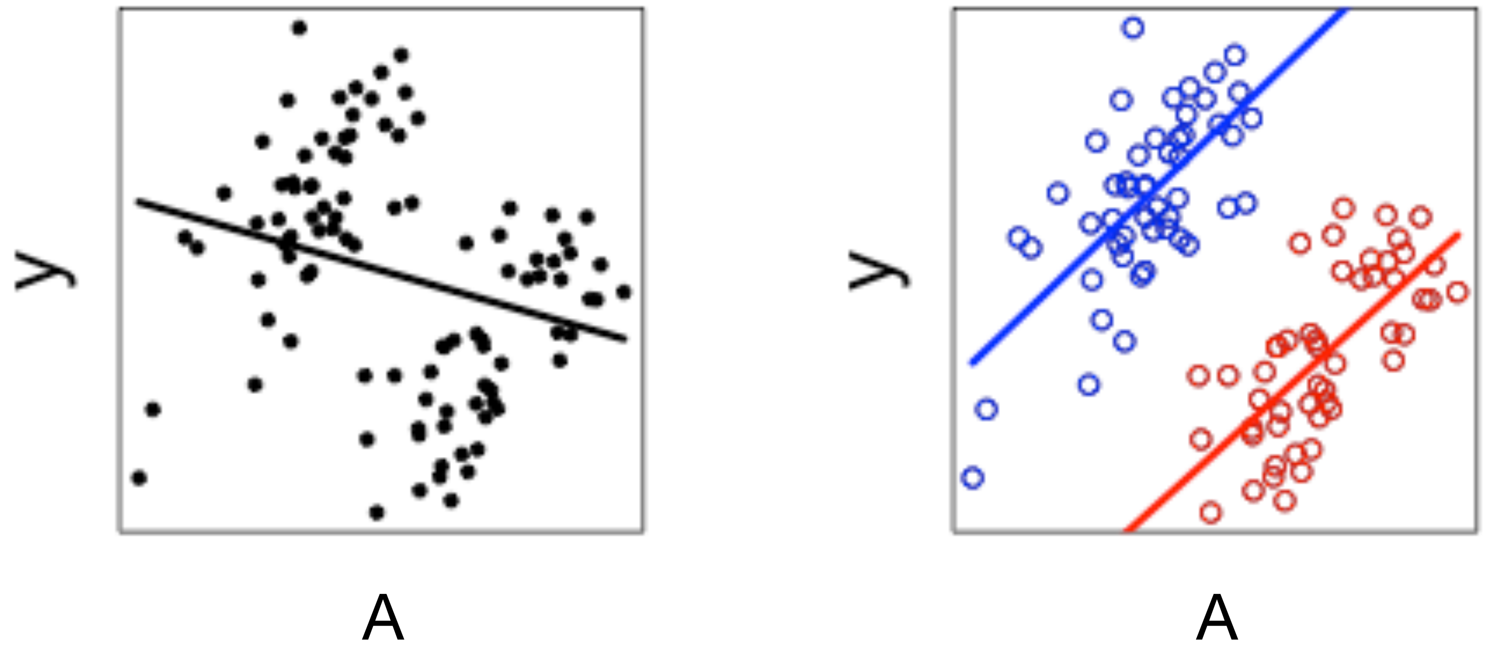

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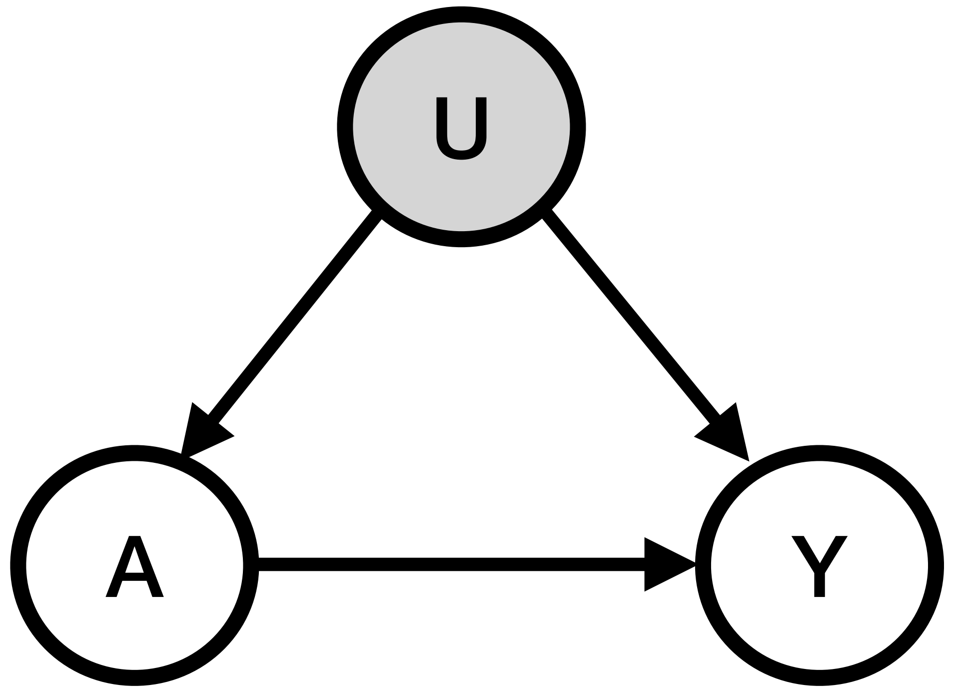

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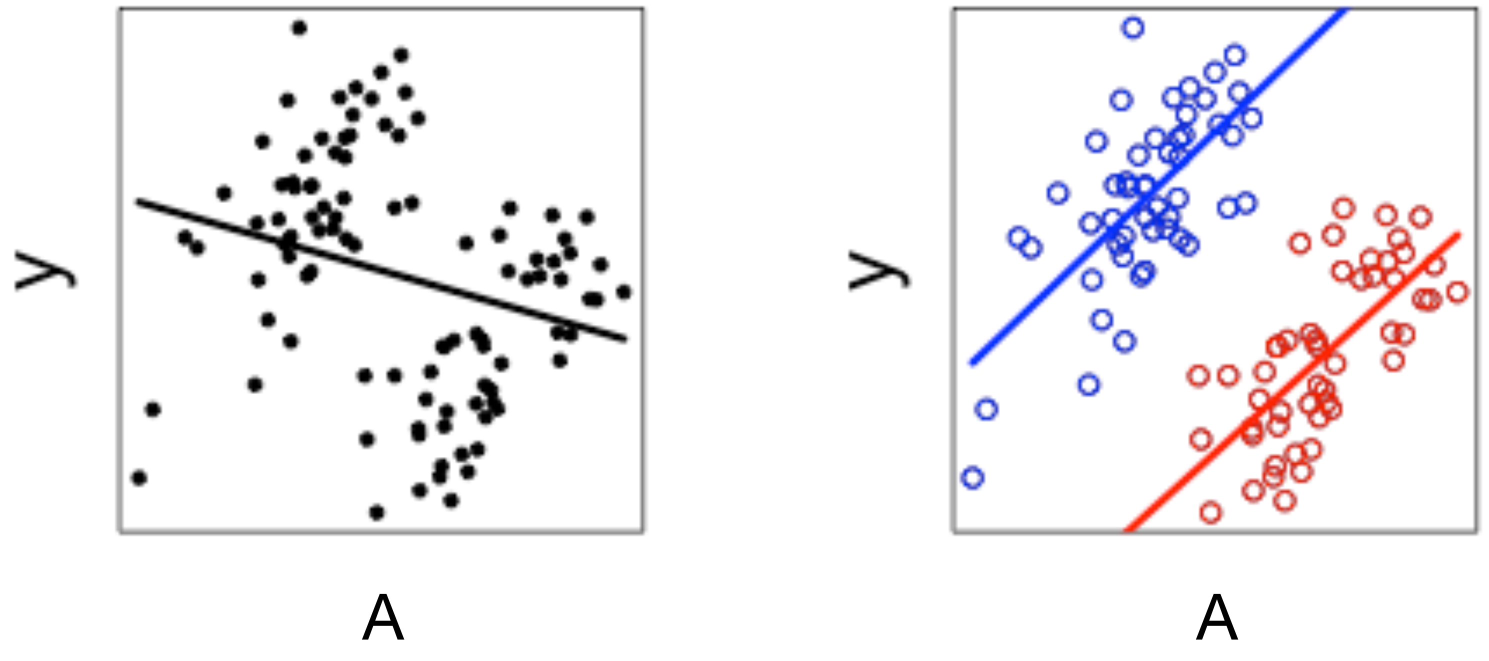
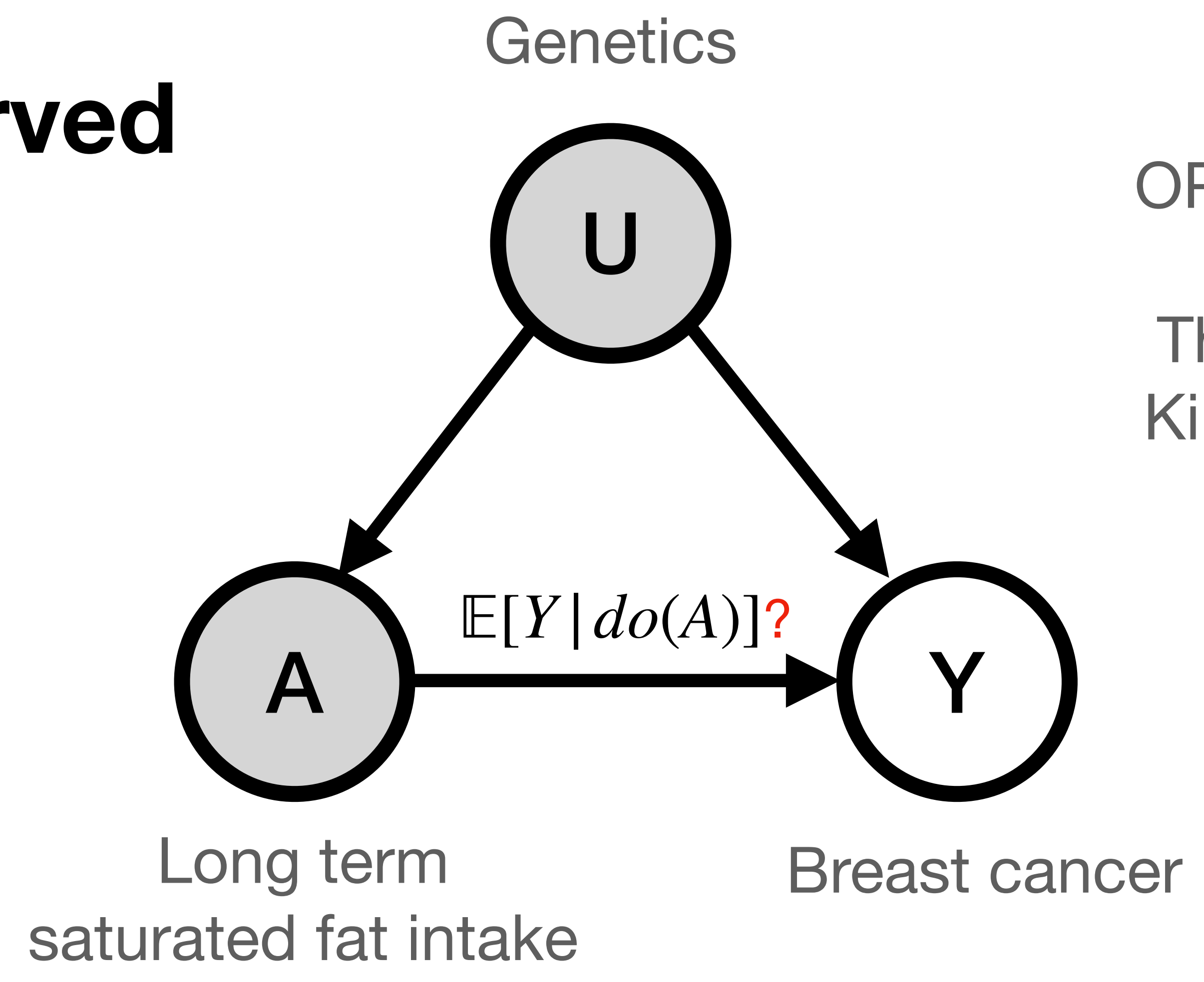


