Wireless Communication Systems
@CS.NCTU

Lecture 7: 802.11 PHY and OFDM

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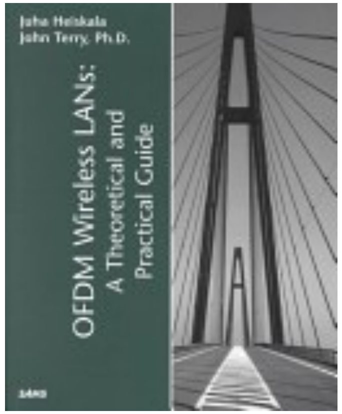
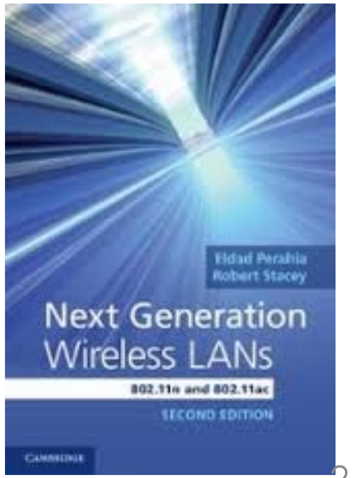
Reference

1. OFDM Tutorial
   online:
   http://home.iitj.ac.in/~ramana/ofdm-tutorial.pdf

2. OFDM Wireless LWNs: A Theoretical and Practical Guide
   By John Terry, Juha Heiskala



3. Next Generation Wireless LANs: 802.11n and 802.11ac
   By Eldad Perahia



Agenda

- **OFDM (Orthogonal Frequency Division Modulation)**
- Multipath Effect
- Packet Detection
- Channel Estimation and Decoding
- Synchronization
- OFDM vs. OFDMA

Narrow-Band Channel Model

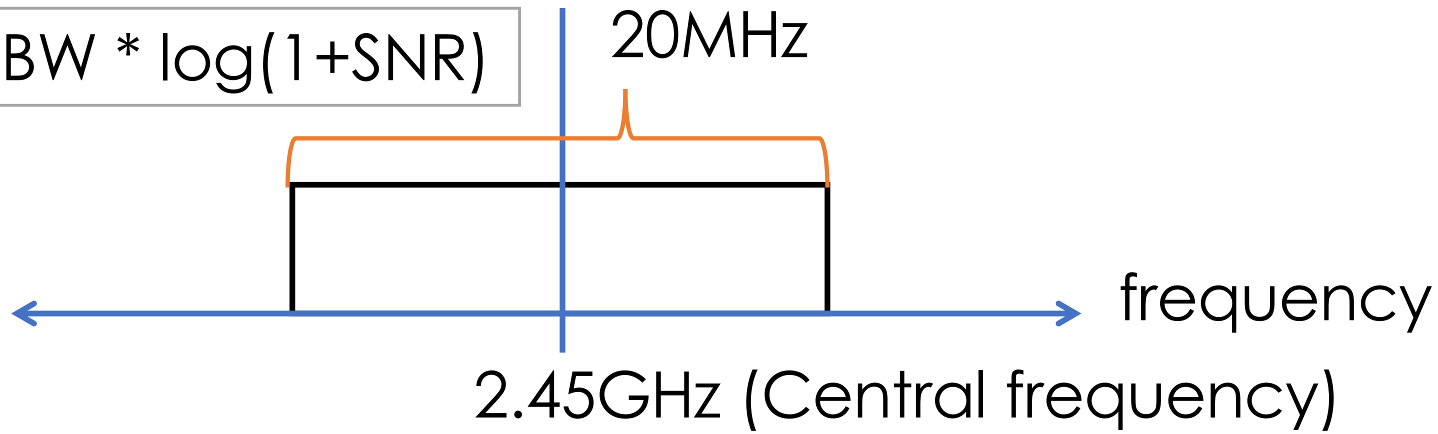
- Signal over wireless channels
  - $y = hx + n$
- $h = a*exp^{2j\pi f\delta}$ is the channel between Tx and Rx
  - a: received amplitude, δ: propagation delay
- How to decode x?
  - $x = y/h + n$
  - How to learn h?
  - Re-use the **known** preamble to learn h
    → since $y = hp + n$, we get $h' = y/p$

The procedure of finding H is called **channel estimation**

Why OFDM?

- Signal over wireless channels
  - $y = hx + n$ → Decoding: $x' = y/h$
- Work only for narrow-band channels, but not for wide-band channels, e.g., 20 MHz for 802.11
  - Channels of different narrow bands will be different!

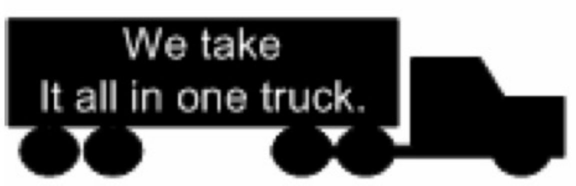
Capacity = BW * log(1+SNR)

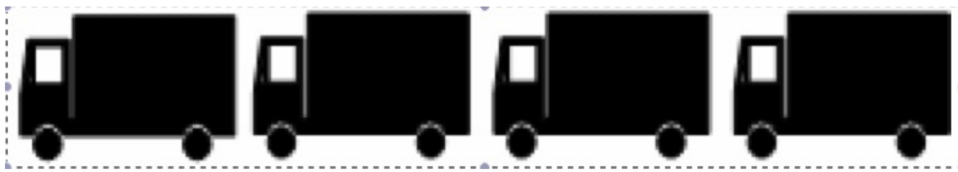


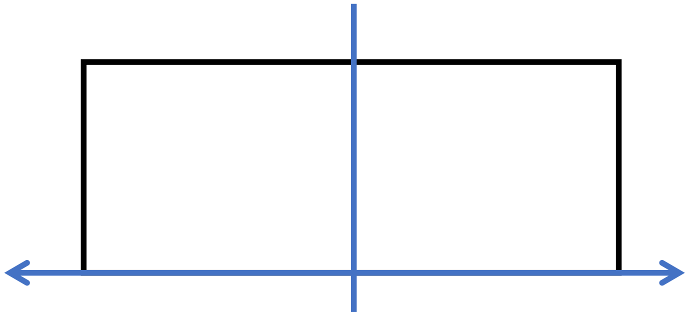
Basic Concept of OFDM

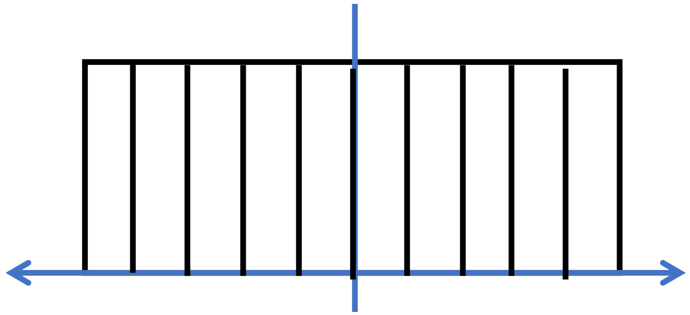
Wide-band channel

Multiple narrow-band channels





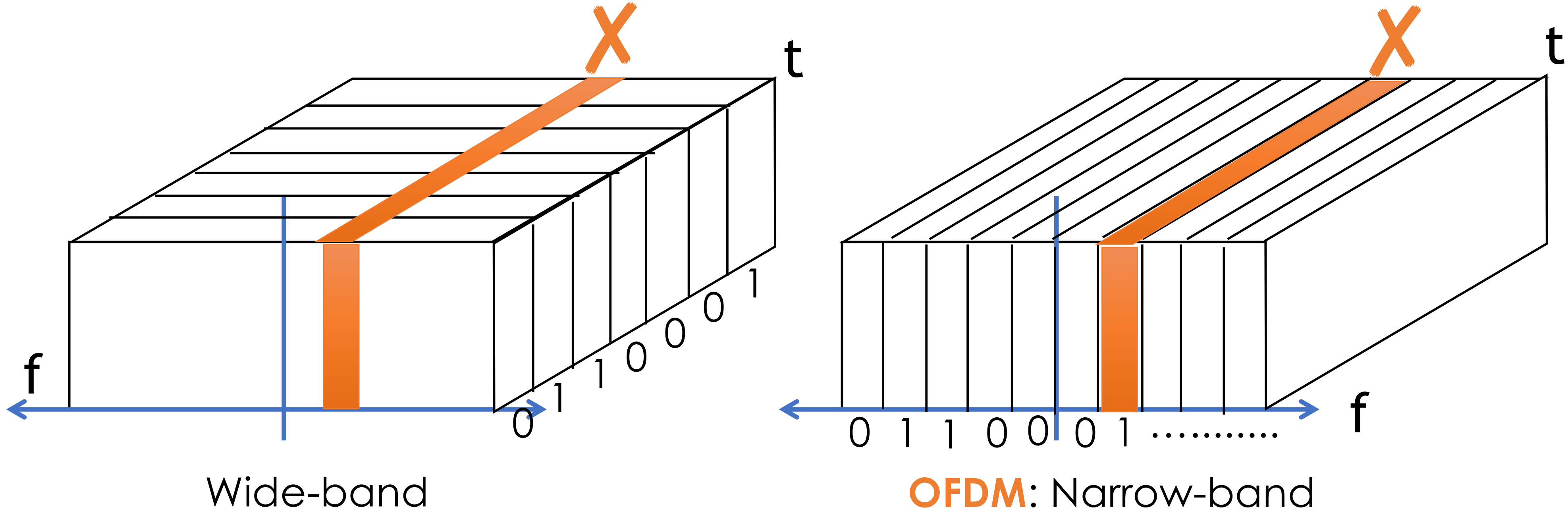




Send a sample using the entire band

Send samples concurrently using multiple orthogonal sub-channels

Why OFDM is Better?

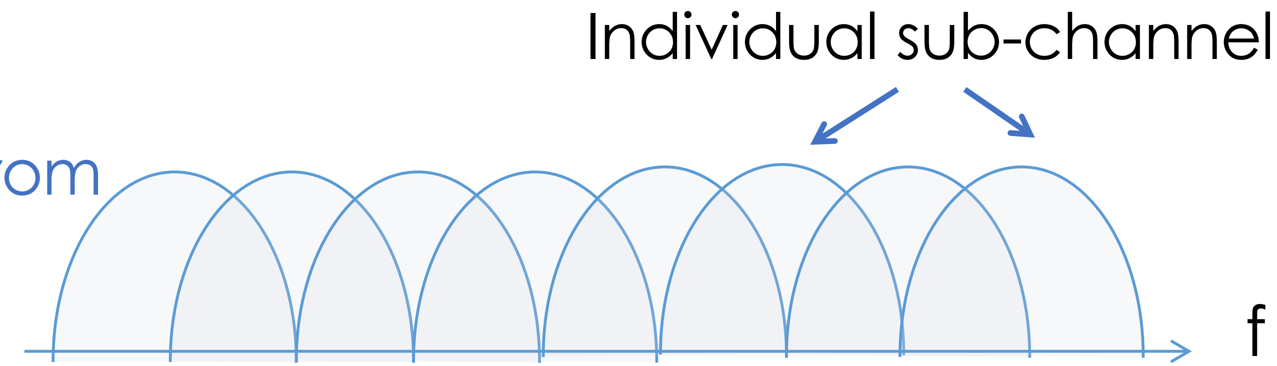


- Multiple sub-channels (sub-carriers) carry samples sent at a lower rate
  - Almost same bandwidth with wide-band channel
- Only some of the sub-channels are affected by interferers or multi-path effect

Importance of Orthogonality

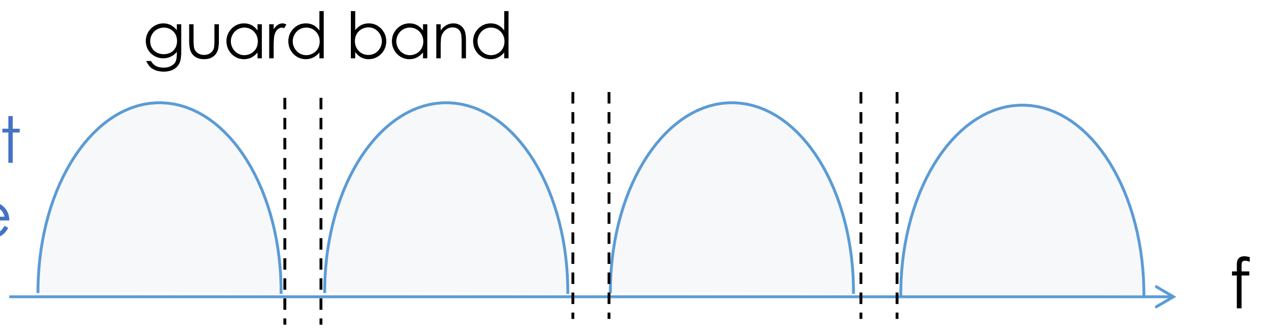
- Why not just use FDM (frequency division multiplexing)
  - Not orthogonal

Leakage interference from adjacent sub-channels

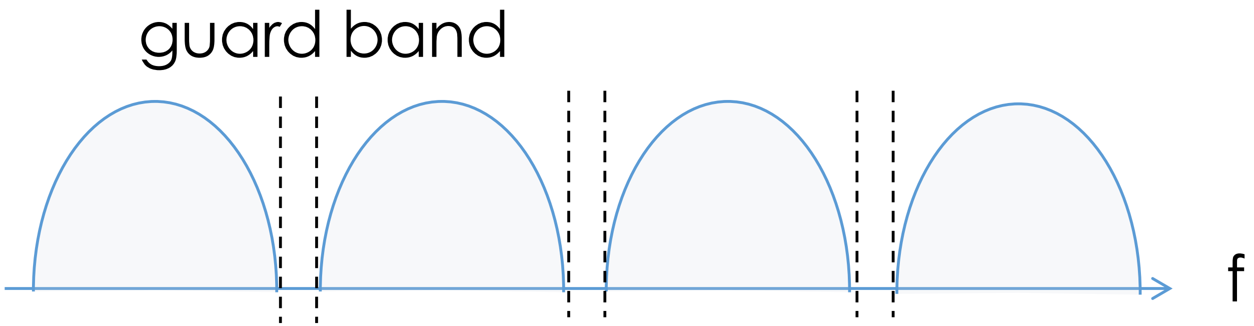


- Need **guard bands** between adjacent frequency bands → extra overhead and lower utilization

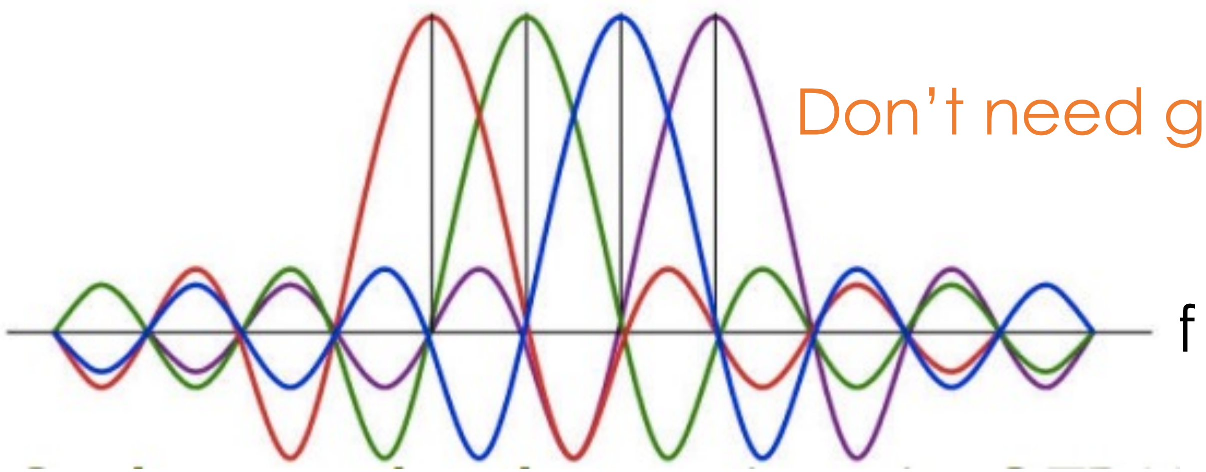
Guard bands protect leakage interference



