

# Hierarchy Integrated Signature and Encryption

(Key Separation vs. Key Reuse: Enjoy the Best of Both Worlds)



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# Outline

- ① Background
- ② Hierarchy Integrated Signature and Encryption
  - HISE from (Constrained) IBE
  - HISE from PKE and NIZKPoK (HI conversion)
- ③ Global Escrow Property
  - Global Escrow PKE from NIZK and PKE (GE conversion)
  - Global Escrow PKE from 3-party NIKE
- ④ Efficient Instantiations
- ⑤ Summary

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PKE and SIG are workhorse typically used simultaneously to secure communication

- PKE  $\Rightarrow$  protect confidentiality
- SIG  $\Rightarrow$  protect authenticity: data integrity & authenticated data source

### Classical examples

- Secure communication software: PGP, WhatsApp
- Privacy-preserving cryptocurrency: Zcash, Zether, PGC

### Joint security (akin to UC)

- EUF-CMA security for SIG: holds even in the presence of  $\mathcal{O}_{\text{dec}}$
- IND-CCA security for PKE: holds even in the presence of  $\mathcal{O}_{\text{sign}}$

## A Subtle Point or a Dilemma

Key Separation vs. Key Reuse

## Key Separation: Cartesian-Product Combined Public-Key Scheme



Engineering folklore: using different keypairs for different cryptographic operations

### Pros

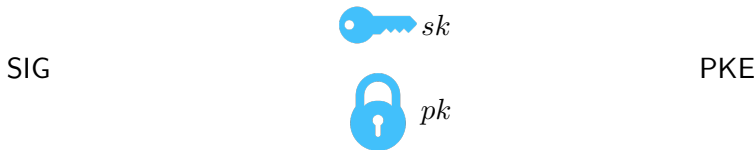
- joint security is immediate & construction is off-the-shelf
- naturally admits **individual key escrow**: achieve a balance between user's authenticity requirement and society's auditing requirement

### Cons

- double key management complexity and certificate cost<sup>1</sup>
- complicate the design of high-level protocol: tricky address derivation

<sup>1</sup>Certificate costs include but not limit to registration, issuing, storage, transmission, verification, and building/recurring fees.

## Key Reuse: Integrated Signature and Encryption



### Pros

- reduce key management complexity, certificate cost, and cryptographic footprint
- simplify the design of high-level protocol

### Cons

- joint security is not immediate (consider textbook RSA) & require careful design
- does not admit **individual key escrow**
- does not admit classified protection

Deployed in EMV standard, Ping Identity, Zether and PGC

## Motivation

We are facing a dilemma between key reuse that brings performance benefit and key separation that supports individual key escrow.



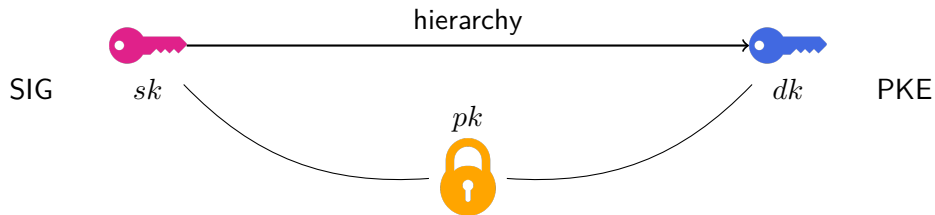
*Can we enable individual key escrow mechanism while retaining the merits of key reuse?*



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## Hierarchy Integrated Signature and Encryption



- $\text{Setup}(1^\lambda) \rightarrow pp$
- $\text{KeyGen}(1^\lambda) \rightarrow (pk, sk)$ .  $pk$  serves as encryption and verification key;  $sk$  is the signing key, serving as master secret key.
- $\text{Derive}(sk) \rightarrow dk$  used only for decryption
- $\text{Enc}(pk, m) \rightarrow c$
- $\text{Dec}(dk, c) \rightarrow m$
- $\text{Sign}(sk, \tilde{m}) \rightarrow \sigma$
- $\text{Vefy}(pk, \tilde{m}, \sigma) \rightarrow 0/1$