Action observed with error:



OPEN study: Subar, Thompson, Kipnis, et al. 2001

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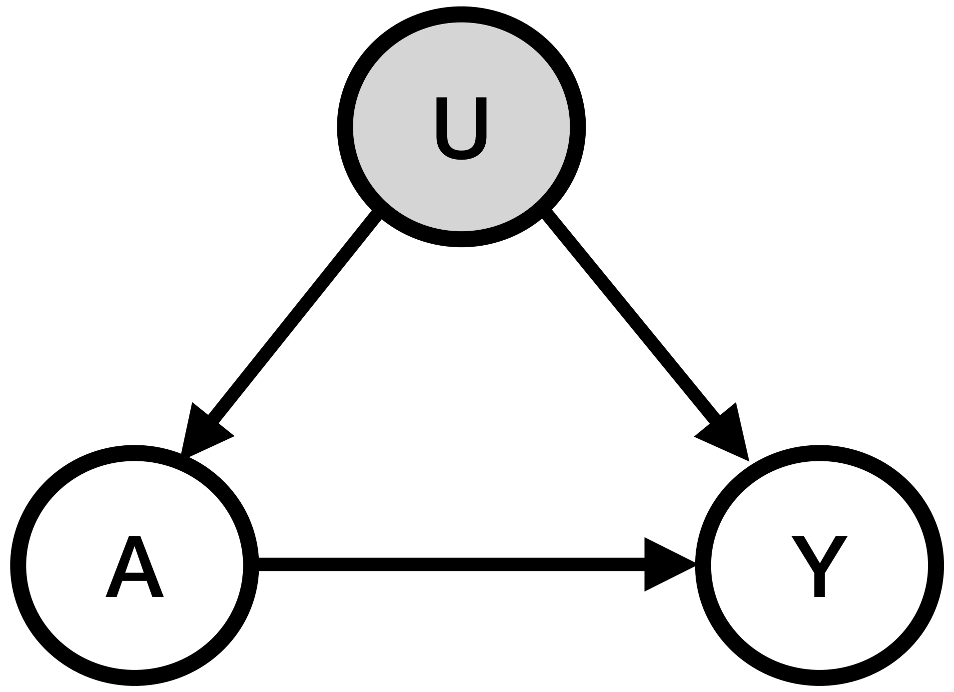

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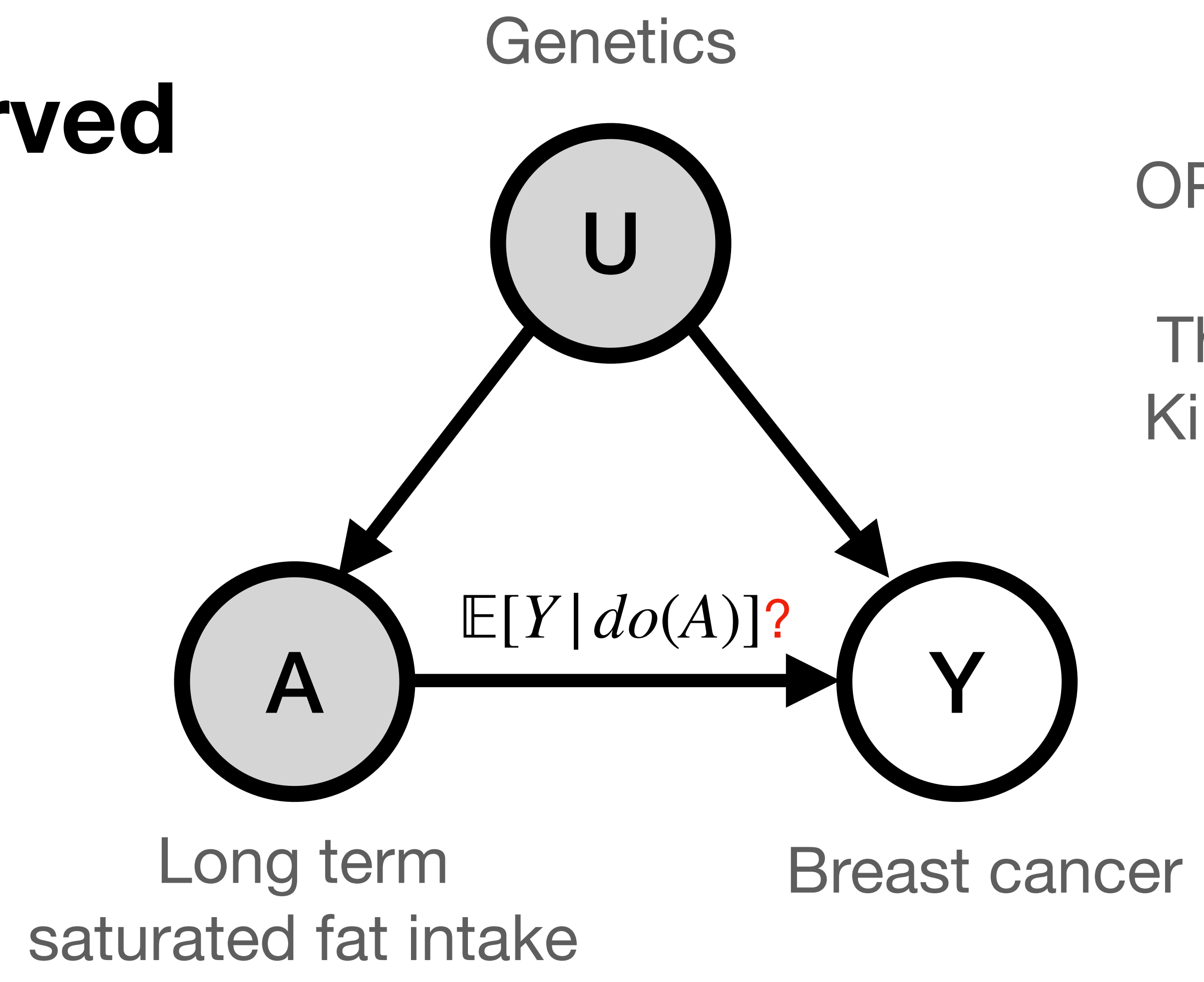

