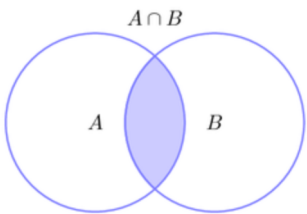
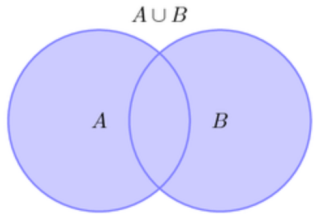
Private Set Operations





Yu Chen
Shandong University

Outline

Secure Multi-party Computation (MPC)

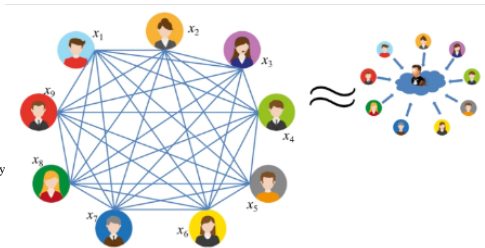
[Yao82]: Protocols for Secure Computations

ANDREW CHI-CHIH YAO

China – 2000

CITATION

In recognition of his fundamental contributions to the theory of computation, including the complexity-based theory of pseudorandom number generation, cryptography and communication complexity.



MPC enable a group of independent data owners who do not trust each other or any common third party

- jointly compute a function that depends on all of their private inputs
- without learn anything else beyond output and its own input

Short History of MPC

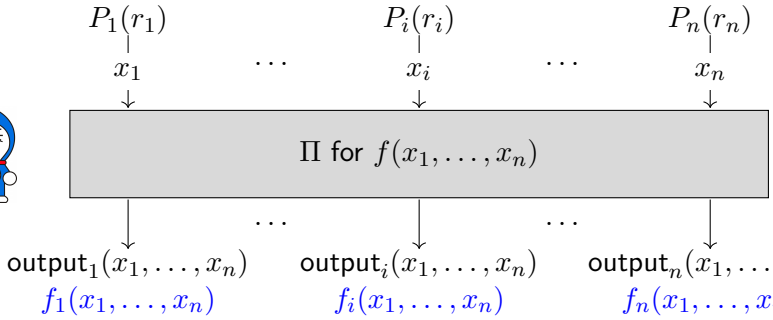
General-purpose MPC (a.k.a. can compute arbitraty function) is possible!
This makes MPC extremely powerful

Celebrated Paradigms of Generic MPC

Approach	Round Complexity	Adversarial Behavior	Party	Corruption Threshold	Protocol	Technique	Circuits
Garbled Circuit	$O(1)$	Semi-honest	$n = 2$	—	Yao [Yao86]	standard GC	Boolean
			$n \geq 2$	$t \geq n/2$	BMR [BMR90]	Distributed Garbling	
		Malicious	$n = 2$	—	LP [LP07], LEGO [NO09]	Cut-and-Choose	
					WRK [WRK17]	IT-MAC	
			$n \geq 2$	$t \geq n/2$	CKMZ [CKMZ14]	Cut-and-Choose	
					YWZ [YWZ20]	IT-MAC	
Secret Sharing	$O(d)$	Semi-honest	$n \geq 2$	$t \geq n/2$	GMW [GMW87]	Additive SS	Arithmetic
				$t < n/2$	BGW [BGW88]	Shamir SS	
		Malicious	$n \geq 2$	$t \geq n/2$	GMW [GMW87]	ZKP	
					SPDZ [DPSZ12]	IT-MAC	
				$t < n/2$	LN [LN17]	Triple Verification	
					CGHIKLN [CGH+18]	Dual Execution	
					BGIN [BGIN20]	Distributed ZKP	

d is the depth of C, typically $\log |C|$.

MPC with Semi-Honest Security



P_i's view includes
- private input
- random tape
- incoming messages

MPC with Semi-Honest Security

$P_1(r_1)$ $\quad \cdots \quad$ $P_i(r_i)$ $\quad \cdots \quad$ $P_n(r_n)$

$x_1 \quad \cdots \quad x_i \quad \cdots \quad x_n$

Π for $f(x_1, \ldots, x_n)$

$\cdots \quad \cdots$

$\mathsf{output}_1(x_1, \ldots, x_n) \quad \mathsf{output}_i(x_1, \ldots, x_n) \quad \mathsf{output}_n(x_1, \ldots, x_n)$