### IND-CCA security in the presence of a signing oracle (unrestricted access)

$$\Pr \left[ \begin{array}{l} pp \leftarrow \text{Setup}(1^\lambda); \\ (pk, sk) \leftarrow \text{KeyGen}(pp); \\ b = b' : (m_0, m_1) \leftarrow \mathcal{A}^{\mathcal{O}_{\text{dec}}, \mathcal{O}_{\text{sign}}}(pp, pk); \\ b \xleftarrow{R} \{0, 1\}, c^* \leftarrow \text{Enc}(pk, m_b); \\ b' \leftarrow \mathcal{A}^{\mathcal{O}_{\text{dec}}, \mathcal{O}_{\text{sign}}}(c^*); \end{array} \right] - \frac{1}{2} \leq \text{negl}(\lambda).$$

### EUFCMA security in the presence of a decryption key

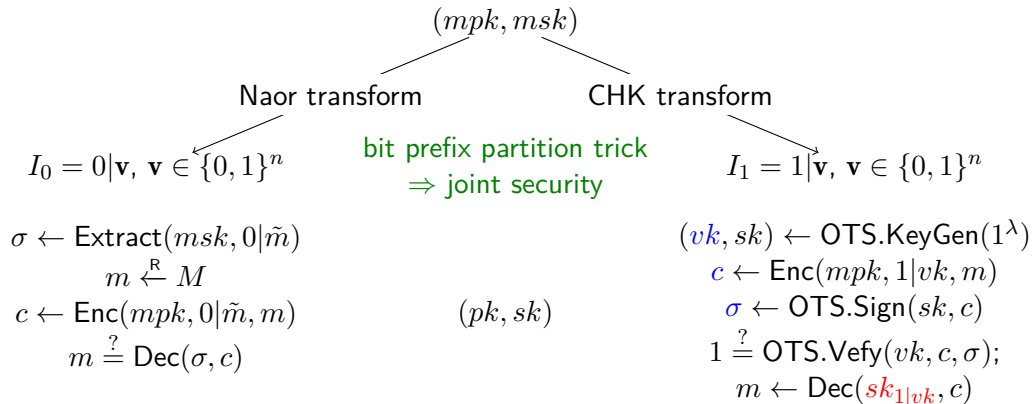
$$\Pr \left[ \begin{array}{l} \text{Vrfy}(pk, m^*, \sigma^*) = 1 \\ \wedge m^* \notin \mathcal{Q} \end{array} : \begin{array}{l} pp \leftarrow \text{Setup}(1^\lambda); \\ (pk, sk) \leftarrow \text{KeyGen}(pp); \\ dk \leftarrow \text{Derive}(sk); \\ (m^*, \sigma^*) \leftarrow \mathcal{A}^{\mathcal{O}_{\text{sign}}}(pp, pk, \boxed{dk}); \end{array} \right] \leq \text{negl}(\lambda).$$

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## Starting Point: ISE from IBE

Paterson et al. [PSST11] give an elegant ISE construction from IBE.



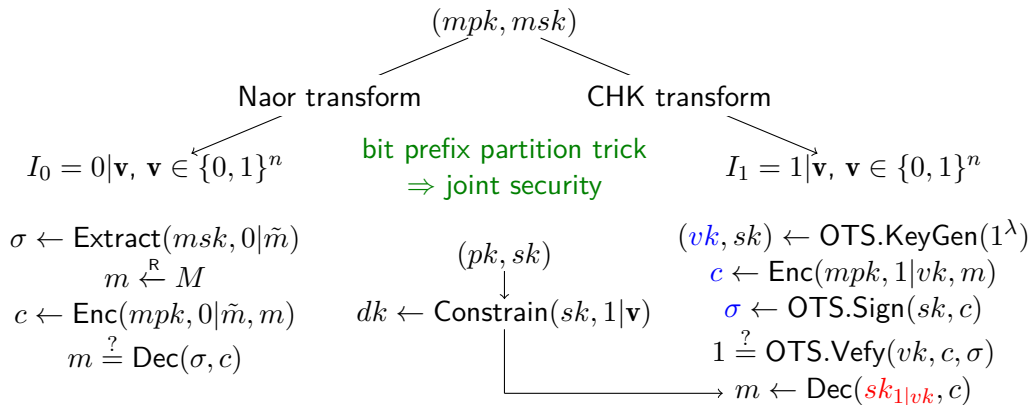
ISE from IBE does not lend itself to HISE

$msk$  plays the role of both  $sk$  and  $dk \leadsto$  compromise strong joint security

