Wireless Communication Systems
@CS.NCTU

Lecture 2: Modulation and Demodulation

Reference: Chap. 5 in Goldsmith's book

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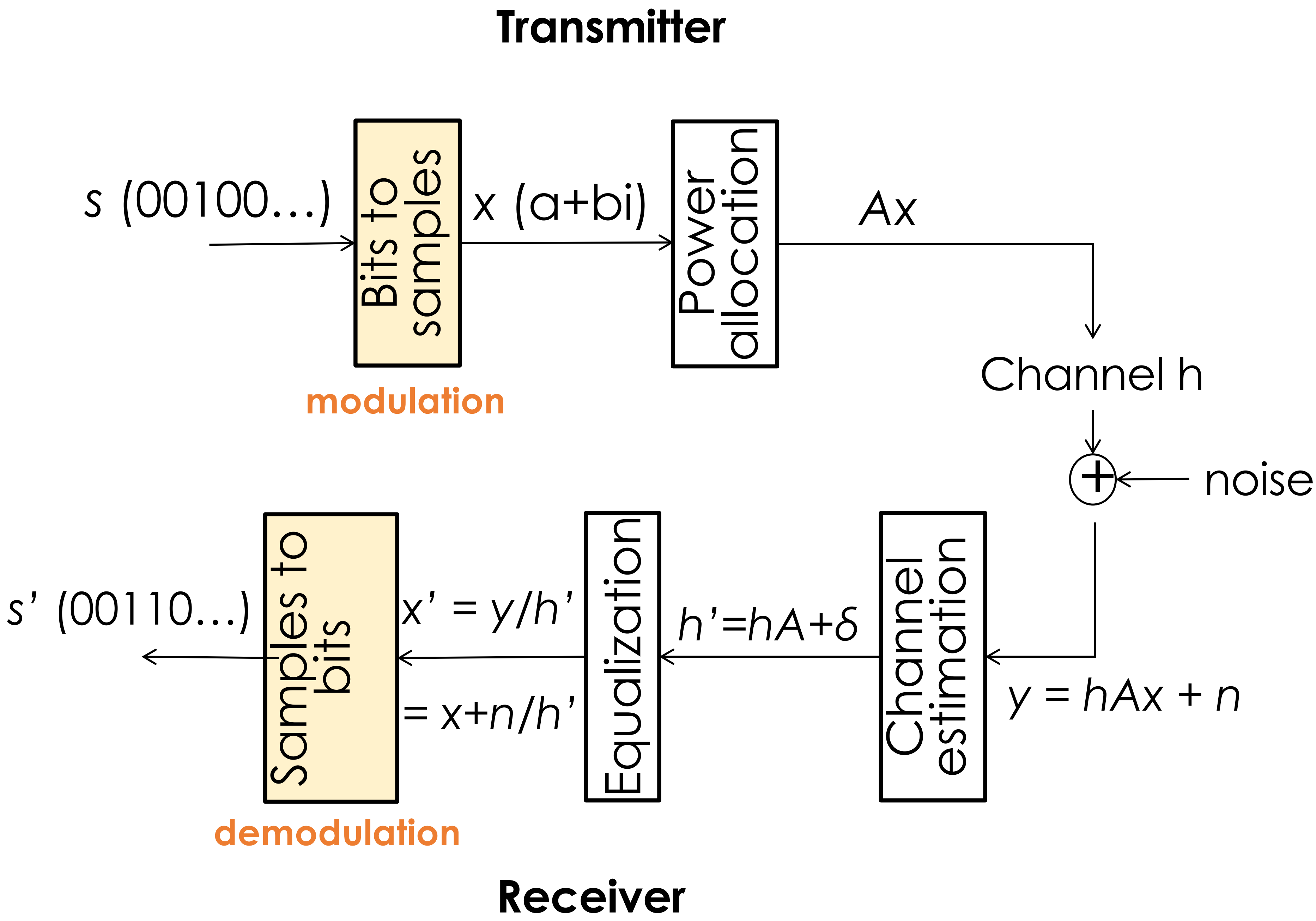
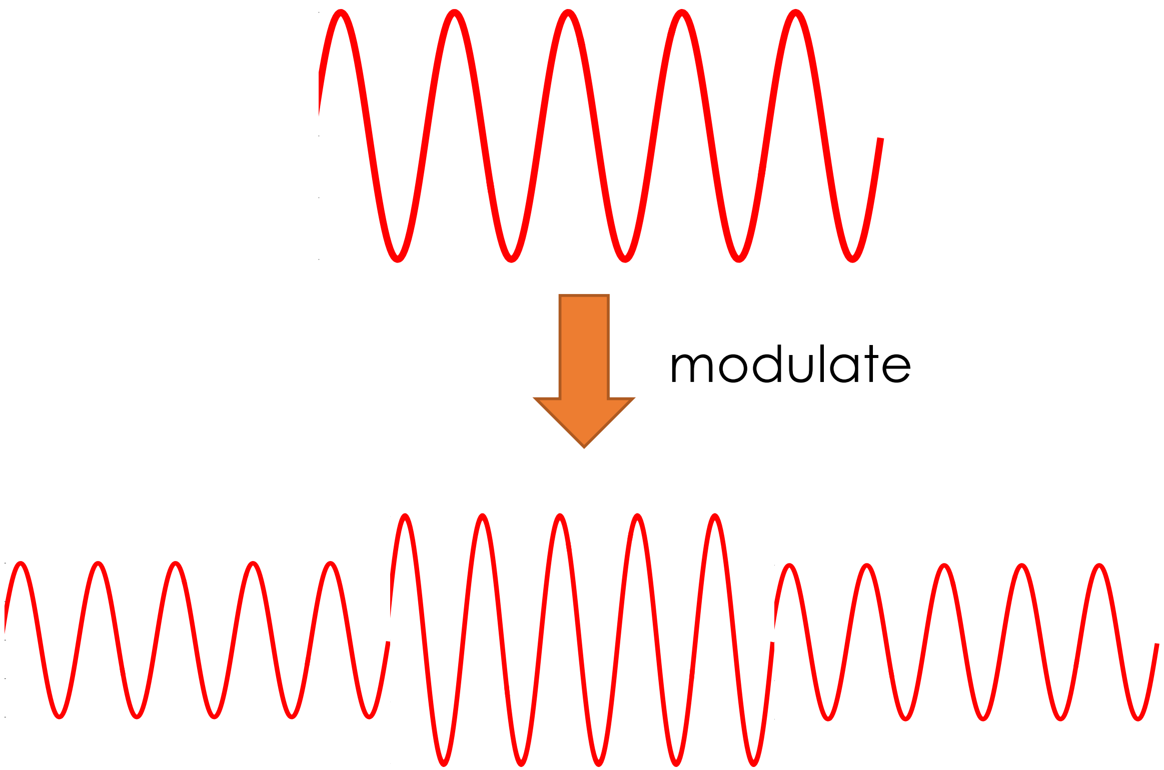

Transmitter
s (00100…)
Bits to samples
x (a+bi)
Power allocation
Ax
Channel h
noise
modulation
s' (00110…)
Samples to bits
x' = y/h'
= x+n/h'
Equalization
h'=hA+δ
Channel estimation
y = hAx + n
demodulation
Receiver

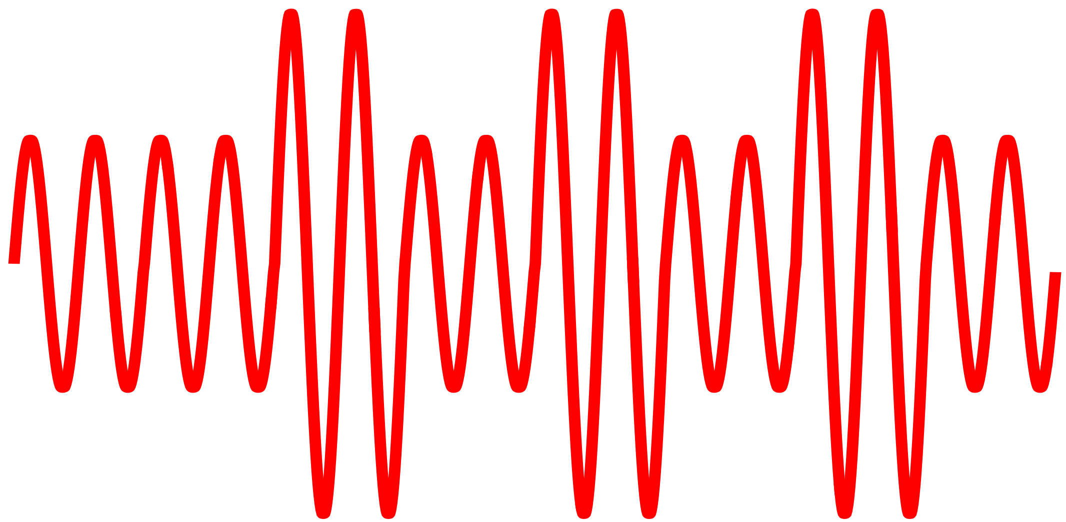
Modulation

From Wikipedia:

The process of varying one or more properties of a periodic waveform with a modulating signal that typically contains information to be transmitted.



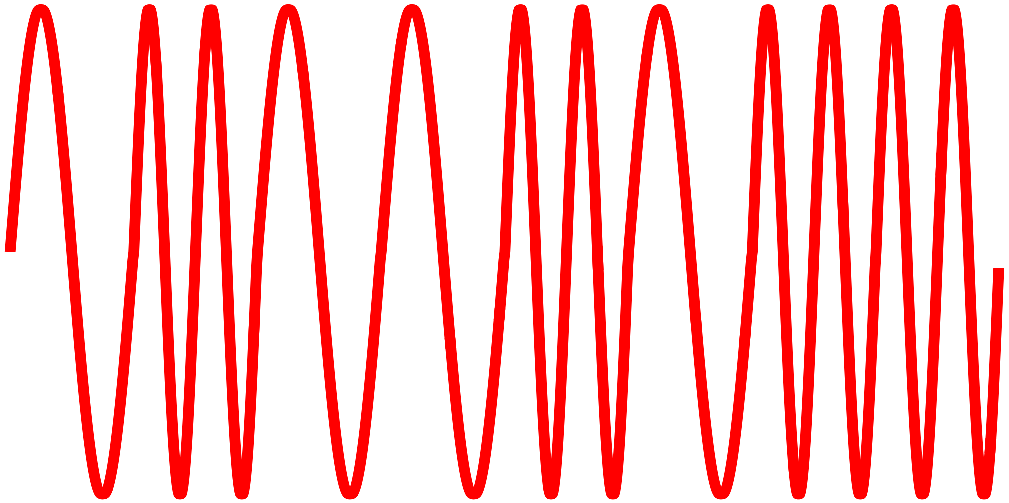
Example 1



= bit-stream?

(a) 10110011 (b) 00101010 (c) 10010101

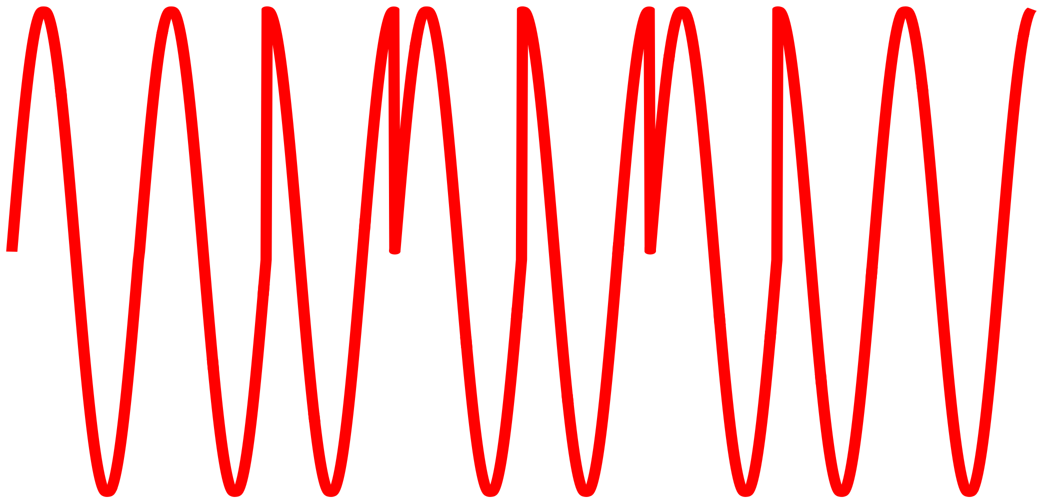
Example 2



= bit-stream?