$f_1(x_1, \ldots, x_n) \quad f_i(x_1, \ldots, x_n) \quad f_n(x_1, \ldots, x_n)$

P_i's view includes
private input
random tape
incoming messages

- all P_i are semi-honest (honest but curious)
- P_i learns no more information other than his output and private input

Definition 1 (Semi-honest Security)

Π securely realizes probabilistic f in the presence of semi-honest adversaries if there exists a PPT simulator Sim such that for all inputs $x_1, \ldots, x_n$ and all $i \in [n]$:

$(\mathsf{View}_{P_i}(x_1, \ldots, x_n), \mathsf{output}(x_1, \ldots, x_n)) \approx_{c,s} (\mathsf{Sim}(i, x_i, f_i(x_1, \ldots, x_n)), f(x_1, \ldots, x_n))$

MPC with Semi-Honest Security

$P_1(r_1)$ $\quad \cdots \quad$ $P_i(r_i)$ $\quad \cdots \quad$ $P_n(r_n)$

$x_1 \quad \cdots \quad x_i \quad \cdots \quad x_n$

Π for $f(x_1, \ldots, x_n)$

$\cdots \quad \cdots$

$\mathsf{output}_1(x_1, \ldots, x_n) \quad \mathsf{output}_i(x_1, \ldots, x_n) \quad \mathsf{output}_n(x_1, \ldots, x_n)$

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Definition 1 (Semi-honest Security)

Π securely realizes deterministic f in the presence of semi-honest adversaries if there exists a PPT simulator Sim such that for all inputs $x_1, \ldots, x_n$ and all $i \in [n]$:

$$\mathsf{View}_{P_i}(x_1, \ldots, x_n) \approx_{c,s} \mathsf{Sim}(i, x_i, f_i(x_1, \ldots, x_n))$$

Short History of MPC

MPC was primarily of only theoretical interest for the first twenty years.

- After 2000s, algorithmic improvements and computing costs reached a point where generic MPC is practical for real-world applications.

But, generic MPC is still relatively heavy and thus not very fast!

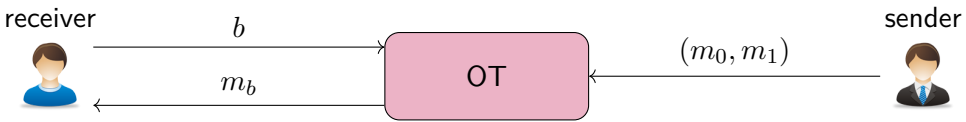
One important sub-area of MPC focuses on specific functionalities

- For specific functionalities, there maybe custom protocols that are much more efficient than the best generic protocols.
- Specific functionalities can be interesting in their own right, but also can be natural building blocks for use in other applications.


这个就叫专业

Oblivious Transfer

1-out-of 2 OT [Rab05] enables the receiver learns only one messages from sender, while sender learns nothing.



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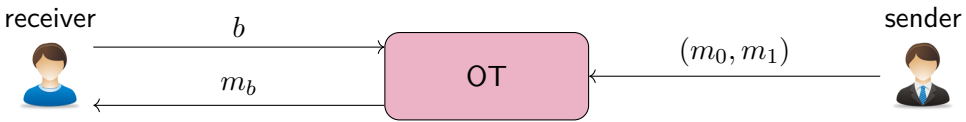

OT is complete for MPC [Kil88].

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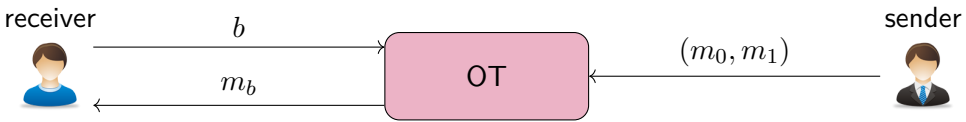

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- Private-information retrieval (PIR) is weaker than OT: it only cares privacy of receiver

OT does not belong to Minicrypt $\rightsquigarrow$ expensive public-key operations are unavoidable, while real applications need a large number of OT

- [IKNP03] proposed Ishai-Kilian-Nissim-Petrank OT extension: generate many OT efficiently from $O(\kappa)$ number of base OT $\Rightarrow$ OTe is cheap

Private Equality Test Protocol

PEQT [PSSZ15] enables P_1 and P_2 check if their ℓ-bits elements x and y are equal.