Difference between FDM and OFDM



Frequency division multiplexing

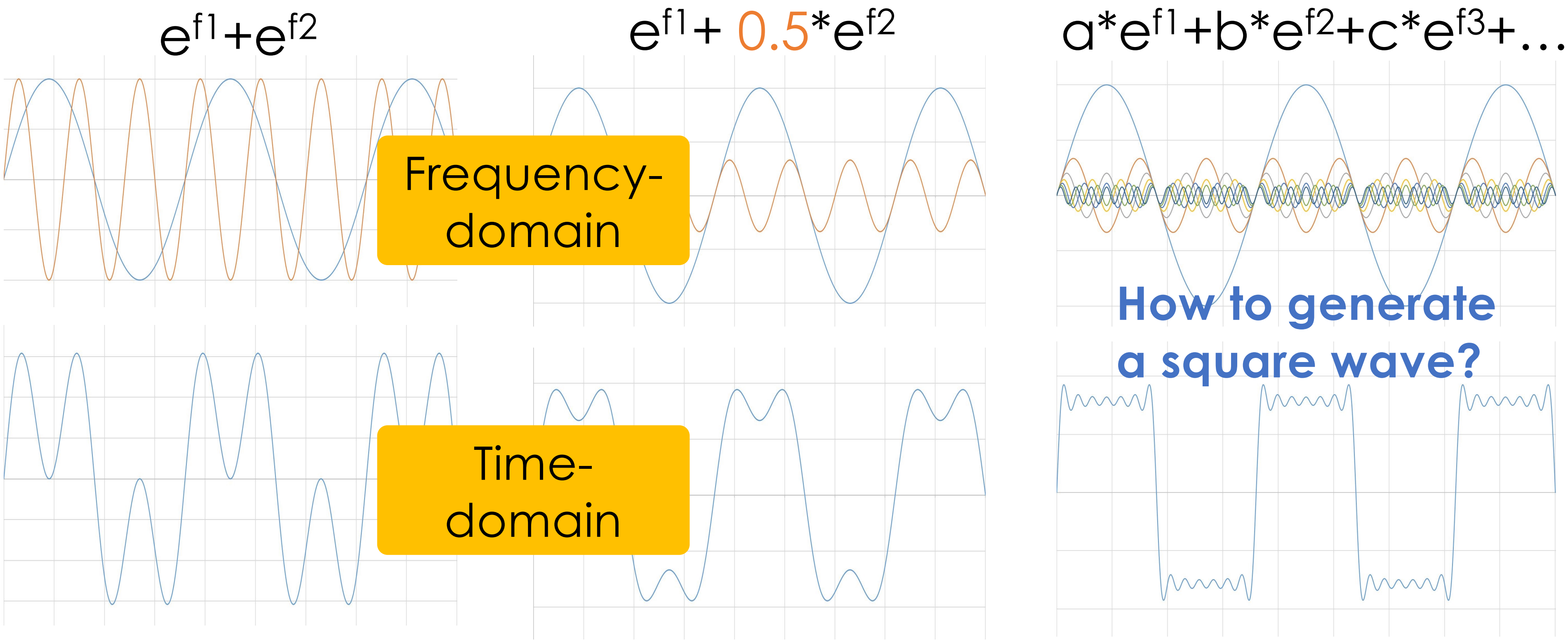


Don’t need guard bands

Orthogonal sub-carriers in OFDM

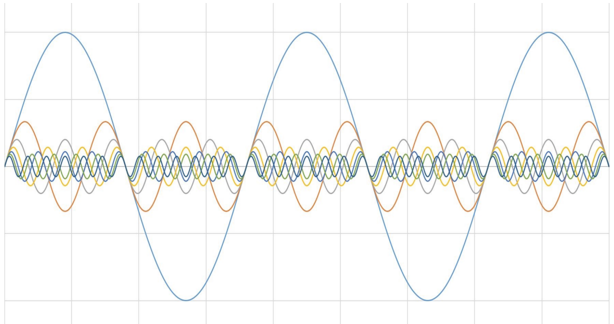
Key to Achieve Orthogonality: FFT

- Fast Fourier Transform (FFT)
- **Any waveform is the Sum of Sines**
  - Fourier's theorem: **ANY** waveform in the time domain can be represented by the **weighted sum of sines**



Primer of FFT/iFFT

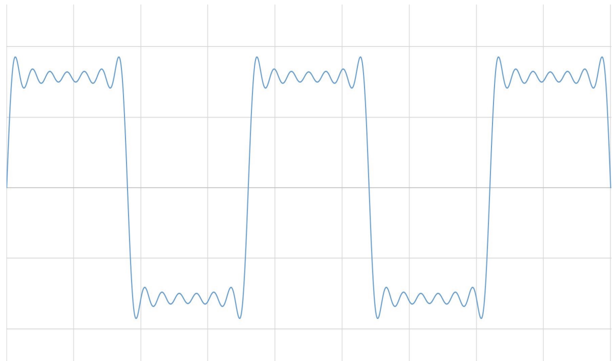
- iFFT: from frequency-domain signals to time-domain signals
- FFT: from time-domain signals to frequency-domain signals



Frequency-domain signal:
Amplitude of each freq.
a, b, c, d, …

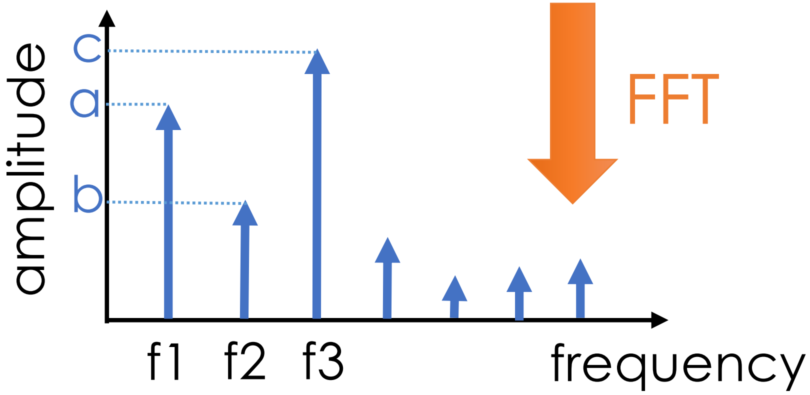
iFFT

time-domain signal



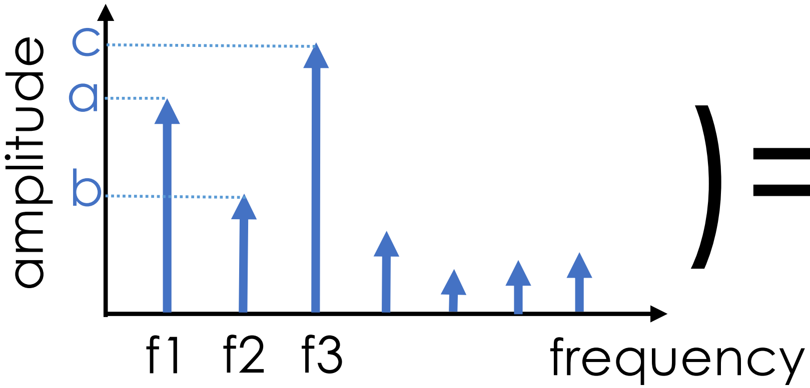
FFT

How can we know the frequency-domain components **(a, b, c ,…)** from this time-domain signal?



Primer of FFT/iFFT

- iFFT: from frequency to time
  - Use periodical waveforms to generate signals

iFFT(


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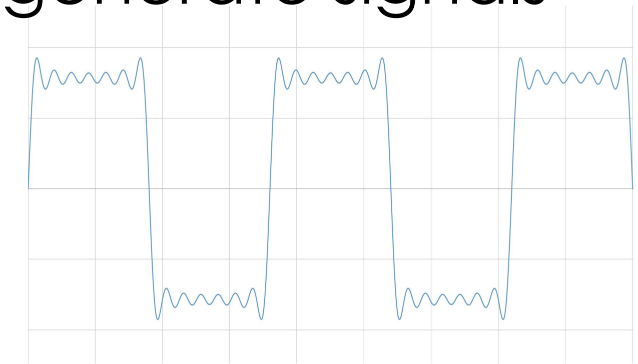
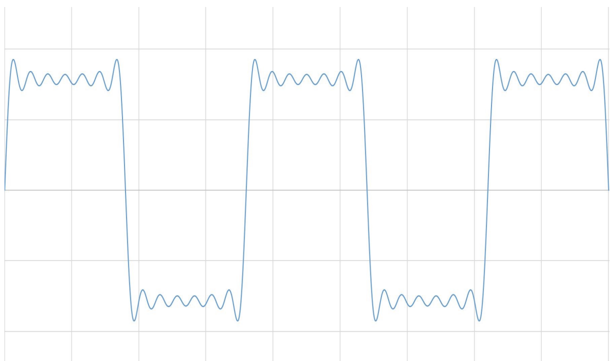


- FFT: from time to frequency
  - Extract frequency components of any signal

FFT(

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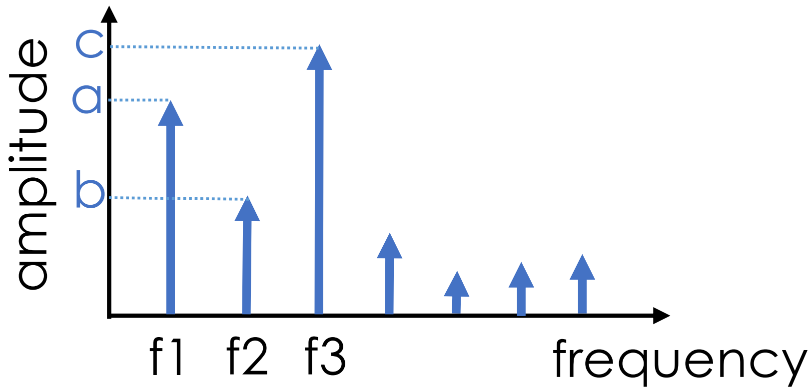
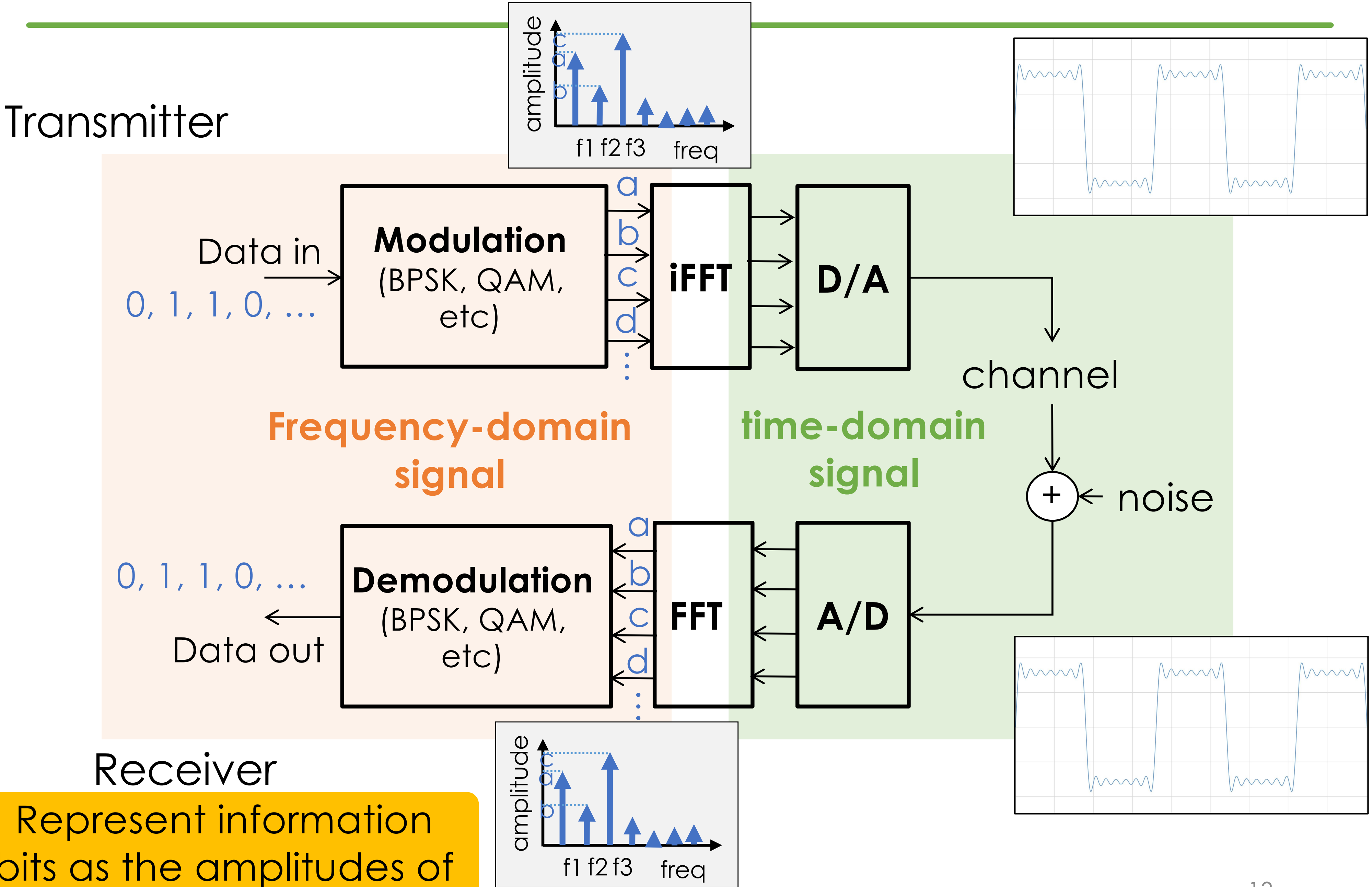


OFDM Transmitter and Receiver

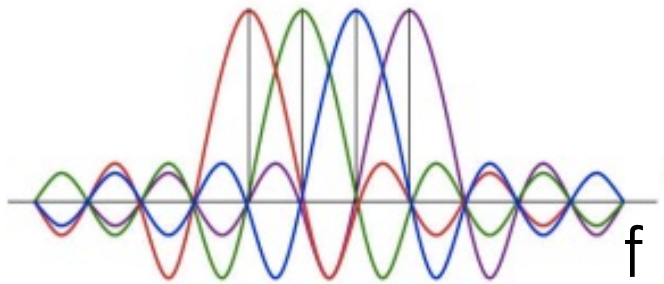


Represent information bits as the amplitudes of **orthogonal subcarriers**

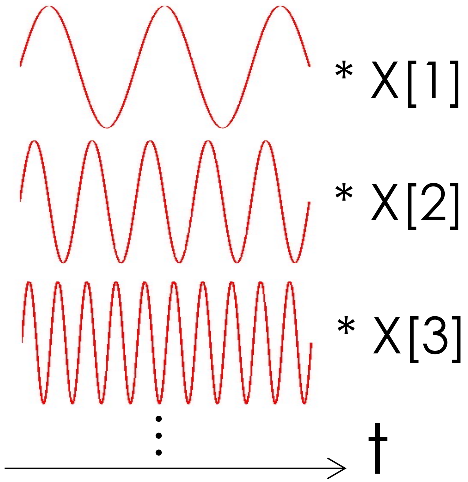
OFDM Basic

1. Partition the wide band to multiple narrow sub-carriers f_1, f_2, f_3, …, f_N
2. Represent information bits as the **frequency-domain signal** (amplitude of each sub-carrier)
   - Example: if we want to send 1, -1, 1, 1, we let 1, -1, 1, 1 be the frequency-domain signals
3. Use iFFT to convert the information to the **time-domain sent over the air**
   - Ex: Transmit iFFT(1,-1,1,1) = **1***e^{f1} + **(-1)***e^{f2} + **1***e^{f3} + **1***e^{f4}
4. Rx uses FFT to extract information
   - Ex: [1 -1 1 1] = FFT(**1***e^{f1} + **(-1)***e^{f2} + **1***e^{f3} + **1***e^{f4})

