Homework 2

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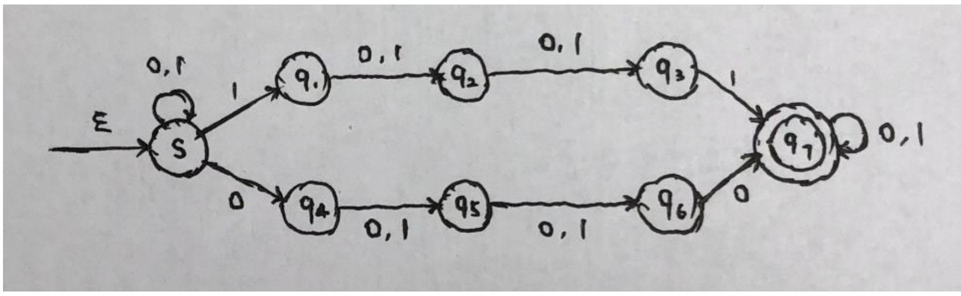
Problem 1.

(a) The NFA is drawn as below.



s is the start state.

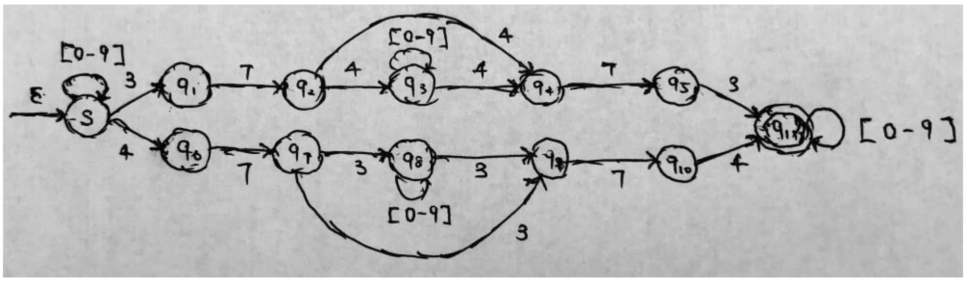
q1, q2, q3 are the states which represent strings starting with 1.

q4, q5, q6 are the states which represent strings starting with 0.

q7 is the accepting state which represents strings that have two of the same characters at a distance 3.

Because in the NFA, the state receives 1 in the first transition also receives 1 in the final transition at a distance 3 and the state receives 0 in the first transition also receives 0 in the final transition at a distance 3, in the accepting state two of the same characters are at a distance 3 from each other, the NFA accepts the correct language.

(b) The NFA is drawn as below.



s is the start state.

q1 is the state which represents strings that contain 3.

q2 is the state which represents strings that contain 37.

q3 is the state which represents strings that contain 374 as a substring.

q4 is the state which represents strings that contain 374 as a substring and 4.

q5 is the state which represents strings that contain 374 as a substring and 47.

q6 is the state which represents strings that contain 4.

q7 is the state which represents strings that contain 47.

q8 is the state which represents strings that contain 473 as a substring.

q9 is the state which represents strings that contain 473 as a substring and 3.

q10 is the state which represents strings that contain 473 as a substring and 37.

q11 is the accepting state which represents strings that contain 374 and 473 as substrings.

If a string contains 374 first, then 473, by q1, q2, q3, q4, q5, then q11 accepts the case.

If a string contains 473 first, then 374, by q6, q7, q8, q9, q10, then q11 accepts the case.

Because there are only two cases for the correct language, the NFA accepts the correct language.

Problem 2.

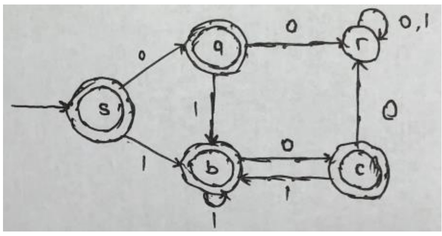
(a)

The NFA is drawn as below.

The DFA which uses the incremental subset construction is drawn as below.

q'	ε - reach(q')	q' ∈ A'	δ(q', 0)	δ (q', 1)
s	abcdfghikp	Yes	e	jl
e	fghikp	Yes	∅	jl
jl	ghikmnop	Yes	n	jl
n	ghikop	Yes	∅	jl

Another DFA which is with fewer states is drawn as below.



State a: 0.

State b: (0+ ε)*(1+10)*1*.

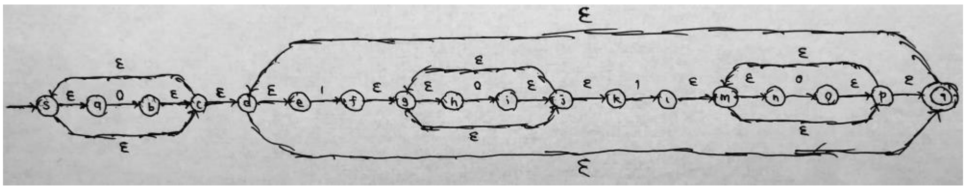
State c: (0+ ε)*(1+10)*(10)*.