(a) 01001011 (b) 00101011 (c) 11110100

Example 3



= bit-stream?

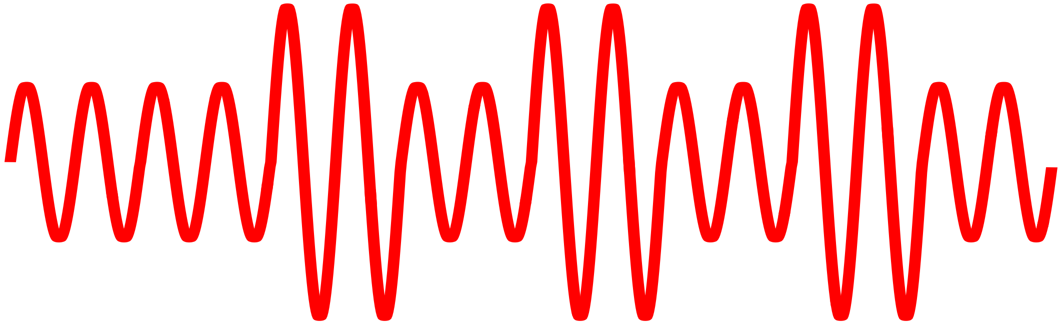
(a) 11010100

(b) 00101011

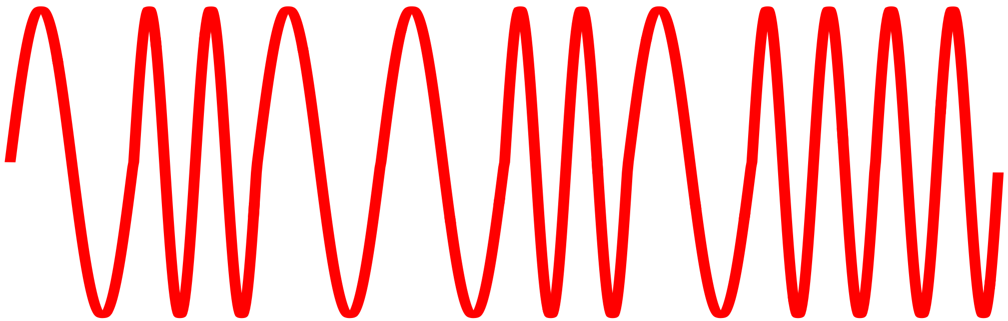
(c) 01010011

(d) 11010100 or 00101011

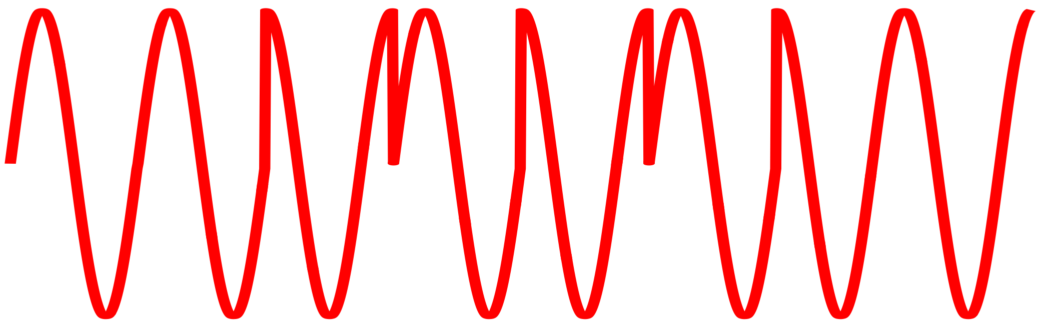
Types of Modulation



Amplitude
ASK



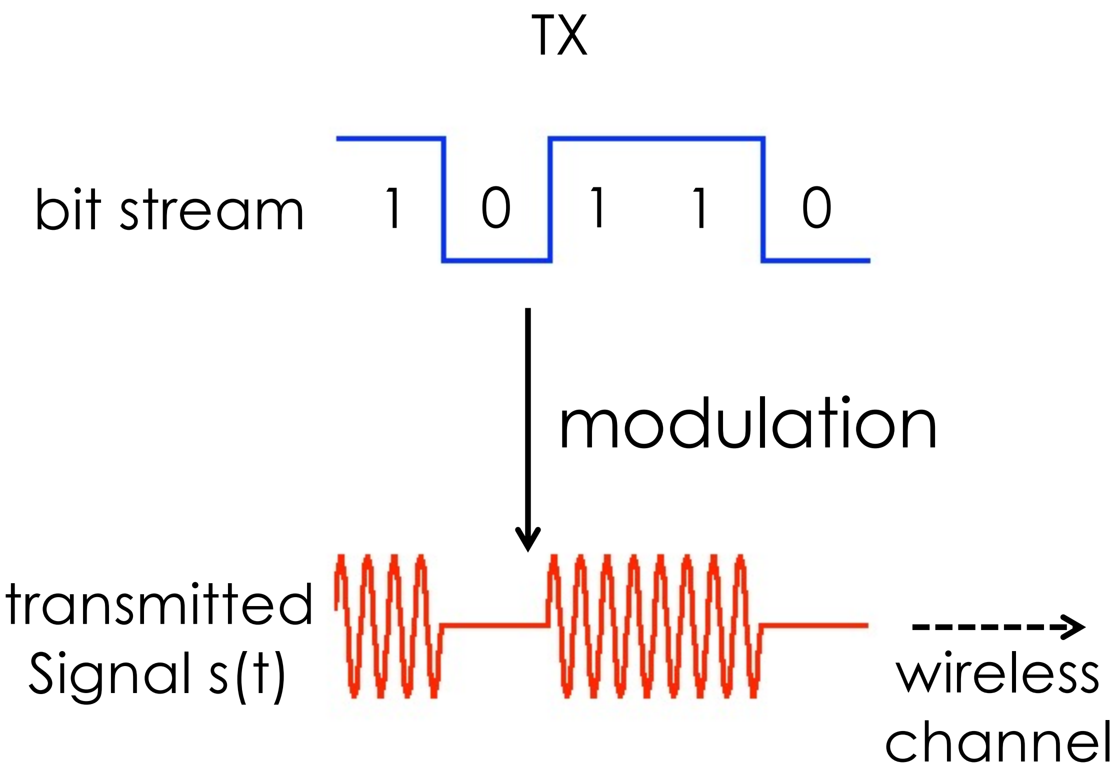
Frequency
FSK



Phase
PSK

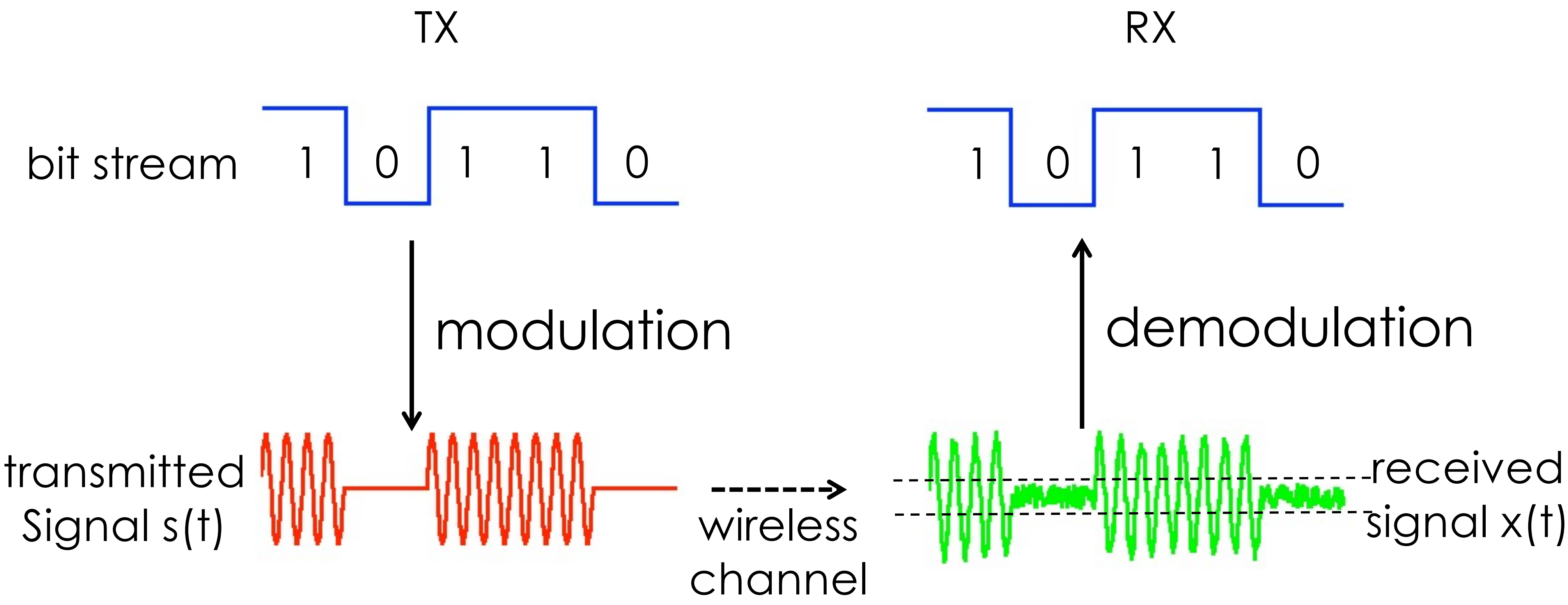
Modulation

- Map bits to signals



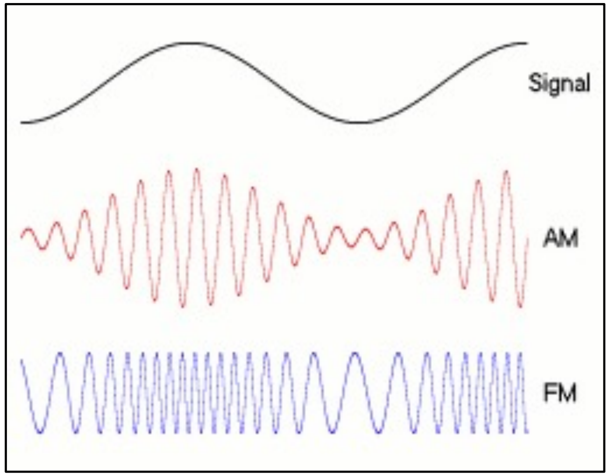
Demodulation

- Map signals to bits

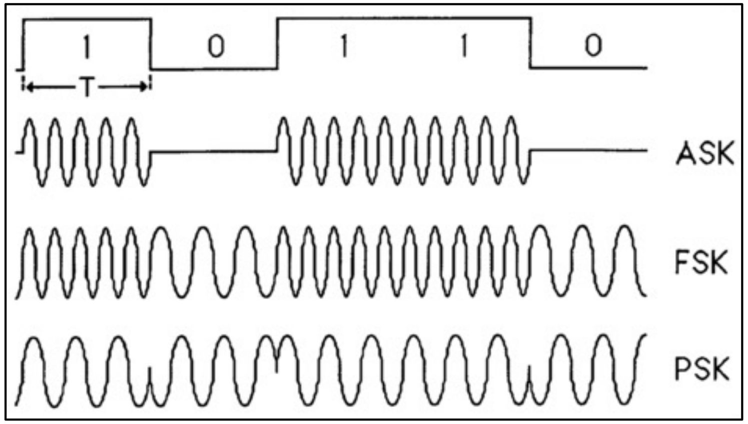


Analog and Digital Modulation

- Analog modulation
  - Modulation is applied continuously
  - Amplitude modulation (AM)
  - Frequency modulation (FM)