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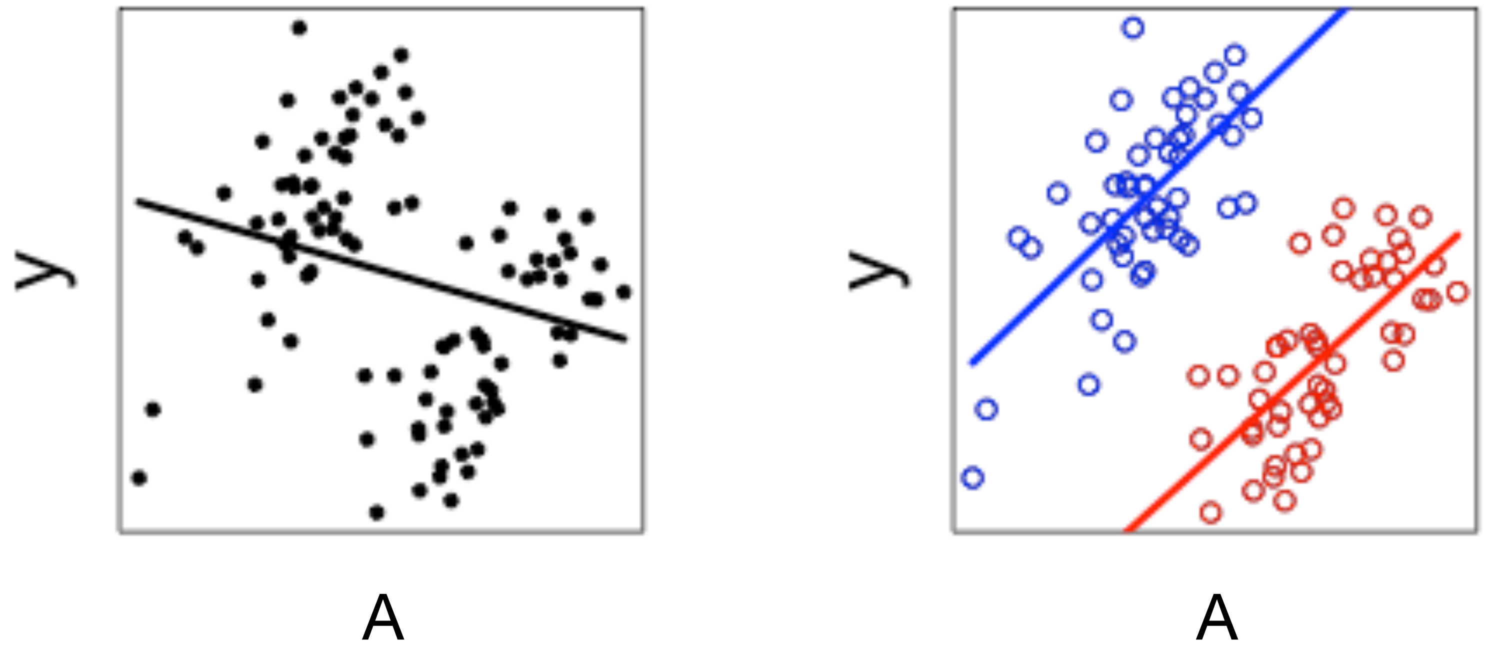
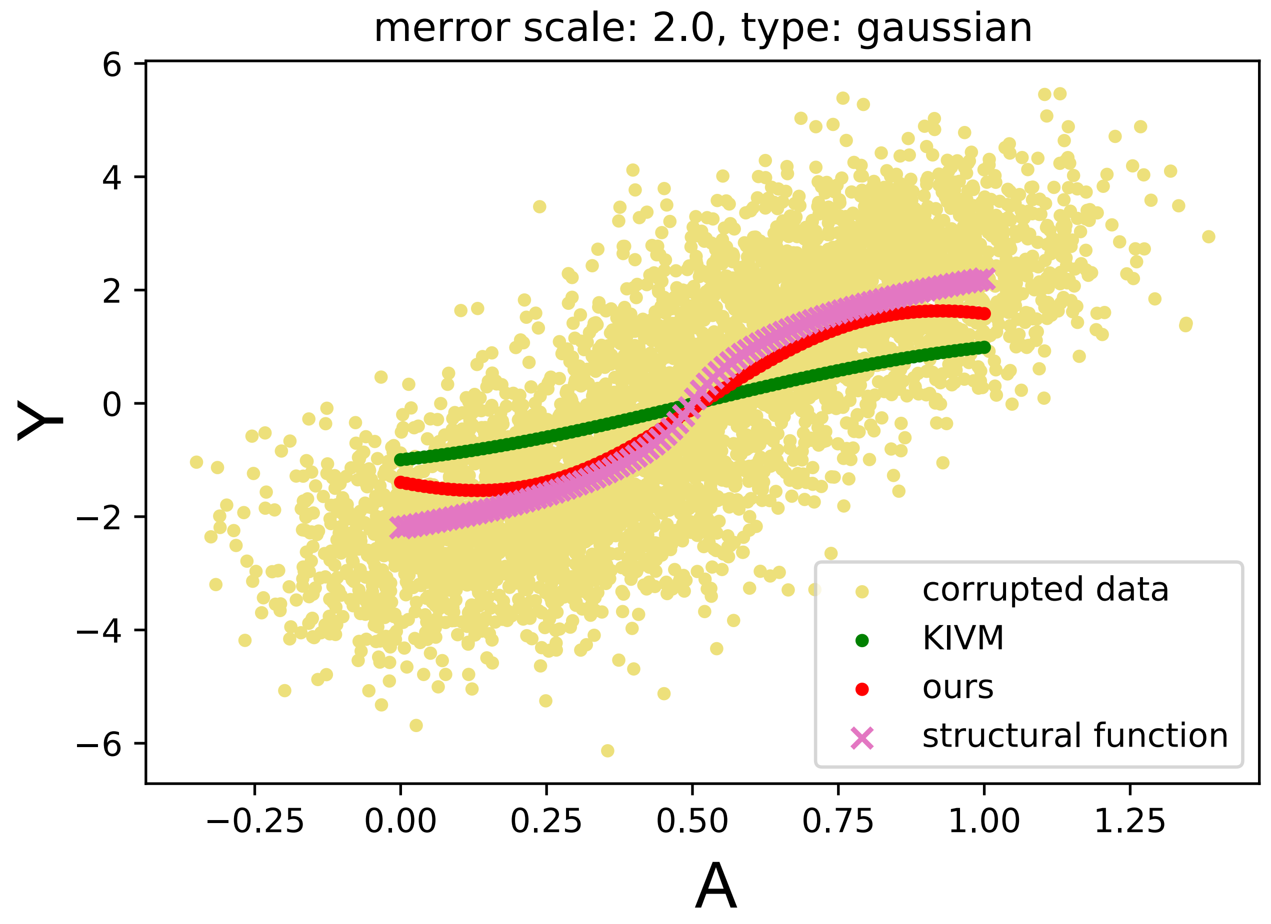


Mask interesting relationships:



Measurement error on action variables - overview

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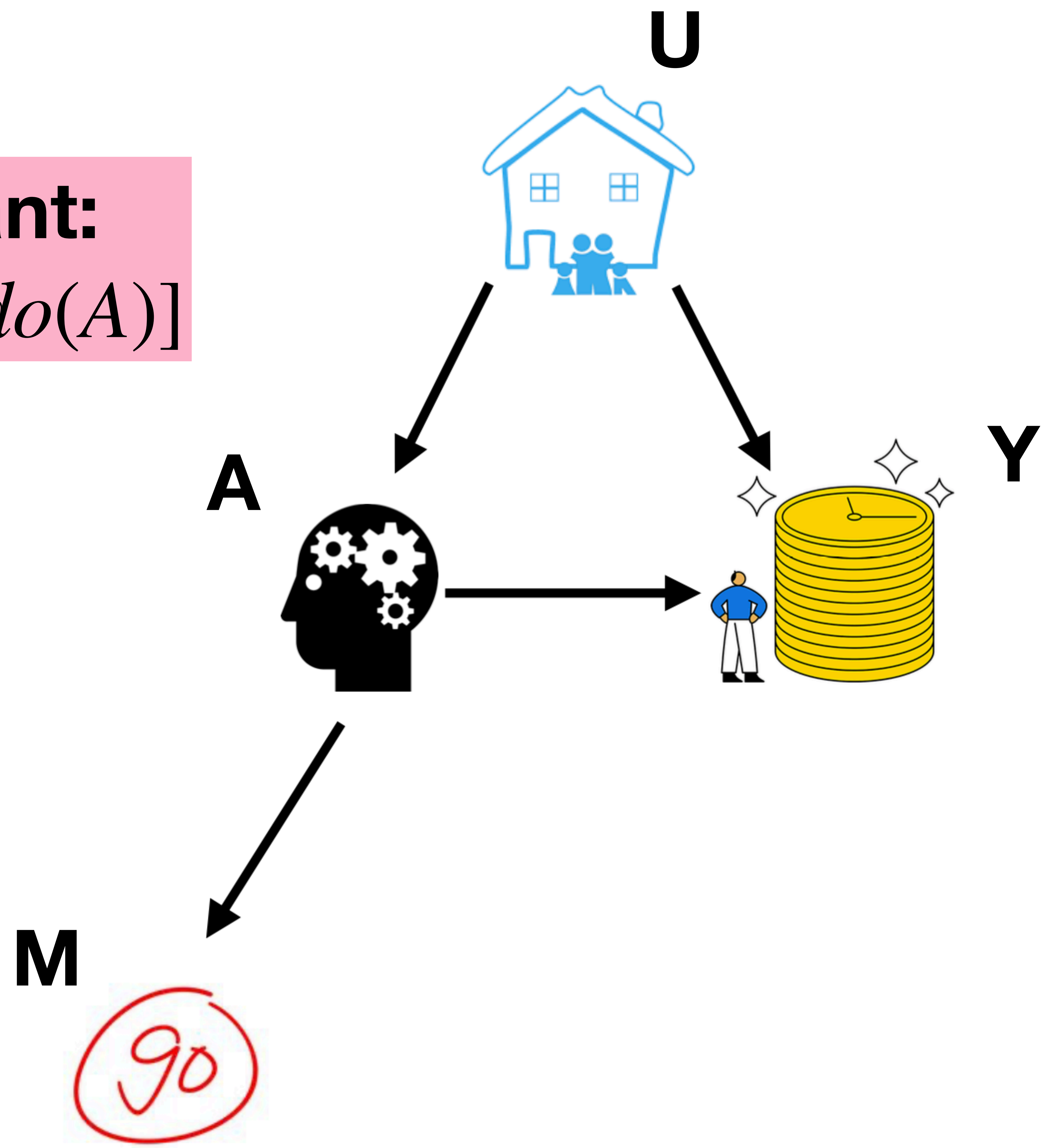

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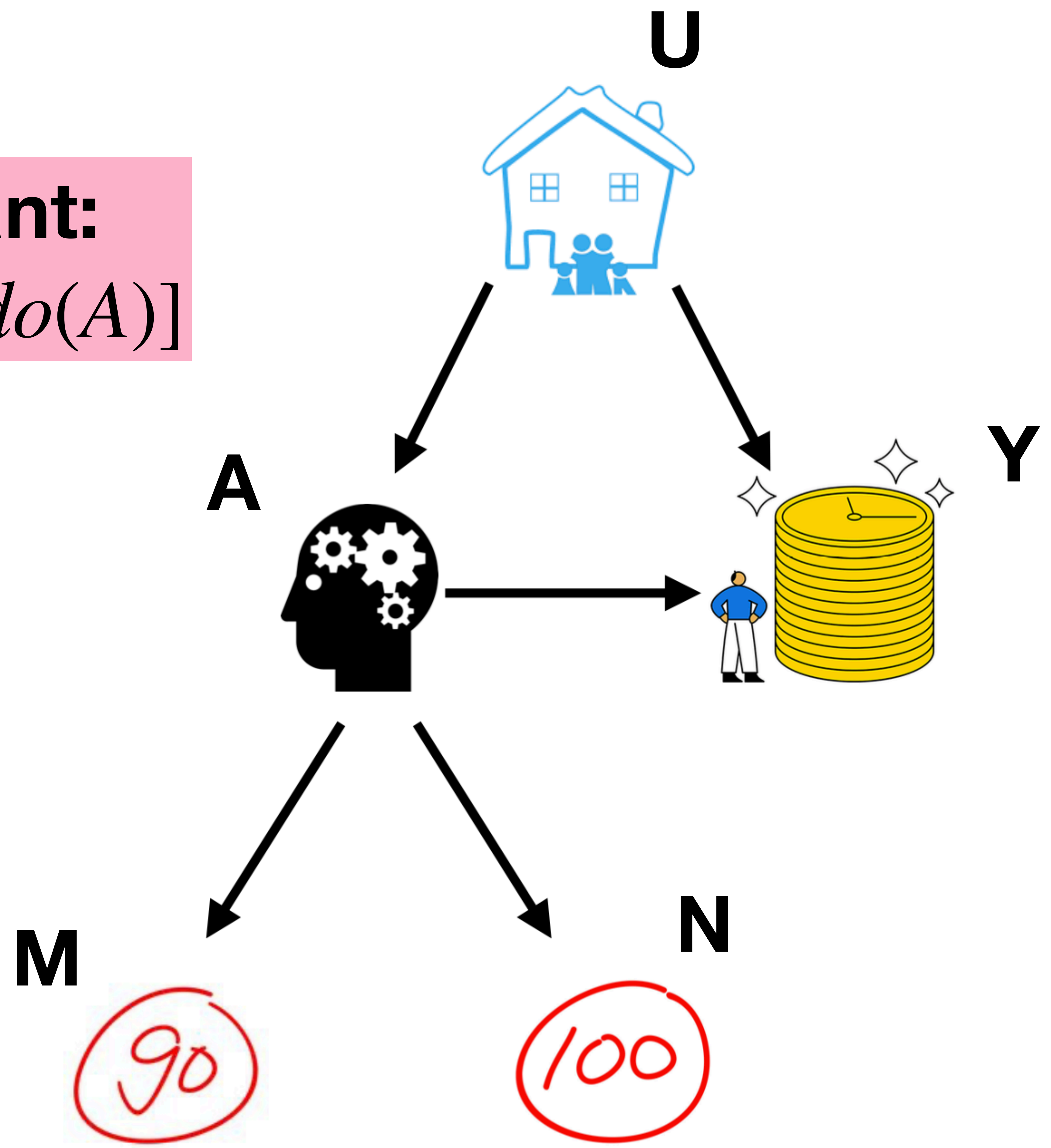

