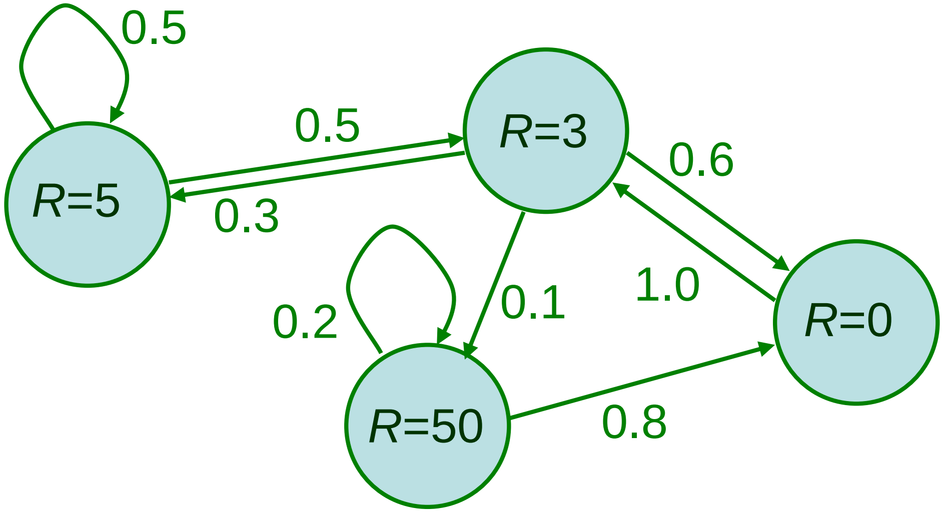
Reinforcement Learning (RL)

Why Reinforcement Learning?

- The task involves a series of actions.
- For a given state, there is no teacher to tell the agent what the "correct" or "optimal" action is from that state.
- External feedbacks (reward/penalty) are available, but not after every action.
- Example scenario: Playing a game without a given evaluation function. Can the agent learn an evaluation function from its experience?

Markov Decision Processes

- We talk about **Markov Decision Processes** (MDPs) because these are how the tasks of RL are represented.
- The content of a MDP consists of:
  - States.
  - Allowed actions of the states.
  - Immediate rewards of the states.
  - Transition function $P(s'|s,a)$.



Markov Decision Processes

- In a MDP, we can not find an optimal "path" because it is stochastic.
- What we try to find is a **policy**, $\pi(s)$, which is a function from states to actions.
- A **state-value function** (called "utility" in the textbook, and often called just "value function") of a state, $V^{\pi}(s)$, represents the "expected total rewards" when we start from state s with policy π.
  - This is like the averaged reward of all the possible paths from the given state to the terminal states, weighted by probabilities of the paths.
  - We represent the optimal policy as π^* and the corresponding optimal state-value function as $V^*(s)$.

Rewards

- The total reward of a path with state sequence s_0, s_1, s_2, … is given by

$$R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \cdots$$

- **Discounted Reward**:.
  - Far-away future rewards are less important than immediate rewards (when $\gamma < 1$).
  - Here γ is called the discount factor.
  - When $\gamma = 1$, we say we have additive rewards.

Policy, Reward and State-Values

Bellman's equation (fixed policy):

Reward

Probabilistic transition function

$$V^{\pi}(s) = R(s) + \gamma \sum_{s'} P(s' \mid s, \pi(s)) V^{\pi}(s')$$

State-values

Discount Factor

Policy (action at s)

Bellman's equation (optimal policy):

$$V^{*}(s) = R(s) + \gamma \max_{a} \sum_{s'} P(s' \mid s, a) V^{*}(s')$$

Value Iteration

- **Bellman update**: We take the Bellman's equation for optimal policy and do right-to-left assignment iteratively to estimate the state-value function (when the policy is optimal):

$$V(s) \leftarrow R(s) + \gamma \max_a \sum_{s'} P(s' \mid s, a) V(s')$$

- We start with all-zero state values.
- The optimal policy is linked to the estimated state-value function as

$$\pi^*(s) = \arg\max_a \sum_{s'} P(s' \mid s, a) V(s')$$

Policy Iteration

Alternating iteration of these two steps:

- Policy Evaluation: Estimate the state-value function using the current policy:

$$V^{\pi}(s) = R(s) + \gamma \sum_{s'} P(s' | s, \pi(s)) V^{\pi}(s')$$

  - For small MDPs, this can be solved exactly.
  - For larger MDPs, we can just apply several iterations of Bellman update.
  - The policy is randomly initialized.

