

Countability Practice

- (a). Yes, they have the same cardinality.

$$f(x) = \frac{1}{x} - 1$$

Suppose $f(x) = f(y)$

$$\begin{aligned}\frac{1}{x} - 1 &= \frac{1}{y} - 1 \\ x &= y\end{aligned}$$

Hence f is one to one.

Take any $y \in (0, \infty)$, let $x \in (0, 1)$, $x = 1/(1 + y)$. Then,

$$\begin{aligned}f(x) &= \frac{1}{x} - 1 \\ f(x) &= \frac{1}{\frac{1}{1+y}} - 1 \\ f(x) &= y\end{aligned}$$

So f maps x to y , hence onto.

- (b). Yes, it is countable. We can enumerate the strings in such a way that each string appears exactly once in the list. Firstly, list all strings of length 1 in lexicographic order, then all strings of length 2, and then length 3, and so on. Since each step, there are only finite number of strings of a particular length, any string of finite length appears in the list, and it is clear that each string appears exactly once in this list.