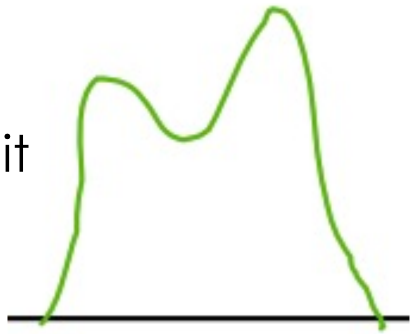
Orthogonal Frequency Division Modulation



IFFT



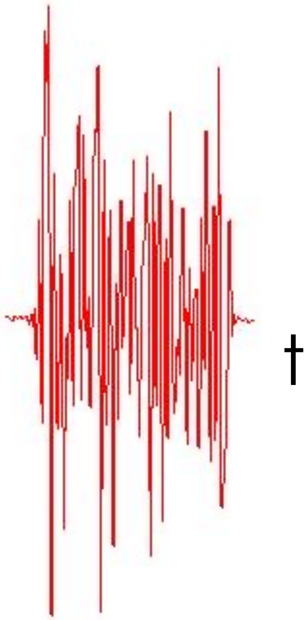
transmit



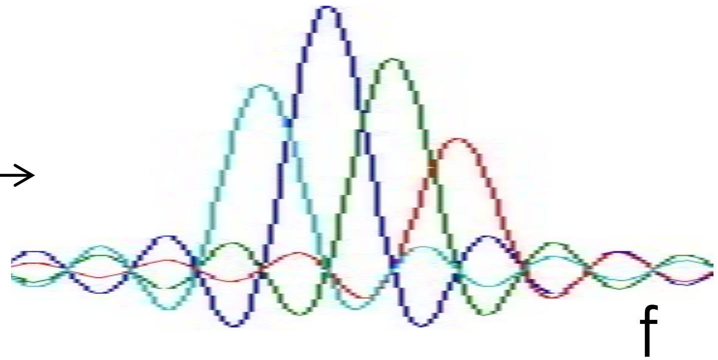
Data X[n] coded in frequency domain

Transformation to time domain: each frequency is a sine wave in time, all added up

receive



FFT



Decode each subcarrier separately

X’[N] = amplitude of each sub-carrier

Time domain signal

Frequency domain signal

Orthogonality of Sub-carriers

Time-domain signals: x(t)   Frequency-domain signals: X[k]

IFFT

Encode: frequency-domain samples → time-domain samples

k-th subcarrier

$$x(t) = \frac{1}{N} \sum_{k=-N/2}^{N/2-1} X[k] e^{j2\pi kt/N}$$

FFT

Decode: time-domain samples → frequency-domain sample

$$X[k] = \sum_{t=-N/2}^{N/2-1} x(t) e^{-2j\pi kt/N}$$

Orthogonal → inner product = 0

Orthogonality of any two bins : $\sum_{k=-N/2}^{N/2-1} e^{j2\pi kt/N} e^{-j2\pi pt/N} = 0, \forall p \neq k$

Orthogonality between Subcarriers

- Subcarrier frequencies (**k/N**, k=-N/2,…, N/2-1) are chosen so that the subcarriers are **orthogonal** to each other
  - No guard band is required
- Two signals are orthogonal if their inner product equals zero

$$\sum_{k=-N/2}^{N/2-1} e^{j2\pi kt/N} e^{-j2\pi pt/N} = \sum_{k=-N/2}^{N/2-1} e^{2j\pi(k-p)t/N}$$

$$= N\delta(k,p) = \begin{cases} N & \text{if } p = k \\ 0 & \text{if } p \neq k \end{cases}$$

$$X[k] \perp X[p], k \neq p$$

Serial to Parallel Conversion

- Say we use BPSK and 4 sub-carriers to transmit a stream of samples

1, 1, -1, -1, 1, 1, 1, -1, 1, -1, -1, -1, -1, 1, -1, -1, -1, 1, 1, -1, -1, -1, 1, 1

- Serial-to-parallel conversion of samples

Frequency-domain signal $\xrightarrow{\text{IFFT}}$ Time-domain signal

	c1	c2	c3	c4
symbol1	1	1	-1	-1
symbol2	1	1	1	-1
symbol3	1	-1	-1	-1
symbol4	-1	1	-1	-1
symbol5	-1	1	1	-1
symbol6	-1	-1	1	1

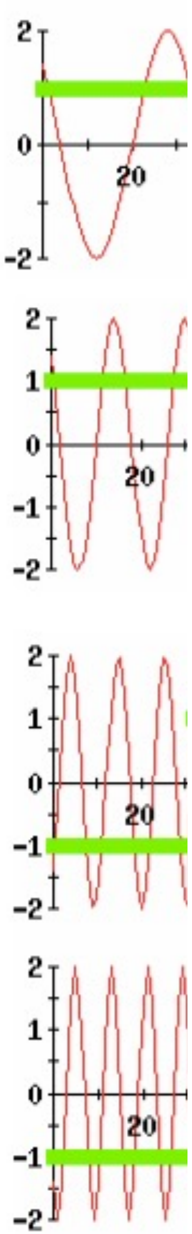
0	2 - 2i	0	2 + 2i
2	0 - 2i	2	0 + 2i
-2	2	2	2
-2	0 - 2i	-2	0 + 2i
0	-2 - 2i	0	-2 + 2i
0	-2 + 2i	0	-2 - 2i

- Send time-domain samples after parallel-to-serial conversion

0, 2 - 2i, 0, 2 + 2i, 2, 0 - 2i, 2, 0 + 2i, -2, 2, 2, 2, -2, 0 - 2i, -2, 0 + 2i, 0, -2 - 2i, 0, -2 + 2i, 0, -2 + 2i, 0, -2 - 2i, …

t1-4

symbol1	1	1	-1	-1
symbol2	1	1	1	-1
symbol3	1	-1	-1	-1
symbol4	-1	1	-1	-1
symbol5	-1	1	1	-1
symbol6	-1	-1	1	1



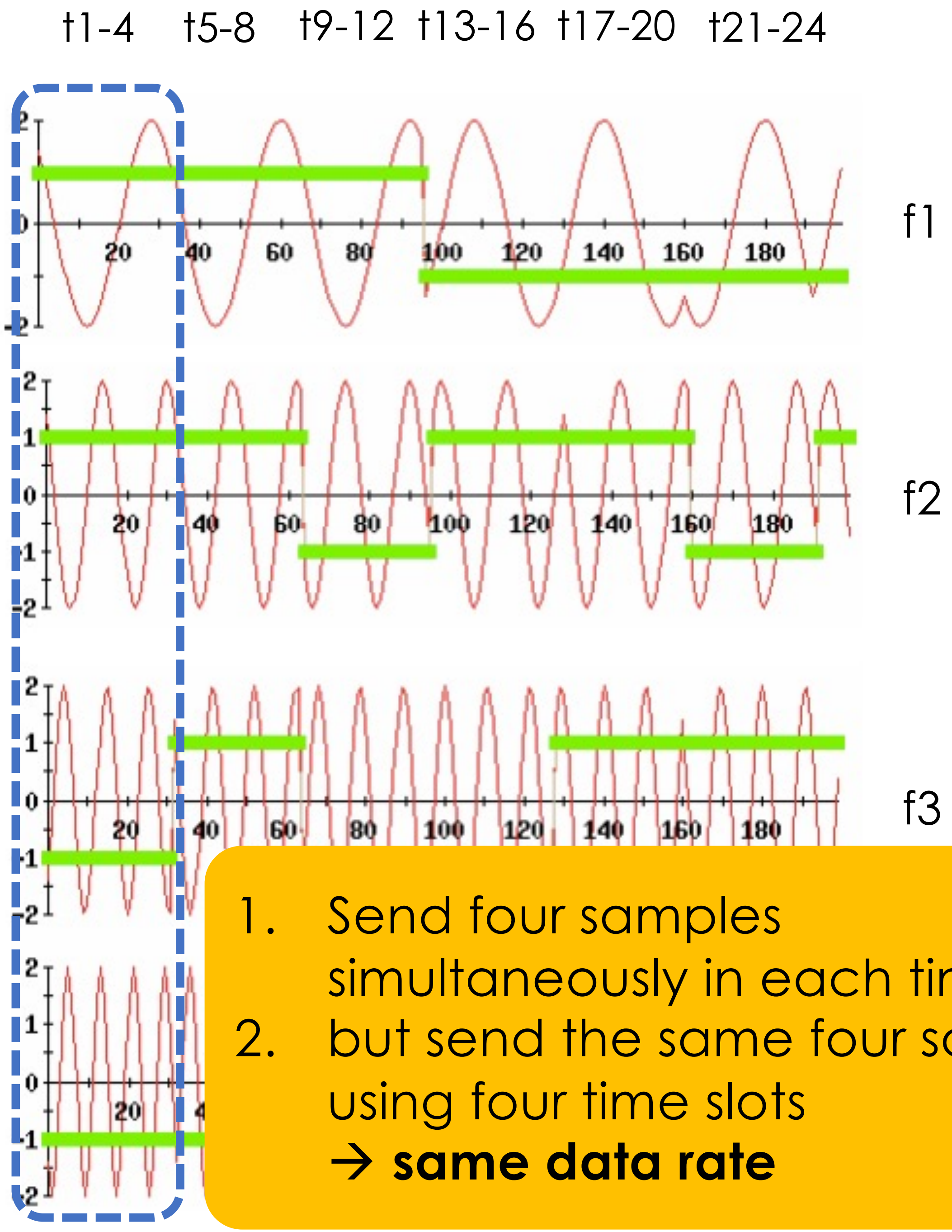
f1

f2

f3

f4

symbol1	1	1	-1	-1
symbol2	1	1	1	-1
symbol3	1	-1	-1	-1
symbol4	-1	1	-1	-1
symbol5	-1	1	1	-1
symbol6	-1	-1	1	1



1. Send four samples simultaneously in each time-slot
2. but send the same four samples using four time slots
   → **same data rate**

Send the combined signal as the time-domain signal

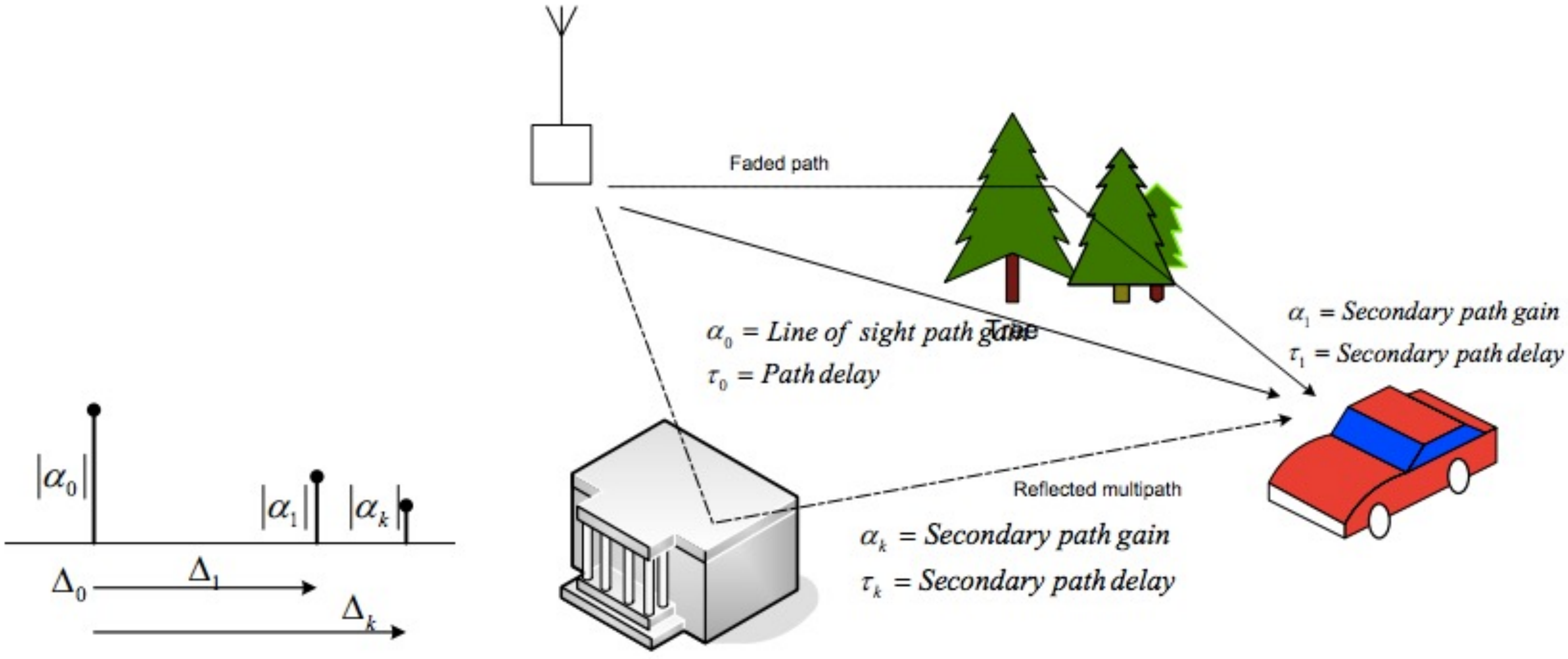
Agenda

- OFDM
  (Orthogonal Frequency Division Modulation)
- **Multipath Effect**
- Packet Detection
- Channel Estimation and Decoding
- Synchronization
- OFDM vs. OFDMA

Why OFDM?