- (Greedy) Policy Improvement: Update the policy so that it is optimal with the current state-value function:

$$\pi(s) \leftarrow \arg\max_{a} \sum_{s'} P(s' | s, a) V(s')$$

Policy Parameterization

- While we have a very large state space, the policy can no longer expressed as a look-up table (like we have assumed so far).
- What we can do is to represent the policy with a set of parameters, which themselves are functions of the states.
- Example: Represent the policy as a neural network (input: state; output: action; parameters: network weights)
  - Use a soft representation of policy: $\pi^{\theta}(a|s)$, as normalized probabilities of actions for the given state.

Policy Parameterization

- Policy optimization becomes the optimization of the parameters.
- **Policy Gradient**: Without getting into the mathematical details, just understand this as local-search based methods to optimize the policy.
- Optimization methods that do not rely on gradients (e.g., genetic algorithms or simulated annealing) are also useful.

Active Learning

- The discussion so far has assumed that the transition functions and reward functions are known.
  - There is no trial-and-errors.
  - The iterative methods are needed only because direct solutions are too computationally expensive.
- In most real-world problems, the MDP is not well specified, and the agent needs to learn from experiences to build the (inexact) knowledge of the MDP.

Active Learning

- So far we have assumed that the MDP is known. This is not the case for many real-world problems.
- We need methods to learn a policy without knowing or estimating the MDP, so such methods are considered model-free.
- Such learning is achieved by **sampling** paths through the MDP in order to collect information.
- Monte-Carlo Tree Search is a kind of such sampling.
- Monte-Carlo Policy Gradient (**REINFORCE** algorithm): An update to the policy parameters after a path is sampled.
  - Updates are favored for the parameters that enhance the probabilities of actions along paths with larger accumulated rewards.

Temporal Difference (TD) Learning

- It is not necessary to do updates only after complete sample paths (episodes).
- Idea: Use "estimated evaluations at future states" (which tend to be more accurate) to update the evaluation of the current state.
- A basic TD learning step: $\boxed{V(s) \leftarrow V(s) + \alpha\left[r + \gamma V(s') - V(s)\right]}$
- This is an actor-critic method in RL. (The "critic" is the supposedly better estimation from a subsequent state.)

Q-Learning

- A really popular approach of reinforcement learning.
- **Q-functions** are **action-value functions**: Each entry, $Q(s,a)$, represents the expected total reward of taking action a at state s.
- The goal: To learn the Q-functions
  - When the learning converges, the best policy is to follow the action with the best Q value.
  - The "expected total reward" assumes that the best-Q-value action is taken for all subsequent steps until a terminal state is reached.
- The algorithm follows the idea of TD learning.
- Q functions, like state-value functions, can be parameterized.

The Q-Learning Algorithm

- Initialize all the Q values (for example, to zero).
- A typical Q-learning procedure is to repeat the following many times (learning episodes):
  - Start at any valid initial state.
  - Repeat until a terminal state is reached:
    - Choose a valid action a from the current state s. Let s' be the resulting new state, and let r be the reward incurred for this action.
    - Update $Q(s,a)$: (This occurs only for non-terminal s.)

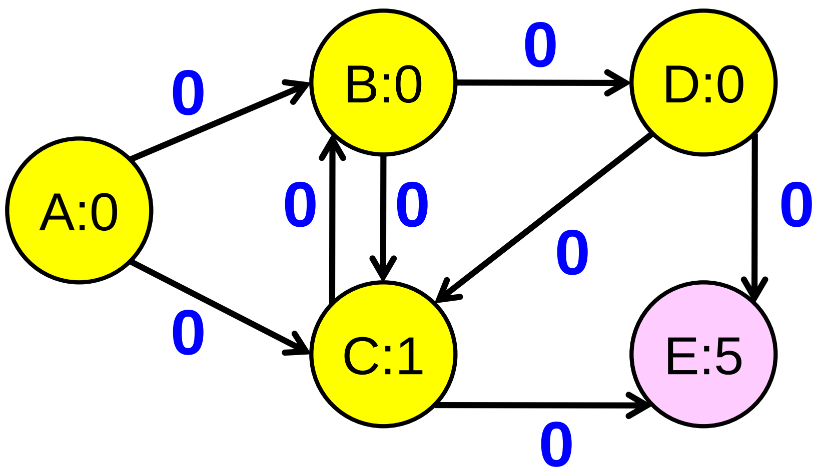
$$Q(s,a) \leftarrow Q(s,a) + \alpha \left[r + \gamma \max_{a'} Q(s',a') - Q(s,a) \right]$$