## HISE from Constrained IBE for Prefix Predicate

**Main idea:**  $msk$  acts as  $sk$ , secret keys for identities in  $I_1$  as decryption key

**Technical hurdle:** decryption key should be short  $\leadsto$  we need a **succinct representation** for all secret keys for identities in  $I_1 \Leftarrow$  **constrained IBE for prefix predicates**  $\Leftarrow$  BTE



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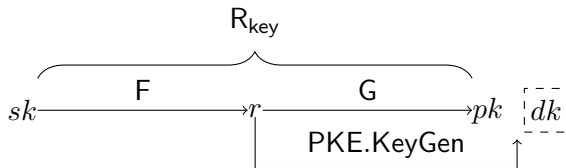
## HISE from PKE and NIZKPoK

**Goal:** add signing functionality to PKE in a generic manner

- bootstrap PKE in-use to HISE  $\leadsto$  enables a seamless upgrade

**Idea:** create hierarchical key structure via OWF

- 1 picks  $sk \xleftarrow{R} \{0, 1\}^n$  as signing key
- 2 maps  $sk$  to randomness  $r$  via uniform OWF:  $F(sk) \rightarrow r$
- 3 runs  $\text{PKE.KeyGen}(r) \rightarrow (pk, dk)$



**Figure:** The hierarchical key structure



## Design of HISE from PKE

The encryption component of HISE is simple: same as that of the underlying PKE.

But, we are facing the following technical hurdle when designing signature:

- $sk$  is unstructured bit string, how to design signature?
- the signature should remain secure even in the presence of  $dk$  (partial leakage of  $sk$ )  $\Rightarrow$  strong joint security

### Solution

- using general-purpose public-coin ZKPoK to prove knowledge of  $sk$
- prove  $R_{\text{key}}$  is leakage-resilient one-way w.r.t. leakage  $dk$ 
  - minimum requirement on  $G$ : target-collision resistant

### Strong joint security:

- SIG component: Sigma protocol for leakage-resilient one-way relation  $\leadsto$  leakage-resilient SIG
- PKE component: zero-knowledge property  $\leadsto \mathcal{O}_{\text{sign}}$  is useless + uniformity of  $F$  admits security reduction

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## Motivation of Global Escrow

**Motivating example:** large-scale collaborative working Apps such as Slack is getting popular  $\leadsto$  encrypted communication may contain proprietary information

- employer may have the right to get access to all private communications for various reasons
  - naive solution: collect individual decryption key one by one  $\Rightarrow$  impractical and inefficient
- employees need to be assured that even a malicious employer cannot slander them by forging signatures for fabricated communications

We further expect global escrow property

- there is a “super” key that can decrypt any ciphertext under any public key
- signature remains secure even in the presence of the “super” key

To attain global escrow property for HISE in a generic manner, we take a detour to revisit global escrow PKE

## Global Escrow PKE

**Global escrow PKE:** an escrow agent holds a global escrow decryption key that can decrypt ciphertexts encrypted under any public key



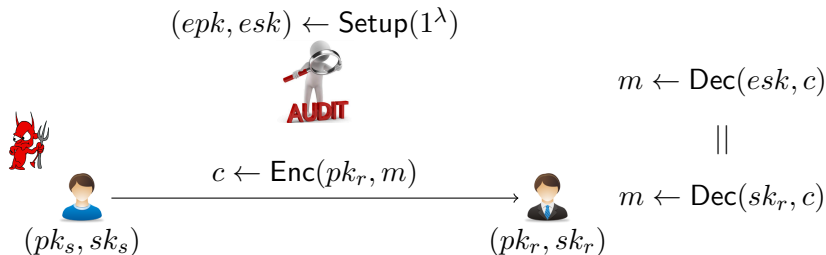
The state of the art of global escrow PKE is less satisfactory

- long overdue for formal definition and generic construction
- the only known practical scheme based on standard assumption is the escrow ElGamal PKE proposed by Boneh and Franklin from bilinear maps

## Formal Definition

### Failure attempts

- 1 Identity-based encryption: does not know how to extend to the public-key setting (users must be able to generate keypairs themselves)
- 2 Broadcast encryption: sender could be malicious especially when he has incentive to evade oversight



