CS/ECE 374 B Fall 2019

Homework 2 Problem 3

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Solution:

Let $M = (\Sigma, Q, s, A, \delta)$ be an arbitrary DFA that accepts L. We define a new NFA $N = (\Sigma, Q^{'}, s^{'}, A^{'}, \delta^{'})$ with ϵ-transitions that accepts palindromes in $L(M)$.

$$L(N) = (x \in L(M) \mid x = x^R)$$
$$Q^{'} = (Q \times Q \times Q) \cup s^{'}$$
$$s^{'} \text{ is an explicit state in } Q^{'}$$
$$A^{'} = \{(h,h,h) \mid h \in Q\}$$
$$\delta^{'}(s^{'}, \epsilon) = \{(s,h,m) \mid h \in Q, m \in A\}$$
$$\delta^{'}((p,h,q),a) = \{(\delta(p,a), h, \delta^{-1}(q,a))\}$$

Assume $x = ww^R$ is an arbitrary string over Σ, and x is in $L(M)$. By definition, x is a palindrome, we want to prove that $x \in L(N)$:

$$\delta(s^{'}, x) \in A^{'}$$

In the definition above, we denote that:

$$\delta(s, ww^R) = q,\ q \in A$$
$$\delta^*(s,w) = h \text{ and } \delta^*(s,w^R) = h,\ h \in Q$$
$$\delta^*(h, w^R) = q,\ q \in A$$
$$\delta^{'}((a,b,c),x) = \{\delta(a,x), b, \delta^{-1}(c,x)\},\ a,b,c \in Q$$

There are two cases to consider,

- if $x = \epsilon$, then the empty string is accepted by definition:

$$\delta^{'}(s^{'}, \epsilon) = \{(s,h,m) \mid h \in Q, m \in A\}$$

- if x is not empty, we need to prove there exists an accepting state (h,h,h), such that:

$$(h,h,h) \in \delta^{'*}((s^{'}), x)$$

  The proof is as follows:

$$\text{it is equivalent to prove } (h,h,h) \in \{(\delta^{'*}((s,h,q),w) \mid h \in Q, q \in A)\}$$
$$\delta^{'*}((s,h,q),w) = \{(\delta^*(s,w), h, \delta^{-1*}(q,w))\}$$
$$\text{since } \delta^*(s,w) = h,\ h \in Q \text{ and } \delta^*(h,w) = q \text{ (because we can replace } w^R \text{ with } w),\ q \in A,$$
$$\text{therefore } h \in \delta^{-1*}(q,w)$$

  So $(h,h,h) \in \{(\delta^{'*}((s,h,q),w) \mid h \in Q, q \in A)\}$.

In conclusion, N is the NFA that accepts all palindromes in $L(M)$.

Sorry that my friends reminded me of adding proof of the NFA rejects any string that is not in $L(M)$.

Let $w = v \cdot v^R$ be an arbitrary string not in $L(M)$:

$$\delta^*(s, v \cdot v^R) = q, \ q \notin A$$

We want to prove that $w \notin L(N)$ by contradiction.

Assume $w \in L(N)$:

$$(h, h, h) \in \delta'^* \{(s, h, q), v \mid h \in Q, q \in A\}$$

Since $\delta'(s', \epsilon) = \{(s, h, q) \mid h \in Q, q \in A\}$

$$\delta'^*((s, h, q), v) = \{(\delta^*(s, v), h, \delta^{-1*}(q, v)) \mid h \in Q, q \in A\}$$

So to make $w \in L(N)$, we must have:

$$\delta^*(s, v) = h \text{ and } h \in \delta^{-1*}(q, v) \text{ or}$$

$$\delta^*(s, v) = h \text{ and } \delta^*(h, v^R) = q$$

Since $\delta^*(s, v) = h$ and $\delta^*(h, v^R) = q$,

So $\delta^*(s, v \cdot v^R) = q$, where $q \in A$,

and $v \cdot v^R \in L(M)$, which conflicts our assumption.

Therefore, any other strings that not in $L(M)$ are rejected.

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