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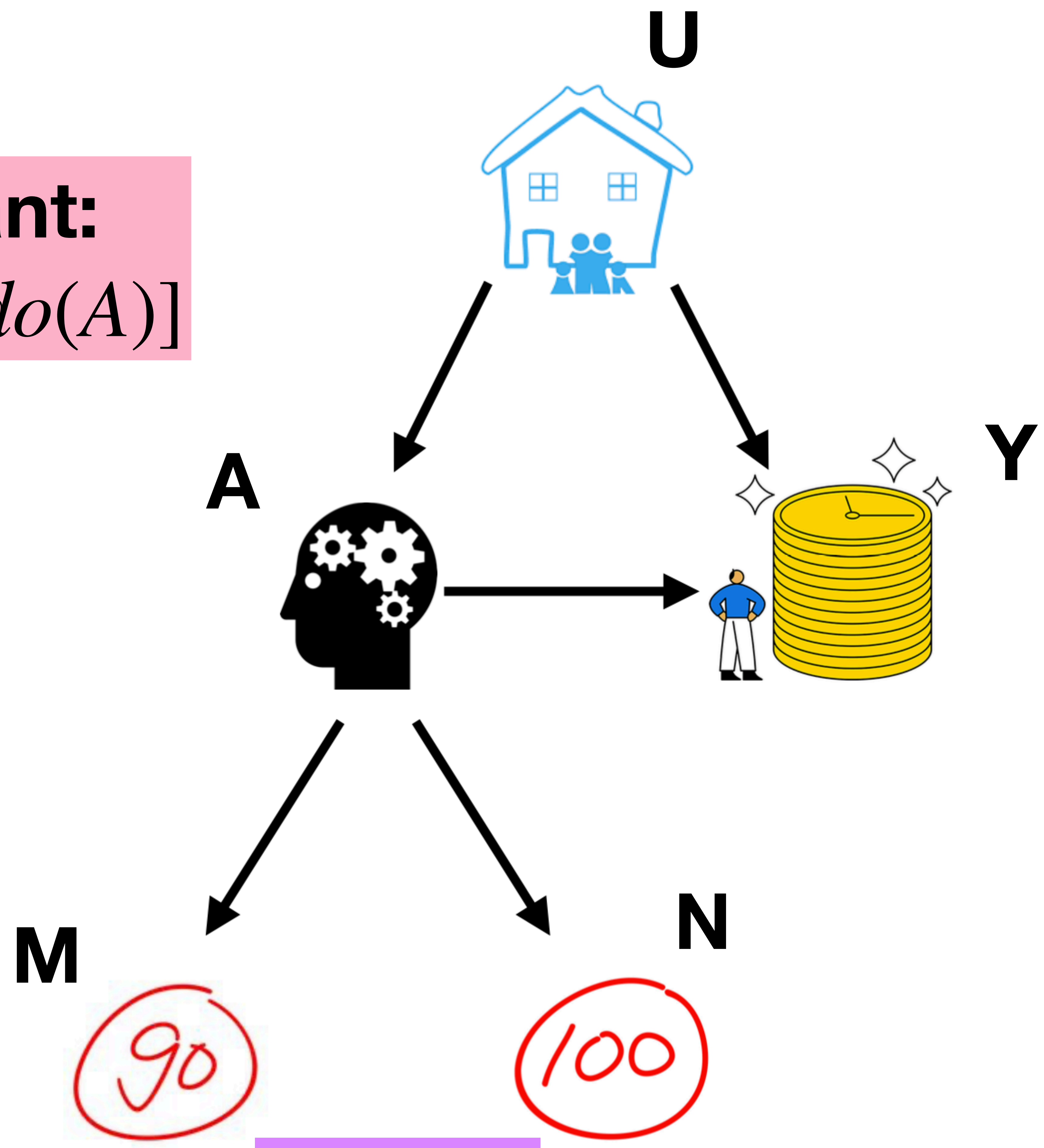

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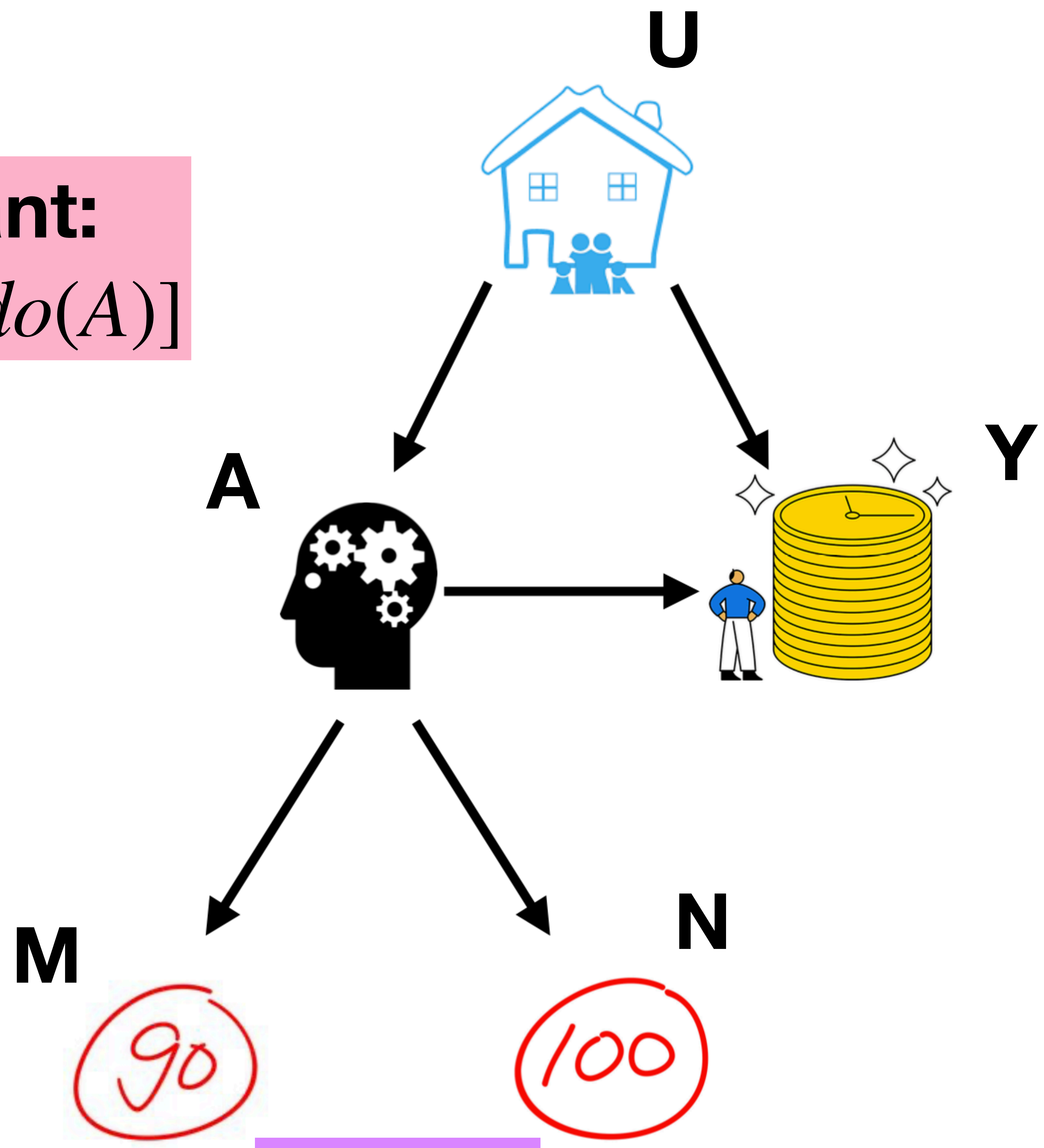

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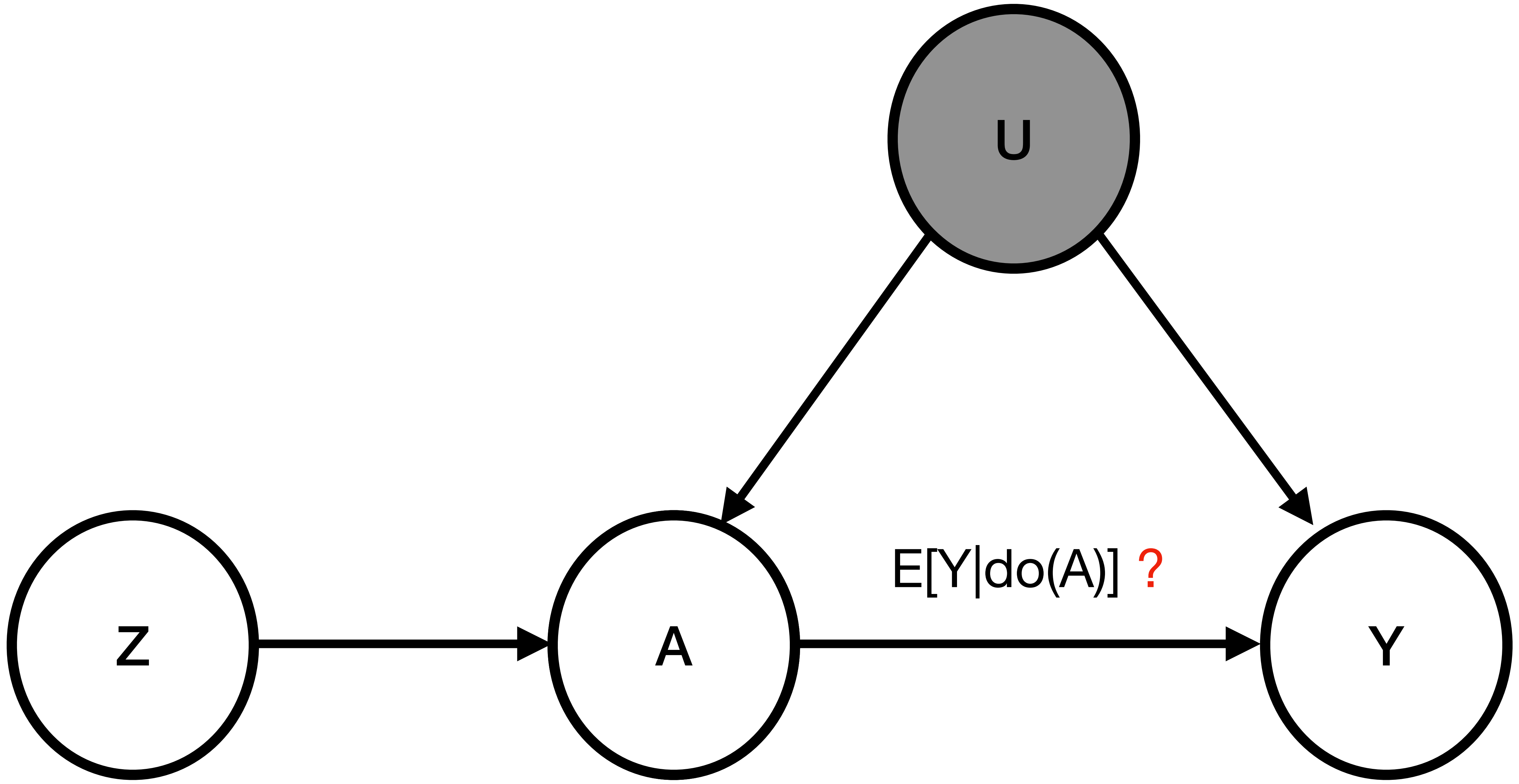
Identification with instrumental variables