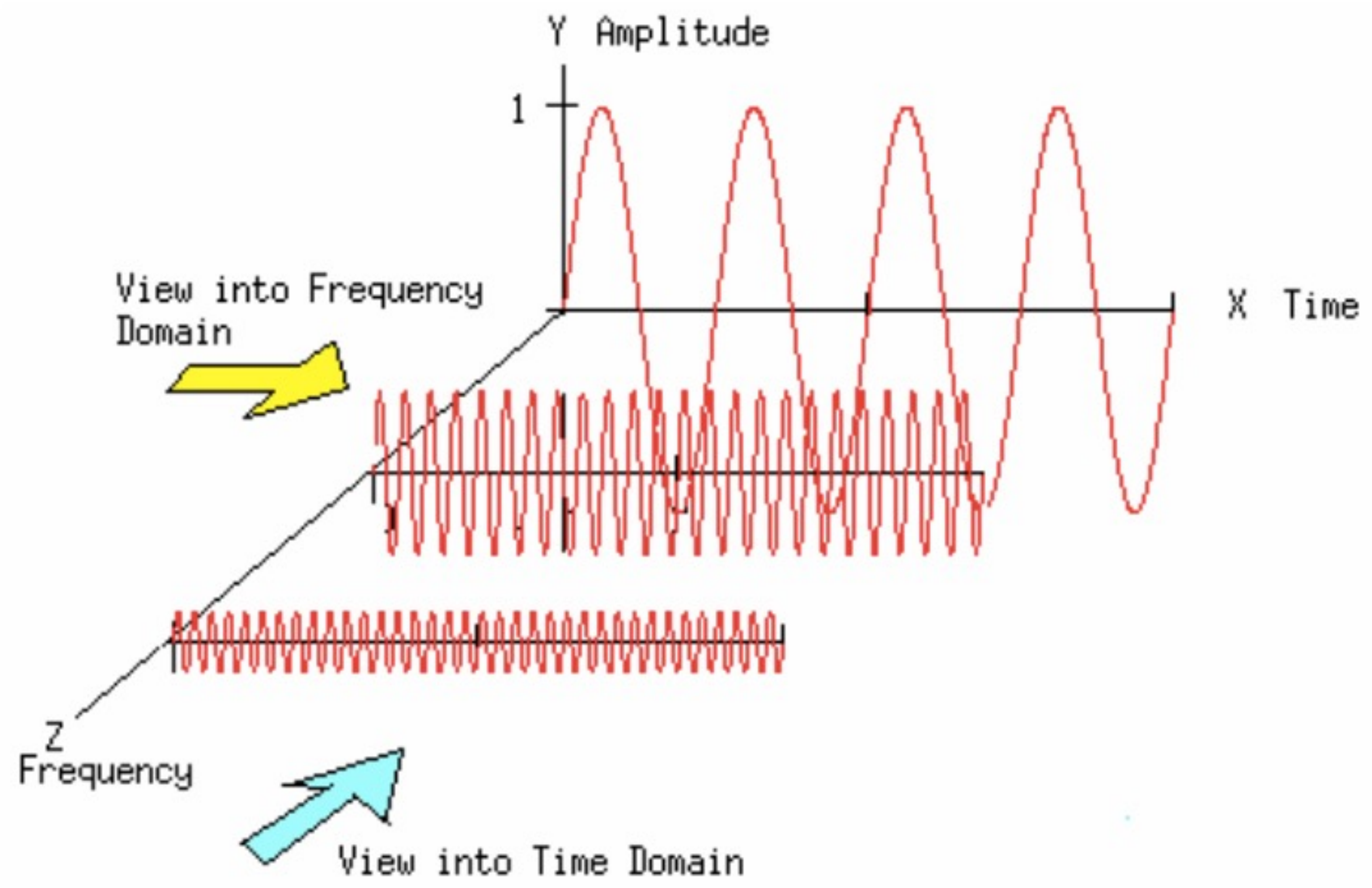
combat **multipath** fading

Multi-Path Effect



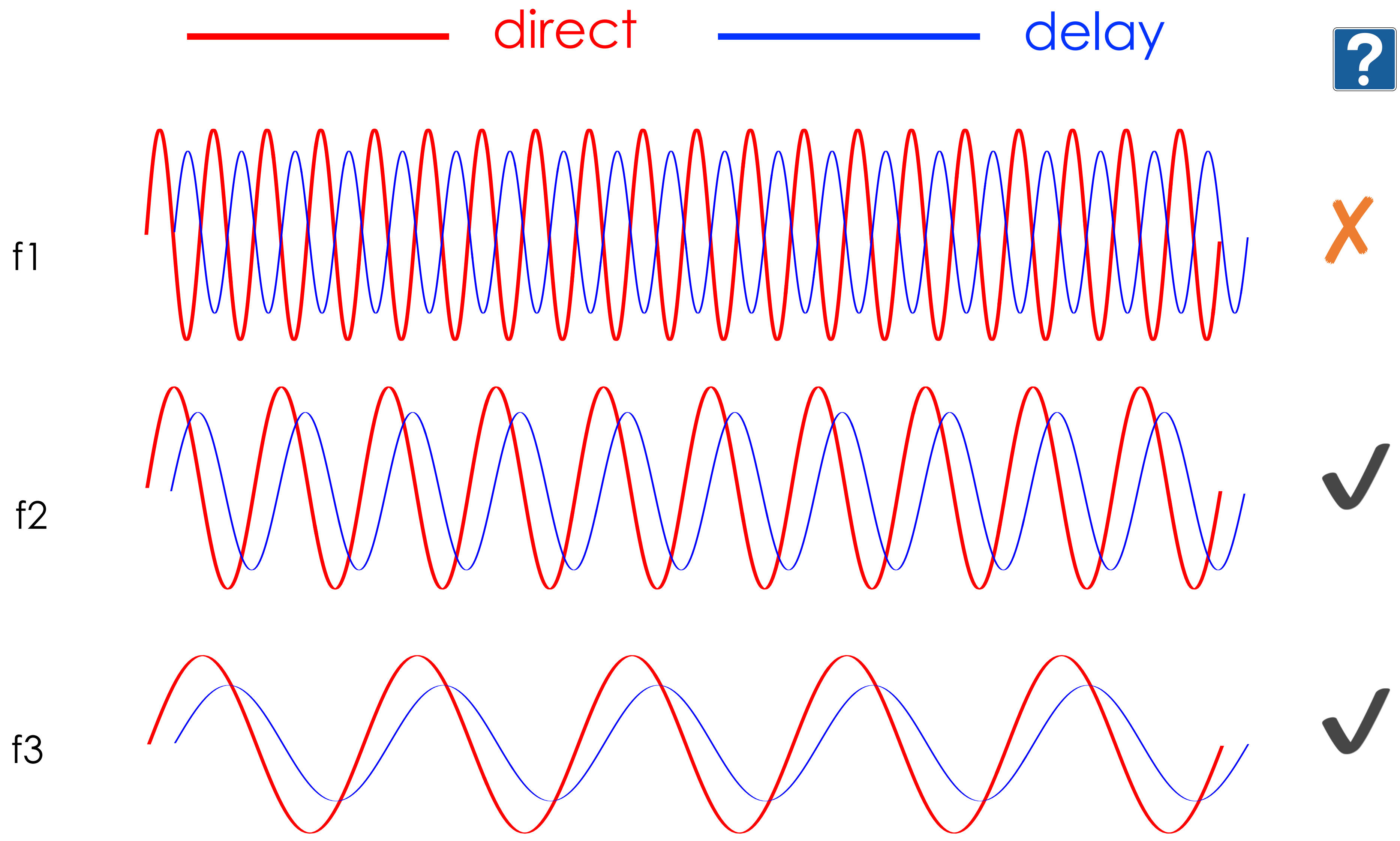
$$y(t) = h(0)x(t) + h(1)x(t-1) + h(2)x(t-2) + \cdots$$
$$= \sum_{\triangle} h(\triangle)x(t-\triangle) = h(t) \otimes x(t) \quad \Leftrightarrow \quad Y(f) = H(f)X(f)$$

time-domain convolution frequency-domain



Current symbol + delayed-version symbol

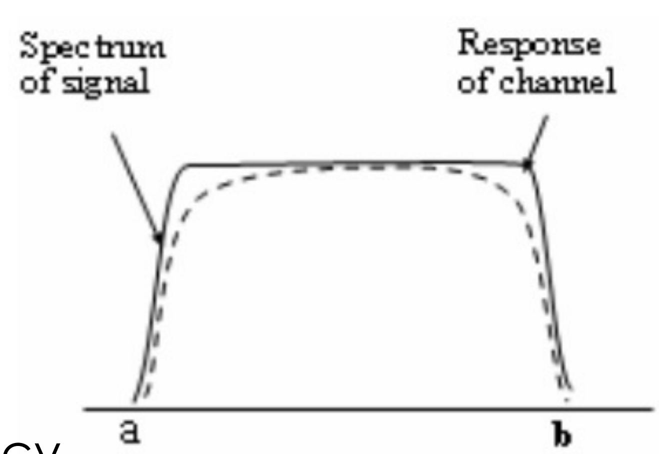
→ Signals are destructive in only certain frequencies



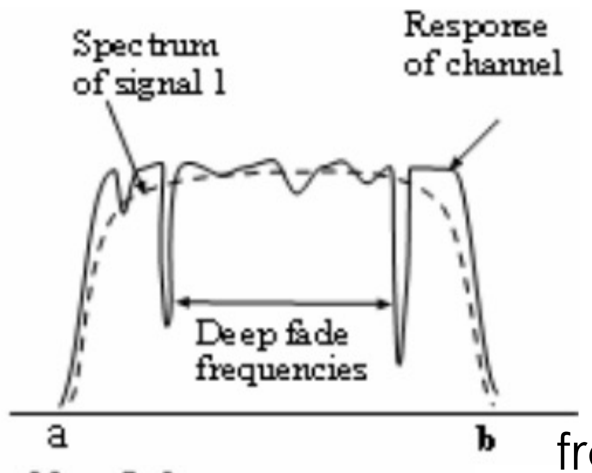
Current symbol + delayed-version symbol

→ Signals are destructive in only certain frequencies

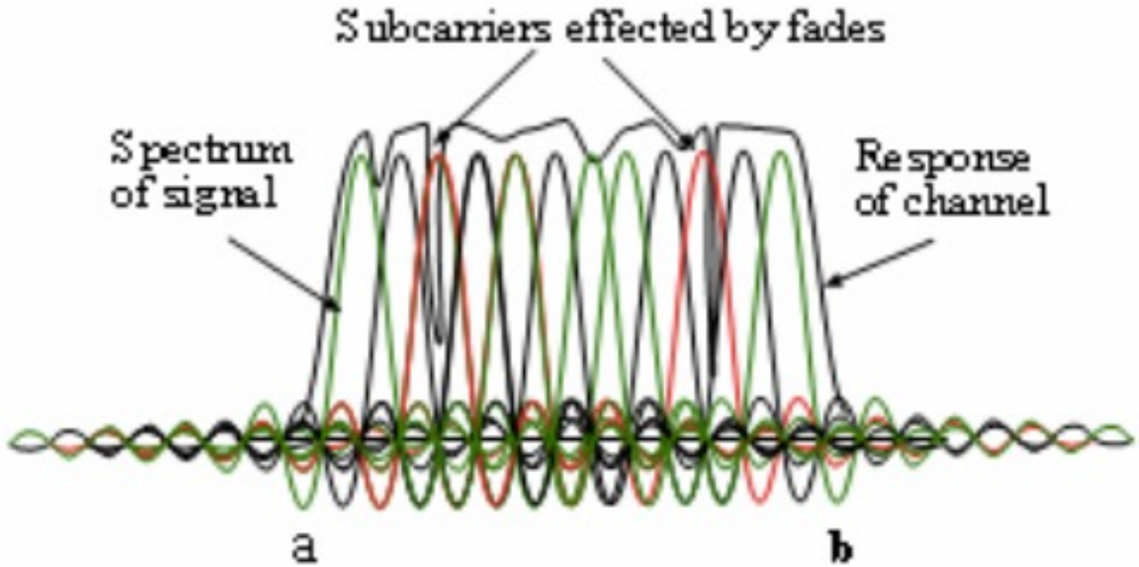
Frequency Selective Fading



frequency



frequency

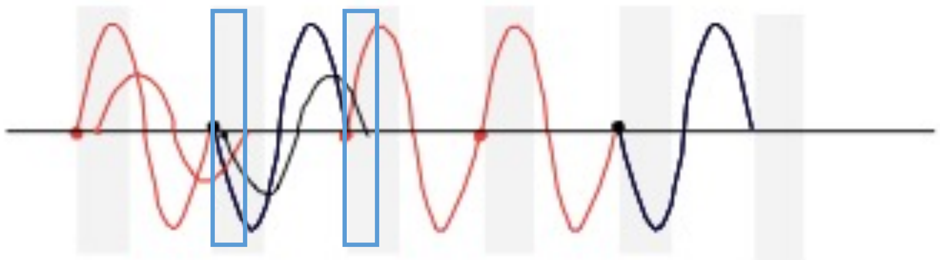


Frequency selective fading: Only some sub-carriers get affected

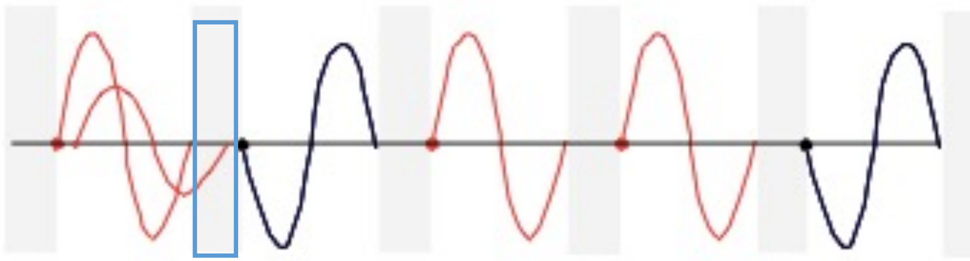
Can be recovered by proper coding!

Inter Symbol Interference (ISI)

- The delayed version of a symbol overlaps with the adjacent symbol



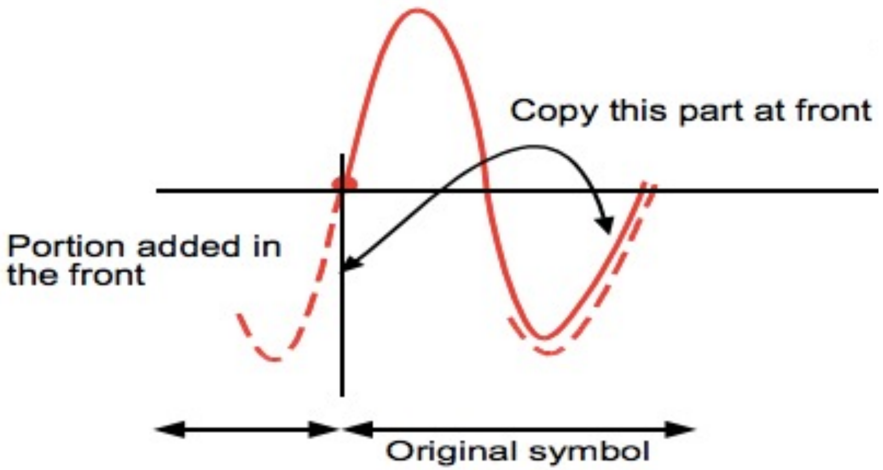
- One simple solution to avoid this is to introduce a guard-band



Guard band

Cyclic Prefix (CP)

- However, we don't know the delay spread exactly
  - The hardware doesn't allow blank space because it needs to send out signals continuously
- Solution: Cyclic Prefix
  - Make the symbol period longer by copying **the tail of time-domain samples** and glue them in the front

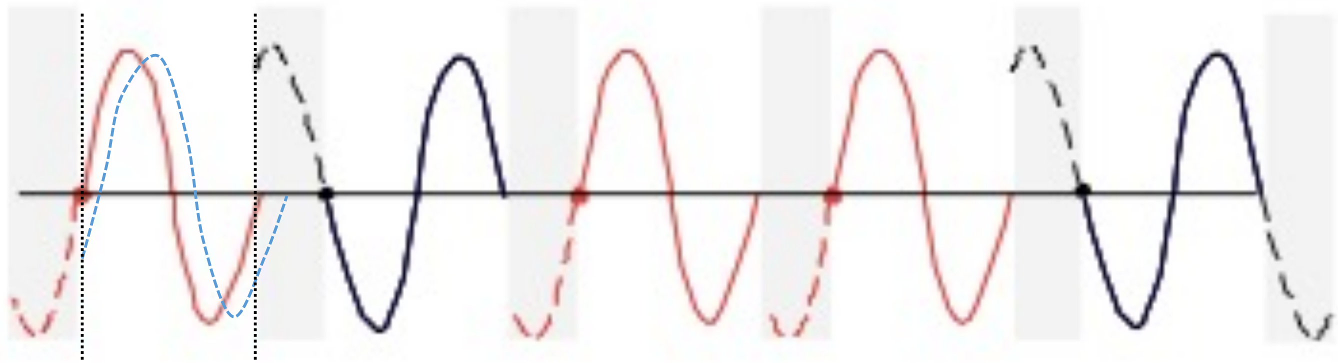


CP	Symbol i	CP	Symbol i+1	…

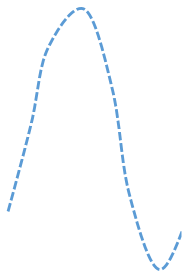
In 802.11, each symbol with 64 samples

CP:data = 1:4
→ CP: last 16 samples

Cyclic Prefix (CP)