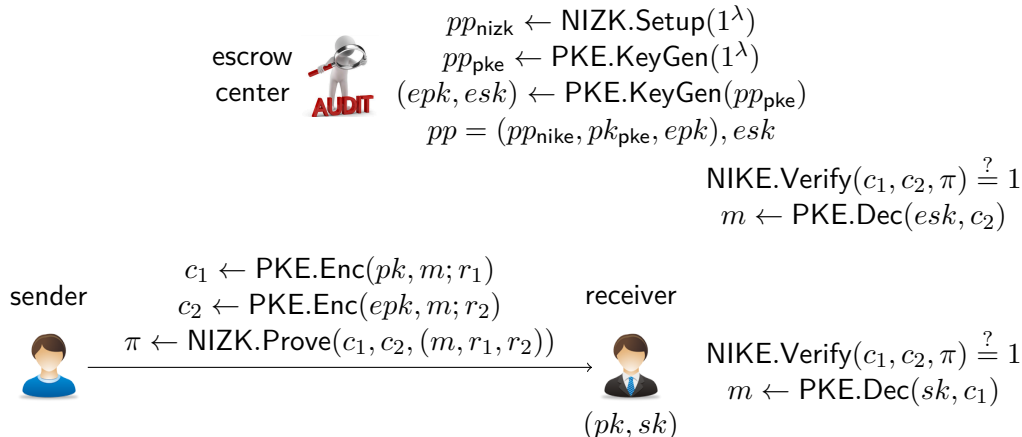
Correctness: honestly generated CTs decrypting to the same result under  $esk$  and  $sk_s$

**Consistency:** no PPT adversary can generate an ill-formed CT decrypting different results under  $esk$  and  $sk_s$

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## Global Escrow PKE from NIZK and PKE



Give a generic approach to compile any PKE into global escrow PKE

- enrich the application scope of the Naor-Yung transform beyond CCA security
- achieve CCA security with no overhead

## Instantiation of the First Approach

### Choices of primitives

- PKE: ElGamal PKE in EC groups
- NIZK: Groth-Sahai proof in standard model or Sigma proof in random oracle model

### Improvement

- When PKE satisfies the “[randomness fusion](#)” property [BMV16], we can safely reuse the randomness and then apply twisted Naor-Yung transform  $\Rightarrow$  better efficiency

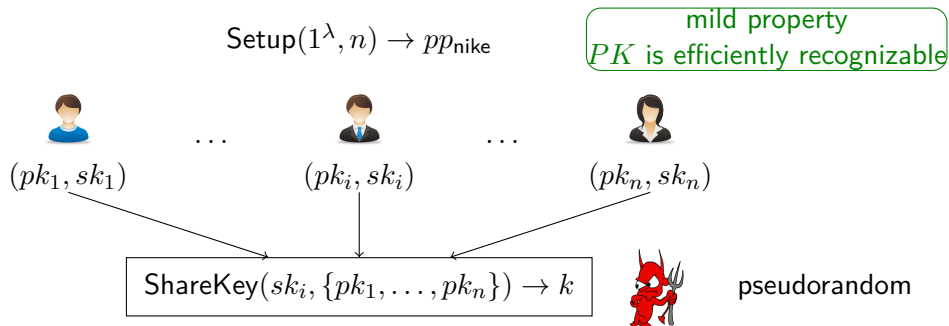
plenty of PKE schemes from the DDH, quadratic residuosity, and subset sum assumptions satisfy randomness fusion property.



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## Multiparty NIKE



- $n = 2$ : Diffie-Hellman key exchange [DH76]
- $n = 3$ : Joux's key exchange [Jou04] from bilinear maps
- $n$  is any positive integer
  - Boneh and Silverberg [BS02] using multilinear maps
  - Alamati et al. [AMPR19] using composable input homomorphic weak PRF

## Global Escrow PKE from 3-party NIKE

escrow  
center



$$\begin{aligned} pp_{\text{nike}} &\leftarrow \text{NIKE.Setup}(1^\lambda, 3) \\ (pk_\gamma, sk_\gamma) &\leftarrow \text{NIKE.KeyGen}(pp_{\text{nike}}) \\ pp &= (pp_{\text{nike}}, pk_\gamma), esk = sk_\gamma \end{aligned}$$

running 3-party NIKE in-the-head

$$\begin{aligned} k &\leftarrow \text{NIKE.ShareKey}(sk_\gamma, S) \\ m &\leftarrow \text{SKE.Dec}(k, c) \end{aligned}$$