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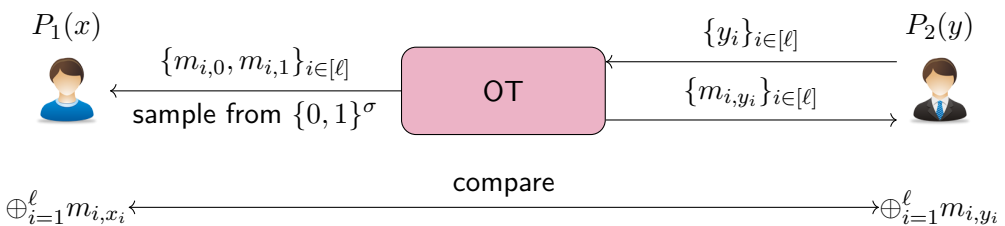
[PSSZ15] showed how to build PEQT by invoking 1-out-of-2 random OT ℓ times



Oblivious Pseudorandom Functions

OPRF [FIPR05] enables server obtain a key k and client evaluate obliviously.

Oblivious Pseudorandom Functions

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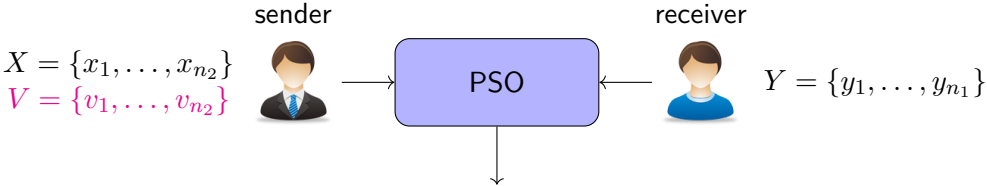
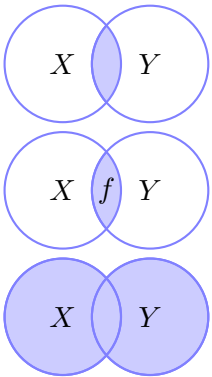
OPRF is a powerful tool in MPC (see [CHL22] for a good survey)

- many variants: batch/programmable/permuted/distributed OPRF
- fast construction from OT or VOLE

Specific Functionality: Private Set Operations (high frequency and high value)

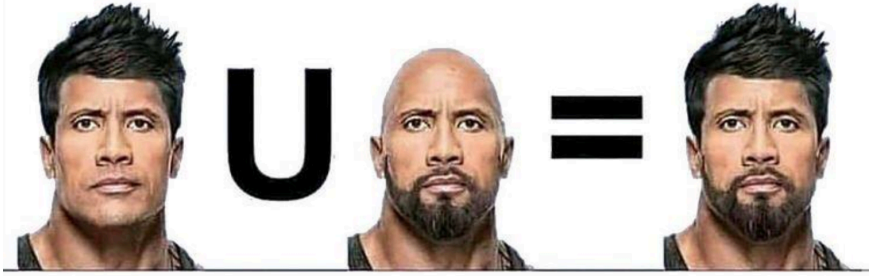




$$\text{PSI} = X \cap Y$$

$$\text{PCSI} = \begin{cases} |X \cap Y| & \text{cardinality} \\ |X \cap Y|, \sum_{x_i \in X \cap Y} v_i & \text{cardinality-sum} \\ f(X \cap Y) & \text{general computation} \end{cases}$$

$$\text{PSU} = X \cup Y$$



Applications of PSI

Contact discovery (when signing up an App)

- X: address book in my phone
- Y: App user database

Private scheduling

- X: avaiable timeslots on my calendar
- Y: avaiable timeslots on your calendar

Credit risk profiling

- X: bank's credit risk watchlist
- Y: prospective borrowers

Password checkup

- X: Google's database of breached passwords
- Y: client's passwords

Applications of PSI-card-sum and PSU

Ad conversion rate (widely used in Microsoft and Google)

- X: users who saw the advertisement
- Y: customers who bought the product

IP blacklist and vulnerability data aggregation

- X: blacklist in organization A
- Y: blacklist in organization B

Private DB supporting full join

- X: data from table A
- Y: data from table B

SOTA of PSI

PSI has been extensively studied in the last four decades

According to the techniques

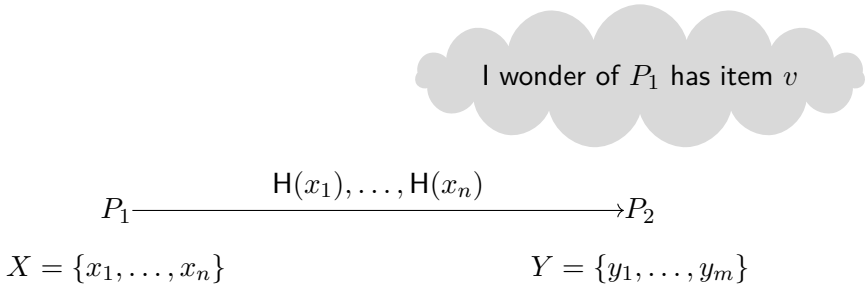
- symmetric-key: OPRF-based [KKRT16, CM20, RR22]
- public-key: communication-efficient DH-PSI [Mea86]