- Because of the usage of FFT, the signal is periodic

FFT() = $\exp(-2j\pi\Delta_f)$*FFT()

delayed version

original signal

- Delay in the time domain corresponds to phase shift in the frequency domain
  - Can still obtain the correct signal in the frequency domain by compensating this rotation

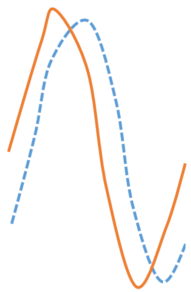
Cyclic Prefix (CP)

w/o multipath

$y(t) \rightarrow \text{FFT}($  $) \rightarrow Y[k] = H[k]X[k]$

original signal

w multipath

$y(t) \rightarrow \text{FFT}($  $) \rightarrow Y[k]$

$$= (H[k] + \exp(-2j\pi\Delta_k)H[k])X[k]$$
$$= (H[k] + H_2[k])X[k]$$
$$= \mathbf{H'[k]}X[k]$$

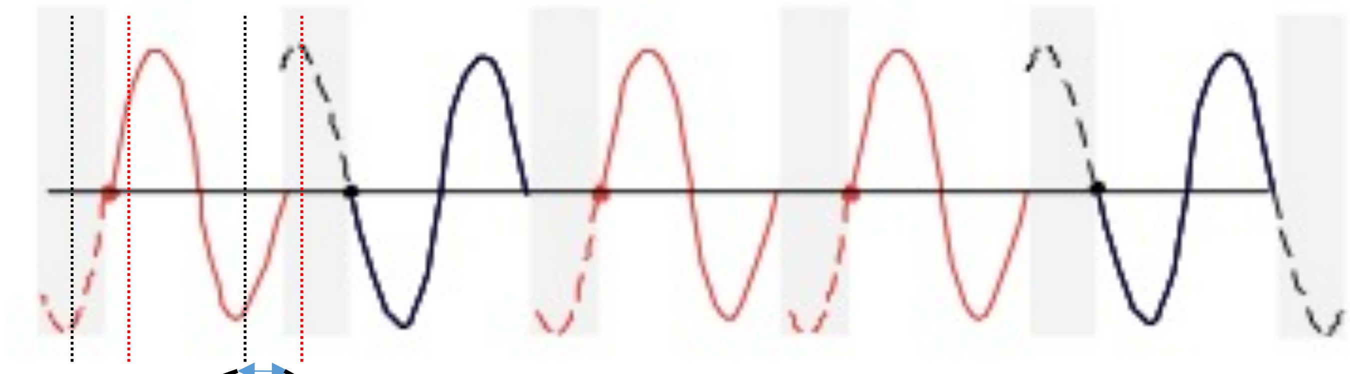
original signal
+ delayed-version signal

Lump the
phase shift in H

Side Benefit of CP

- Allow the signal to be decoded even if the packet is detected not that accurately

decodable undecodable



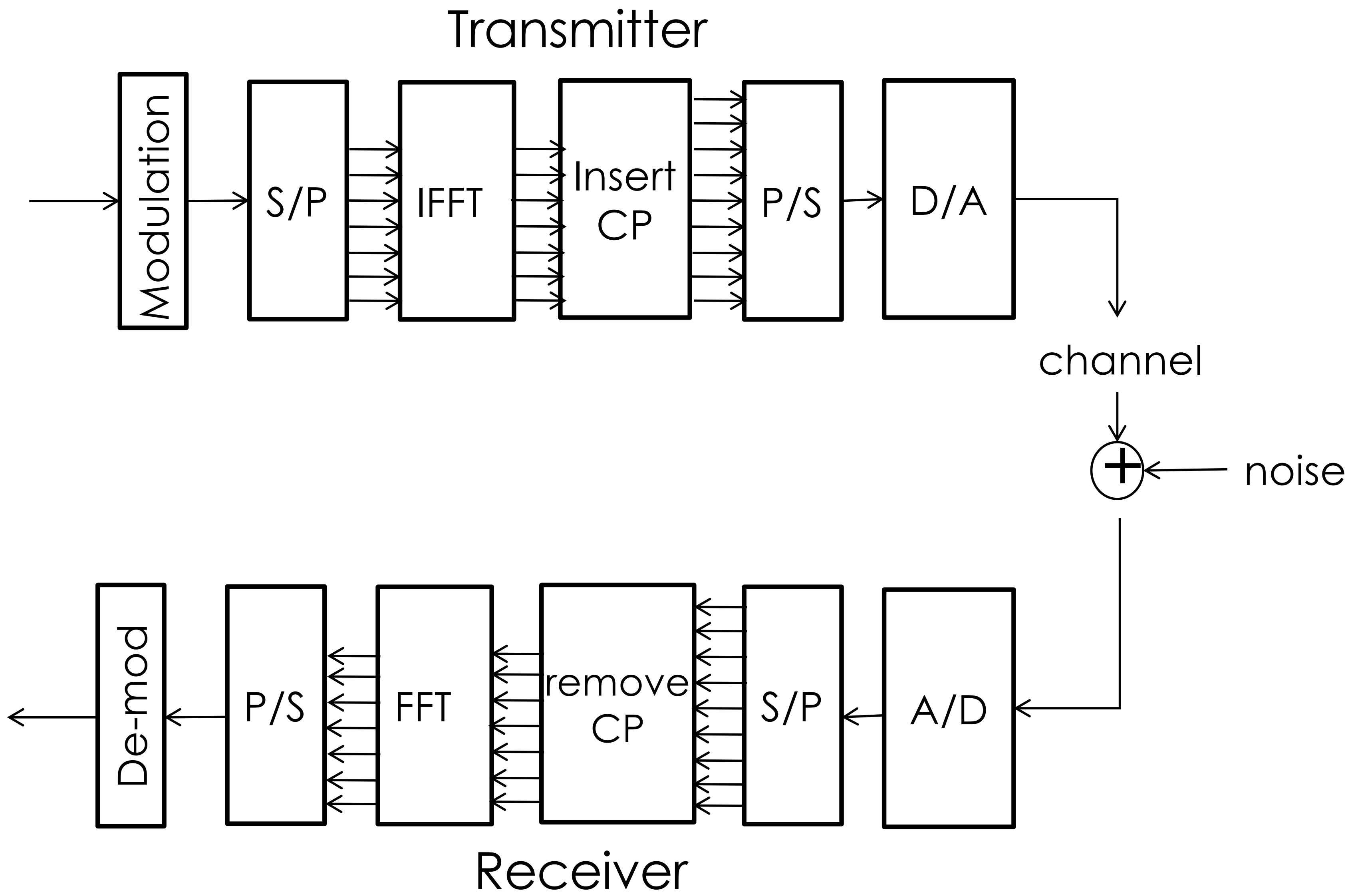
FFT_OFFSET

The last sample you actually use for FFT

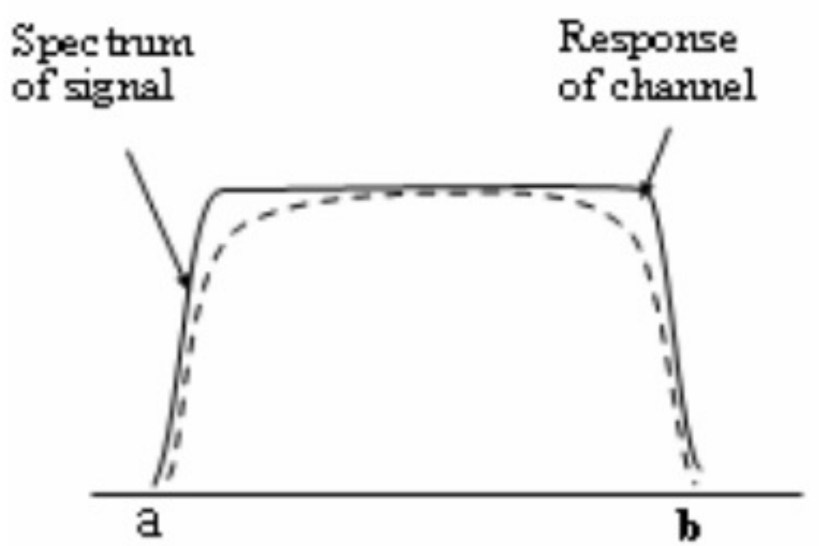
The point you think the first symbol ends

Check the parameter **FFT_OFFSET** in the WARP code. Try to modify it!

OFDM Diagram



Unoccupied Subcarriers



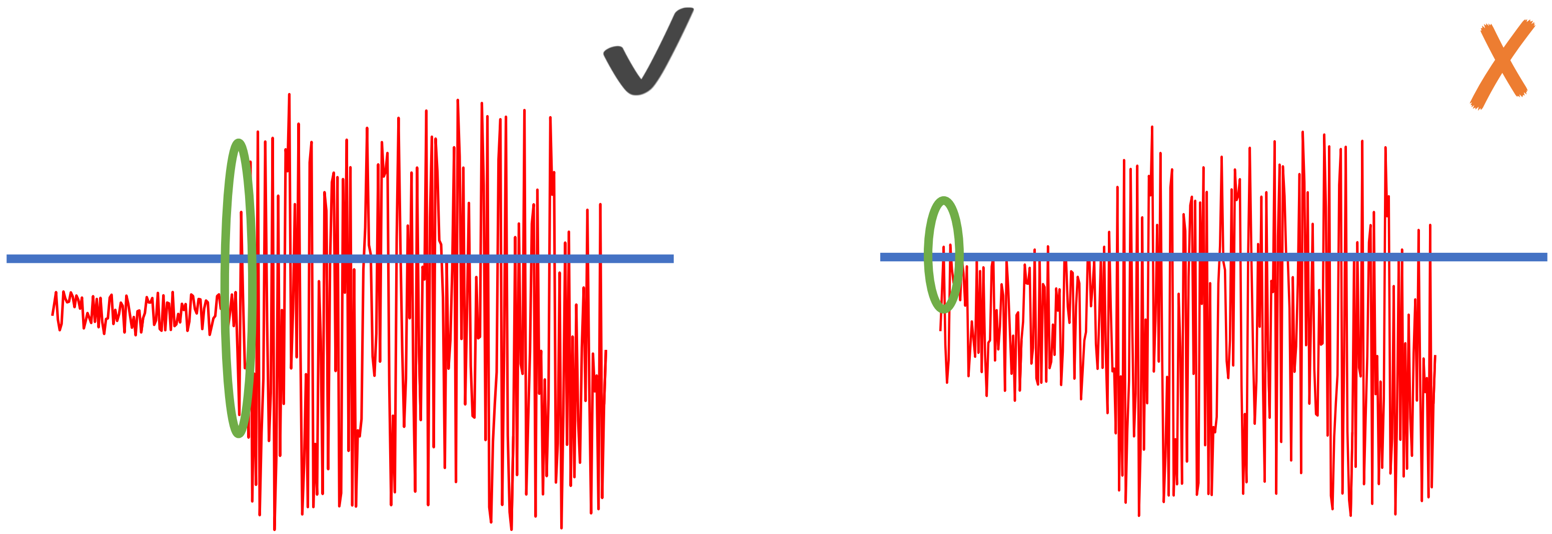
- Edge sub-carriers are more vulnerable
  - Frequency might be shifted due to noise or multi-path
- Leave them unused
  - In 802.11, only 48 of 64 bins are occupied bins
- Is it really worth to use OFDM when it costs so many overheads (CP, unoccupied bins)?

Agenda

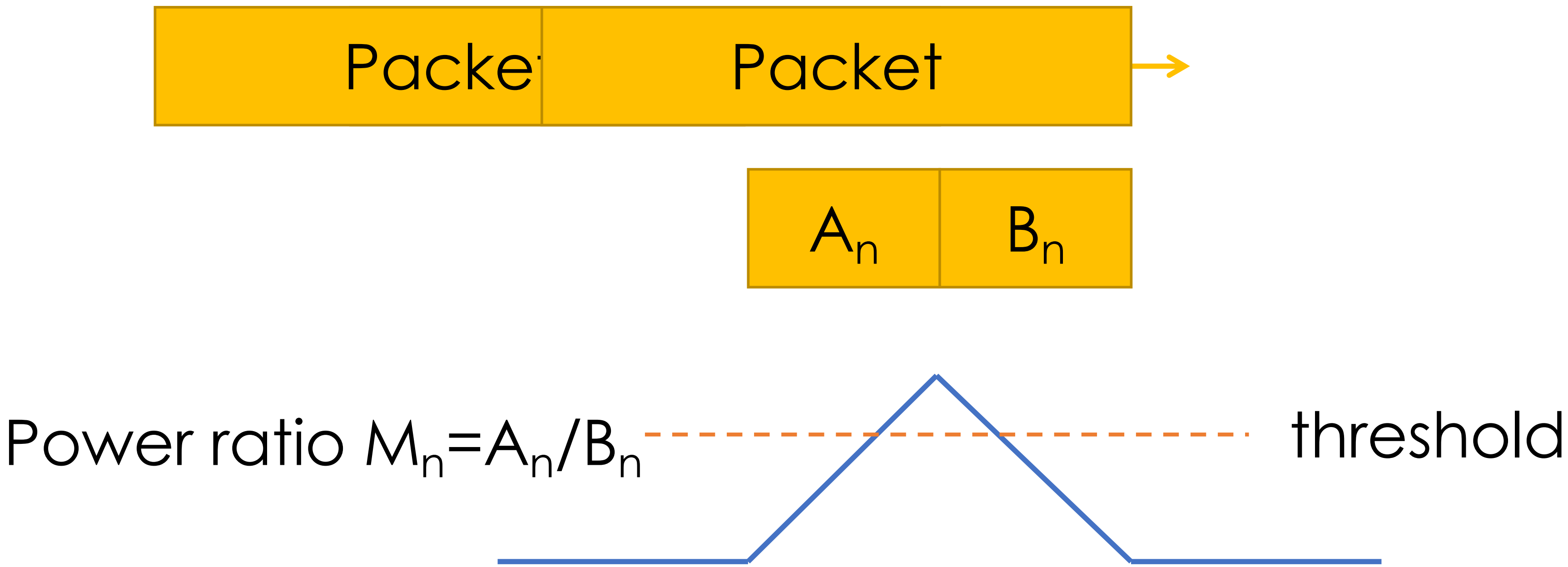
- OFDM
  (Orthogonal Frequency Division Modulation)
- Multipath Effect
- **Packet Detection**
- Channel Estimation and Decoding
- Synchronization
- OFDM vs. OFDMA

What is Packet Detection

- Detect where is the starting time of a packet
- It might be easy to detect visually, but how can a device automatically find it?
  - Simplest way: find the energy burst using a threshold
  - Difficulty: hard to determine a good threshold