State r: reject state.

Problem 2.

(b)

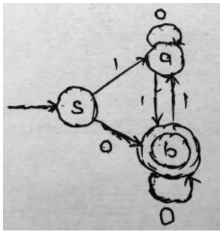
The NFA is drawn as below.



The DFA which uses the incremental subset construction is drawn as below.

q'	ε - reach(q')	q' ∈ A'	δ(q', 0)	δ (q', 1)
s	acdeqs	Yes	b	∅
b	acdeq	Yes	b	f
f	ghjk	No	i	l
i	ghjk	No	i	l
j	edmnpq	Yes	o	f
o	edmnpq	Yes	o	f

Another DFA which is with fewer states is drawn as below.



State a: string contains odd #1s.

State b: string contains even #1s.

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Solution:

Let $M = (\Sigma, Q, s, A, \delta)$ be an arbitrary DFA that accepts L. We define a new NFA $N = (\Sigma, Q^{'}, s^{'}, A^{'}, \delta^{'})$ with ϵ-transitions that accepts palindromes in $L(M)$.

$$L(N) = (x \in L(M) \mid x = x^R)$$
$$Q^{'} = (Q \times Q \times Q) \cup s^{'}$$
$$s^{'} \text{ is an explicit state in } Q^{'}$$
$$A^{'} = \{(h,h,h) \mid h \in Q\}$$
$$\delta^{'}(s^{'}, \epsilon) = \{(s,h,m) \mid h \in Q, m \in A\}$$
$$\delta^{'}((p,h,q),a) = \{(\delta(p,a), h, \delta^{-1}(q,a))\}$$

Assume $x = ww^R$ is an arbitrary string over Σ, and x is in $L(M)$. By definition, x is a palindrome, we want to prove that $x \in L(N)$:

$$\delta(s^{'}, x) \in A^{'}$$

In the definition above, we denote that:

$$\delta(s, ww^R) = q,\ q \in A$$
$$\delta^*(s,w) = h \text{ and } \delta^*(s, w^[R]) = h,\ h \in Q$$
$$\delta^*(h, w^R) = q,\ q \in A$$
$$\delta^{'}((a,b,c),x) = \{\delta(a,x), b, \delta^{-1}(c,x)\},\ a,b,c \in Q$$

There are two cases to consider,

- if $x = \epsilon$, then the empty string is accepted by definition:

$$\delta^{'}(s^{'}, \epsilon) = \{(s,h,m) \mid h \in Q, m \in A\}$$

- if x is not empty, we need to prove there exists an accepting state (h,h,h), such that:

$$(h,h,h) \in \delta^{'*}((s^{'}), x)$$

  The proof is as follows:

$$\text{it is equivalent to prove } (h,h,h) \in \{(\delta^{'*}((s,h,q),w) \mid h \in Q, q \in A)\}$$
$$\delta^{'*}((s,h,q),w) = \{(\delta^*(s,w), h, \delta^{-1*}(q,w))\}$$
$$\text{since } \delta^*(s,w) = h,\ h \in Q \text{ and } \delta^*(h,w) = q \text{ (because we can replace } w^R \text{ with } w),\ q \in A,$$
$$\text{therefore } h \in \delta^{-1*}(q,w)$$

  So $(h,h,h) \in \{(\delta^{'*}((s,h,q),w) \mid h \in Q, q \in A)\}$.

In conclusion, N is the NFA that accepts all palindromes in $L(M)$.

Sorry that my friends reminded me of adding proof of the NFA rejects any string that is not in $L(M)$.

Let $w = v \cdot v^R$ be an arbitrary string not in $L(M)$:

$$\delta^*(s, v \cdot v^R) = q, \quad q \notin A$$

We want to prove that $w \notin L(N)$ by contradiction.

Assume $w \in L(N)$:

$$(h, h, h) \in \delta'^* \{(s, h, q), v \mid h \in Q, q \in A\}$$

Since $\delta'(s', \varepsilon) = \{(s, h, q) \mid h \in Q, q \in A\}$

$$\delta'^*((s, h, q), v) = \{(\delta^*(s, v), h, \delta^{-1*}(q, v)) \mid h \in Q, q \in A\}$$

So to make $w \in L(N)$, we must have:

$$\delta^*(s, v) = h \text{ and } h \in \delta^{-1*}(q, v) \text{ or}$$

$$\delta^*(s, v) = h \text{ and } \delta^*(h, v^R) = q$$

Since $\delta^*(s, v) = h$ and $\delta^*(h, v^R) = q$,

So $\delta^*(s, v \cdot v^R) = q$, where $q \in A$,

and $v \cdot v^R \in L(M)$, which conflicts our assumption.

Therefore, any other strings that not in $L(M)$ are rejected.

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