According to the scenarios

- balanced setting: [KKRT16, CM20, RR22] achieves linear complexity
- unbalanced setting: [CLR17, CHLR18, CMdG+21] achieves sub-linear complexity of large set

the SOTA [RR22] s almost as efficient as insecure hash-based protocol
million size set: [0.16s, 31 Mb]

Naive Insecure Hash-based Protocol

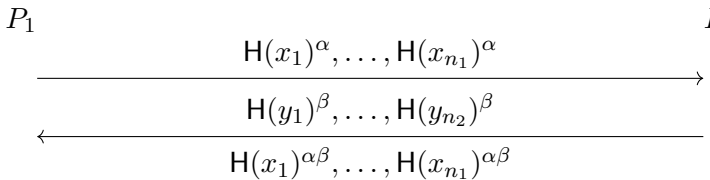


INSECURE: $\mathsf{H}(X)$ reveal too much information $\rightsquigarrow$ P_2 can test any $v? \in X$ offline

- Especially problematic if items have low entropy (e.g., phone numbers)

Recap of Classic DH-PSI

$X = \{x_1, \ldots, x_{n_1}\}$ $\qquad$ $Y = \{y_1, \ldots, y_{n_2}\}$



compute $X \cup Y$ by comparing
$\mathsf{H}(x_i)^{\alpha\beta} \in \{\mathsf{H}(y_i)^{\beta\alpha}\}_{i\in[n_2]}$

Idea:

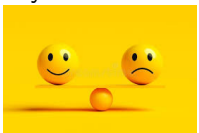
- If $x_i \in Y$, then $\mathsf{H}(x_i)^{\alpha\beta} = \mathsf{H}(y_i)^{\beta\alpha}$ for some $y_j \in Y$
- If $x_i \notin Y$, messages are independently random by modeling H as random oracle

Exercise

Prove the classic DH-PSI protocol in the semi-honest security model.

SOTA of PSO

In sharp contrast, the study of PCSI and PSU are not satisfying.



PCSI

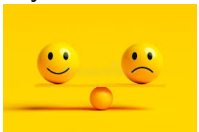
- [HFH99, IKN+20, PSTY19] achieve linear complexity

concretely $20\times$ slower in timing and $30\times$ more communication than PSI

Somewhat counter-intuitive, less is harder!

SOTA of PSO

In sharp contrast, the study of PCSI and PSU are not satisfying.



PCSI

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Somewhat counter-intuitive, less is harder!

PSU

- [KS05, Fri07, HN10, KRTW19, JSZ+22] have superlinear complexity
- [DC17] achieve linear complexity, but not strict (communication or computation complexity additionally depends on statistical parameter $\lambda \approx 40$)

concretely $20\times$ slower in timing and $25\times$ more communication than PSI

Need to fetch information belong to other parties.

Motivation

Different approaches are used for different private set operations $\rightsquigarrow$ require much more engineering effort and maintaining cost

- Goal: a unified framework of PSO

[1] [GMR+21] presented a PSO framework from permuted characteristic. However, its oblivious shuffle functionality is not necessary for PSO, and incurs superlinear complexity.

Motivation

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There exists huge efficiency gap between PSI and other PSO protocols

- Goal: efficient instantiations to close the gap[1]

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- Goal: efficient instantiations to close the gap[1]

After $\approx$ 40 years, DH-PSI [Mea86] is still the most easily understood and implemented one among numerous PSI protocols. Surprisingly, no counterpart is known in the PSU setting yet. Existing protocols are very complicated.

- Goal: build DDH-based PSU protocol as simple as DH-PSI

[1] [GMR+21] presented a PSO framework from permuted characteristic. However, its oblivious shuffle functionality is not necessary for PSO, and incurs superlinear complexity.

Questions in Mind

1. *Is there a central building block that enables a unified framework for PSO?*
2. *How to give instantiations with optimal asymptotic complexity and good concrete efficiency?*
3. *Can the DDH assumption strike back with efficient PSU protocol?*

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