Learning Rate

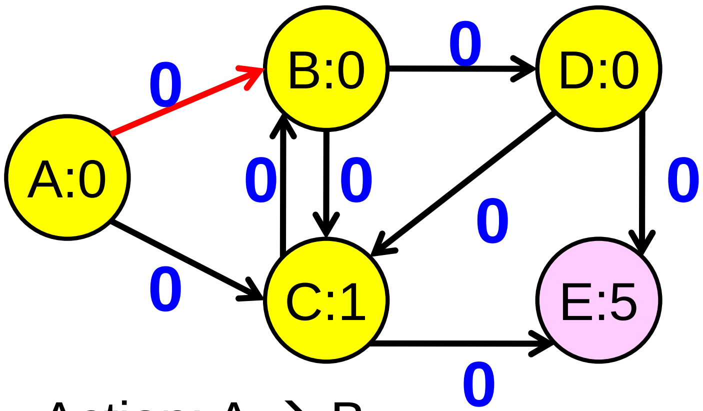
Discount Factor

Q-Learning Example

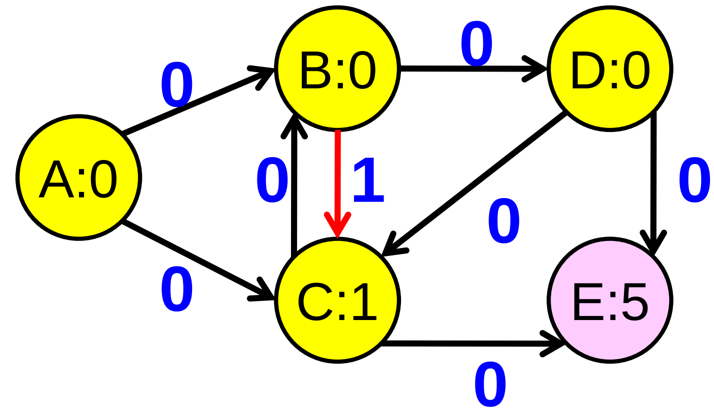
- For simplicity, we will consider only a deterministic environment here, and all the Q values are initialized to zero.
- Settings (state space given below): Start states = {A}. Learning rate = 1. Discount factor = 1. Terminal states = {E}.



