Solution:

(a) Let x and y be arbitrary strings.

Assume for any string w where $|w| < |x|$ that $digsum(w \bullet y) = digsum(w) + digsum(y)$.

There are two cases to consider:

- If $x = \epsilon$, then

$$\begin{aligned}
digsum(x \bullet y) &= digsum(\epsilon \bullet y) && \text{because } x = \epsilon \\
&= digsum(y) && \text{by Lemma 1} \\
&= 0 + digsum(y) \\
&= digsum(\epsilon) + digsum(y) && \text{because } digsum(\epsilon) = 0 \\
&= digsum(x) + digsum(y) && \text{because } x = \epsilon
\end{aligned}$$

- Otherwise, $x = aw$ for some symbol $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and some string w.

$$\begin{aligned}
digsum(x \bullet y) &= digsum(aw \bullet y) && \text{because } x = aw \\
&= digsum(a \cdot (w \bullet y)) && \text{by definition of } \bullet \\
&= a + digsum(w \bullet y) && \text{by definition } digsum(ax) = a + digsum(x) \\
&= a + digsum(w) + digsum(y) && \text{by the induction hypothesis} \\
&= digsum(aw) + digsum(y) && \text{by definition } digsum(ax) = a + digsum(x) \\
&= digsum(x) + digsum(y) && \text{because } x = aw
\end{aligned}$$

Also, since $digsum(y \bullet x) = digsum(y) + digsum(x) = digsum(x) + digsum(y)$, we conclude that,

$$digsum(x \bullet y) = digsum(y \bullet x) = digsum(x) + digsum(y).$$

(b) Let x be an arbitrary string.

Assume for any string w where $|w| < |x|$ that $digsum(w^R) = digsum(w)$.

There are two cases to consider:

- If $x = \epsilon$, then

$$\begin{aligned}
digsum(x^R) &= digsum(\epsilon) && \text{because } x = \epsilon \text{ and } \epsilon^R = \epsilon \\
&= digsum(x) && \text{because } x = \epsilon
\end{aligned}$$

- Otherwise, $x = aw$ for some symbol $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and some string w.

$$\begin{aligned}
digsum(x^R) &= digsum((aw)^R) && \text{because } x = aw \\
&= digsum(w^R a) && \text{by definition of } {}^R \\
&= digsum(w^R \bullet a) && \text{because } a \text{ can be seen as a string of just one symbol} \\
&= digsum(w^R) + digsum(a) && \text{by conclusion of (a)} \\
&= digsum(w) + digsum(a) && \text{by the induction hypothesis} \\
&= digsum(a) + digsum(w) && \\
&= digsum(a \bullet w) && \text{by conclusion of (a)} \\
&= digsum(x) && \text{because } x = aw
\end{aligned}$$

In both cases, we conclude that $digsum(x^R) = digsum(x)$.

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CS/ECE 374 B Fall 2019 Jin Yucheng (yucheng9)
Homework 0 Problem 2

Solution:

(a) By definition ① $a \in L_{odd}$ for $a \in \{1,3,5,7,9\}$, the string 374 obviously fails to match since it contains 3 symbols.
By definition ② $ax \in L_{odd}$ for $a \in \{0,2,4,6,8\}$ and $x \in L_{odd}$, the string 374 obviously fails to match since its first symbol is 3.
By definition ③ $axb \in L_{odd}$ for $a, b \in \{1,3,5,7,9\}$ and $x \in L_{odd}$, the string 374 can be written as $3 \cdot 7 \cdot 4$, $3 \in \{1,3,5,7,9\}$ but $4 \notin \{1,3,5,7,9\}$. 374 fails to match definition ③, therefore, $374 \notin L_{odd}$.

(b) x is an arbitrary string that x $\in L_{odd}$.
Assume for any string w where $|w| < |x|$ that w $\in L_{odd}$ and digsum(w) is odd.
There are 3 cases to consider:

- If x is in the form of "a", by definition ① $a \in L_{odd}$ for $a \in \{1,3,5,7,9\}$, x is either 1 or 3 or 5 or 7 or 9, $digsum(x)$ is also either 1 or 3 or 5 or 7 or 9, so $digsum(x)$ is odd.
- If x is in the form of "aw", by definition ② $aw \in L_{odd}$ for $a \in \{0,2,4,6,8\}$ and $w \in L_{odd}$, we have:

$$\begin{aligned}
digsum(x) &= digsum(aw) && \text{because } x = aw \\
&= a + digsum(w) && \text{by definition } digsum(ax) = a + digsum(x) \\
&a \text{ is even} && a \in \{0,2,4,6,8\} \\
&digsum(w) \text{ is odd} && \text{by the induction hypothesis} \\
&a + digsum(w) \text{ is odd}
\end{aligned}$$

- If x is in the form of "awb", by definition ③ $awb \in L_{odd}$ for $a, b \in \{1,3,5,7,9\}$ and $w \in L_{odd}$, we have:

$$\begin{aligned}
digsum(x) &= digsum(awb) && \text{because } x = awb \\
&= digsum(a \bullet (wb)) && \text{by definition of } \bullet \\
&= a + digsum(wb) && \text{by definition } digsum(ax) = a + digsum(x) \\
&= a + digsum(w \bullet b) && \text{by definition of } \bullet \\
&= a + b + digsum(w) && \text{by definition } digsum(ax) = a + digsum(x) \\
&a, b \text{ are odd} && a, b \in \{1,3,5,7,9\} \\
&digsum(w) \text{ is odd} && \text{by the induction hypothesis} \\
&a + b + digsum(w) \text{ is odd}
\end{aligned}$$

In conclusion, for any $x \in L_{odd}, digsum(x)$ is odd. ■

CS/ECE 374 B Fall 2019 Jin Yucheng (yucheng9)
Homework 0 Problem 3

Solution: Assume w is an arbitrary string over $\{1,0\}$, L_{bad} is defined as follows:

- $w \in L_{bad}$ for $|w| < 3$
- for $|w| \geq 3$ and $w = ax$, $w \in L_{bad}$ for $a \in \{1,0\}$ if $x \in L_{bad}$ and ax_1x_2 is not 000 or 111

The idea is the same as Problem(2). First we define base case, that if the string is only consisted of less than 3 digits, then it belongs to L_{bad} since it can not contain 000 or 111. Second, if the string has 3 or more digits, we can express the string as the concatenation of its leading digit and the remaining string. The remaining string is a substring of the string to be examined, if it not belong to L_{bad}, then the superstring must have 000 or 111 and not belong to L_{bad} either. If it belongs to L_{bad}, we guarantee that from x_3 to the last digit x_n, there is no 000 or 111, but we must take a step further to examine whether ax_1x_2 is 000 or 111, if it is not 000 or 111, then the superstring belongs to L_{odd}.

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