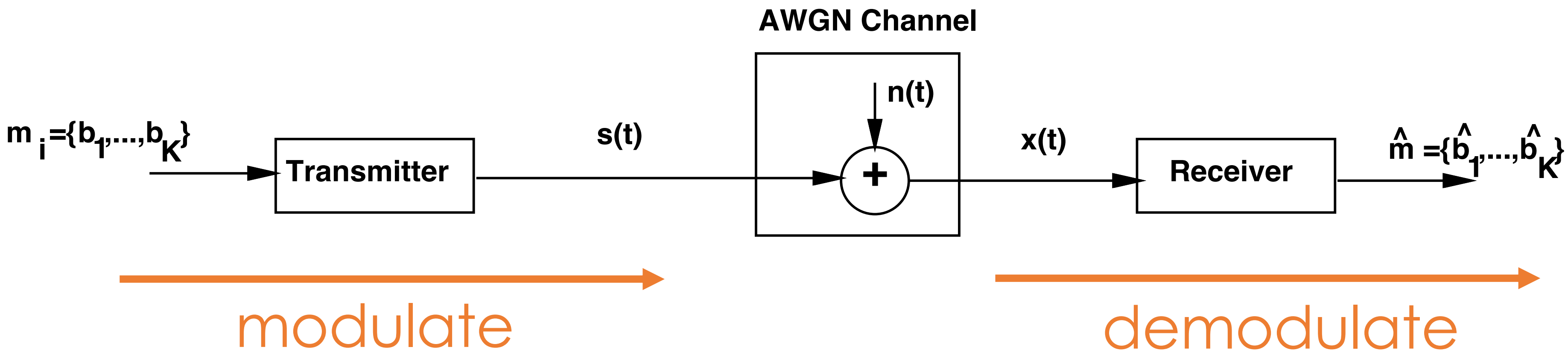
- Digital modulation
  - An analog carrier signal is modulated by a discrete signal
  - Amplitude-Shift Keying (ASK)
  - Frequency-Shift Keying (FSK)
  - Phase-Shift Keying (PSK)
  - Quadrature Amplitude Modulation (QAM)



Advantages of Digital Modulation

- Higher data rate (given a fixed bandwidth)
- More robust to channel impairment
  - Advanced coding/decoding can be applied to make signals less susceptible to noise and fading
  - Spread spectrum techniques can be applied to deal with multipath and resist interference
- Suitable to multiple access
  - Become possible to detect multiple users simultaneously
- Better security and privacy
  - Easier to encrypt

Modulation and Demodulation



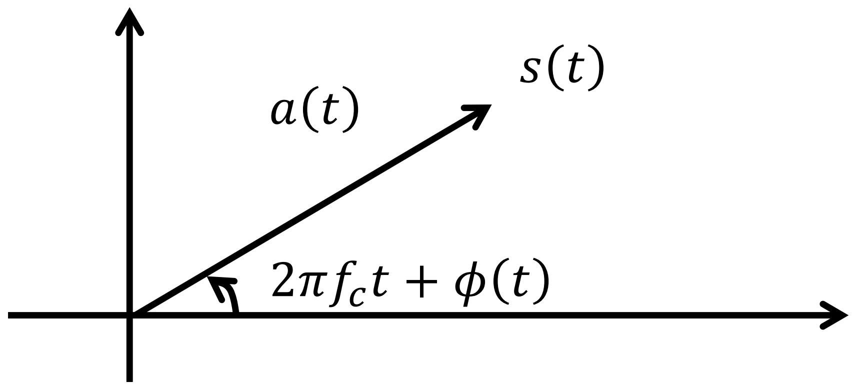
- Modulation
  - Encode a bit stream of finite length to one of several possible signals
- Delivery over the air
  - Signals experience fading and are combined with AWGN (additive white Gaussian noise)
- Demodulation
  - Decode the received signal by mapping it to the closest one in the set of possible transmitted signals

Band-pass Signal Representation

- General form

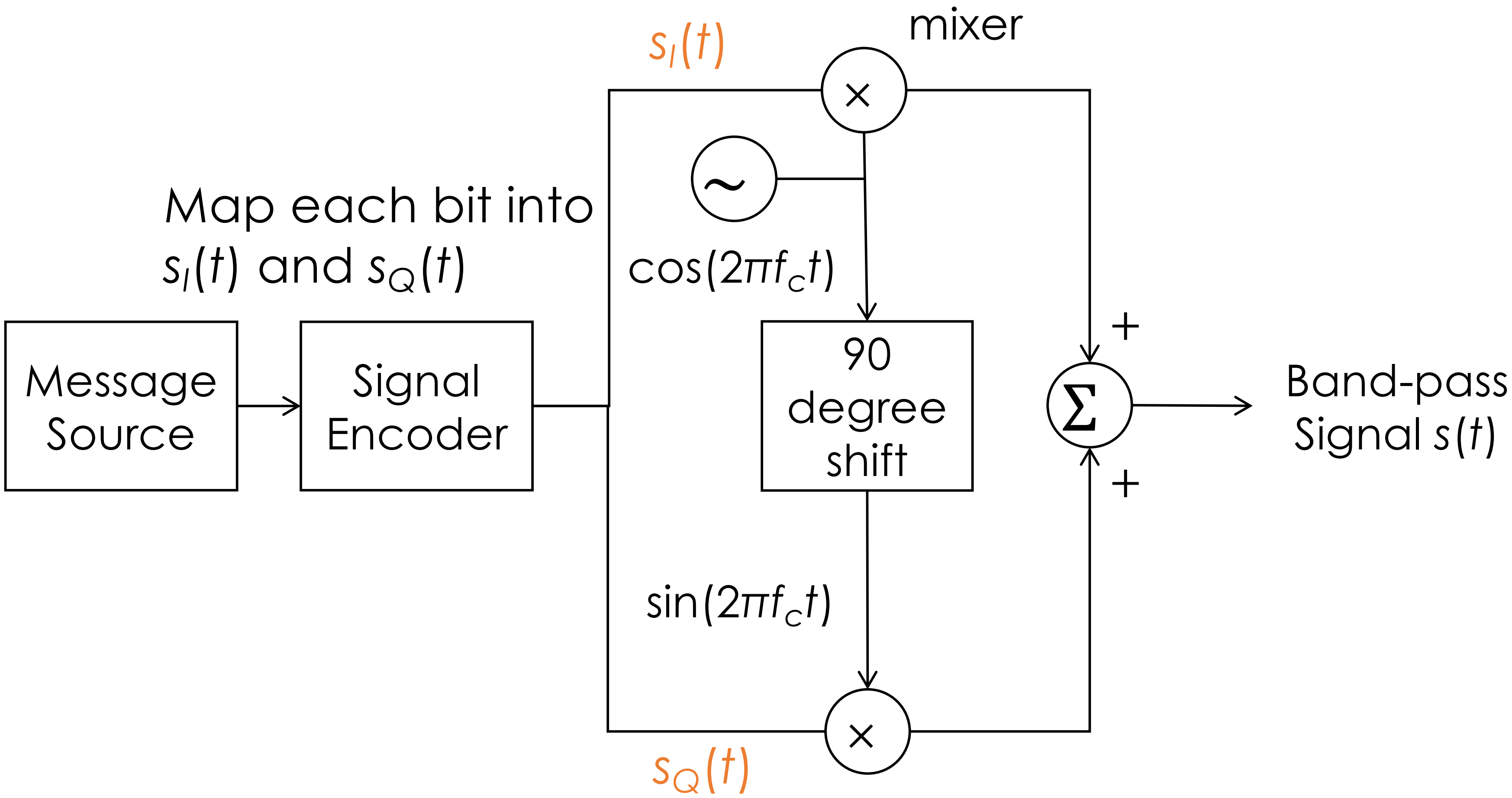
$$s(t) = \underbrace{a(t)}_{\text{amplitude}} cos(2\pi \underbrace{f_c}_{\text{frequency}} t + \underbrace{\phi(t)}_{\text{phase}})$$

- Amplitude is always non-negative
  - Or we can switch the phase by 180 degrees
- Called the canonical representation of a band-pass signal

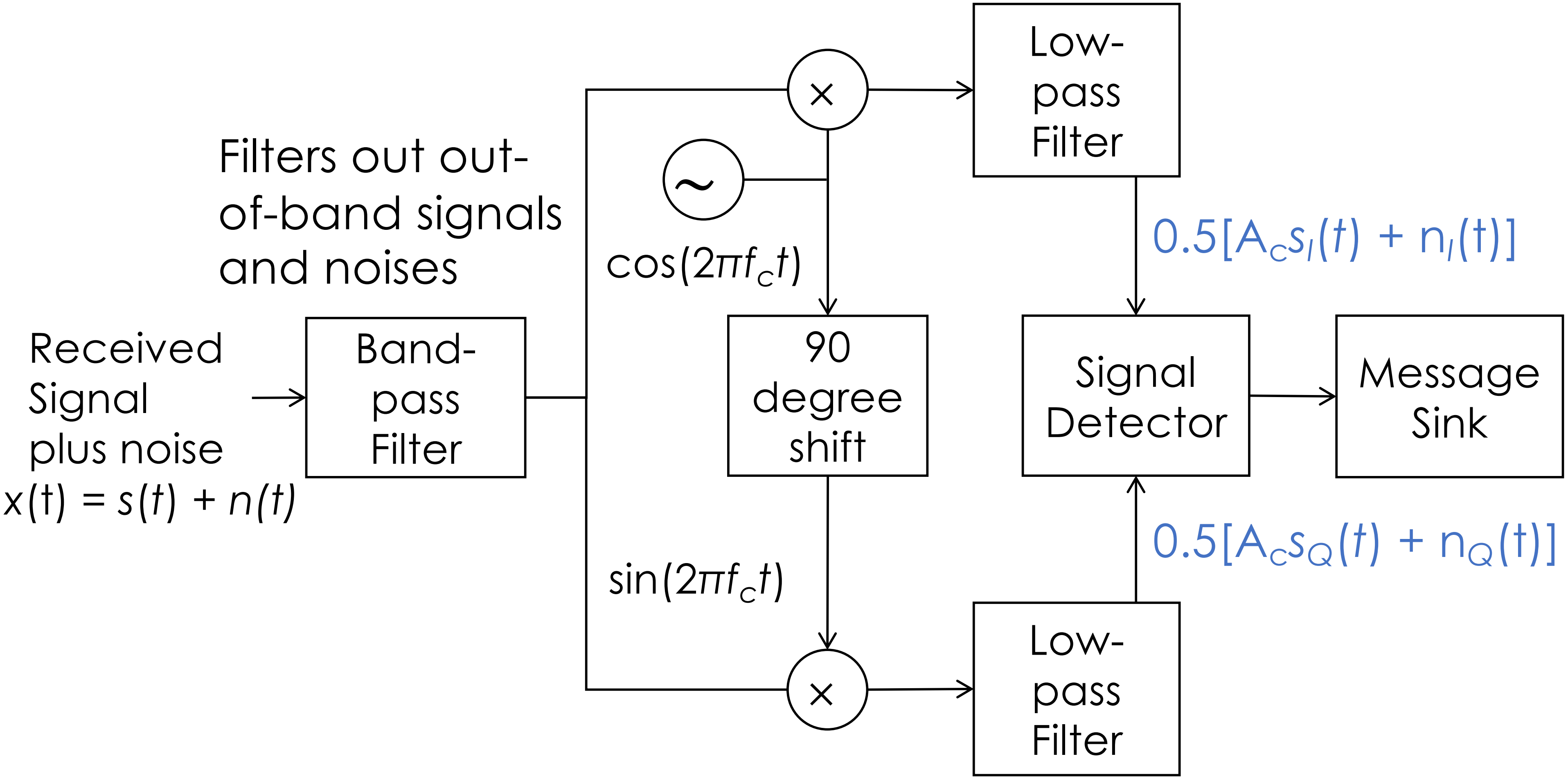


Band-Pass Signal Transmitter

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$



Band-Pass Signal Receiver



In-phase and Quadrature Components

$$\begin{aligned} s(t) &= a(t)\cos(2\pi f_c t + \phi(t)) \\ &= a(t)[\cos(2\pi f_c t)\cos(\phi(t)) - \sin(2\pi f_c t)\sin(\phi(t))] \\ &= \boxed{s_I(t)}\cos(2\pi f_c t) - \boxed{s_Q(t)}\sin(2\pi f_c t) \end{aligned}$$