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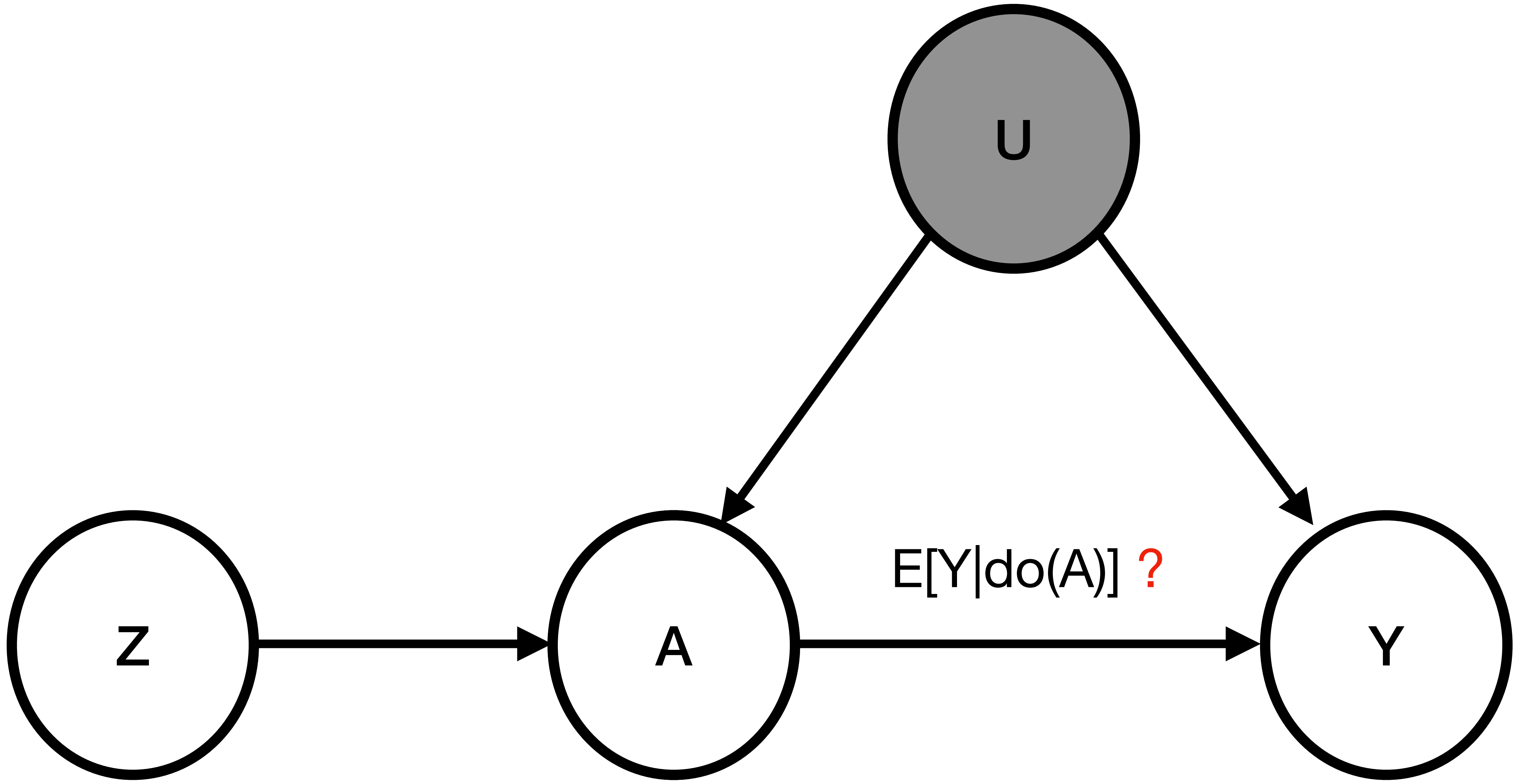

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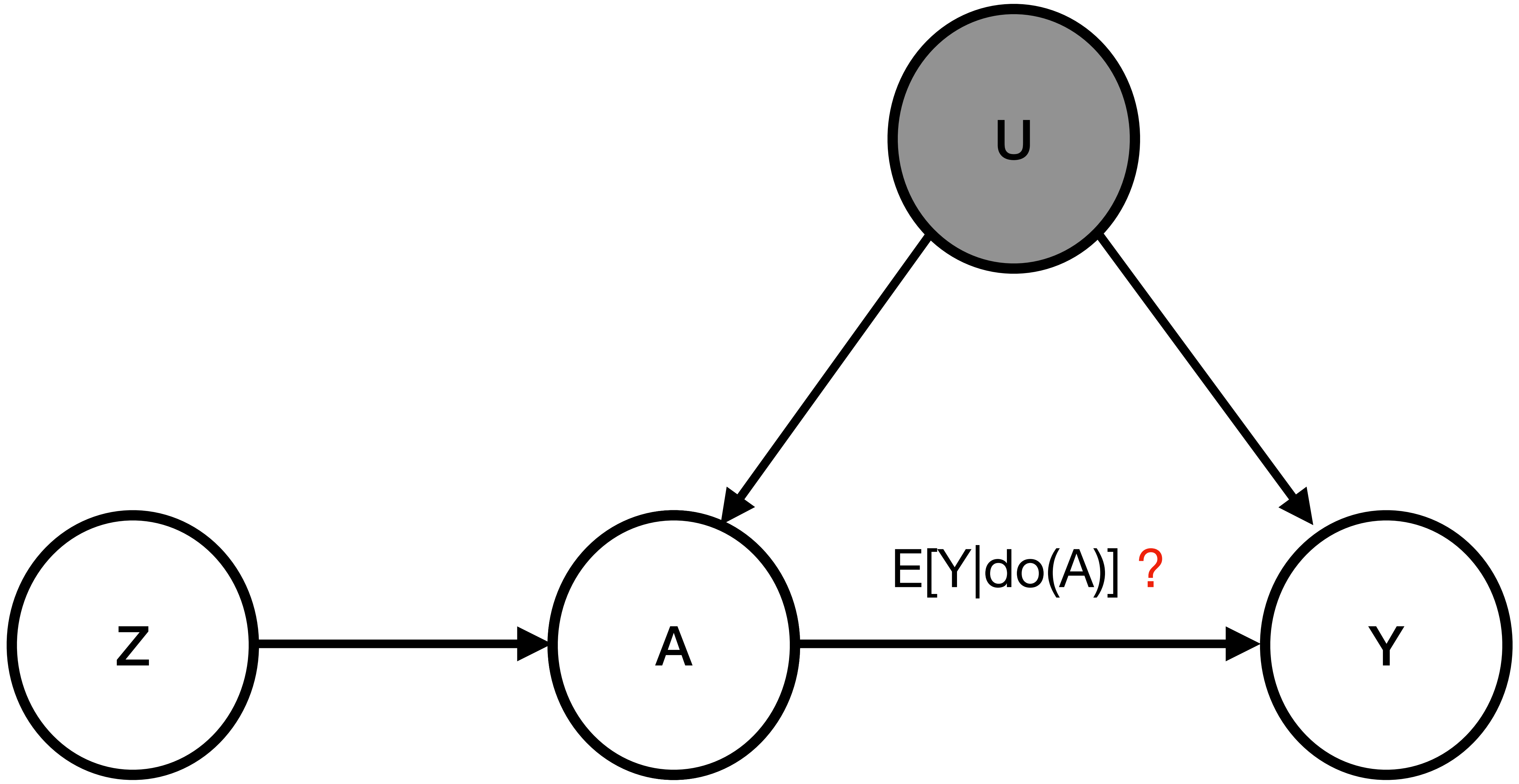

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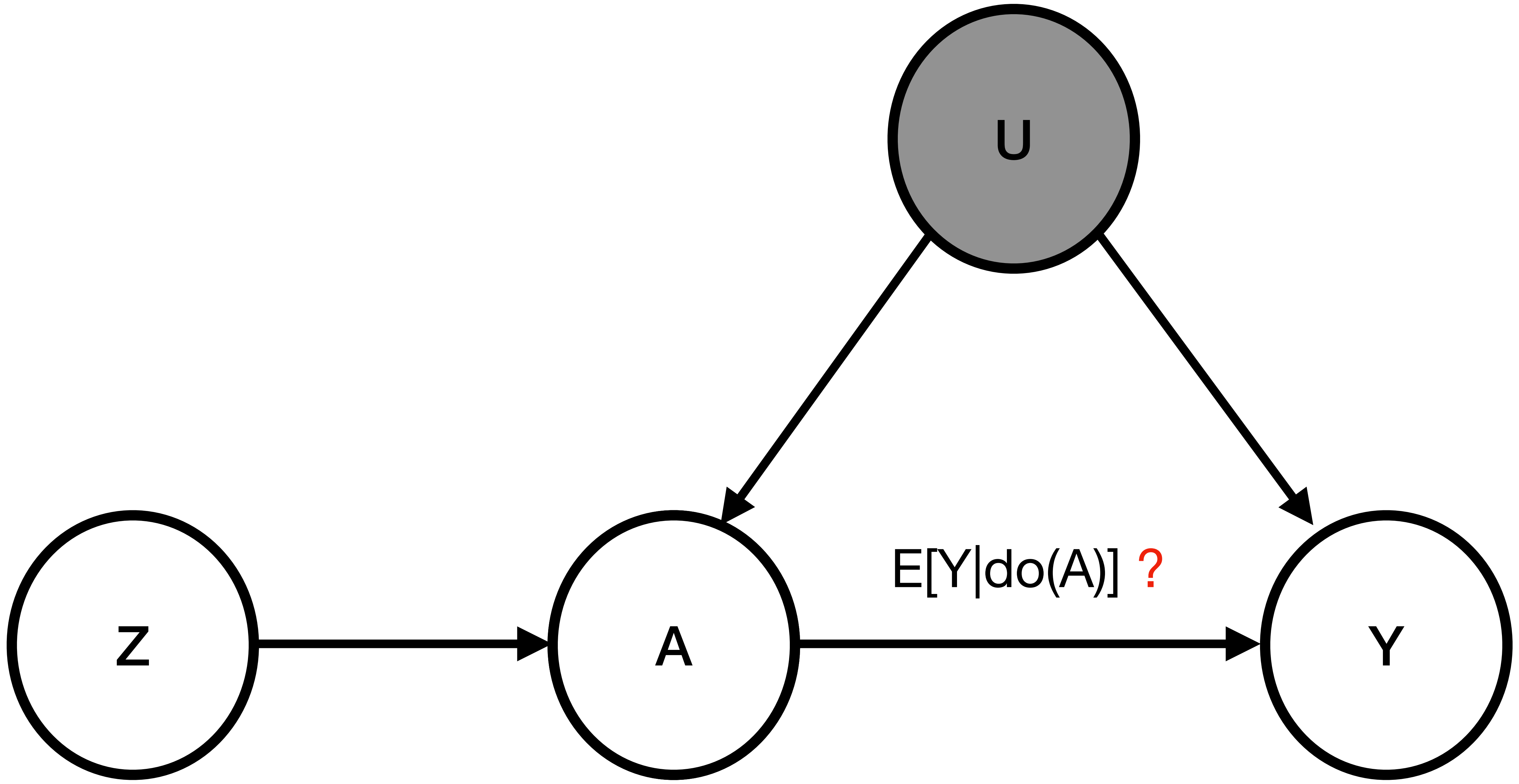