$$S = \{pk_\alpha, pk_\beta, pk_\gamma\}$$

sender



$$(pk_\alpha, sk_\alpha) \leftarrow \text{NIKE.KeyGen}(pp_{\text{nike}})$$

$$k \leftarrow \text{NIKE.ShareKey}(sk_\alpha, S)$$

$$c \leftarrow \text{SKE.Enc}(k, m)$$

receiver



$$k \leftarrow \text{NIKE.ShareKey}(sk_\beta, S)$$

$$m \leftarrow \text{SKE.Dec}(k, c)$$

$$(pk_\beta, sk_\beta)$$

- pseudorandomness of shared key  $k \Rightarrow$  IND-CPA/CCA security
- $PK$  is efficiently recognizable  $\Rightarrow$  consistency

## Instantiation of the Second Approach

Joux's 3-party NIZK from symmetric pairing



$(b, g^b)$

$$pp = (\mathbb{G}, \mathbb{G}_T, e, g)$$
$$k \leftarrow e(g, g)^{abc}$$



$(c, g^c)$

supersingular curve ss-1536

$$|\mathbb{G}| = 1536$$

$$|\mathbb{G}_T| = 1536$$

$$|\mathbb{Z}_p| = 256$$

too slow

dirty little secret

Boneh-Franklin escrow ElGamal PKE

- $\text{Setup}(1^\lambda)$ :  $esk \xleftarrow{\mathbb{R}} \mathbb{Z}_p$ ,  $epk \leftarrow g^{esk}$ .
- $\text{KeyGen}(pp)$ :  $sk \xleftarrow{\mathbb{R}} \mathbb{Z}_p$ ,  $pk \leftarrow g^{sk}$ .
- $\text{Enc}(pk, m)$ :  $sk_t \xleftarrow{\mathbb{R}} \mathbb{Z}_p$ ,  $pk_t \leftarrow g^{sk_t}$ ;  $k \leftarrow \text{ShareKey}(sk_t, S = \{pk_t, pk, epk\})$ ,  
 $c = (pk_t, m \oplus k)$
- $\text{Dec}(sk, c)$ :  $k \leftarrow \text{ShareKey}(sk, S = \{pk_t, pk, epk\})$ ,  $m \leftarrow c_2 \oplus k$ .
- $\text{Dec}'(esk, c)$ :  $k \leftarrow \text{ShareKey}(esk, S = \{pk_t, pk, epk\})$ ,  $m \leftarrow c_2 \oplus k$ .

## Improved Scheme based on a Relaxed Version of NIKÉ

3-party NIKÉ from asymmetric pairing



AUDIT

type A:  $(a, g_1^a, g_2^a)$



$pp = (\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2)$   
 $k \leftarrow e(g_1, g_2)^{abc}$