- $s_I(t) = a(t)\cos(\phi(t))$: In-phase component of $s(t)$
- $s_Q(t) = a(t)\sin(\phi(t))$: Quadrature component of $s(t)$

Amplitude: $a(t) = \sqrt{s_I^2(t) + s_Q^2(t)}$

Phase: $\phi(t) = \tan^{-1}(\frac{s_Q(t)}{s_I(t)})$

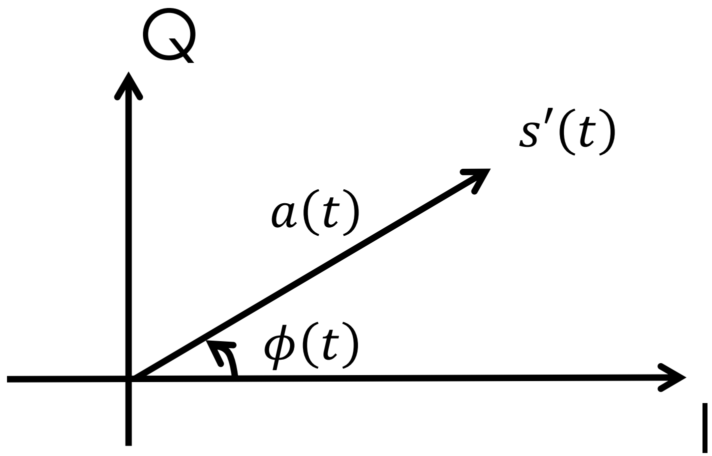
Band-Pass Signal Representation

$$s(t) = s_I(t)\cos(2\pi f(t)t) - s_Q(t)\sin(2\pi f(t)t)$$

- We can also represent s(t) as

$$s(t) = \Re[s'(t)e^{2j\pi f_c t}]$$

$$\boxed{exp(i\theta) = cos(\theta)+jsin(\theta)}$$

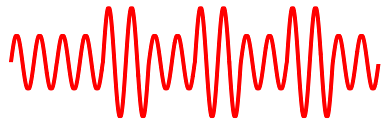


- $s'(t) = s_I(t) + js_Q(t)$
- s'(*t*) is called the complex envelope of the band-pass signal
- This is to remove the annoying $e^{2j\pi f_c t}$ in the analysis

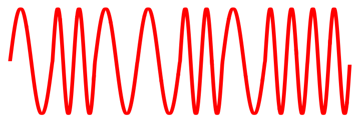
Types of Modulation

$$s(t) = A\cos(2\pi f_c t + \phi)$$

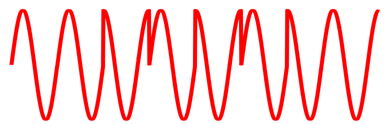
- *Amplitude*
  - *M-ASK: Amplitude Shift Keying*



- *Frequency*
  - *M-FSK: Frequency Shift Keying*



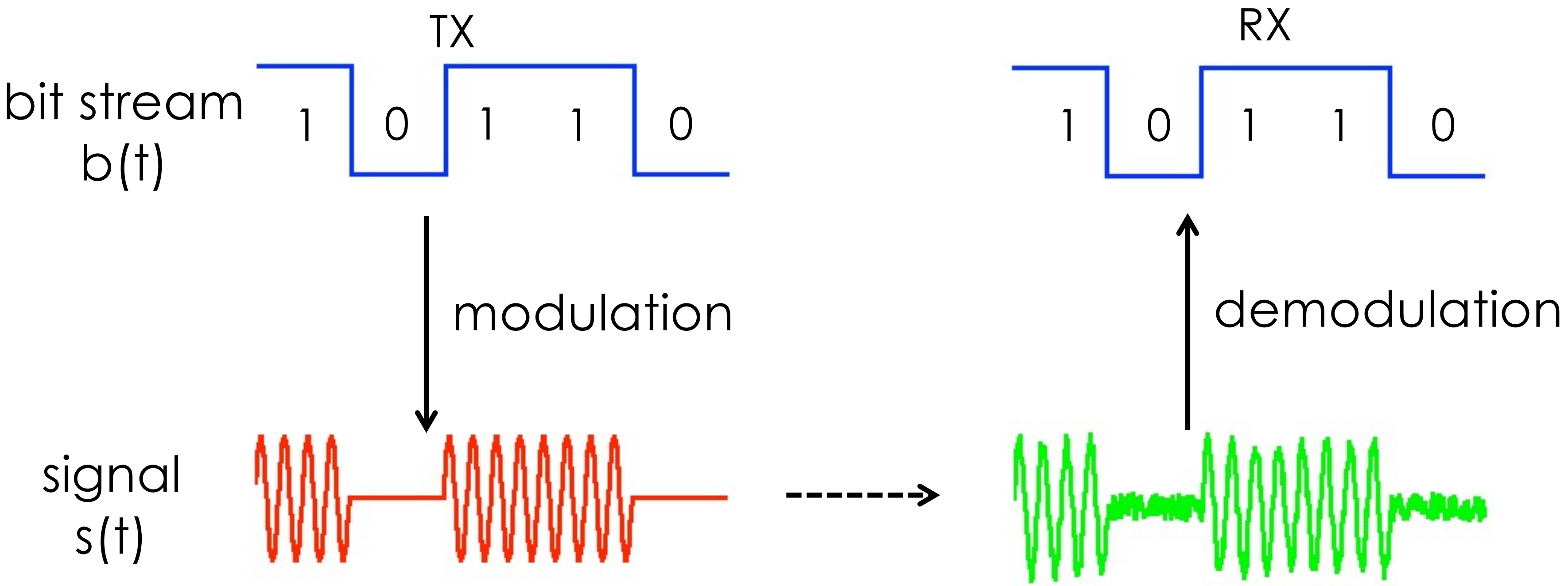
- *Phase*
  - *M-PSK: Phase Shift Keying*



- *Amplitude* + *Phase*
  - *M-QAM: Quadrature Amplitude Modulation*

Amplitude Shift Keying (ASK)

- A bit stream is encoded in the amplitude of the transmitted signal
- Simplest form: On-Off Keying (OOK)
  - ‘1’→A=1, ‘0’→A=0



M-ASK

- M-ary amplitude-shift keying (M-ASK)

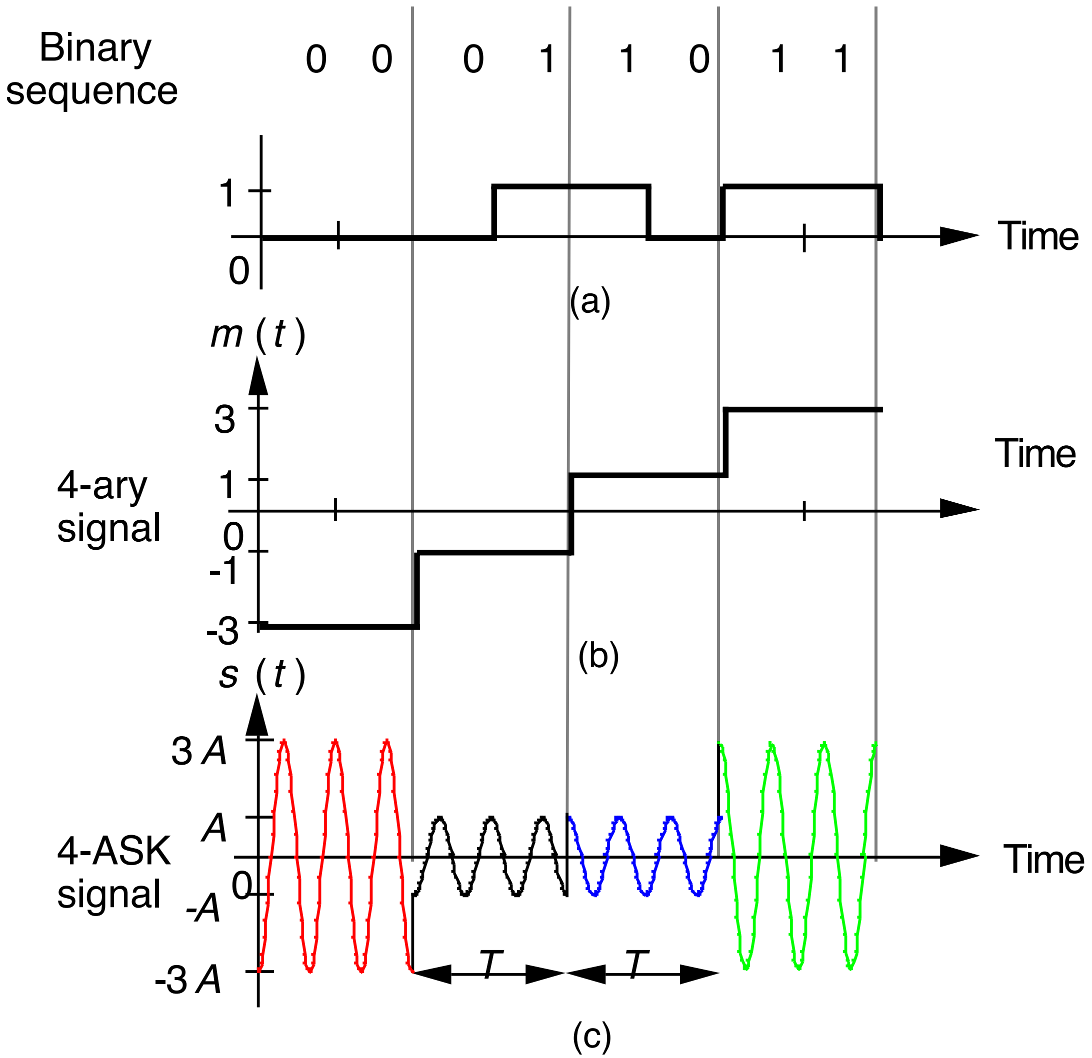
$$s(t) = \begin{cases} A_i \cos(2\pi f_c t) & \text{, if } 0 \le t \le T \\ 0 & \text{, otherwise,} \end{cases}$$

where $i = 1, 2, \cdots, M$

A_i is the amplitude corresponding to bit pattern i

Example: 4-ASK

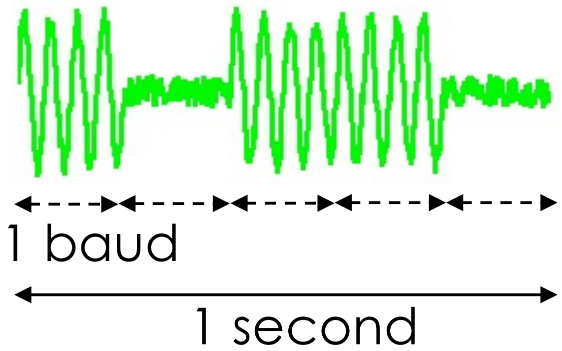
- Map '00', '01', '10', '11' to four different amplitudes