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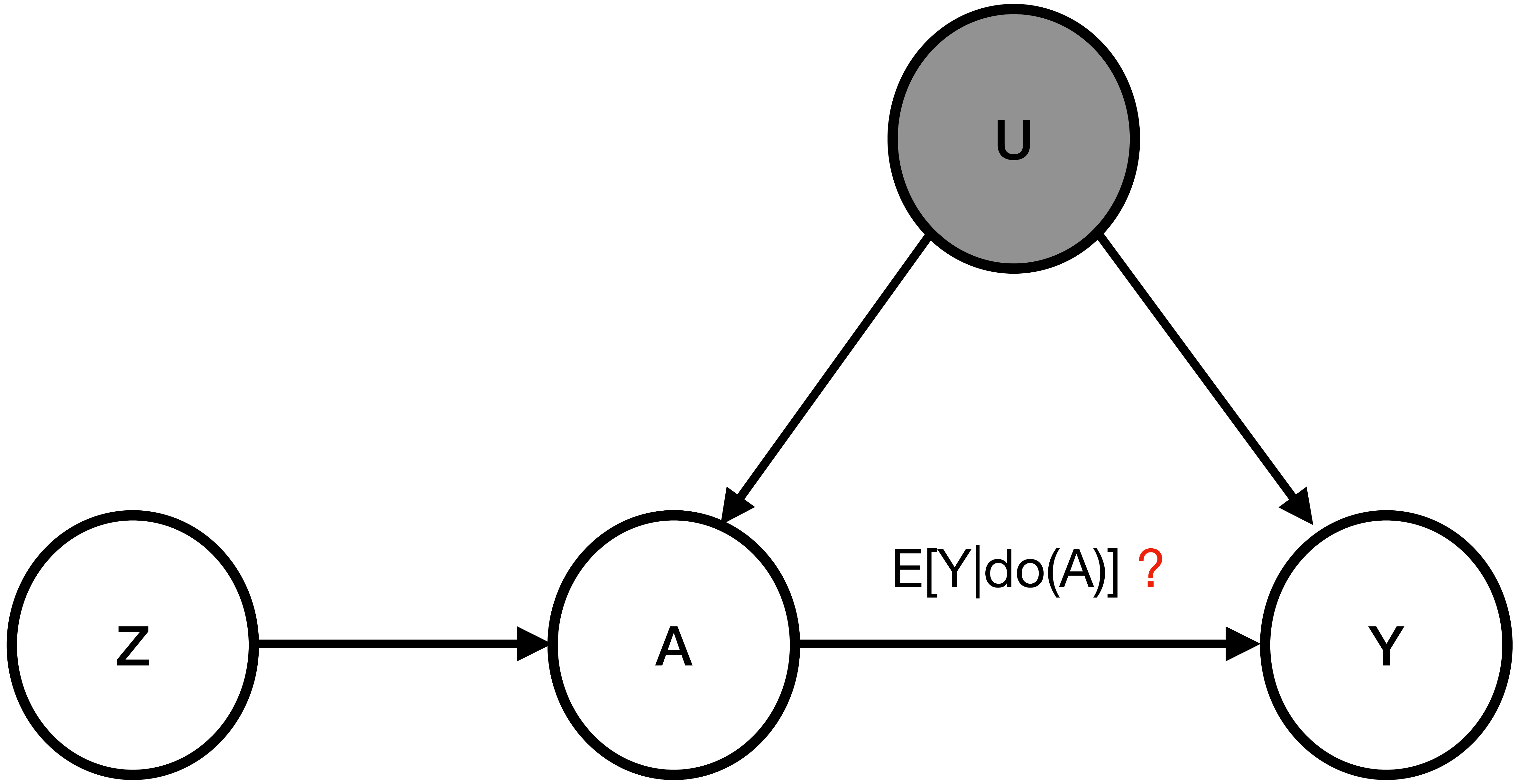

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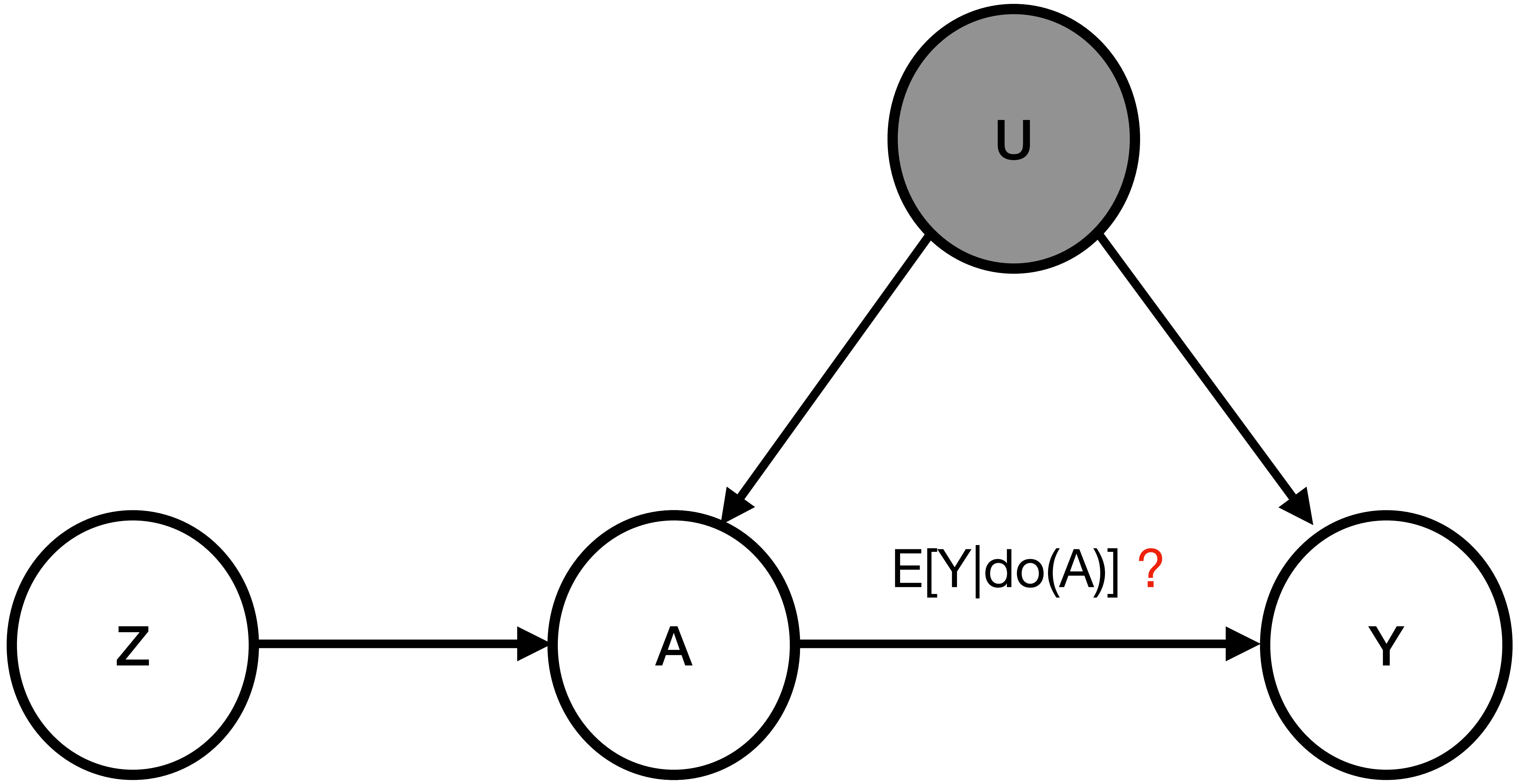