Packet Detection



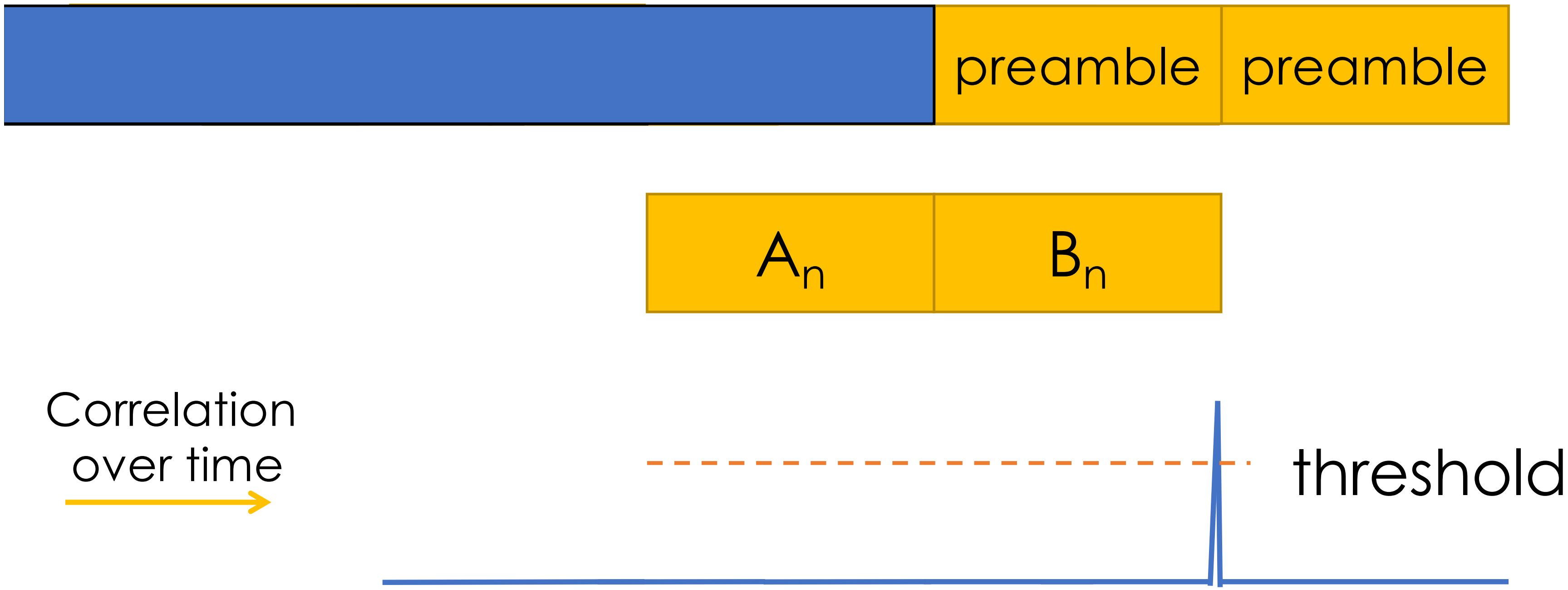
- Double sliding window packet detection
- Optimal threshold depends on the receiving power

Packet Detection in 802.11

- Each packet starts with a preamble
  - First part of the preamble is exactly the same with the second part

- Use cross-correlation to detect the preamble
  - Use double sliding window to calculate the auto-correlation of the signals received in two windows
  - Leverage the key properties: 1) noise is uncorrelated with the preamble, and 2) data payload is also uncorrelated with the preamble

Packet Detection in 802.11



- Noise is uncorrelated with noise
- Noise is uncorrelated with preamble
- Get a peak exactly when the double windows receives the entire preamble
- Data is again uncorrelated with noise

Agenda

- OFDM
  (Orthogonal Frequency Division Modulation)
- Multipath Effect
- Packet Detection
- **Channel Estimation and Decoding**
- Synchronization
- OFDM vs. OFDMA

Channel Estimation

$$y(t) = h(0)x(t) + h(1)x(t-1) + h(2)x(t-2) + \cdots$$
$$= \sum_{\triangle} h(\triangle)x(t-\triangle) = h(t) \otimes x(t) \quad \Leftrightarrow \quad Y(f) = H(f)X(f)$$

time-domain frequency-domain

- Y[k] = H[k]X[k] + N[k]
  - Subcarriers have different channels H[k]
- Equalize X[k] by
  - X'[k] = Y[k] / H[k] = X[k] + N[k] / H[k]
- How to learn the channels H[k]?
  - Preamble is known by both Tx and Rx
  - Y[k] = H[k]P[k] + N[k]
    ➔ H'[k] = Y[k]/ P[k] = H[k] + N[k]/ P[k]
  - How to minimize the impact of noise? Send multiple preambles and take average to zero out the noise

Agenda

- OFDM
  (Orthogonal Frequency Division Modulation)
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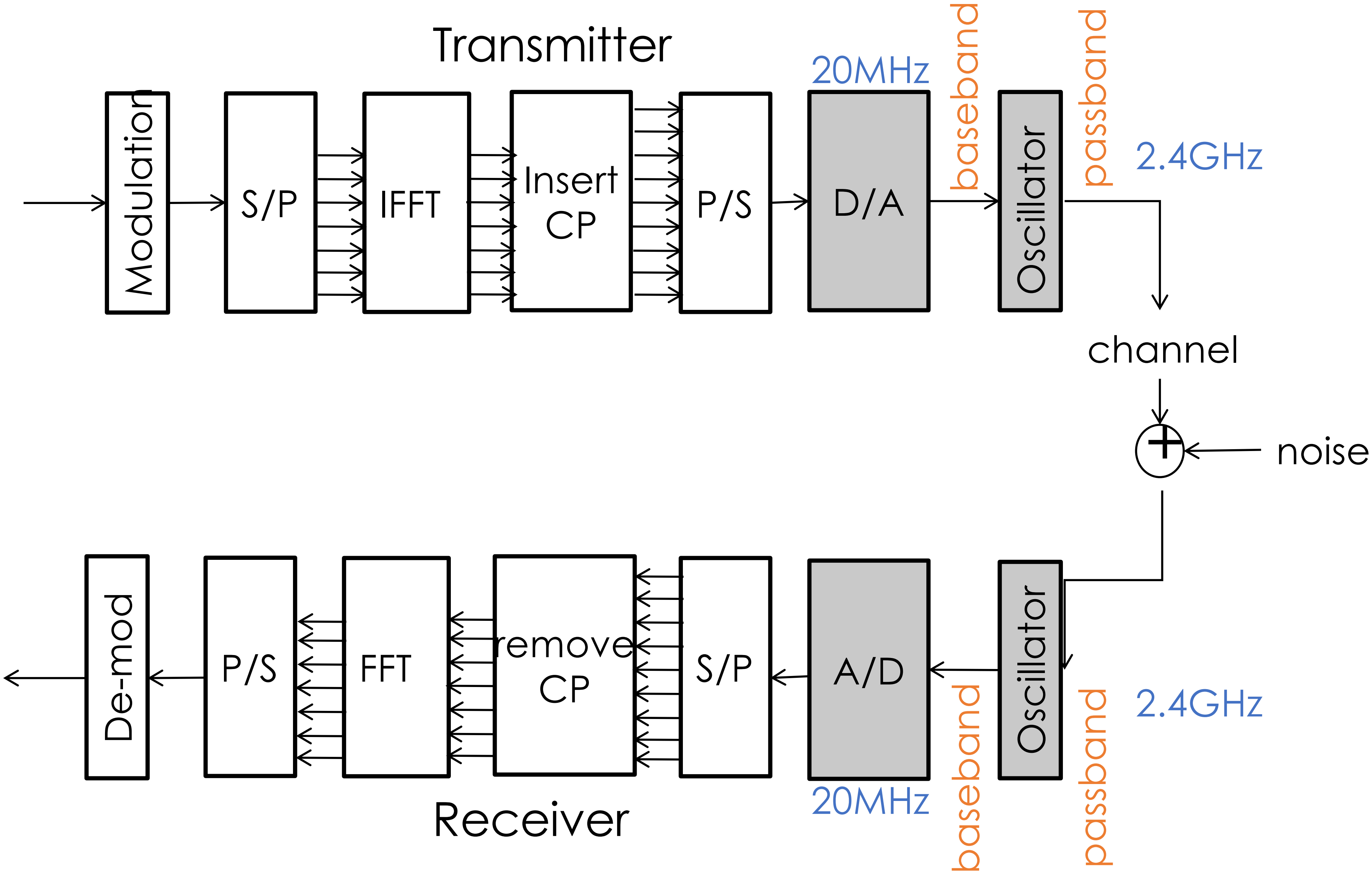
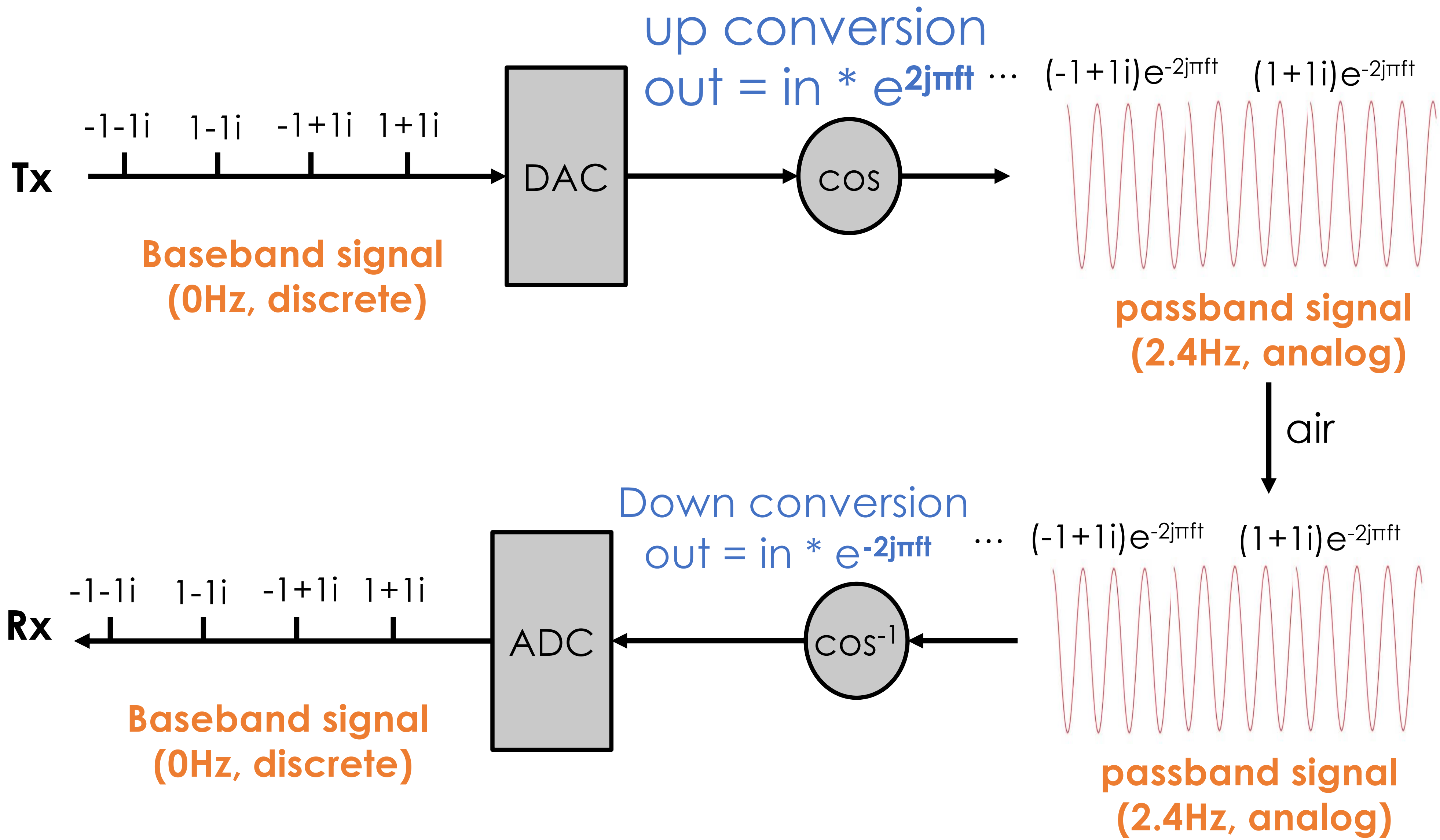
- **Synchronization**
- OFDM vs. OFDMA