type B:  $(b, g_1^b)$



type C:  $(c, g_2^c)$

curve bls12-381

$$|\mathbb{G}_1| = 381$$

$$|\mathbb{G}_2| = 762$$

$$|\mathbb{G}_T| = 1524$$

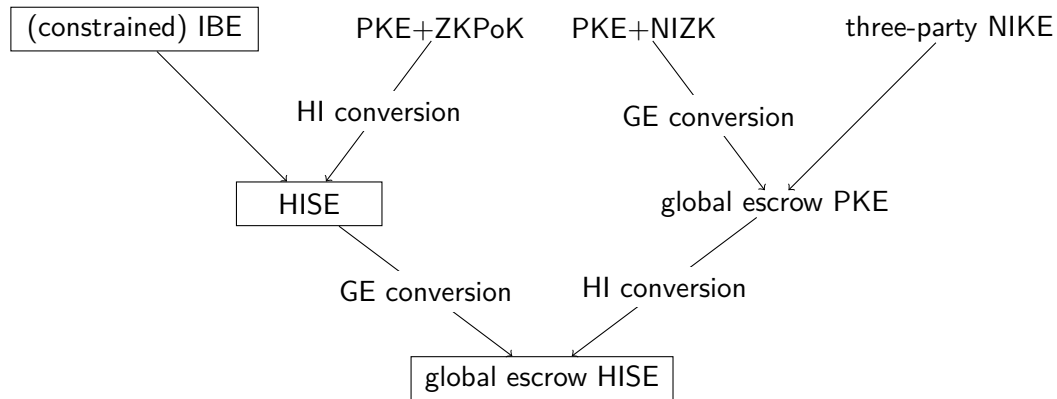
$$|\mathbb{Z}_p| = 256$$

much faster

### New Global Escrow PKE

- $\text{Setup}(1^\lambda)$ :  $esk \xleftarrow{R} \mathbb{Z}_p$ ,  $epk = (g_1^{esk}, g_2^{esk})$  (type A)
- $\text{KeyGen}(pp)$ :  $sk \xleftarrow{R} \mathbb{Z}_p$ ,  $pk \leftarrow g_2^{sk}$  (type B)
- $\text{Enc}(pk, m)$ :  $sk_t \xleftarrow{R} \mathbb{Z}_p$ ,  $pk_t \leftarrow g_1^{sk_t}$  (type C);  
 $k \leftarrow \text{ShareKey}(sk_t, S = \{pk_t, pk, epk\})$ ,  $c = (pk_t, m \oplus k)$
- $\text{Dec}(sk, c)$ :  $k \leftarrow \text{ShareKey}(sk, S = \{pk_t, pk, epk\})$ ,  $m \leftarrow c_2 \oplus k$
- $\text{Dec}'(esk, c)$ :  $k \leftarrow \text{ShareKey}(esk, S = \{pk_t, pk, epk\})$ ,  $m \leftarrow c_2 \oplus k$

## Global Escrow HISE



**Figure:** Technology roadmap of global escrow HISE. The rectangles denote our newly introduced cryptographic schemes.

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## Comparison with Cartesian-Product CPK and ISE

Table: Comparison between CP-CPK, ISE, and our (global escrow) HISE

Scheme	strong joint security	individual escrow	global escrow	key reuse	certificate cost
CP-CPK [PSST11]	✓	✓	✗	✗	×2
ISE [PSST11]	✗	✗	✗	✓	×1
HISE	✓	✓	✗	✓	×1
global escrow HISE	✓	✓	✓	✓	×1

For certificate cost, ×1 (resp. ×2) means the cost associated with one (resp. two) certificate(s). As aforementioned, certificate costs include but not limit to registration, issuing, storage, transmission, verification, and building/recurring fees. Take SSL certificate as an example, one certificate is roughly 1KB, takes roughly 200~300ms to transmit in WAN setting with 50Mbps network bandwidth and 8ms to verify. The monetary cost for an SSL certificate varies depending on features and business needs. While the cost of an SSL certificate for common usage is \$10~\$2000/year, the banks and large financial institutions could spend up to \$500,000/year on an SSL certificate with high-level security guarantee.



## Instantiation of (Global Escrow) HISE

### Instantiation of HISE

- HISE scheme 1: Naor+CHK transform (Boneh-Franklin IBE)
  - HISE scheme 2: HI conversion (ElGamal PKE+Poseidon hash+Spartan)
- 

### Instantiation of global escrow HISE

- global escrow HISE scheme 1: twsited Naor-Yung transform (HISE scheme 1)
- global escrow HISE scheme 2: HI conversion (global escrow PKE from 3-party NIKE+Poseidon hash+Spartan)

## Experimental Results

Table: Efficiency comparison of CPK and our proposed (global escrow) HISE schemes

Scheme	efficiency (ms) [# exp, #pairing]							sizes (bits) [# $\mathbb{G}$ , # $\mathbb{Z}_p$ ]			
	KGen	Sign	Vrfy	Enc	Dec	Der	Dec'	$ pk $	$ sk $	$ c $	$ \sigma $
CP-CPK	0.015 [2, 0]	0.064 [1, 0]	0.120 [2, 0]	0.118 [2, 0]	0.056 [1, 0]	$\emptyset$ $\emptyset$	$\emptyset$ $\emptyset$	512 $2\mathbb{G}$	512 $2\mathbb{Z}_p$	512 $2\mathbb{G}$	512 [ $\mathbb{G}, \mathbb{Z}_p$ ]
HISE scheme 1	0.057 [1, 0]	0.148 [1, 0]	0.733 [0, 2]	0.569 [2, 1]	0.364 [0, 1]	0.148 [1, 0]	$\emptyset$ $\emptyset$	381 $\mathbb{G}_1$	256 $\mathbb{Z}_p$	1905 [ $\mathbb{G}_1, \mathbb{G}_T$ ]	762 $\mathbb{G}_2$
HISE scheme 2	0.058 [1, 0]	3.5s N/A	250 N/A	0.115 [2, 0]	0.056 [1, 0]	0.0004 N/A	$\emptyset$ $\emptyset$	256 $\mathbb{G}$	256 $\mathbb{Z}_p$	512 $2\mathbb{G}$	40K N/A
global escrow HISE scheme 1	0.057 [1, 0]	0.148 [1, 0]	0.733 [0, 2]	1.462 [5, 2]	1.505 [4, 1]	0.148 [1, 0]	1.505 [4, 1]	381 $\mathbb{G}_1$	256 $\mathbb{Z}_p$	5590 [ $2\mathbb{G}_1, 3\mathbb{G}_T, \mathbb{Z}_p$ ]	762 $\mathbb{G}_2$
global escrow HISE scheme 2	0.057 [1, 0]	3.5s N/A	250 N/A	0.629 [2, 1]	0.531 [1, 1]	0.0004 N/A	0.532 [1, 1]	381 $\mathbb{G}_1$	256 $\mathbb{Z}_p$	2286 [ $\mathbb{G}_2, \mathbb{G}_T$ ]	40K N/A