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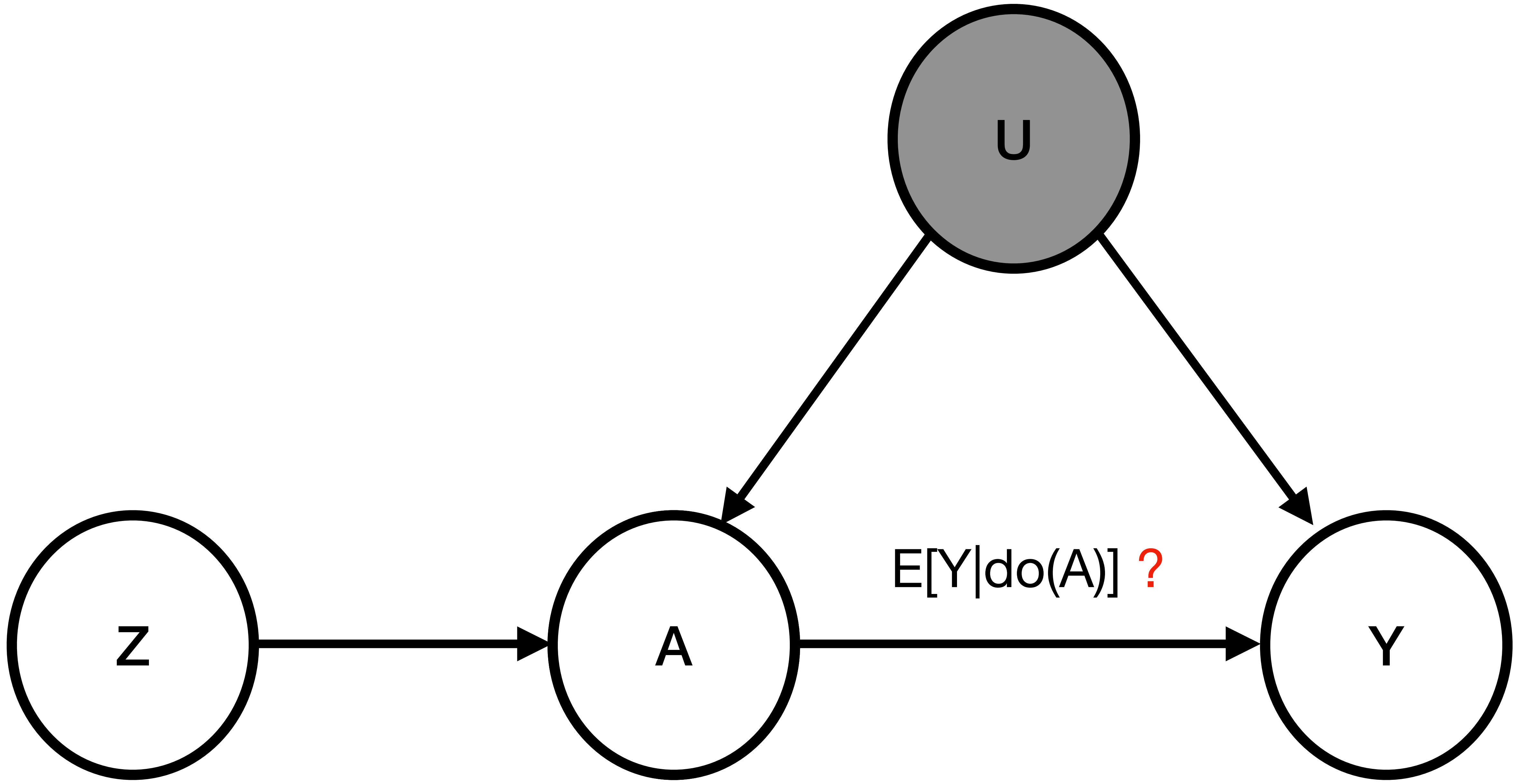

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But if $f(a) = \theta^T \phi(a)$, then rhs simplies to

$$\mathbb{E}[Y | Z] = \theta^T \mathbb{E}[\phi(A) | Z]$$

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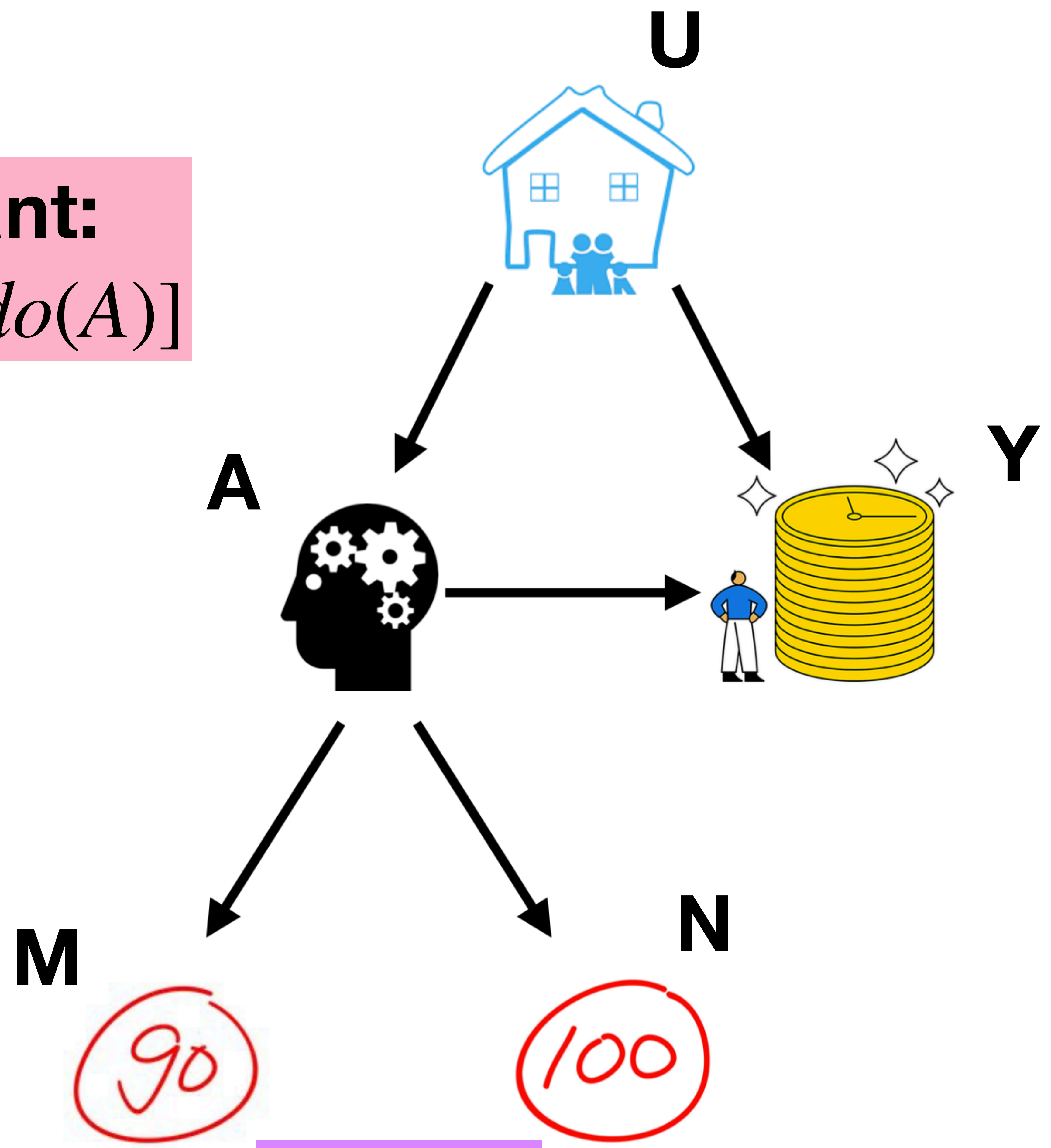

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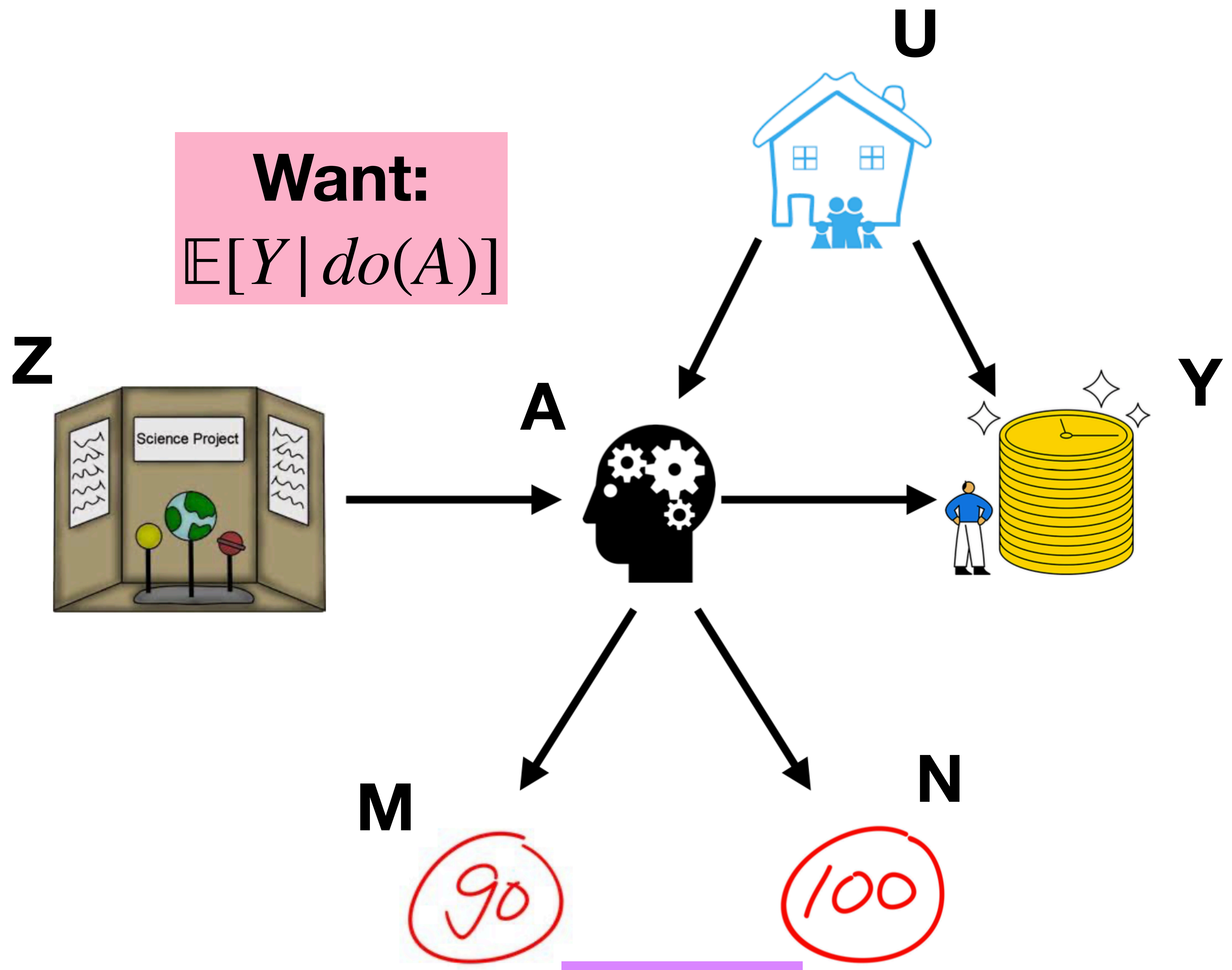