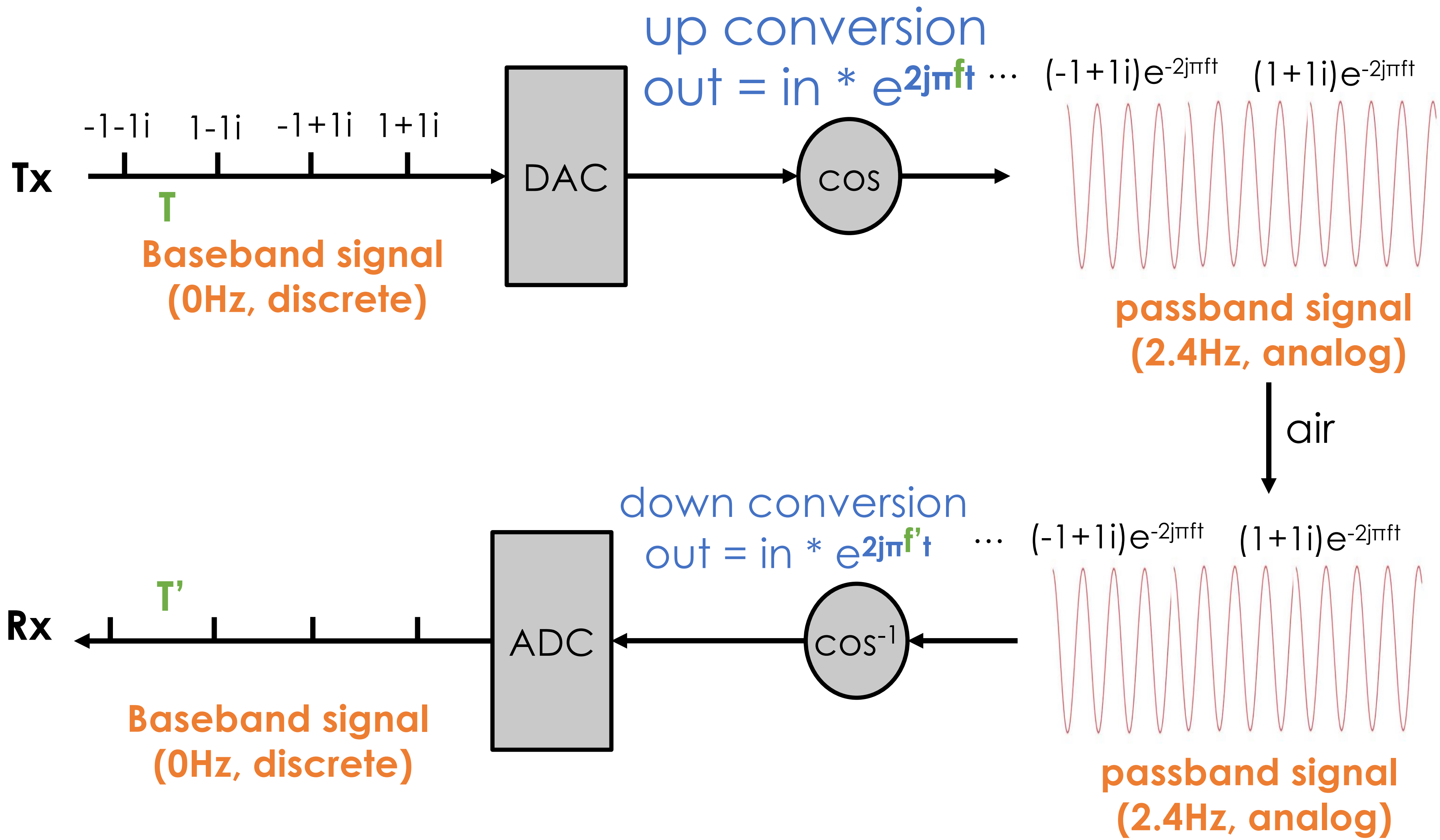
OFDM Diagram



Sampling and Down/Up Conversion



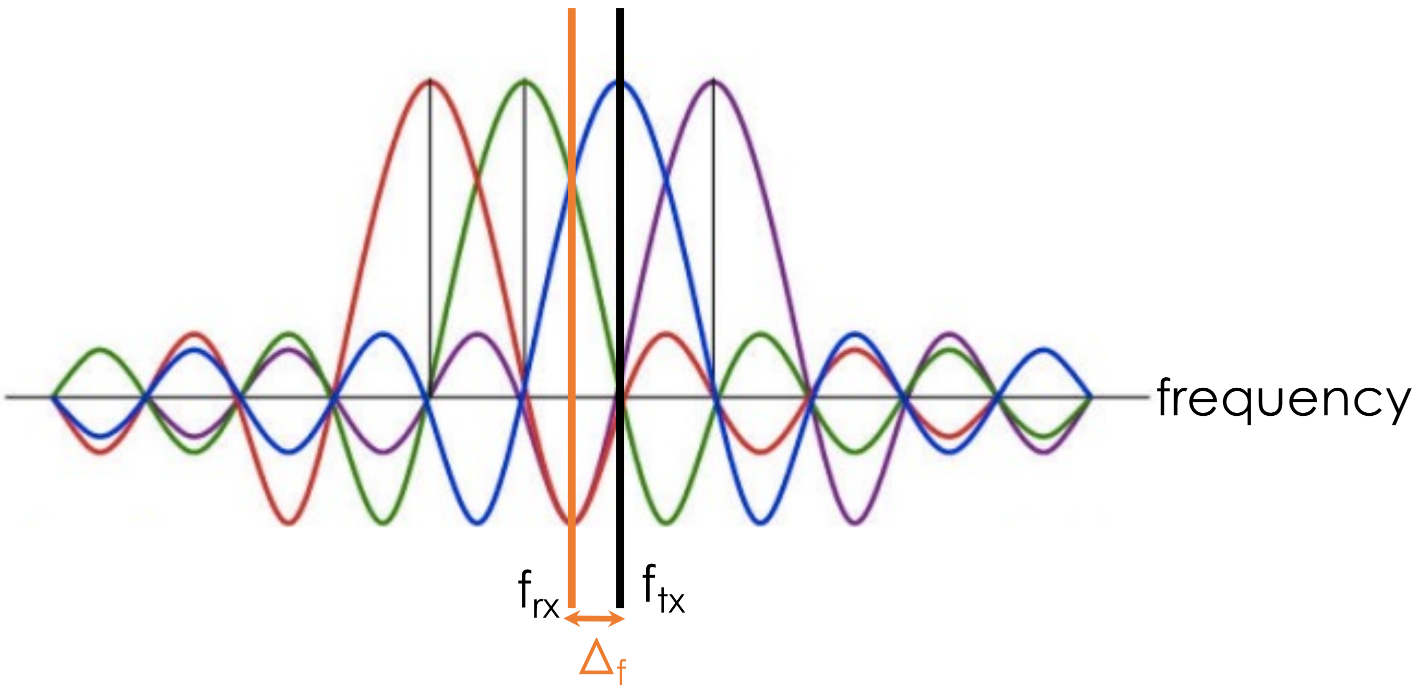
Without Synchronization



f'!=f, T' != T

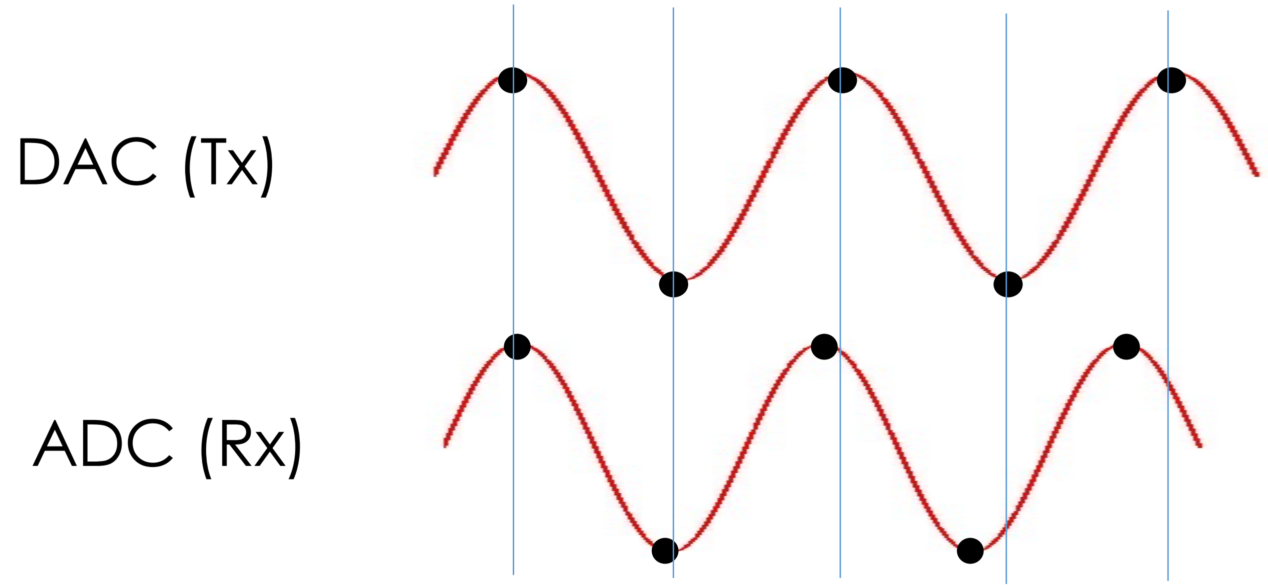
What, How, Why

Carrier Frequency Offset (CFO)



- The oscillators of Tx and Rx are not perfectly synchronized
  - Carrier frequency offset (CFO) $\Delta_f = f_{tx} - f_{rx}$
  - Leading to inter-carrier interference (ICI)
- OFDM is sensitive to CFO

Sampling Frequency Offset (SFO)



- DAC (at Tx) and ADC (at Rx) never have exactly the same sampling period ($T_{tx} \neq T_{rx}$)
  - Tx and Rx may sample the signal at slightly different timing offset

$$\text{SFO} : \delta = \frac{T_{rx} - T_{tx}}{T_{tx}}$$

Overview

- Carrier Frequency Offset (CFO)
  - $f^c_{tx} \neq f^c_{rx}$ (e.g., TX: 2.45001GHz, RX: 2.44998GHz)
  - CFO: $\Delta_f = f_{tx} - f_{rx}$
  - Time-domain signals:
    y'(t) = y(t) * exp(2jπΔ_ft)

    real theoretical

Error accumulates over time

- Sample Frequency Offset (SFO)
  - Sampling rates in Tx and Rx are slightly different (e.g., TX: 20.0001MHz, RX: 19.99997MHz)
  - $$\mathrm{SFO} : \delta = \frac{T_{rx} - T_{tx}}{T_{tx}}$$
  - Freq.-domain signals: Y'[k] = Y[k] * exp(2jπδkφ)

    constant

Phase rotates **2jπδkφ** in the k-th subcarrier

What, How, Why

How to Calibrate

- Carrier Frequency Offset (CFO)
  - Calibrate in time-domain
  - $y'(t)$ **$* \exp(-2j\pi\Delta_f t)$**
    $= y(t) * \exp(2j\pi\Delta_f t)$ **$* \exp(-2j\pi\Delta_f t)$**
    $= y(t)$
  - How: Use the preamble

- Sample Frequency Offset (SFO)
  - Calibrate in frequency-domain
  - $Y'[k]$ **$* \exp(-2j\pi\delta k\varphi)$**
    $= Y[k] * \exp(2j\pi\delta k\varphi)$ **$* \exp(-2j\pi\delta k\varphi)$**
    $= Y[k]$
  - How: Use the pilot subcarriers

CFO Correction in 802.11

- Reuse the preamble to calibrate CFO
- The first half part of the preamble is identical to the second half part
  - The two transmitted signals are identical: $s_n = s_{n+N}$
  - But, the received signals contain different errors

$$y_n = (s_n \otimes h)e^{j2\pi\Delta_f nT_s}$$ → Additional phase rotation $\Delta_f nT_s$

$$y_{n+N} = (s_n \otimes h)e^{j2\pi\Delta_f (n+N)T_s}$$ → Additional phase rotation $\Delta_f(n+N)T_s$

Find $\mathbf{\Delta_f}$ by taking $\mathbf{y_{n+N}}$ / $\mathbf{y_n}$

CFO Correction in 802.11

$$y_n y^*_{n+N} = (s_n \otimes h) e^{j2\pi\Delta_f nT_s} (s_n \otimes h) e^{-j2\pi\Delta_f (n+N)T_s}$$
$$= e^{-j2\pi\Delta_f NT_s} |(s_n \otimes h)|^2$$

- To learn CFO Δ_f, find the angle of $(y_n y^*_{n+N})$

$$\angle\left(\sum_n y_n y^*_{N+n}\right) = -2\pi\Delta_f NT_s$$

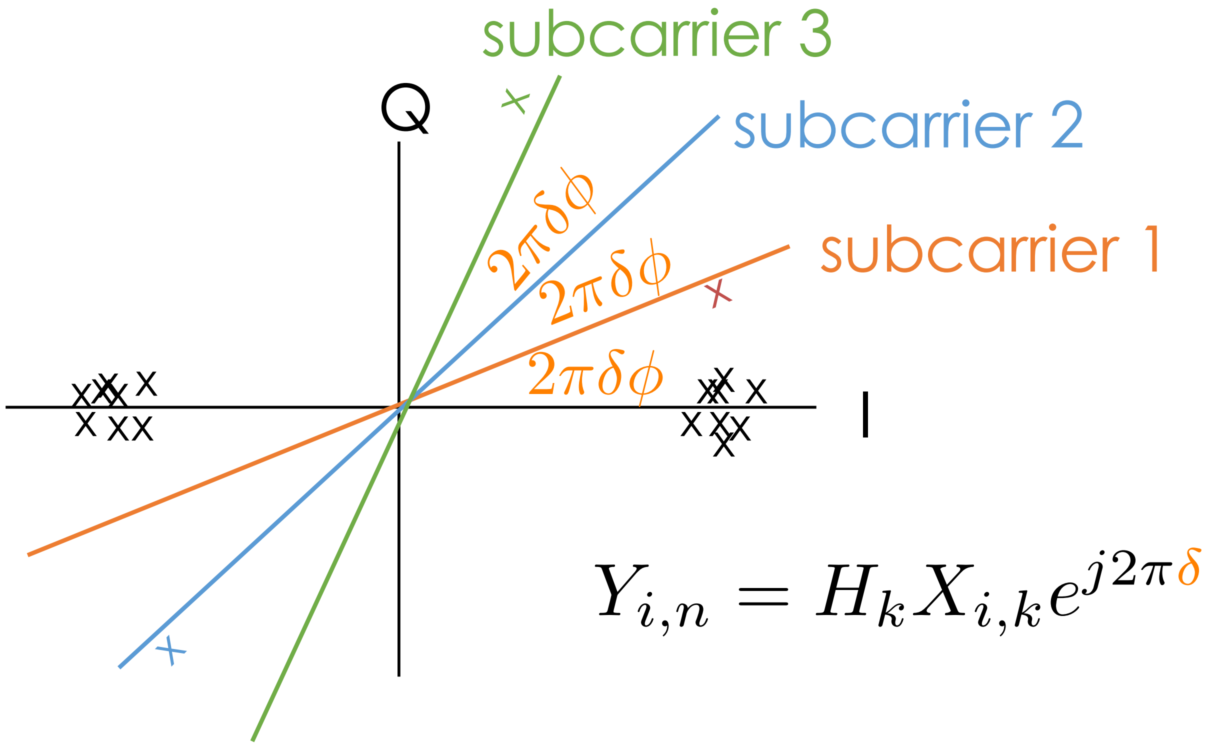
$$\Rightarrow \quad \tilde{\Delta}_f T_s = \frac{-1}{2\pi N}\angle\left(\sum_n y_n y^*_{N+n}\right)$$

- Calibrate the signals to remove phase rotation

$$y_n e^{-j2\pi\tilde{\Delta}_f nT_s} = \underbrace{\boxed{(s_n \otimes h)\cancel{e^{j2\pi\Delta_f nT_s}}}}_{\text{Received signals}} \underbrace{\boxed{\cancel{e^{-j2\pi\tilde{\Delta}_f nT_s}}}}_{\text{calibration}} \approx (s_n \otimes h)$$

Sample Rotation due to SFO

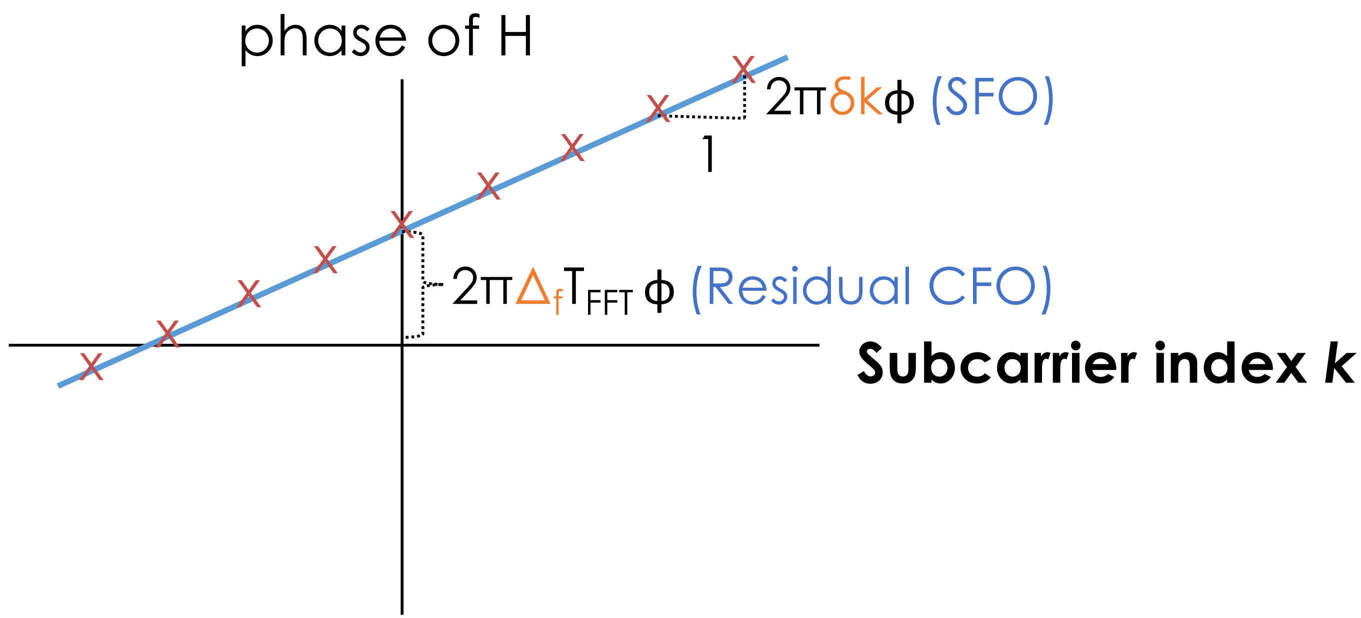
Incremental phase errors in different subcarriers
→ Signals keep rotating in the I-Q plane



$$Y_{i,n} = H_k X_{i,k} e^{j2\pi\delta k\phi}$$

Ideal BPSK signals (No rotation)

