ECE 198 Fall 2019 Jin Yucheng (yucheng9)
Homework 2 Problem 1-4

Solution:

(1) How many strings with 5 or more characters can be formed from the letters in SEERESS?

We have 1 R, 3 E's and 3 S's. There are 3 cases to consider:

- for strings with 7 characters, there are:

$$\frac{7!}{1!3!3!}$$

  Totally 140 strings.

- for strings with 6 characters, we may remove either R, E, or S, there are:

$$\frac{6!}{3!3!}+\frac{6!}{1!2!3!}+\frac{6!}{1!2!3!}$$

  Totally 140 strings.

- for strings with 5 characters, we may remove either R and E, R and S, E and S, E and E, or S and S, there are:

$$\frac{5!}{2!3!}+\frac{5!}{2!3!}+\frac{5!}{1!2!2!}+\frac{5!}{1!1!3!}+\frac{5!}{1!1!3!}$$

  Totally 90 strings.

Therefore, sum up the results of the above 3 cases, there are 140 + 140 + 90 = 370 strings with 5 or more characters that can be formed from SEERESS.

(2) Show that in Z_m with addition modulo $m, m \geq 2, m$ is integer, satisfies closure, associative, commutative properties.

- **Closure** If a and b belong to Z_m, then $a +_m b$ belongs to Z_m.
  By definition, $a +_m b = (a+b)$ **mod** m
  By the Division Theorem, there are unique integers q, r, such that $(a+b) = qm + r$, where $0 \leq r < m$
  Therefore, for all $0 \leq r < m$, $(a+b) \equiv r$ **mod** m
  Hence, from the definition of integers modulo m, $a +_m b \in Z_m$

- **Associativity** If a, b, and c belong to Z_m, then $(a +_m b) +_m c = a +_m (b +_m c)$.

$$
\begin{aligned}
(a +_m b) +_m c &= [(a+b) \textbf{ mod } m] +_m c && \text{by definition of } +_m \\
&= [(a+b)+c] \textbf{ mod } m && \text{by definition of } +_m \\
&= [a+(b+c)] \textbf{ mod } m && \text{Associative Law of Addition} \\
&= a +_m [(b+c) \textbf{ mod } m] && \text{by definition of } +_m \\
&= a +_m (b +_m c) && \text{by definition of } +_m
\end{aligned}
$$

- **Commutativity** If a, b belong to Z_m, then $a +_m b = b +_m a$.

$$
\begin{aligned}
a +_m b &= (a+b) \textbf{ mod } m && \text{by definition of } +_m \\
&= (b+a) \textbf{ mod } m && \text{Commutative Law of Addition} \\
&= b +_m a && \text{by definition of } +_m
\end{aligned}
$$

(3) How many zeros are there at the end of 100!?

The answer is 24.

Because $10 = 2 \times 5$, $10^2 = 2^2 \times 5^2$, and $10^n = 2^n \times 5^n$, we need to find the number of 2 and 5 factors to determine the number of trailing 0's.

There are $\lfloor 100/5 \rfloor$ = 20 terms divisible by 5^1, $\lfloor 100/5^2 \rfloor$ = 4 terms divisible by 5^2, $\lfloor 100/5^3 \rfloor$ = 0 terms divisible by 5^3, therefore, the number of 5 factors is 24.

There are $\lfloor 100/2 \rfloor$ = 50 terms divisible by 2^1, the number of 2 factors is at least 50 and is larger than 24. Therefore, 100! is divisible by 10^{24} and no greater power of 10.

Hence there are 24 0's at the end of 100!.

(4) Prove or disprove that $n^2 - 79n + 1601$ is prime whenever n is a positive integer.

$n = 1601$ is just a counterexample, since if $n = 1601$, then $n^2 - 79n + 1601 = 1601^2 - 79 \times 1601 + 1601 = 1601 \times (1601 - 79 + 1) = 1601 \times 1523$. So $n^2 - 79n + 1601$ is not a prime when $n = 1601$.

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