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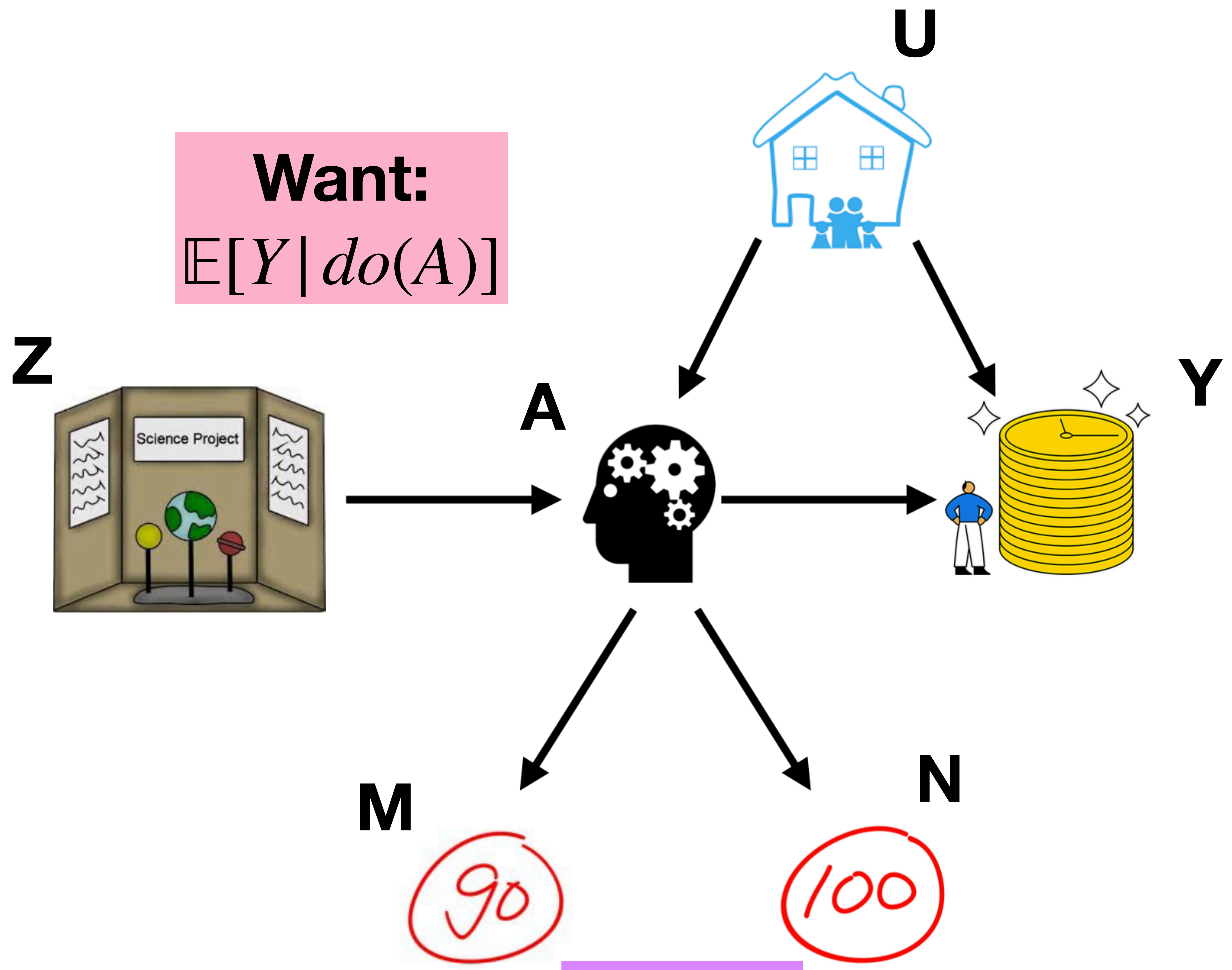

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What about
$\mu_{A|z} := \mathbb{E}[\phi(A) \mid z]$**?**

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Theorem 1. With translation-invariant, characteristic kernel:

$\hat{\mu}^n_{X|Z} \to^n \mu_{X|Z}$ iff $\hat{\psi}^n_{X|Z} \to^n \psi_{X|Z}$ in IFT of kernel.

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Measurement Error KIV

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(Replace with sample estimates.)

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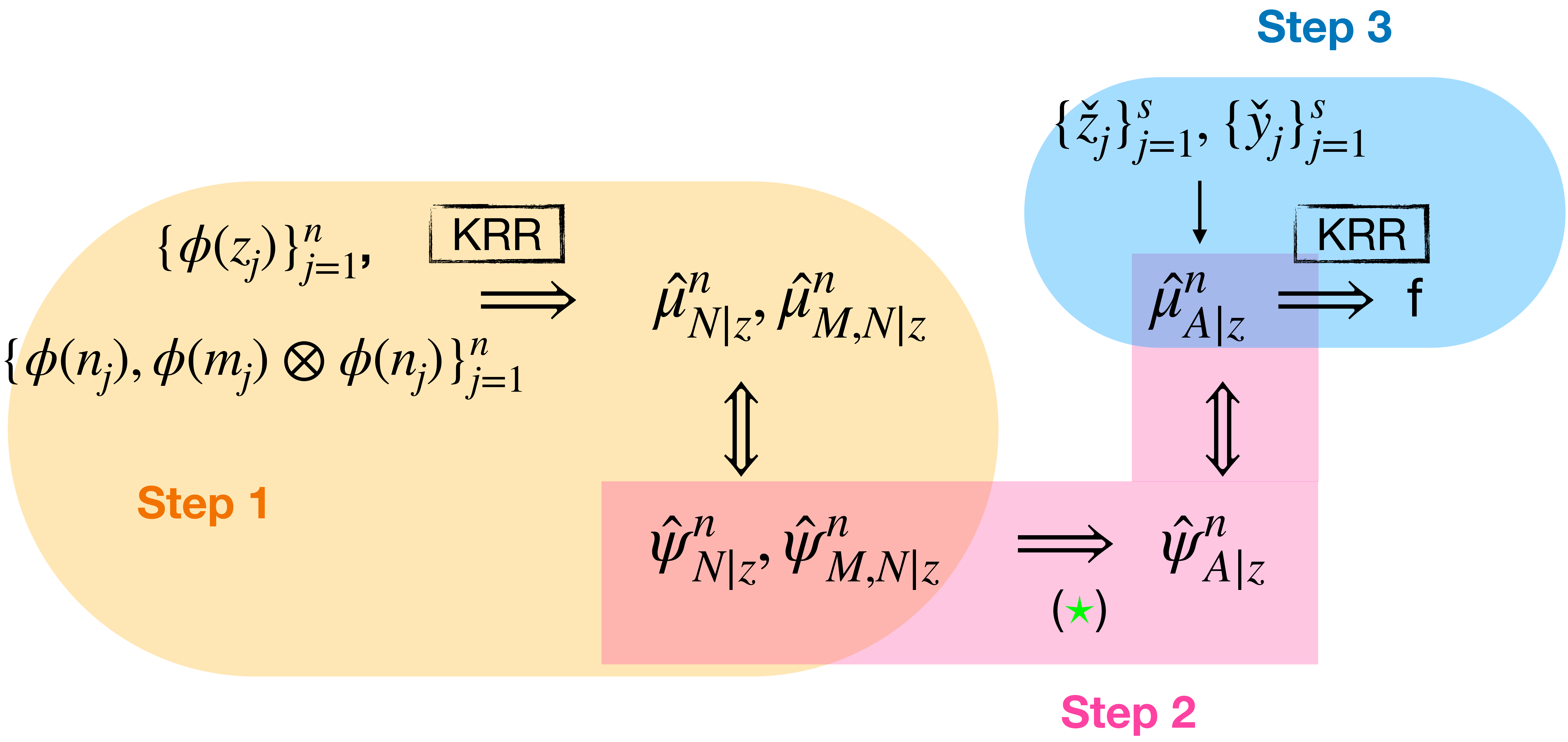
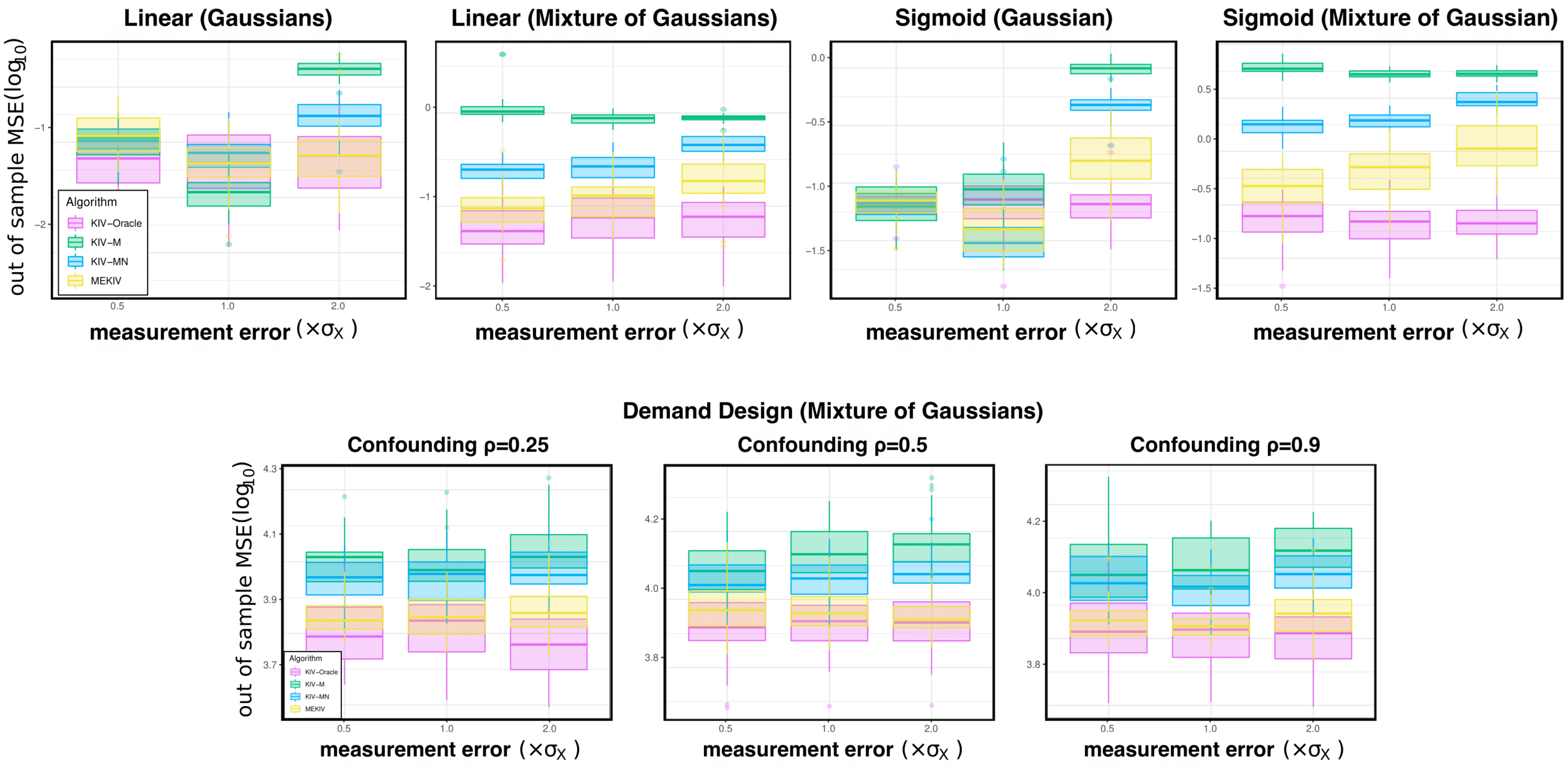


MEKIV results



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