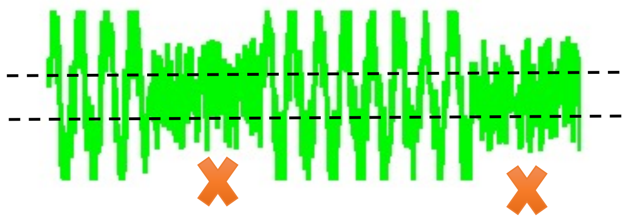
Pros and Cons of ASK

> *Bandwidth is the difference between the upper and lower frequencies in a continuous set of frequencies.*

- Pros
  - Easy to implement
  - Energy efficient
  - Low bandwidth requirement
- Cons
  - Low data rate
    - bit-rate = baud rate



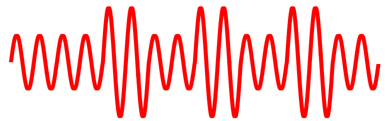
  - High error probability
    - Hard to pick a right threshold



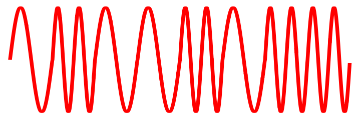
Types of Modulation

$$s(t) = A\cos(2\pi f_c t + \phi)$$

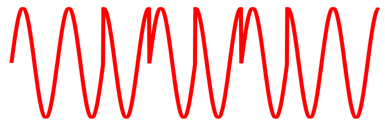
- Amplitude
  - M-ASK: Amplitude Shift Keying



- Frequency
  - M-FSK: Frequency Shift Keying



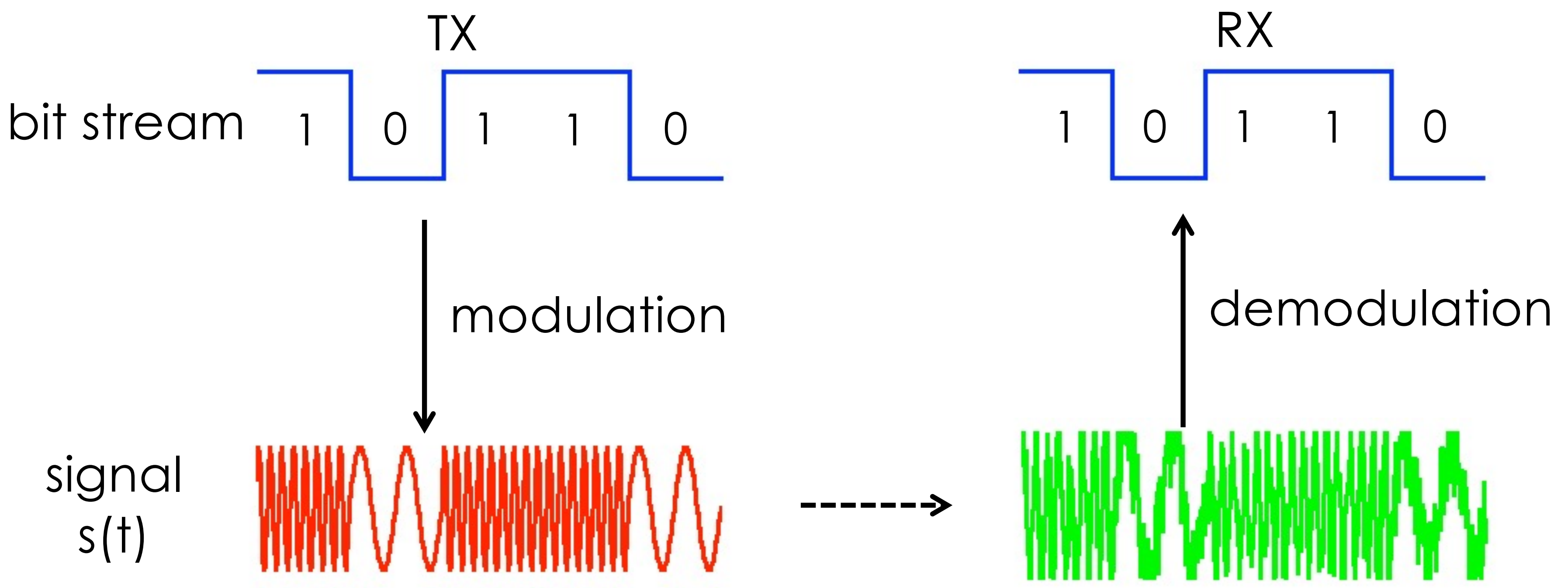
- Phase
  - M-PSK: Phase Shift Keying



- Amplitude + Phase
  - M-QAM: Quadrature Amplitude Modulation

Frequency Shift Keying (FSK)

- A bit stream is encoded in the frequency of the transmitted signal
- Simplest form: Binary FSK (BFSK)
  - '1'→$f=f_1$, '0'→$f=f_2$



M-FSK

- M-ary frequency-shift keying (M-FSK)

$$s(t) = \begin{cases} A\cos(2\pi f_{c,i} t) & \text{, if } 0 \leq t \leq T \\ 0 & \text{, otherwise,} \end{cases}$$

where $i = 1, 2, \cdots, M$

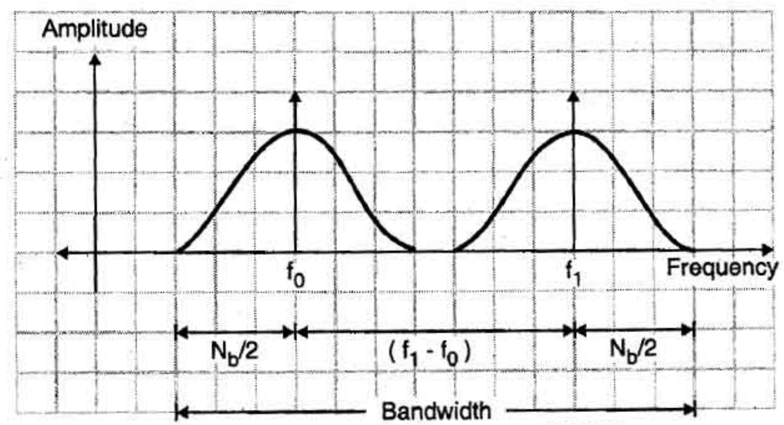
$f_{c,i}$ is the center frequency corresponding to bit pattern i

- Example: Quaternary Frequency Shift Keying (QFSK)
  - Map '00', '01', '10', '11' to four different frequencies

Pros and Cons of FSK

- Pros
  - Easy to implement
  - Better noise immunity than ASK

- Cons
  - Low data rate
    - Bit-rate = baud rate
  - Require higher bandwidth
    - $BW(min) = N_b + N_b$

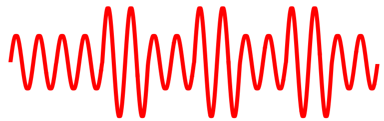


Spectrum of FSK

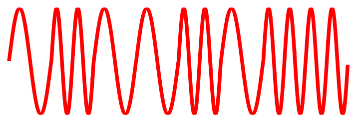
Types of Modulation

$$s(t) = A\cos(2\pi f_c t + \phi)$$

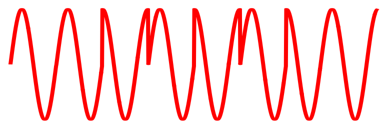
- *Amplitude*
  - M-ASK: Amplitude Shift Keying



- *Frequency*
  - M-FSK: Frequency Shift Keying



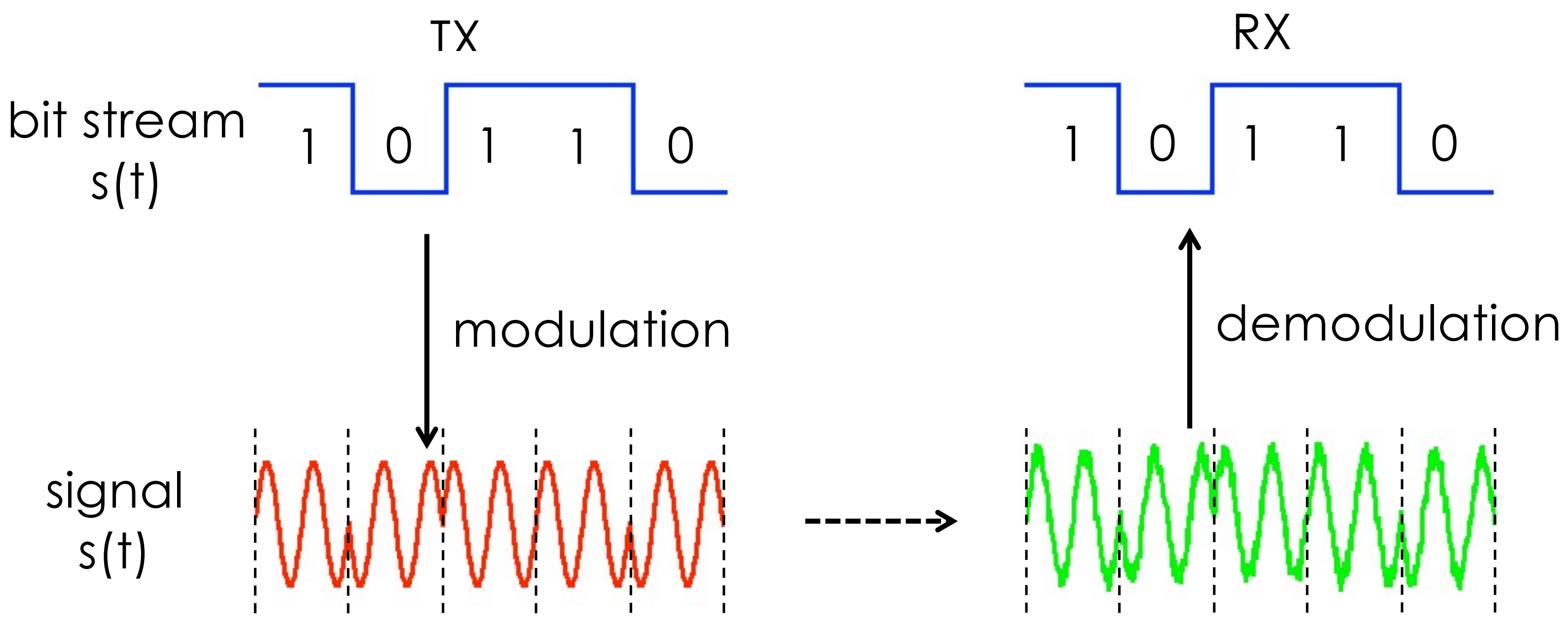
- *Phase*
  - M-PSK: Phase Shift Keying



- *Amplitude* + *Phase*
  - M-QAM: Quadrature Amplitude Modulation

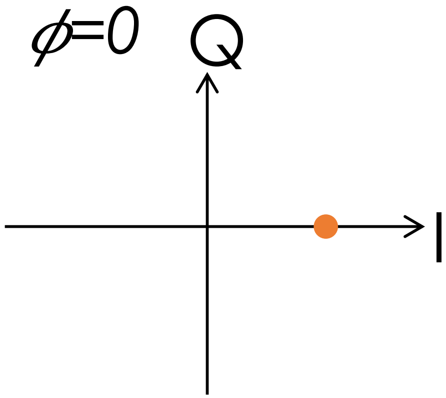
Phase Shift Keying (PSK)

- A bit stream is encoded in the phase of the transmitted signal
- Simplest form: Binary PSK (BPSK)
  - '1'→ϕ=0, '0'→ϕ=π



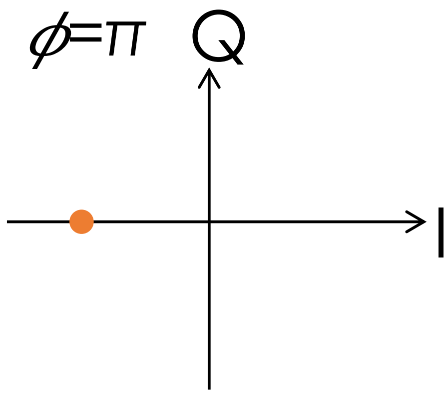
Constellation Points for BPSK

- '1' → $\phi=0$
- $\cos(2\pi f_c t+0)$
  $= \color{blue}{\cos(0)}\cos(2\pi f_c t) - \color{orange}{\sin(0)}\sin(2\pi f_c t)$
  $= \color{blue}{s_I}\cos(2\pi f_c t) - \color{orange}{s_Q}\sin(2\pi f_c t)$