Performance of Cartesian product CPK and (global escrow) HISE schemes with 128-bit security level. ( $\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T$ ) refers to asymmetric pairing groups.  $\mathbb{G}$  refers to ordinary elliptic group. The symbol  $\emptyset$  indicates that there is no corresponding algorithm. The symbol N/A indicates that the efficiency (or bandwidth) is hard to measure by algebra operations (or elements).

## A Byproduct: Global Escrow PKE

Table: Comparison of escrow ElGamal PKE [BF03] and our global escrow PKE

Scheme	efficiency (ms) [# exp, #pairing]					sizes (bits) [# $\mathbb{G}$ , # $\mathbb{Z}_p$ ]				
	Setup	KGen	Enc	Dec	Dec'	$ pp $	$ edk $	$ pk $	$ sk $	$ c $
Boneh-Franklin escrow ElGamal PKE	2.879 [2, 0]	2.014 [1, 0]	8.723 [2, 1]	6.654 [1, 1]	6.745 [1, 1]	3072 $2\mathbb{G}$	256 $\mathbb{Z}_p$	1536 $\mathbb{G}$	256 $\mathbb{Z}_p$	3072 [ $\mathbb{G}, \mathbb{G}_T$ ]
our proposed global escrow PKE	0.243 [4, 0]	0.058 [1, 0]	0.680 [2, 1]	0.579 [1, 1]	0.586 [1, 1]	2286 [ $2\mathbb{G}_1, 2\mathbb{G}_2$ ]	256 $\mathbb{Z}_p$	381 $\mathbb{G}_1$	256 $\mathbb{Z}_p$	2286 [ $\mathbb{G}_2, \mathbb{G}_T$ ]

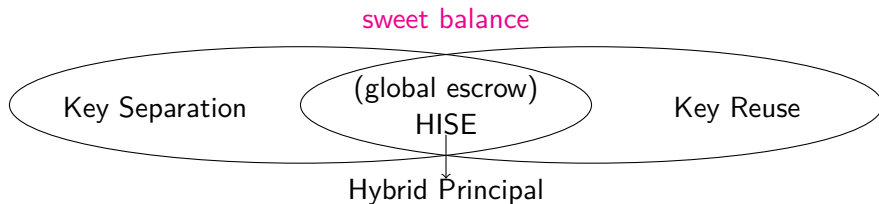
Performance of global escrow PKE schemes with 128-bit security level.  $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T)$  refers to asymmetric pairing groups.  $(\mathbb{G}, \mathbb{G}_T)$  refers to symmetric pairing groups. We report times for setup, key generation, encryption, and (escrow) decryption, as well as the sizes of public parameters  $pp$ , global escrow decryption key  $edk$ , public key  $pk$ , secret key  $sk$ , and ciphertext  $c$ .

12 ~ 30× speed up

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HISE (formal definition + generic constructions)

- reconcile the apparent conflict between key separation and key reuse
- resolve the problem left open in Verheul [\[Ver01\]](#)
- can be used as a drop-in replacement of PKE+SIG in scenarios that requires authenticity, confidentiality and auditability simultaneously
- both users and authority have incentives to deploy

Global escrow PKE revisit (formal definition + generic constructions)

- indicate a new application of Naor-Yung paradigm
- establish a connection from 3-party NIKE

Thanks for Your Attention!

Any Questions?

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