Phase Errors due to SFO and CFO



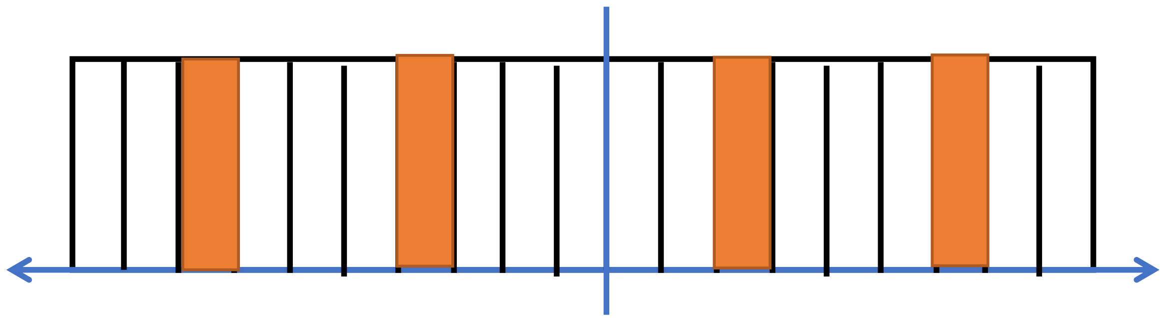
- Subcarrier *i* of the received frequency domain signals in symbol *n*

$$Y_{i,k} = H_k X_{i,k} e^{j2\pi(\Delta_f T_{FFT} + \delta k)\phi}$$

- SFO: slope; residual CFO: intersection of y-axis

Pilot Subcarriers

- 4 subcarriers are reserved as pilot subcarriers
  - Subcarrier:
- Known by both Tx/Rx (Similar to preamble)
- Used for SFO calibration

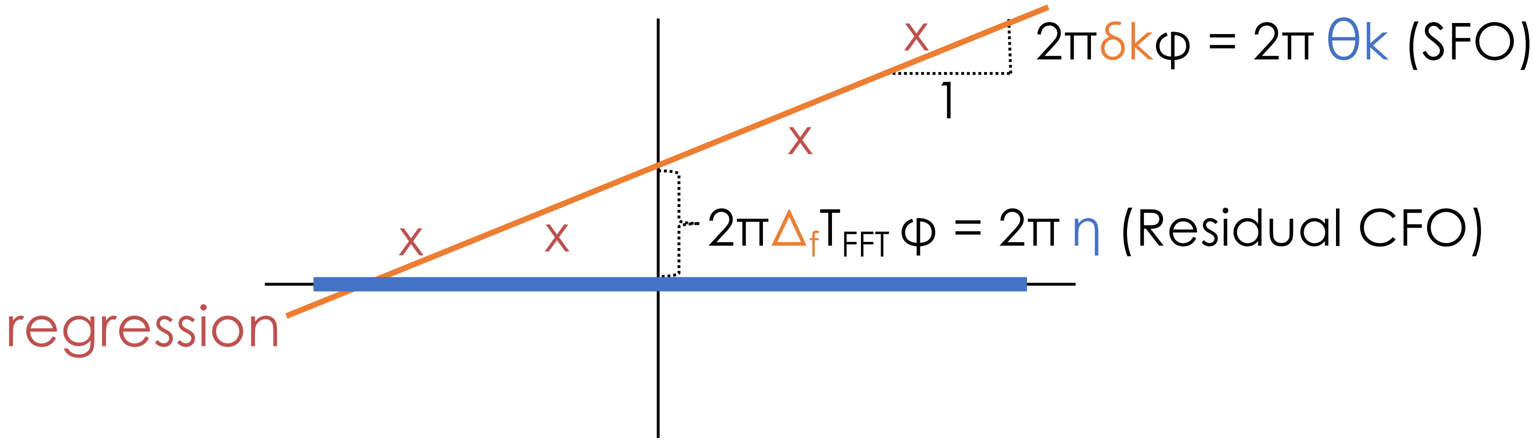


$Y'[k] = H[k] \underline{\mathbf{\ast \exp(-2j\pi\delta k\varphi)}} \ast P[k]$

$Y'[k] / P[k] = H[k] \underline{\mathbf{\ast \exp(-2j\pi\delta k\varphi)}}$

$\underline{\mathbf{2j\pi\delta\varphi}}$ = angle(Y'[k] / P[k]) / k

Data-aided Phase Tracking

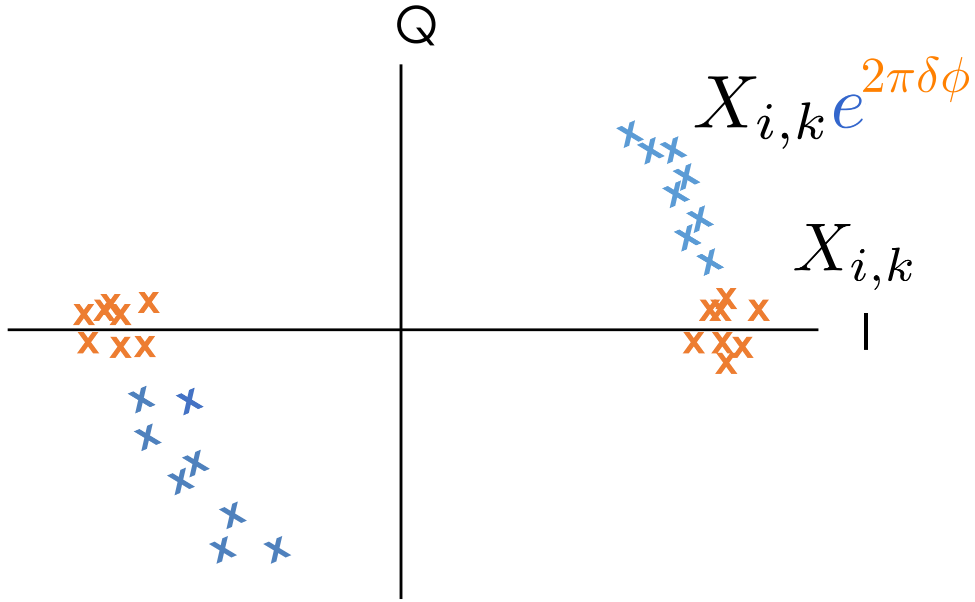


- WiFi reserves 4 known pilot bits (subcarriers) to compute $H_k e^{j2\pi(\eta+\theta k)} = Y_k/X_k$
- Estimate SFO θ_k and CFO η by finding the linear regression of the phase changes experienced by the pilot bits
- Update the channel by $H'_k = H_k e^{2j\pi(\eta+\theta k)}$ for every symbol k, and then decode the remaining non-pilot subcarriers

$$Y_{i,k} = H_k X_{i,k} e^{j2\pi(\eta+\theta k)} = H'_k X_{i,k}$$

$$\Rightarrow \hat{X}_{i,k} = Y_{i,k}/H'_k$$

After Phase Tracking



Decoded signals in the I-Q plane after phase tracking

What, How, Why

CFO Estimation

- Up/Down conversion at Tx/Rx
  - Up-convert baseband signal *s*(*t*) to passband signal

$$r(t) = s(t)e^{j2\pi f_{tx} t} \otimes h(t,\tau)$$

  - Down-convert passband signal *r*(*t*) back to

$$\begin{aligned} y_n &= r(nT_s)e^{-j2\pi f_{rx} t} \\ &= s(nT_s)e^{j2\pi f_{tx} t}e^{-j2\pi f_{rx} t} \otimes h(nT_s,\tau) \\ &= s(nT_s)\boxed{e^{j2\pi \Delta_f nT_s}} \otimes h(nT_s,\tau) \end{aligned}$$

Error caused by CFO, accumulated with time nT_s

Phase errors due to SFO

- Assuming no residual CFO, the k-th subcarrier in the received symbol i becomes

$$Y_{i,n} = H_k X_{i,k} e^{j2\pi \delta k \phi}$$

See proof in the next slide

- All subcarriers experience the same sampling offset, but applied on different frequencies *k*
  - ϕ is a constant
  - Each subcarrier is rotated by a constant phase shift $2\pi\delta\phi$
  - Lead to Inter Carrier interference (ICI), which causes loss of the orthogonality of the subcarriers
  - **Delay in time-domain = phase rotation in frequency-domain**

Proof of phase errors due to SFO

Time-domain

Up-convert: $r(t) = s(t)e^{j2\pi f_{tx} t} \otimes h(t,\tau) + n(t)$

Down-convert: $y_{i,n} = r(t)e^{-j2\pi f_{rx} t}|_{t=(iN_S+N_{CP}+n)T_{rx}}$

Frequency-domain

Residual CFO

SFO

$$\xrightarrow{\text{FFT}} \qquad Y_{i,k} = H_k X_{i,k} e^{j2\pi(\Delta_f T_{FFT} + \delta k)\phi}$$

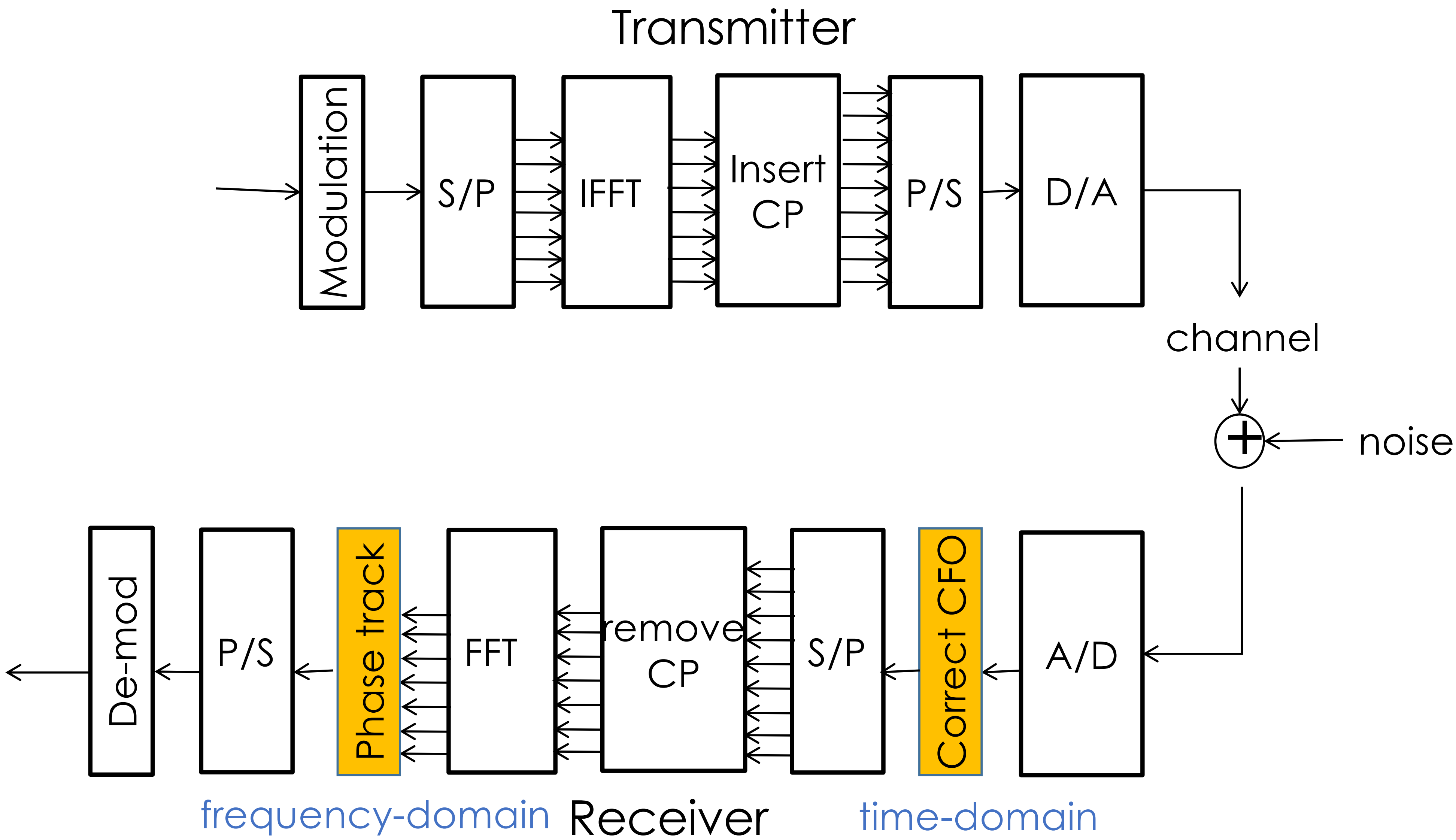
N_{CP} : Number of samples in CP

N_{FFT} : FFT window size

$N_S = N_{FFT} + N_{CP}$: Symbol size

$\phi = 0.5 + \dfrac{iN_S + N_{CP}}{N_{FFT}}$: a constant indicating the initial phase error of symbol i

OFDM Diagram



Agenda

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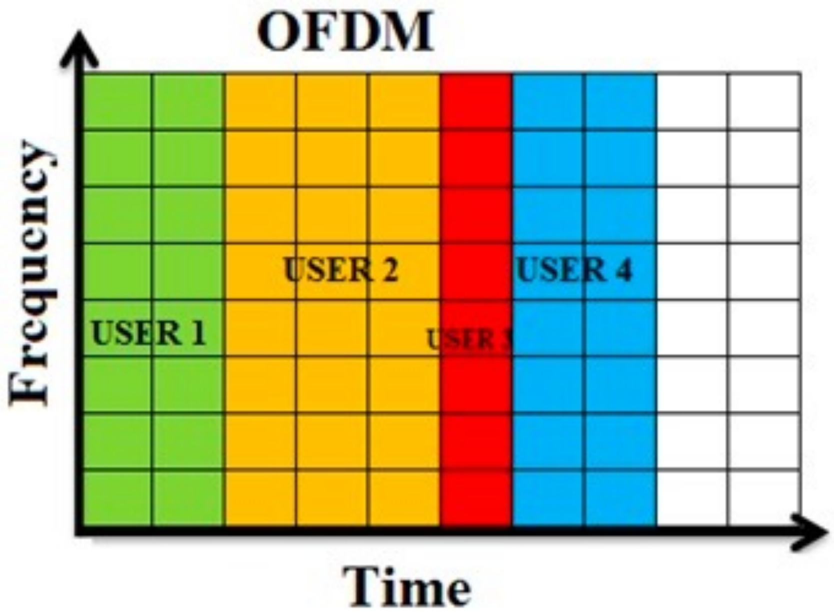
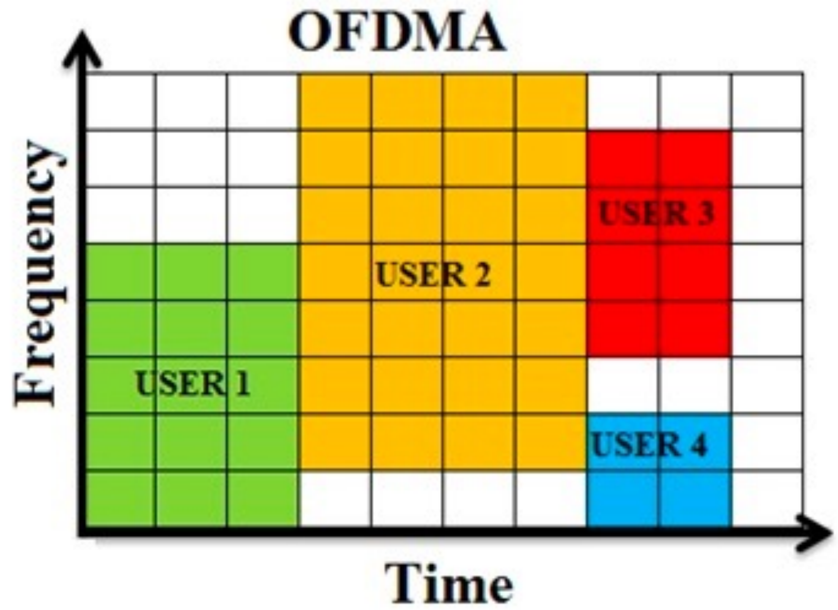
OFDM and OFDMA





WiFi

LTE