$(s_I,s_Q) = (1, 0)$
'1' → $1+0i$

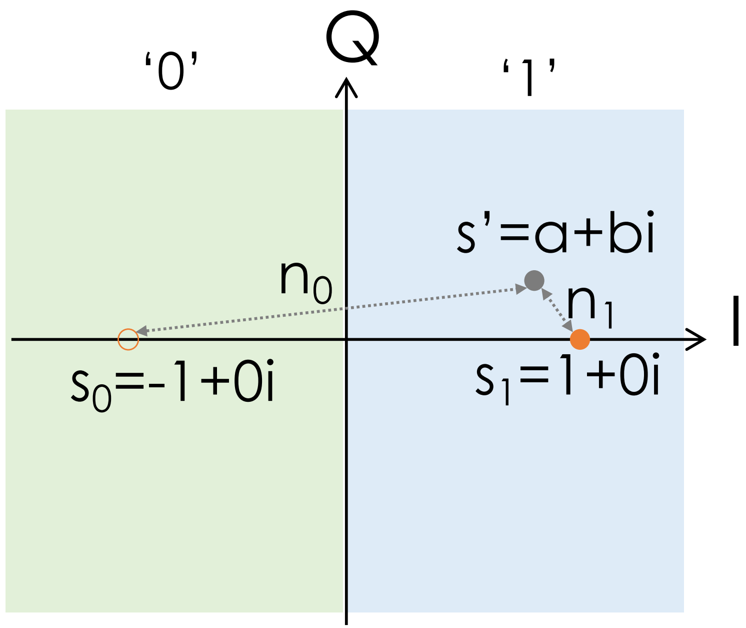
- '0' → $\phi=\pi$
- $\cos(2\pi f_c t+\pi)$
  $= \color{blue}{\cos(\pi)}\cos(2\pi f_c t) - \color{orange}{\sin(\pi)}\sin(2\pi f_c t)$
  $= \color{blue}{s_I}\cos(2\pi f_c t) - \color{orange}{s_Q}\sin(2\pi f_c t)$



$(s_I,s_Q) = (-1, 0)$
'0' → $-1+0i$

Demodulate BPSK

- Map to the closest constellation point
- Quantitative measure of the distance between the received signal *s'* and any possible signal *s*
  - Find |s'-s| in the I-Q plane



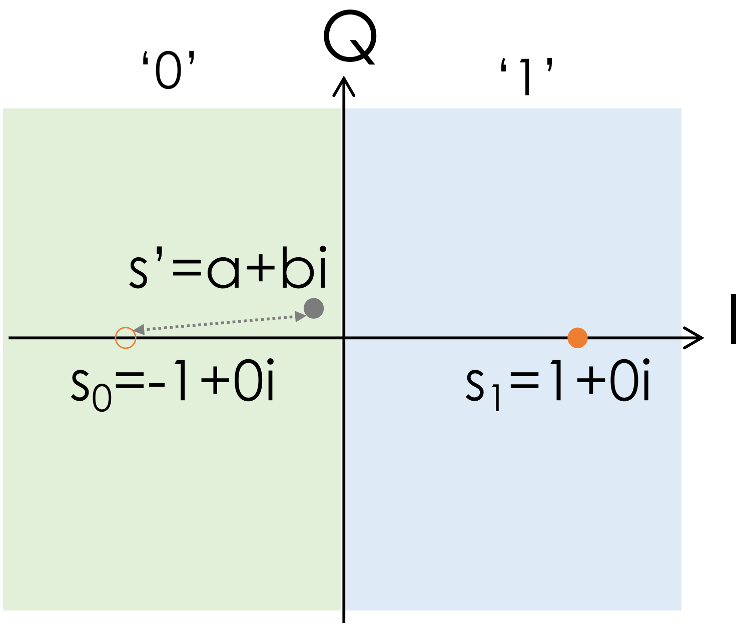
$n_1 = |s' - s_1| = |s' - (1+0i)|$

$n_0 = |s' - s_0| = | \ |s' - (-1+0i)|$

since $n_1 < n_0$, map *s'* to (1+0i)→'1'

Demodulate BPSK

- Decoding error
  - When the received signal is mapped to an incorrect symbol (constellation point) due to a large error
- Symbol error rate
  - P(mapping to a symbol s_j, $j \neq i$ | s_i is sent)



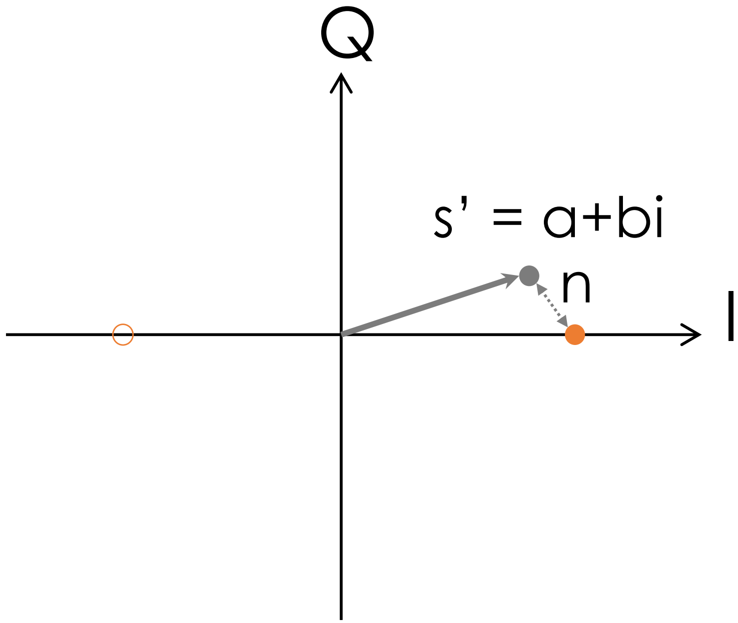
Given the transmitted symbol s_1

→ incorrectly map s' to s_0=(-1+0)→'0', when the error is too large

SNR of BPSK

- SNR: Signal-to-Noise Ratio

$$SNR = \frac{|s|^2}{n^2} = \frac{|s|^2}{|s' - s|^2}$$
$$= \frac{|1+0i|^2}{|(a+bi)-(1+0i)|^2}$$
$$SNR_{dB} = 10\log_{10}(SNR)$$



- Example:
  - Say Tx sends (1+0i) and Rx receives (1.1 – 0.01i)
  - SNR?

SER/BER of BPSK

- BER (Bit Error Rate) = SER (Symbol Error Rate)

$$SER = BER = P_b$$

Minimum distance of any two cancellation points

$$= Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q(\sqrt{2SNR})$$

From Wikipedia:

Q(x) is the probability that a normal (Gaussian) random variable will obtain a value larger than x standard deviations above the mean.

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{u^2}{2}\right) du.$$

https://www.unilim.fr/pages_perso/vahid/notes/ber_awgn.pdf

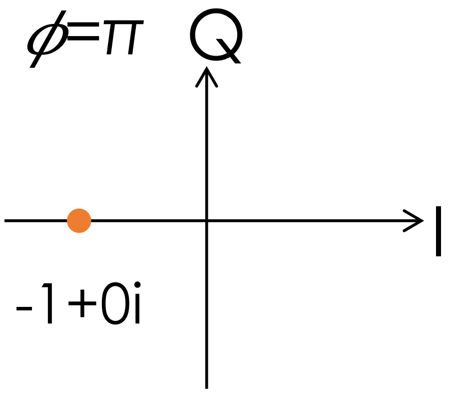
Constellation point for BPSK

- Say we send the signal with phase delay π

$$
\begin{aligned}
&\cos(2j\pi f_c t + \pi) \\
=&\cos(2j\pi f_c t)\cos(\pi) - \sin(2j\pi f_c t)\sin(\pi) \\
=&-1 * \cos(2j\pi f_c t) - 0 * \sin(2j\pi f_c t) \\
=&(-1 + 0i)e^{2j\pi f_c t}
\end{aligned}
$$

$\longrightarrow$ **Band-pass representation**

Illustrate this by the **<u>constellation point</u> (-1 + 0i)** in an I-Q plane

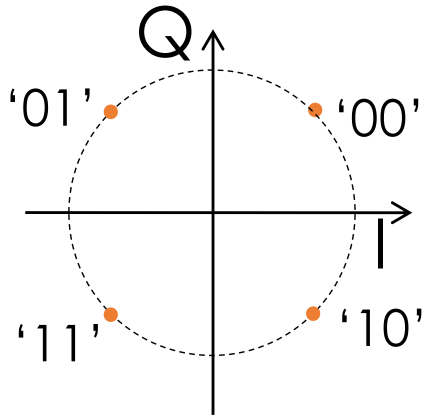


Quadrature PSK (QPSK)

- Use four phase rotations 1/4π, 3/4π, 5/4π, 7/4π to represent '00', '01', '11', 10'

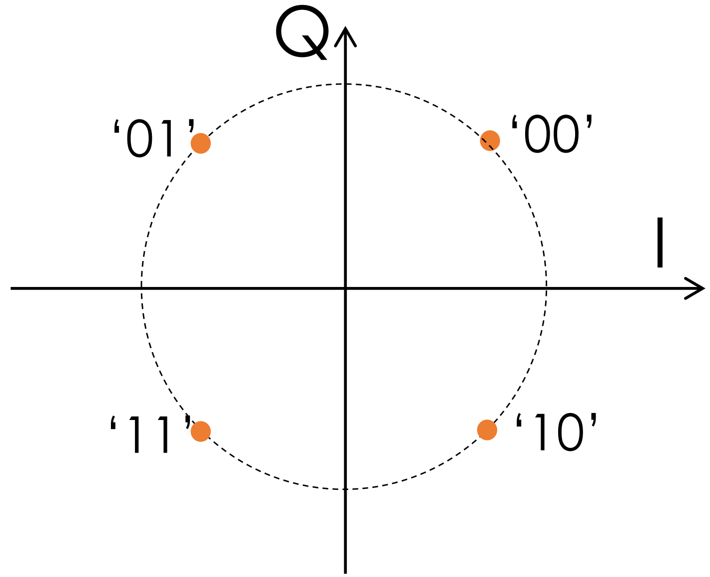
$$
\begin{aligned}
&A\cos(2j\pi f_c t + \pi/4)\\
=&A\cos(2j\pi f_c t)\cos(\pi/4) - A\sin(2j\pi f_c t)\sin(\pi/4)\\
=&1 * \cos(2j\pi f_c t) - 1 * \sin(2j\pi f_c t)\\
=&(1 + 1i)e^{2j\pi f_c t}
\end{aligned}
$$

$$
\begin{aligned}
&A\cos(2j\pi f_c t + 3\pi/4)\\
=&A\cos(2j\pi f_c t)\cos(3\pi/4) - A\sin(2j\pi f_c t)\sin(3\pi/4)\\
=&-1 * \cos(2j\pi f_c t) - 1 * \sin(2j\pi f_c t)\\
=&(-1 + 1i)e^{2j\pi f_c t}
\end{aligned}
$$



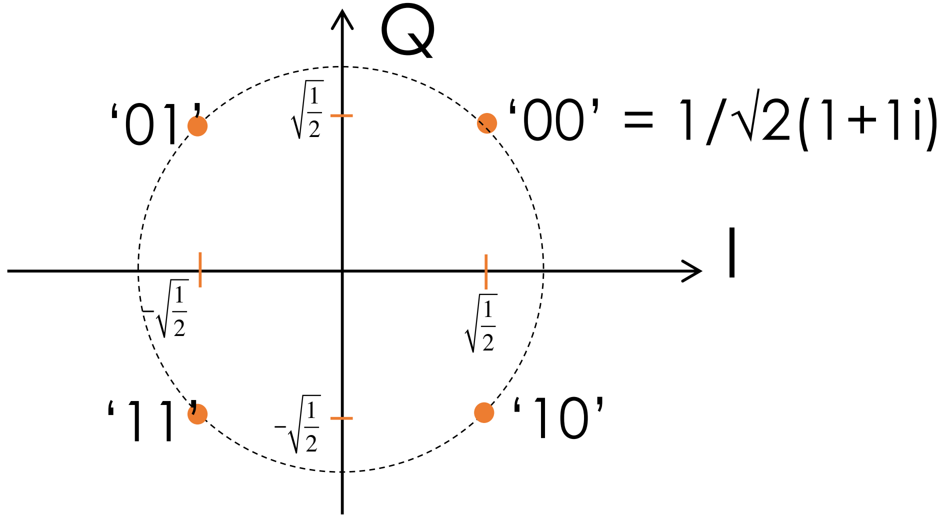
Quadrature PSK (QPSK)

- Use 2 degrees of freedom in I-Q plane
- Represent two bits as a constellation point
  - Rotate the constellations by $\pi/2$
  - Demodulation by mapping the received signal to the closest constellation point
  - Double the bit-rate
- No free lunch:
  - Higher error probability (Why?)



Quadrature PSK (QPSK)

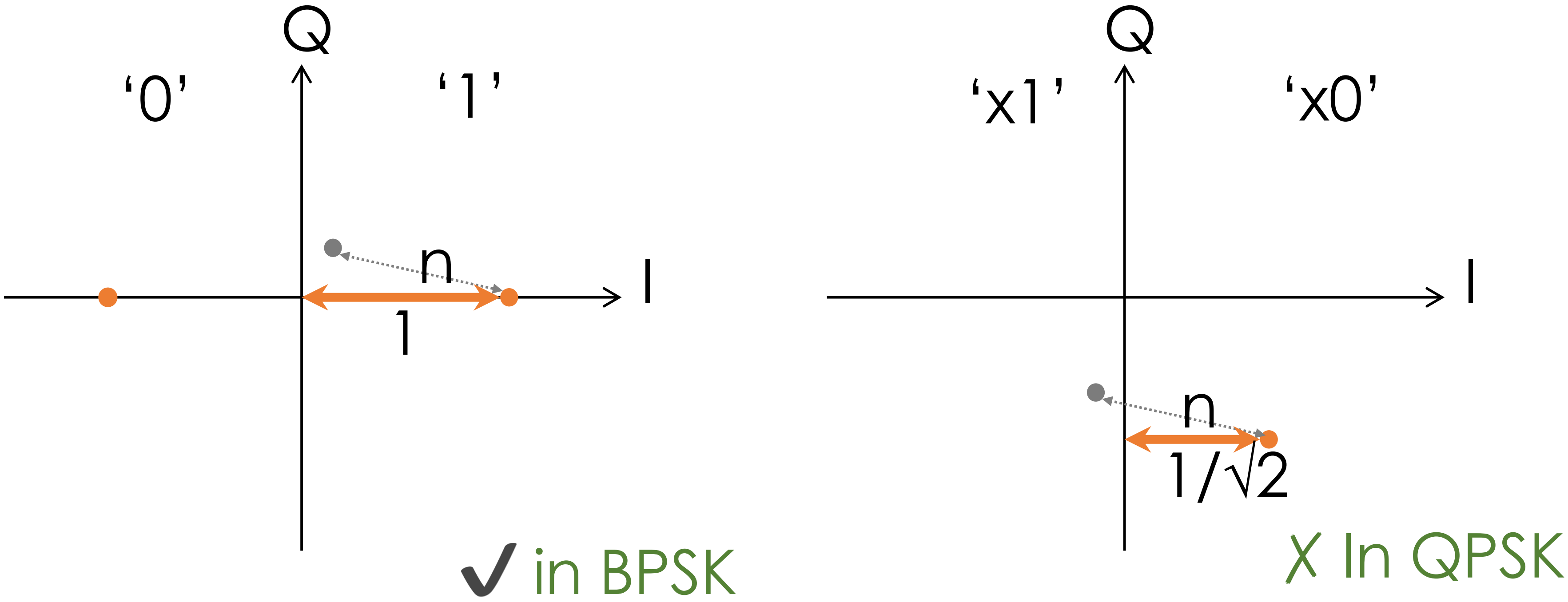
- Maximum power is bounded
  - Amplitude of each constellation point should still be 1



Bits	Symbols
'00'	$1/\sqrt{2}+1/\sqrt{2}i$
'01'	$-1/\sqrt{2}+1/\sqrt{2}i$
'10'	$1/\sqrt{2}-1/\sqrt{2}i$
'11'	$-1/\sqrt{2}-1/\sqrt{2}i$

Higher Error Probability in QPSK

- For a particular error *n*, the symbol could be decoded correctly in BPSK, but not in QPSK
  - Why? Each sample only gets half power



Trade-off between Rate and SER

- Trade-off between the data rate and the symbol error rate
  - Denser constellation points
    → More bits encoded in each symbol
    → Higher data rate
  - Denser constellation points
    → Smaller distance between any two points
    → Higher decoding error probability

SEN and BER of QPSK

- SNR_s: SNR per symbol; SNR_b: SNR per bit

$$SNR_b \approx \frac{SNR_s}{\log_2 M}, P_b \approx \frac{P_s}{\log_2 M}$$

QPSK: M=4

- SER: The probability that each branch has a bit error

$$SER = P_s = 1 - [1 - Q(\sqrt{2SNR_b})]^2 = 1 - [1 - Q(\sqrt{\frac{2E_b}{N_0}})]^2$$

$$= 1 - [1 - Q(\sqrt{SNR_s})]^2 = 1 - [1 - Q(\sqrt{\frac{E_s}{N_0}})]^2$$

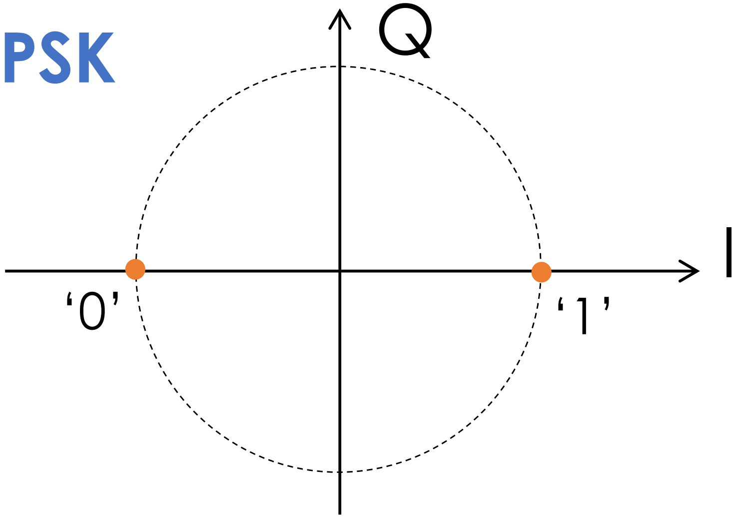
- BER

$$BER = P_b \approx \frac{P_s}{2}$$

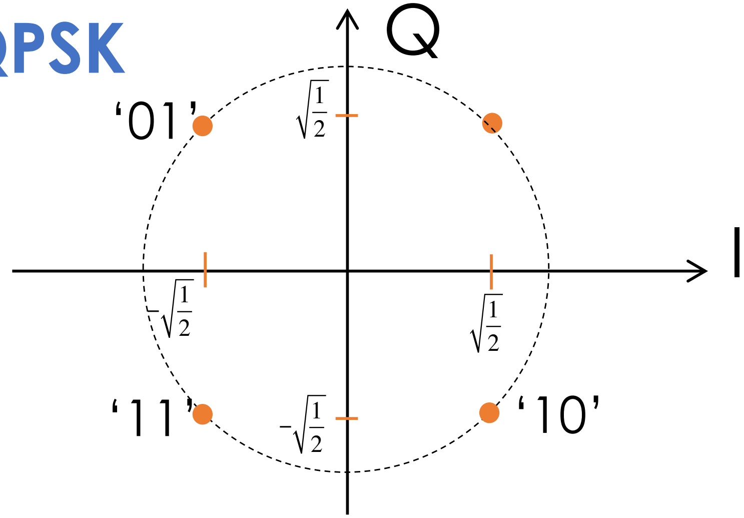
E_s is the bounded maximum power

M-PSK

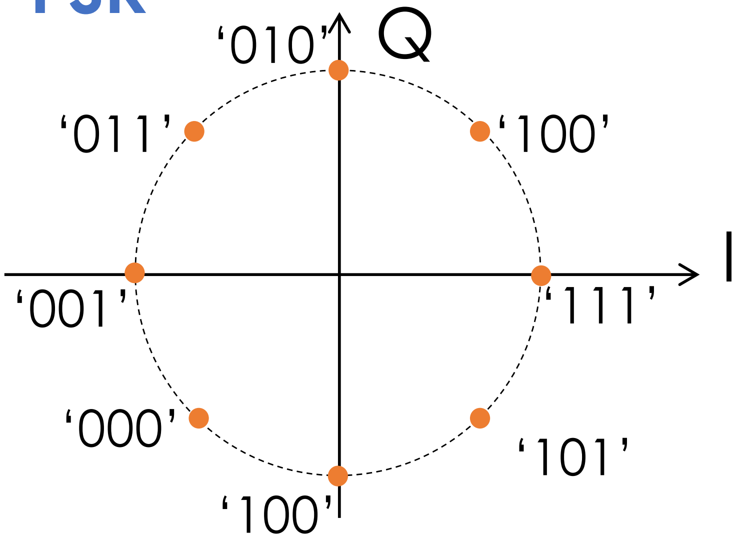
BPSK



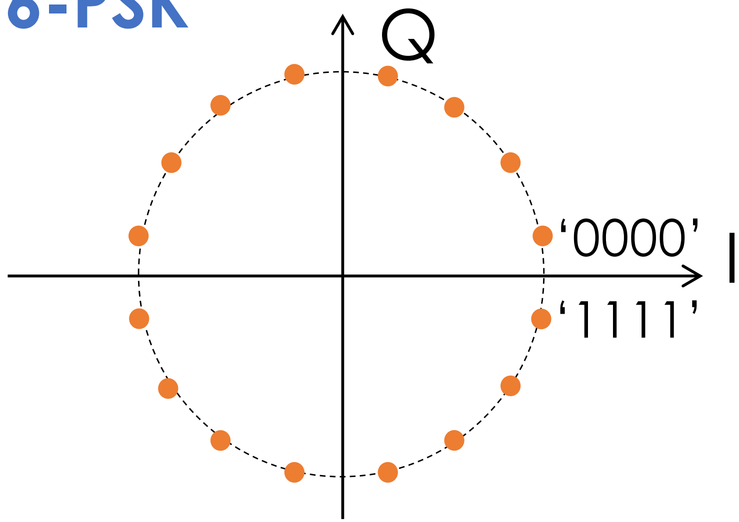
QPSK



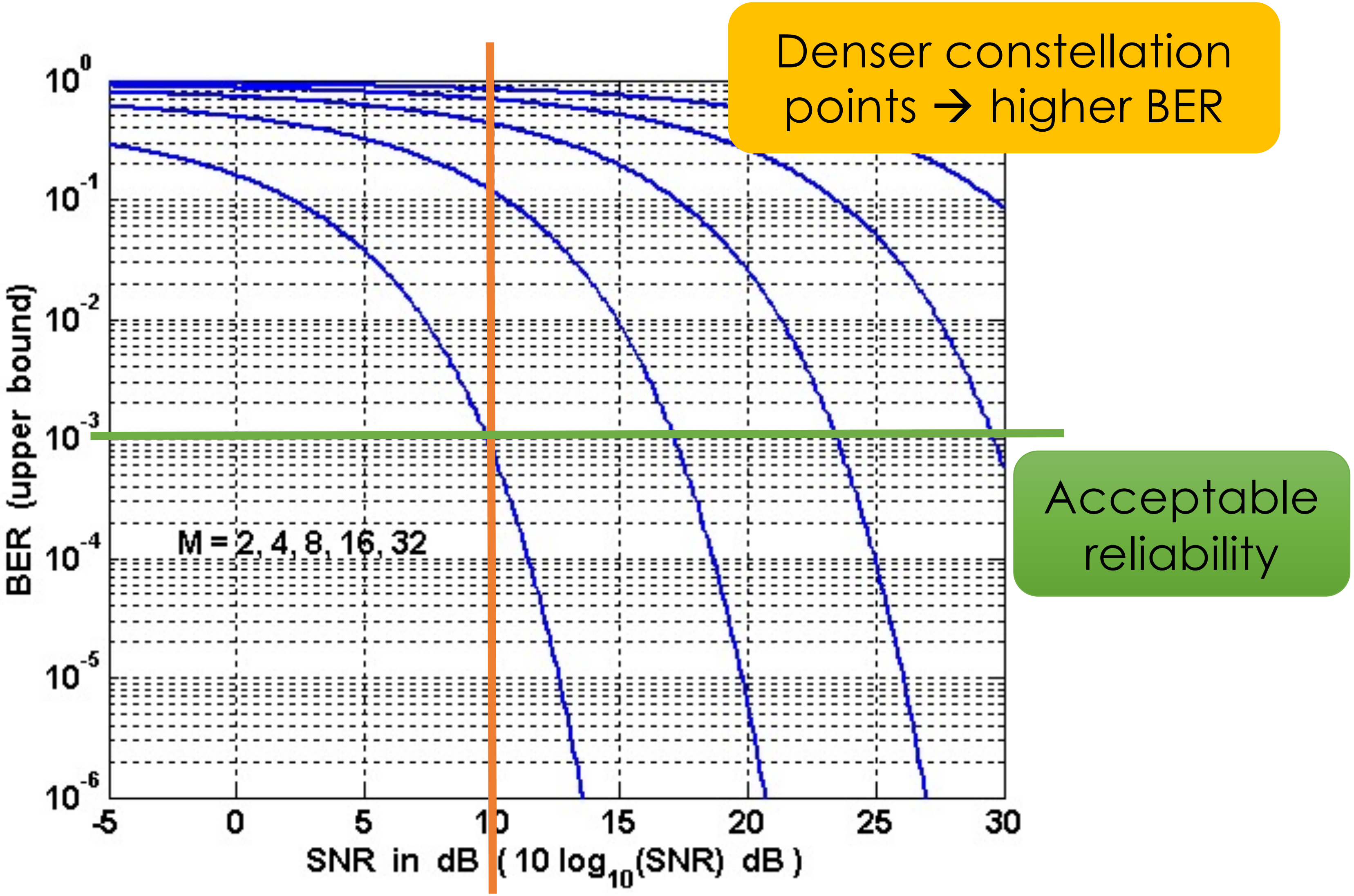
8-PSK



16-PSK



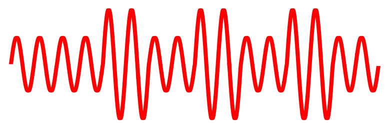
M-PSK BER versus SNR



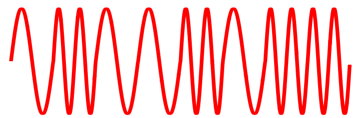
Types of Modulation

$$s(t) = A\cos(2\pi f_c t + \phi)$$

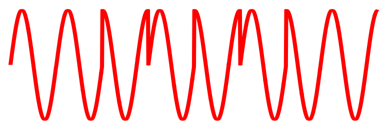
- *Amplitude*
  - *M-ASK: Amplitude Shift Keying*



- *Frequency*
  - *M-FSK: Frequency Shift Keying*



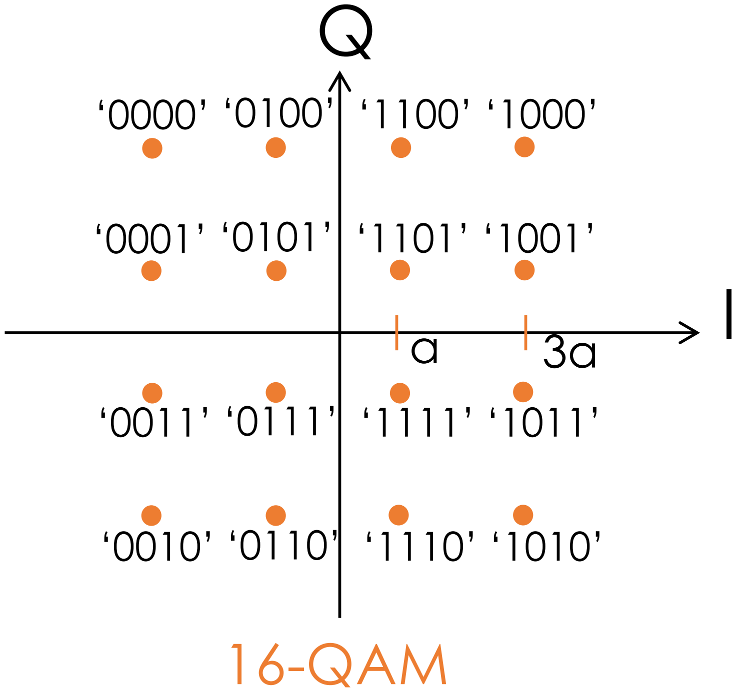
- *Phase*
  - *M-PSK: Phase Shift Keying*



- *Amplitude* + *Phase*
  - *M-QAM: Quadrature Amplitude Modulation*

Quadrature Amplitude Modulation

- Change both amplitude and phase
- s(t)=Acos($2\pi f_c t+\phi$)

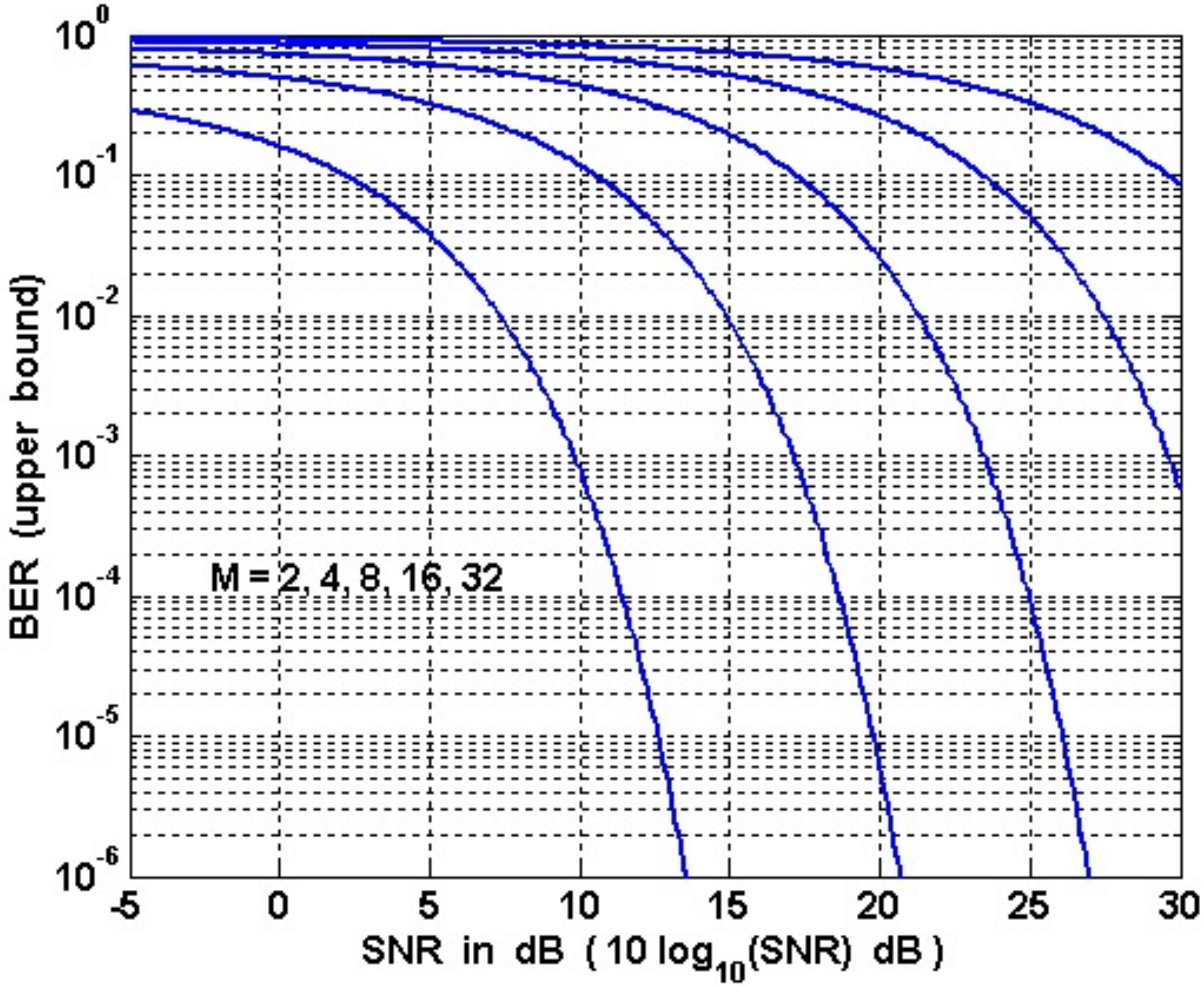


Bits	Symbols
'1000'	s_1=3a+3ai
'1001'	s_2=3a+ai
'1100'	s_3=a+3ai
'1101'	s_4=a+ai

expected power: $E\left[\left|s_i\right|^2\right]=1$

- 64-QAM: 64 constellation points, each with 8 bits

M-QAM BER versus SNR



Modulation in 802.11

- 802.11a
  - 6 mb/s: BPSK + ½ code rate
  - 9 mb/s: BPSK + ¾ code rate
  - 12 mb/s: QPSK + ½ code rate
  - 18 mb/s: QPSK + ¾ code rate
  - 24 mb/s: 16-QAM + ½ code rate
  - 36 mb/s: 16-QAM + ¾ code rate
  - 48 mb/s: 64-QAM + ⅔ code rate
  - 54 mb/s: 64-QAM + ¾ code rate
- FEC (forward error correction)
  - k/n: k-bits useful information among n-bits of data
  - Decodable if any k bits among n transmitted bits are correct

FEC

- Encode bit stream in a **redundant** way by using an **error-correcting code**
  - Redundancy allows Rx to recover a limited number of bit errors even without retransmissions
- **Code rate:** proportion of bits that is useful (non-redundant)
  - **k/n**: for every ***k*** bits useful information, Tx should generate in total ***n*** bits; ***n-k*** bits are redundant
  - For example, ½ code rate means
    - For every single bit, should send two bits
    - For a 100-bit packet, should send 200 bits
- **Correcting capability:** the number of errors that can be corrected by the code

Detection

- Map the received signal to one of the possible transmitted signal with the *minimum distance*
- Find the corresponding bit streams

received signal

$$[s_I(t)+n(t)] + j[s_Q(t)+n(t)]$$

closest

possible transmitted signals		corresponding bit streams
$s_I^1(t) + js_Q^1(t)$	$\rightarrow$	$'000..00'$
$s_I^2(t) + js_Q^2(t)$	$\rightarrow$	$'000..01'$
$\vdots$	$\vdots$	$\vdots$
$s_I^k(t) + js_Q^k(t)$	$\rightarrow$	$'001..10'$
$\vdots$	$\vdots$	$\vdots$
$s_I^K(t) + js_Q^K(t)$	$\rightarrow$	$'111..11'$