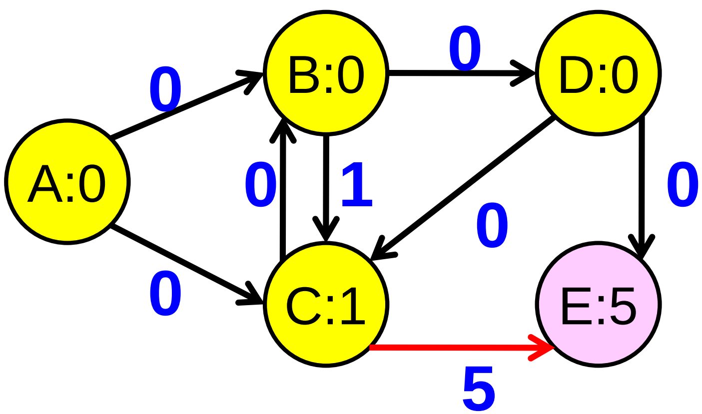
Q-Learning Example



Action: A → B
Q(A, A→B) ← 0

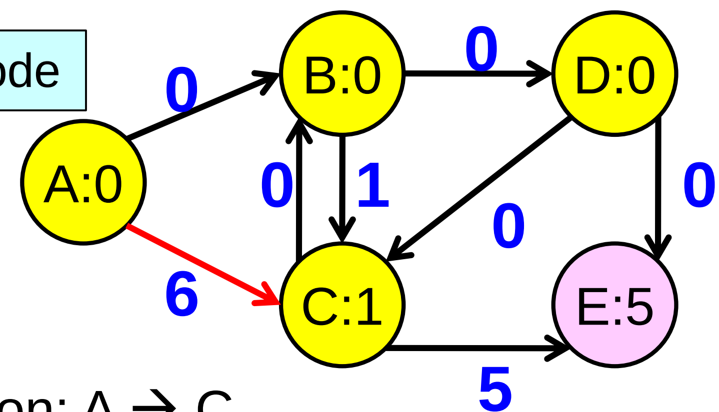


Action: B → C
Q(B, B→C) ← 1 + max(0,0) = 1



Action: C → E
Q(C, C→E) ← 5

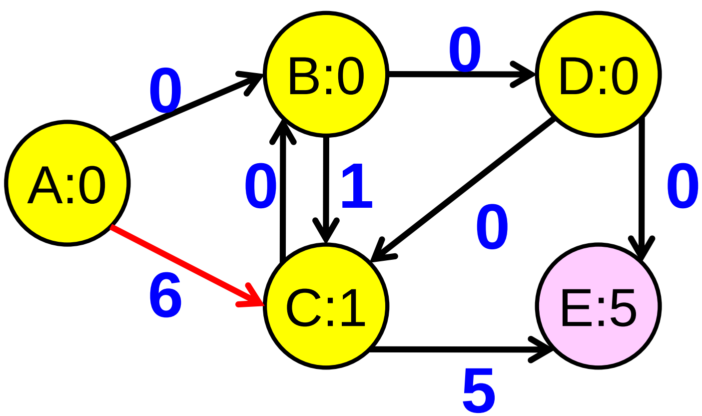
new episode



Action: A → C
Q(A, A→C) ← 1+ max(0,5) = 6

Q Tables

- Q-table (the most common representation): A table of values of each combination of a state and a valid action from that state.
- Convenient for problems with finite states and finite valid actions per state.
- Quantization can be used for environments with continuous states and/or actions.



	→A	→B	→C	→D	→E
A	-	0	6	-	-
B	-	-	1	0	-
C	-	0	-	-	5
D	-	-	0	-	0
E	-	-	-	-	-

Q-Learning: Exploration vs. Exploitation

- Problem: During a training episode, how to choose the action from a state?
  - Best Q-value (greedy approach): Exploitation
  - Random action: Exploration
- ε-greedy: Use a probability (ε) to choose between the two. (More exploration initially, and more exploitation later to facilitate convergence.)
- (Optional) Adjustment of the discount factor: smaller initially (to avoid propagation of "noise") and larger later.

Deep Q-Learning

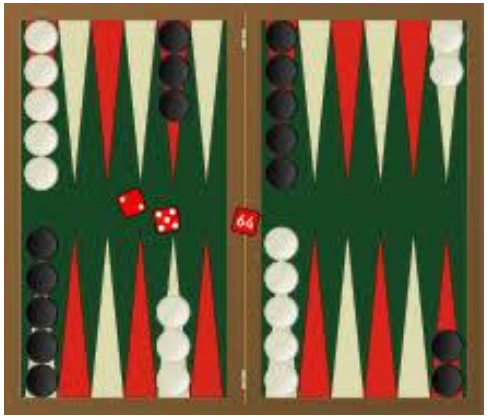
- Use a neural network to represent the Q function.
- The <u>loss function</u> (which leads to TD learning):

$$(1/2)\left[r+\gamma \max_{a'} Q(s',a') - Q(s,a)\right]^2$$

- The network weights are updated via backpropagation.
- DeepMind used DQL to make agents that play many Atari games to top human levels (2016).

Case Study: TD-Gammon

- Two-ply game tree search.
- Classical neural network (single hidden layer) for the evaluation function.
- First version: Pure reinforcement learning (TD-lambda) of the evaluation function without human knowledge. ➔ Good but not top performance.
- Second version: The network learns a representation based on a set of expert-designed attributes. ➔ As good as human world champions.

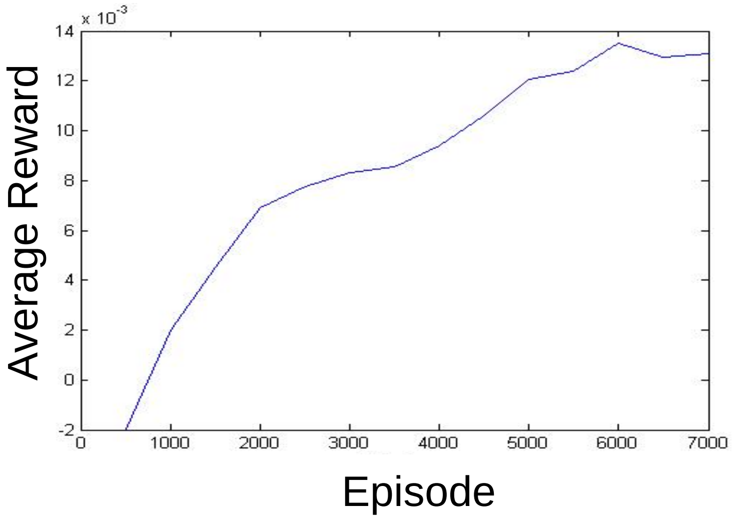


Case Study: Overtaking in Car Racing

- Played on a car racing game simulator.
- Quantization of features and actions:
  - Features: speed difference, cross-track positions of both cars, distance between the cars.
  - Actions: shift-left, shift-right, straight-ahead.
- Reward: success (+1), failure/crash (-1)
- Trained with Q-learning.

Case Study: Overtaking in Car Racing

- Typical learning curve:



- Example of a complex maneuver learned: