Who Gets Hurt?
A Reexamination of Foreign Exchange Intervention

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Abstract

In this paper, I construct a two-country model to reexamine the effect of foreign exchange interventions from a home country when all international trade is flexibly priced in foreign country's currency. The result has two significant policy implications when home country depreciates its own currency against the foreign currency through foreign exchange interventions. Firstly, there is a decrease in prices and an increase in quantities for exports from both countries when interventions are anticipated, which implies intervention is not harmful to exporters in foreign country. Secondly, there is a wealth redistribution from the domestic sector in home country to the rest of the world, making it the only sector hurt by foreign exchange interventions. In the calibration exercise, I find the domestic price increase by 0.34% and the wealth of domestic sector in Taiwan reduces by 0.2143% of real GDP if the central bank in Taiwan increases US reserves purchase by 1% and finances the purchase by seigniorage.

Keywords: exchange rate intervention, international reserves, monetary policy pass-through

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1 Introduction

Foreign exchange intervention (FXI, hereafter) has long been a subject of debate in trade policy. International organizations such as International Monetary Fund (IMF) urges their members to use exchange rates interventions with caution.[1] The United States, a common target of exchange rate intervention, conducts semi-annual investigations of her trading partners on such practices and may impose trade sanctions if necessary to deter exchange rate intervention from trade partners.[2] A quote from the U.S. Department of the Treasury's investigation report in April 2021 provides a vivid example:

> "*Nevertheless, a number of economies have conducted foreign exchange market intervention in a persistent, one-sided manner. ... Three of these economies — Vietnam, Switzerland, and Taiwan — exceeded the two other thresholds established by Treasury to identify potentially unfair currency practices or excessive external imbalances, which could impede U.S. growth or harm U.S. workers and firms.*" [3]

In this paper, I reexamine this conventional view on FXI and verify whether a country can gain competitiveness in the international market by depreciating her own currency against her trade partner. However, we first need to know how such interventions are conducted and whether they alter exchange rates. Although this seems to be a trivial question, central banks around the world usually keep intervention data secretly, which makes such policy evaluation hard. Not until recently, Fratzscher et al. (2019) used confidential data and showed that FXIs effectively smooth the path of nominal exchange rates. Meanwhile, by constructing a proxy, Adler et al. (2021) suggested that FXI practices are asymmetrical, with 76% of observed interventions involving net purchases of foreign currency. These stylized facts indicate that FXIs are primarily conducted through spot market transactions and are able to affect nominal exchange rates. As a result, I take these empirical findings as evidences for the assumptions used in the paper and focus on the case that the central bank conduct FXI only through spot market transaction.

[1] For details, see IMF policy advice report.

[2] The related laws are the Omnibus Trade and Competitiveness Act of 1988 and the Trade Facilitation and Trade Enforcement Act of 2015.

[3] See U.S.Treasury (2021), page 2.

Traditionally, the FXIs are deemed to be effective because currency prices (nominal exchange rates) adjust more quickly than the prices of traded goods, resulting in relative price changes, at least in the short run. Gopinath et al. (2020) analyzed bilateral trade data and identified prominent effects of nominal exchange rates on prices and quantities of trades, particularly against dominant currencies like the U.S. dollar (USD), albeit in an asymmetric manner. Since most international trades are invoiced in dominant currencies, prices tend to be sticky in dominant currencies as well. Given tradable goods are stickily priced in the dominant currency, the price of exported goods from non-dominant currency countries remains insensitive to their own currency depreciation against dominant currencies, whereas imported goods priced in dominant currency become more expensive in terms of non-dominant currency, leading to a substitution toward domestically produced goods. As a result, the only benefit for a non-dominant currency country engaging in FXI is the expenditure switching effect. However, this argument is vulnerable if imported goods are not easily substitutable, and thus, it fails to explain the prevalence of FXI practices across countries as documented in Adler et al. (2021). Furthermore, while inflation theoretically has the potential to alter nominal exchange rates, as Gopinath et al. (2020) showed, the reasons why developing countries prefer to use FXI remain unclear.

To reconcile these empirical findings, I propose a general equilibrium model based on Lagos and Wright (2005) and Rocheteau and Wright (2005), incorporating the dominant currency paradigm (DCP) and flexible price settings in international trade arrangements. The DCP is motivated by the fact that most international trades are invoiced in a few dominant currencies, with the USD being the most significant.[4] The flexible price setting captures the prevalence of long-term relationships and contracts in international trades. In long-term relationships, prices and quantities are predetermined in contracts, and any gains or losses resulting from *ex-post* shocks do not affect the specified contract terms.[5] In other words, prices and quantities are flexible at the moment contracts are signed or are likely renegotiated within contracts. My model predicts that FXIs, if they are expected, lead to a decrease in prices and an increase in quantities of exports from both dominant and non-dominant currency countries due to the higher real return on the dominant

[4]For instance, Gopinath and Rigobon (2008) noted that 90% (97%) of U.S. imports (exports) are priced in dollars, deviating from the conventional symmetric invoiced currency setting.

[5]See Barro (1977) for more details.

currency. The magnitude of this effect depends on the relative size of interventions to the total amount of circulated dominant currency. Additionally, there is a wealth redistribution from the domestic sector in non-dominant currency countries to the rest of the world. Consequently, FXIs only harm the domestic sector in non-dominant currency countries, while benefiting the export sector in the same country and all sectors in the dominant currency country, such as the U.S.

This paper is related to at least three strands of literature. The first one is the exchange rate regime literature. Advocates of floating exchange rates, such as Friedman (1953), argue that the floating exchange rates allow relative prices to adjust more efficiently and reflect fundamental changes across countries. Johnson (1969) had a similar argument but applied it only to major economies, suggesting that developing countries should peg their exchange rates to their major trading partners. In reality, most countries adopt exchange rate regimes somewhere between fully floating and fully fixed, and FXIs are common practices. FXI practices can be seen among advanced countries, e.g. the case of Switzerland in Amador et al. (2020). Fanelli and Straub (2021) provided a general guideline for small open economies to implement optimal FXI policies when segmented fund markets and pecuniary externality on consumption are present. Hassan et al. (2022) developed a risk-based model where small economies optimally choose to stabilize their exchange rates against the world's largest economy, thereby endogenously emerging as the world's "anchor currency", which aligns with Johnson (1969). In this paper, I examine the real effect of FXI on production with the mechanism relying on changes in the real return on currency instead of sticky price.

The second related strand of literature concerns international reserve accumulation. According to IMF data, international reserves across countries grew sixfold from 2000 to 2021, with the USD continuing to dominate at around 60% of the total.[6] Economists are intrigued by the reasons behind this substantial growth. For example, Aizenman and Marion (2004) suggested that central banks may accumulate international reserves to mitigate the deadweight loss resulting from tax collection systems during foreign debt defaults. Similarly, Bacchetta et al. (2013) examined the case of China and argues that central banks can enhance welfare through capital controls and international reserve accumulation when the economy experiences rapid growth and households face credit constraints. Arce et al. (2019) pointed out that central banks can hold international reserves

[6] See IMF's COFER database: COFER database, IMF.

to prevent households from over-borrowing and improve equilibrium allocation when households fail to internalize future credit constraints. In this paper, the central bank can set international reserve balances as one policy goal and adjusts its holdings through FXIs.

Finally, the goal of this paper is to explore the pass-through of FXIs, which is a core question in the New Open Economy Macroeconomics (NOEM) literature. NOEM provides a theoretical framework for analyzing the pass-through of monetary and fiscal policies between countries. Analytical predictions depend on assumptions about price rigidity, consistent with Friedman's argument in Friedman (1953), and invoiced currency settings for exported goods.[7] However, Gopinath et al. (2020) demonstrated that the USD acts as a dominant currency in international trade, and exports from non-dominant currency countries are hardly responsive to exchange rate fluctuations against USD. Thus, the benefits of FXIs for non-dominant currency issuers are limited to expenditure switching effects, rather than boosting exports. The assumption of sticky prices but flexible production is key to generating FXI pass-through effects in Gopinath et al. (2020). However, as international trades predominantly rely on contracts with predetermined prices and quantities, the sticky price assumption alone is insufficient for generating real effects, as highlighted by Barro (1977). Therefore, I adopt a model with flexible prices and show that FXIs can still have real effects through a different mechanism.

My model allows prices to adjust freely while maintaining the DCP arrangement in international trades. These two features shed new light on the effects of FXIs implemented by non-dominant currency-issuing countries to depreciate their own exchange currencies against dominant currencies. In this model, agents operate within three sectors: the final goods sector, the export intermediate goods sector, and the domestic intermediate goods sector. The final goods sectors in both countries utilize intermediate goods from the domestic sector and the foreign export sector as inputs to produce final consumption goods, while intermediate goods sectors only use labor as input. I investigate four types of FXIs: one-time unanticipated interventions, one-time anticipated interventions, persistently active interventions, and persistently passive interventions. The results indicate that one-time unanticipated FXIs do not affect production but solely redistribute wealth from the

[7] For instance, a positive monetary shock causing currency depreciation has a complete pass-through effect on exports in a producer-price model such as Obstfeld and Rogoff (1995), while the same shock has zero pass-through effect in a local-price model such as Devereux et al. (2003).

domestic intermediate goods sector in non-dominant currency countries to the rest of the world. One-time anticipated FXIs reduce production in the domestic intermediate goods sector in non-dominant currency countries but increase the production of goods priced in the dominant currency. The mechanism is as follows: the production of goods priced in the dominant currency increase because the rate of return on means of payment rises, encouraging agents to produce more goods in exchange for the dominant currency. The opposite effect occurs for the production of goods priced in non-dominant currencies. Contrary to the conventional view, these findings suggest that there is no expenditure switching effect as predicted by previous studies. Persistently active FXIs have qualitatively similar effects to one-time anticipated FXIs over multiple periods. Persistently passive FXIs reduce production in the domestic intermediate goods sector and may lower interest rates in non-dominant currency countries.

I calibrate the model by employing Taiwanese and US aggregate data and an estimated FXI proxy from Adler et al. (2021). I chose Taiwan as it was highlighted as a suspect of FXI practices in the U.S. Department of the Treasury's report, and the international trade pricing in Taiwan satisfy two crucial model price settings: USD as the dominant currency and flexible prices. The data indicates that 90% of Taiwan's exports and 80% of its imports were priced in USD between 2019 and 2021.[8] Hence, USD plays a significant role in international trades involving Taiwan and other countries. Moreover, year-over-year absolute changes in import and export price indices in USD for Taiwan demonstrate that prices can react more significantly to major economic events, such as the 2008 financial crisis and the 2020 COVID-19 outbreak (Figure 1). These observations suggest that sluggish price adjustments during normal times are endogenous decisions that should not be taken for granted. In sum, In the calibration exercise, I find the domestic price increase by 0.34% and the wealth of domestic sector in Taiwan reduces by 0.2143% of real GDP if the central bank in Taiwan increases US reserves purchase by 1% and finances FXI by seigniorage.

The remainder of the paper is organized as follows. Section 2 presents the model's structure. Section 3 defines the equilibrium. Section 4 defines and analyzes FXIs and conducts the calibration. Finally, Section 5 concludes. All proofs are provided in the Appendix.

[8] For more details, please refer to the website of the Ministry of Finance (TW): Ministry of Finance (TW).

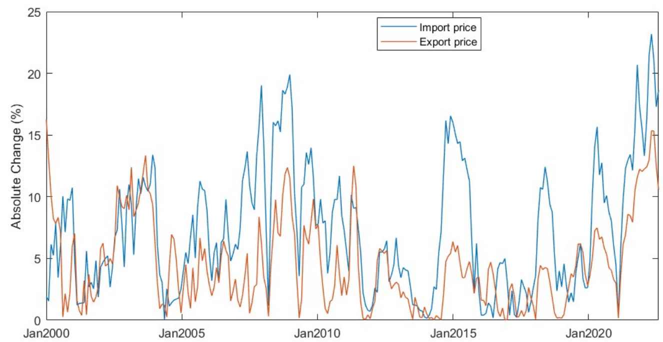

Figure 1: Absolute Import/Export Price Change
Source: Ministry of Finance (TW)

2 The model

The model is based on the framework developed by Lagos and Wright (2005) and Rocheteau and Wright (2005). There are two countries in the economy: the home country, denoted as H, with a population of measure $\mu \in (0,1)$, and the foreign country, denoted as F, with a population of measure 1. Each country has three types of agents with equal share within a country: domestic intermediate goods producers (denoted as A), export intermediate goods producers (denoted as B), and final goods producers (denoted as Z). Throughout the paper, variables and sectors in country F are denoted with an asterisk (*). Besides, the terms "sector" and "producer" are used interchangeably since producers are the only agents residing in their respective sectors.

Time is discrete and indexed by $t = 0, 1, 2, \ldots$, and there are two stages within each period. In stage 1, final goods sectors produce final goods by combining intermediate goods from the domestic sector and the foreign export sector. The production function is given by:

$$F(D,O) = 2[\omega\sqrt{D} + (1-\omega)\sqrt{O}], \tag{1}$$

where $\{D, O\}$ represent the domestic and imported intermediate goods, respectively, and $\omega \in [\frac{1}{2}, 1)$ captures the home bias effect. Thus, the production function for the final goods sector in country H is $F(A, B^*)$, and for country F, it is $F(A^*, B)$.

International trades follow the dominant currency paradigm (DCP) arrangement, where all international trades are priced and settled in the dominant currency issued by country F. Both the export and domestic intermediate goods sectors have the same quadratic cost function, with labor as the sole input for producing intermediate goods in stage 1. The cost of producing ℓ units of intermediate goods is given by:

$$f_i(\ell) = -\frac{\gamma_i}{2}\ell^2, \quad i \in \{H,\ F\}, \tag{2}$$

where the cost parameter γ_i can differ across countries.

In stage 2, all agents participate in a centralized global competitive market. Intermediate goods producers purchase final goods from final goods producers using the revenue earned in stage 1. All agents have the same utility function, which is linear in the consumption of final goods:

$$u(G) = G, \tag{3}$$

where G represents the units of final goods consumed. Agents discount future periods using the time discount rate $\beta \in (0, 1)$. Goods and assets in the model are perfectly divisible, and all goods perish within a period.

The government in each country issues a single perfectly recognizable and uncounterfeitable asset called reserves.[9] Let $M_{i,t}$ denote the nominal reserve supply **per final goods producer**, which grows at a rate $\sigma_{i,t} = \frac{M_{i,t}}{M_{i,t-1}}$ within country i, where $i \in H, F$ and t represents the period. The monetary authorities in both countries use the gross interest rate on reserves, $R_{i,t+1}$, as a monetary policy tool, and it is paid at the beginning of stage 1 in the next period. The fiscal authorities in both countries passively make a lump-sum nominal transfer $T_{i,t}$ (or a lump-sum nominal tax if negative) **per final goods producer** at the beginning of stage 2 each period, where $i \in H, F$. Furthermore, the monetary authority in country H can conduct a foreign exchange intervention (FXI) to depreciate the exchange rate against country F through open market operations in stage 2.

While it is also possible to consider other interventions, e.g. intended appreciation or FXI from country F, this paper focuses solely on the depreciation intervention from country H to examine whether it harms the dominant currency issuer. Additionally, the tendency for depreciation interventions is more significant compared to appreciation interventions, as documented by Fratzscher

[9] To be precise, this should be called as national debt, including reserves, fiat money, and government bonds. In this model, M represents a weighted average of all types of national debt, and the corresponding interest rate R is the weighted average interest rate.

et al. (2019). Hence, whether country F can conduct FXI and the effect of intended appreciation are not the main focus of this paper. Therefore, it is assumed that only the monetary authority of country H can conduct depreciation FXIs.

Trade credit between final goods producers and intermediate goods producers is infeasible.[10] As a result, agents require reserves on the spot to facilitate transactions. Specifically, at stage 1, final goods producers need reserves to pay intermediate goods producers for intermediate inputs purchases, while intermediate goods producers purchase final goods at stage 2 using the revenue they earned. All international trades rely on reserves issued by country F, following the DCP arrangement, while domestic transactions are paid with domestic reserves. Figure 2 illustrates the timeline for agents in both countries.

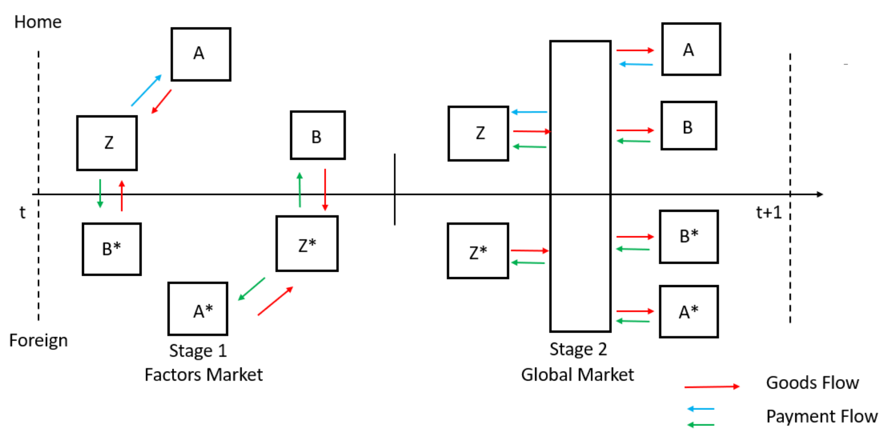

Figure 2: Timeline

3 Equilibrium

In this section, I derive the equilibrium conditions. Since agent's decision making involves intra- and inter-temproal problems, it is convenient to denote variable in the next period as $\widehat{x}$ if it is denoted as x at current period. In the following, I start from country H then country F and solve backwards for both countries.

[10]This is a simplified and ad-hoc assumption. Alternatively, trade credit between agents could be allowed, but due to imperfect enforcement of debt settlement, such as the huge cost of international lawsuits, sellers would require collateral in the form of reserves, leading to the same result.

3.1 Country *H*

Stage 2 problem. Consider an agent in sector $j \in \{A, B, Z\}$ at the beginning of stage 2 with asset portfolio (m_H, m_F, g), which represent Home and Foreign reserve holdings, and final goods produced at stage 1. Agent's value function $W_j(m_H, m_F, g)$ satisfies

$$W_j(m_H, m_F, g) = \max_{G \geq 0,\ \widehat{m}_H \geq 0,\ \widehat{m}_F \geq 0} G + \beta V_j(\widehat{m}_H, \widehat{m}_F)$$
$$s.t. \qquad G = g + \frac{m_H - \widehat{m}_H/\widehat{R}_H + T_H/3}{P_H} + \frac{m_F - \widehat{m}_F/\widehat{R}_F}{P_F}$$

where P_H and P_F are the nominal price for final goods in terms of reserves H and reserves F respectively and $T_H/3$ is the net lump-sum nominal transfer from government H and $V_j(\widehat{m}_H, \widehat{m}_F)$ is the agent's value function at the beginning of next period. The law of one price holds and hence the nominal exchange rate is defined as $e = \frac{P_H}{P_F}$. Substituting the constraint into value function, the maximization problem is rewritten as

$$\begin{aligned} W_j(m_H, m_F, g) &= \frac{m_H}{P_H} + \frac{m_F}{P_F} + g + \max_{\widehat{m}_H \geq 0,\ \widehat{m}_F \geq 0} \left\{ \begin{array}{l} \dfrac{T_H/3}{P_H} - \dfrac{\widehat{m}_H/\widehat{R}_H}{P_H} - \dfrac{\widehat{m}_F/\widehat{R}_F}{P_F} + \\ \beta V_j(\widehat{m}_H, \widehat{m}_F) \end{array} \right\} \\ &= \frac{m_H}{P_H} + \frac{m_F}{P_F} + g + W_j(0,0,0), \end{aligned} \tag{4}$$

which suggests that the optimal asset holding decision for the next period is independent of the current asset holdings. The first order conditions are

$$\frac{-1}{P_H\ \widehat{R}_H} + \beta \frac{\partial V_j(\widehat{m}_H, \widehat{m}_F)}{\partial \widehat{m}_H} \leq 0 \text{ binding if } \widehat{m}_H > 0 \tag{5}$$
$$\frac{-1}{P_F\ \widehat{R}_F} + \beta \frac{\partial V_j(\widehat{m}_H, \widehat{m}_F)}{\partial \widehat{m}_F} \leq 0 \text{ binding if } \widehat{m}_F > 0. \tag{6}$$

Stage 1 problem. At stage 1, there are three types of agent facing their own maximization problem. I examine these problems in the following.

Final goods producer, *Z*. The final goods producer, Z, with portfolio (m_H, m_F) takes prices as given and solves his utility maximizing problem, which is

$$V_Z(m_H, m_F) = \max_{A \geq 0,\ B^* \geq 0} W_Z(m_H - P_A A,\ m_F - P_{B^*} B^*, F(A, B^*))$$
$$s.t.\ P_A A \leq m_H,\ P_{B^*} B^* \leq m_F.$$

Because of the linearity at stage 2 value function, the value function is rewritten as

$$\begin{aligned} V_Z(m_H, m_F) = \max_{A \geq 0,\ B^* \geq 0} & \frac{m_H - P_A A}{P_H} + \frac{m_F - P_{B^*} B^*}{P_F} + F(A, B^*) + W_Z(0,0,0) \\ & + \lambda_H \frac{m_H - P_A A}{P_H} + \lambda_F \frac{m_F - P_{B^*} B^*}{P_F} \end{aligned} \tag{7}$$

where $\{\lambda_H,\ \lambda_F\}$ are the non-negative Lagrange multipliers for two DCP constraints. The first order conditions imply

$$\frac{\omega}{\sqrt{A}} \frac{P_H}{P_A} = 1 + \lambda_H; \quad \frac{1-\omega}{\sqrt{B^*}} \frac{P_F}{P_{B^*}} = 1 + \lambda_F$$

Later it will be clear that final goods producers will use all their asset in intermediate goods purchase and hence constraints are binding when inflation rate is large enough. Furthermore, the final good production function ensures the positive usage for both intermediate goods because the marginal productivity becomes positive infinity when the usage of any intermediate goods approaches to 0. This guarantees positive reserve holdings in the stage 2 problem. The result applies to final goods producers in country F for the same reason. The result is summarized in Proposition 1.

Proposition 1. *The purchases of domestic and foreign exported intermediate goods at stage 1 are positive for final goods producers in both countries. The final goods producers hold positive reserves in the end of stage 2 and hence the corresponding first order conditions for portfolio choices at stage 2 are binding.*

Domestic (A) and export intermediate goods sector (B). The agents in the domestic and export intermediate goods sector in country H with portfolio (m_H, m_F) take prices as given and solve utility maximizing problem to decide how much to produce at stage 1, which are

$$V_A(m_H, m_F) = \max_{\ell_A \geq 0} -\frac{\gamma_H}{2}\ell_A^2 + \frac{m_H + P_A \ell_A}{P_H} + \frac{m_F}{P_F} + W_A(0,0,0) \tag{8}$$

$$V_B(m_H, m_F) = \max_{\ell_B \geq 0} -\frac{\gamma_H}{2}\ell_B^2 + \frac{m_H}{P_H} + \frac{m_F + P_B \ell_B}{P_F} + W_B(0,0,0) \tag{9}$$

where stage 2 value functions are rewritten according to linearity. The first order conditions imply

$$\ell_A = \frac{P_A}{P_H \gamma_H}; \quad \ell_B = \frac{P_B}{P_F \gamma_H}. \tag{10}$$

Optimal portfolio. If we substitute (7) into (4) and take derivative with respect to (m_H, m_F), then the first order conditions for final goods producer Z can be rewritten as

$$\frac{\widehat{\Pi}_H}{\widehat{R}_H} = \beta(1+\widehat{\lambda}_H) \tag{11}$$

$$\frac{\widehat{\Pi}_F}{\widehat{R}_F} = \beta(1+\widehat{\lambda}_F) \tag{12}$$

where $\widehat{\Pi}_i \equiv \frac{\widehat{P}_i}{P_i}$, $i \in \{H,\ F\}$ is the inflation rate measured by the price of final goods at stage 2 and $\{\widehat{\lambda}_H,\ \widehat{\lambda}_F\}$ are liquidity premium paid by the final goods producers. Notice that the equal signs in (11) and (12) are the result from Proposition 1. Therefore, as long as the inflation rate is large enough relative to interest rate (i.e. $\frac{\widehat{\Pi}_j}{\widehat{R}_j} > \beta$, $j \in \{H,\ F\}$), the Lagrange multipliers $(\widehat{\lambda}_H, \widehat{\lambda}_F)$ would be positive, which imply final goods producers would spend all the assets on intermediate goods purchase due to complementary slackness conditions at stage 1. If we divide (11) by (12), we get the modified uncovered interest rate parity (UIP) condition

$$\widehat{\Pi}_e \frac{\widehat{R}_F}{\widehat{R}_H} = \frac{1+\widehat{\lambda}_H}{1+\widehat{\lambda}_F} \tag{13}$$

where $\widehat{\Pi}_e = \frac{\widehat{e}}{e}$.

Two things are noteworthy here. First, the economy in country H achieves the first-best outcome when the Lagrange multipliers $\{\widehat{\lambda}_H,\ \widehat{\lambda}_F\}$ are both 0. These conditions imply that $\frac{1}{\beta} = \frac{\widehat{R}_i}{\widehat{\Pi}_i}$, $i \in \{H,\ F\}$, which is the stylized Friedman Rule result among many monetary economic models. What is novel is that this first-best outcome is not guaranteed even if country H implements the Friedman Rule because the real return of reserve F also depends on monetary policy in country F. Second, the modified UIP condition suggests that the deviation from standard UIP condition is driven by the liquidity premium (or equivalently the real rate of return $\frac{\widehat{R}_i}{\widehat{\Pi}_i}$) differences between these two assets. The results are summarized in the Proposition 2.

Proposition 2. *Both countries need to implement Friedman Rule ($\frac{1}{\beta} = \frac{\widehat{R}_i}{\widehat{\Pi}_i}$, $i \in \{H,\ F\}$) at the same time to achieve the first-best outcome in country H. The modified UIP condition will equal standard UIP condition ($\widehat{\Pi}_e \frac{\widehat{R}_F}{\widehat{R}_H} = 1$) if only if the real rate of return on both reserves are the same.*

Nevertheless, I focus on non-trivial binding reserve requirement constraints case which the

Friedman Rule is not implemented in the analysis.[11] In such case, final goods producers pay liquidity premium to hold reserve for the next period intermediate goods purchase. Leaving reserves idle means paying liquidity premium for nothing. To avoid such scenario, final goods producers use all reserve holdings in the intermediate goods purchase. As a result, the demand for intermediate goods are

$$A = \frac{m_H}{P_A}; \quad B^* = \frac{m_F}{P_{B^*}}.$$

The same argument holds for final goods producers in country F.

The first order conditions for final goods producer, Z, at stage 2 can be further rewritten as

$$\frac{\widehat{\Pi}_H}{\widehat{R}_H} = \beta\left(1 + \frac{\omega \widehat{P}_H}{\sqrt{\widehat{m}_H \widehat{P}_A}}\right) \tag{14}$$

$$\frac{\widehat{\Pi}_F}{\widehat{R}_F} = \beta\left(1 + \frac{(1-\omega) \widehat{P}_F}{\sqrt{\widehat{m}_F \widehat{P}_{B^*}}}\right). \tag{15}$$

Meanwhile, the first order conditions for domestic sector, A and export sector, B, at stage 2 are

$$-\frac{\widehat{\Pi}_H}{\widehat{R}_H} + \beta < 0, \quad -\frac{\widehat{\Pi}_F}{\widehat{R}_F} + \beta < 0$$

implying both types of agent will spend all their revenue at stage 1 on final goods purchase and hold no reserves at stage 2.

Government policy and budget. In the country H, monetary authority has two policy tools, interest rate on reserves and open market operation in FX market.[12] The monetary authority can directly set the nominal interest rate R_H on reserves. On the other hand, open market operation at stage 2 is needed if monetary authority wants to implement certain exchange rate policy. Define the net change in FX reserve holdings of monetary authority through open market operation as $\triangle X = (\frac{\widehat{\sigma}_X}{\widehat{R}_F} - 1)X$, where X is the current reserves F holding per final goods producer inside

[11] In reality, we barely see the example of Friedman Rule. From the theoretical point of view, Friedman Rule might not be incentive-feasible. See the related argument in Andolfatto (2013).

[12] The growth rate of reserve $\widehat{\sigma}_H = \frac{\widehat{M}_H}{M_H}$ can be used as monetary tool as well. However, to satisfy government budget constraint, what matters is the ratio $\frac{\widehat{\sigma}_H}{\widehat{R}_H}$. Therefore, the growth rate of reserve $\widehat{\sigma}_H$ can be taken as given and the monetary authority focuses on interest rate policy. The situation in country F is the same.

government H and $\widehat{\sigma}_X \equiv \frac{\widehat{X}}{X}$. Meanwhile, fiscal authority acts passively and sets the net nominal transfer or tax for every citizen equally to satisfied the government's flow budget, which is

$$e \triangle X + T_H = (\frac{\widehat{\sigma}_H}{\widehat{R}_H} - 1)M_H. \tag{16}$$

3.2 Country F

For the country F, the situation is similar except agents have no incentives to hold Home reserves. The maximizing problems for each stage and result are presented below without the derivation if it is similar to the one in country H section.

Stage 2 problem. Consider an agent in sector $j \in \{A^*, B^*, Z^*\}$ at the beginning of stage 2 with Foreign reserves , m_F^*, and final goods g^* produced at stage 1. Agent's value function satisfies

$$\begin{aligned} W_j(m_F^*, g^*) &= \max_{G^* \geq 0,\ \widehat{m}_F^* \geq 0} G^* + \beta V_j(\widehat{m}_F^*) \\ s.t. \qquad G^* &= g^* + \frac{m_F^* - \widehat{m}_F^*/\widehat{R}_F + T_F/3}{P_F} \end{aligned}$$

where $T_F/3$ is the net lump-sum nominal transfer from government F. The argument that the optimal decision of asset holding is independent of current asset holding is applied and the first order condition is

$$\frac{-1}{P_F\ \widehat{R}_F} + \beta \frac{\partial V_j(\widehat{m}_F^*)}{\partial \widehat{m}_F^*} \leq 0 \text{ binding if } \widehat{m}_F^* > 0. \tag{17}$$

Stage 1 problem.

Final goods producer, Z*. The final goods producer, Z^*, with portfolio (m_F^*) takes prices as given and solves his utility maximizing problem, which is

$$\begin{gathered} V_{Z^*}(m_F^*) = \max_{A^* \geq 0,\ B \geq 0} W_{Z^*}(m_F^* - P_{A^*}A^* - P_B B, F(A^*, B)) \\ s.t.\ P_{A^*}A^* + P_B B \leq m_F^*. \end{gathered}$$

Notice that final goods producer, Z^*, only faces one DCP constraint. Again, due to the linearity at stage 2 value function, the problem is rewritten as

$$V_{Z^*}(m_F^*) = \max_{A^* \geq 0,\, B \geq 0} \; F(A^*, B) + \frac{m_F^* - P_{A^*}A^* - P_B B}{P_F} + W_{Z^*}(0,0) + \lambda^* \frac{m_F^* - P_{A^*}A^* - P_B B}{P_F}$$

where λ^* is the Lagrangian multiplier for DCP constraint. The first order conditions suggest

$$1 + \lambda^* = \frac{\omega}{\sqrt{A^*}} \frac{P_F}{P_{A^*}} = \frac{1-\omega}{\sqrt{B}} \frac{P_F}{P_B}.$$

Following the arguments in country H section, final goods producers in country F purchase positive both intermediate goods and hold positive reserves. Besides, the binding DCP constraint implies all reserves will be spent in intermediate goods purchase. For reserve holdings, m_F^*, the optimal consumption is

$$A^* = \frac{m_F^*}{P_{A^*}[1 + (\frac{1-\omega}{\omega})^2 \frac{P_{A^*}}{P_B}]}; \quad B = \frac{m_F^*}{P_B[1 + (\frac{\omega}{1-\omega})^2 \frac{P_B}{P_{A^*}}]}.$$

domestic (A^*) and traded sector (B^*). The agents in the domestic or traded sector in country F with reserve holdings, m_F^*, take prices as given and solve utility maximizing problem to decide how much to produce at stage 1, which are

$$V_{A^*}(m_F^*) = \max_{\ell_{A^*} \geq 0} -\frac{\gamma_F}{2}\ell_{A^*}^2 + \frac{m_F^* + P_{A^*}\ell_{A^*}}{P_F} + W_{A^*}(0,0,0) \tag{18}$$

$$V_{B^*}(m_F^*) = \max_{\ell_{B^*} \geq 0} -\frac{\gamma_F}{2}\ell_{B^*}^2 + \frac{m_{F^*} + P_{B^*}\ell_{B^*}}{P_F} + W_{B^*}(0,0,0) \tag{19}$$

where stage 2 value functions are rewritten according to linearity. The first order conditions imply

$$\ell_{A^*} = \frac{P_{A^*}}{P_F \gamma_F}; \quad \ell_{B^*} = \frac{P_{B^*}}{P_F \gamma_F}. \tag{20}$$

Optimal asset holding. Follow the same argument in country H section, only final goods producers, Z^*, will hold reserves at stage 2 and others will spend their revenue received from stage 1 at stage 2. With binding budget constraint, the condition for final goods producers' optimal asset holding, m_F^*, is

$$\frac{\widehat{\Pi}_F}{\widehat{R}_F} = \beta \left(1 + \frac{\widehat{P}_F \sqrt{\frac{\omega^2}{\widehat{P}_{A^*}} + \frac{(1-\omega)^2}{\widehat{P}_B}}}{\sqrt{\widehat{m}_F^*}} \right). \tag{21}$$

Government policy and flow budget. In the country F, monetary authority can directly set the nominal interest rate R_F on reserves. Meanwhile, fiscal authority sets the net nominal transfer or tax for every citizen equally. The government's flow budget must be satisfied, which is

$$T_F = (\frac{\widehat{\sigma}_F}{\widehat{R}_F} - 1)M_F. \tag{22}$$

3.3 Competitive equilibrium

The optimal conditions for final goods producers' reserve holding decision and the evolution of nominal exchange rate are below.

$$\begin{aligned}
\frac{\widehat{\Pi}_H}{\widehat{R}_H} &= \beta\left(1 + \frac{\omega \widehat{P}_H}{\sqrt{\widehat{m}_H \widehat{P}_A}}\right) && (23)\\
\frac{\widehat{\Pi}_F}{\widehat{R}_F} &= \beta\left(1 + \frac{(1-\omega)\widehat{P}_F}{\sqrt{\widehat{m}_F \widehat{P}_{B^*}}}\right) && (24)\\
\frac{\widehat{\Pi}_F}{\widehat{R}_F} &= \beta\left(1 + \frac{\widehat{P}_F\sqrt{\frac{\omega^2}{\widehat{P}_{A^*}} + \frac{(1-\omega)^2}{\widehat{P}_B}}}{\sqrt{\widehat{m}^*_F}}\right) && (25)\\
\widehat{\Pi}_e &= \frac{\widehat{\Pi}_H}{\widehat{\Pi}_F} && (26)
\end{aligned}$$

Meanwhile, the market clearing conditions for intermediate goods markets at stage 1 for the next period are listed in order below

$$\begin{aligned}
\frac{\widehat{P}_A}{\widehat{P}_H\gamma_H} &= \frac{\widehat{m}_H}{\widehat{P}_A} && (27)\\
\mu\frac{\widehat{P}_B}{\widehat{P}_F\gamma_H} &= \frac{\widehat{m}^*_F}{\widehat{P}_B[1 + (\frac{\omega}{1-\omega})^2\frac{\widehat{P}_B}{\widehat{P}_{A^*}}]} && (28)\\
\frac{\widehat{P}_{A^*}}{\widehat{P}_F\gamma_F} &= \frac{\widehat{m}^*_F}{\widehat{P}_{A^*}[1 + (\frac{1-\omega}{\omega})^2\frac{\widehat{P}_{A^*}}{\widehat{P}_B}]} && (29)\\
\frac{\widehat{P}_{B^*}}{\widehat{P}_F\gamma_F} &= \mu\frac{\widehat{m}_F}{\widehat{P}_{B^*}} && (30)
\end{aligned}$$

Combine (28) and (30), the terms of trade (TOT) for country F defined as

$$\eta = \frac{\widehat{P}_{B^*}}{\widehat{P}_B} = \mu \left[\frac{\gamma_F}{\gamma_H} \frac{\widehat{m}_F}{\widehat{m}_F^*} \left[1 + (\frac{\omega}{1-\omega})^2 \frac{\widehat{P}_B}{\widehat{P}_{A^*}} \right] \right]^{1/2}. \tag{31}$$

Then substitute (24) and (25) into above equation, we get

$$\eta = \mu^{\frac{2}{3}} (\frac{\gamma_F}{\gamma_H})^{\frac{1}{3}}. \tag{32}$$

The TOT is unresponsive to the changes in nominal exchange rate against dominant currency under DCP pricing aligns with the finding in Gopinath et al. (2020). Define $\chi = \frac{(1-\omega)^2 P_{A^*}}{(1-\omega)^2 P_{A^*} + \omega^2 P_B}$, which is the expenditure share of final goods producers Z^* on foreign exported intermediate goods B. By using (28) and (29), it shows this ratio is unresponsive to nominal exchange rate as well

$$\chi = \frac{(1-\omega)^{4/3} \gamma_F^{1/3}}{(1-\omega)^{4/3} \gamma_F^{1/3} + \omega^{4/3} (\gamma_H/\mu)^{1/3}}. \tag{33}$$

The government nominal flow budget constraints are satisfied such that

$$e \triangle X + T_H = (\frac{\widehat{\sigma}_H}{\widehat{R}_H} - 1) M_H \tag{34}$$

$$T_F = (\frac{\widehat{\sigma}_F}{\widehat{R}_F} - 1) M_F. \tag{35}$$

Finally, reserve supply should meet reserve demand

$$\widehat{M}_H = \widehat{m}_H \tag{36}$$

$$\widehat{M}_F - \mu \widehat{X} = \widehat{m}_F^* + \mu \, \widehat{m}_F. \tag{37}$$

Notice that (37) implies the net supply of Foreign reserves depends both on monetary policy in country F and FXI in country H.

The definition of equilibrium is following.

Definition 1. *Given initial conditions* $\{P_{H,0},\ P_{F,0},\ P_{A,0},\ P_{B,0},\ P_{A^*,0},\ P_{B^*,0},\ m_{H,0},\ m_{F,0},\ m^*_{F,0},\ X_0,\ M_{H,0},\ M_{F,0}\}$ *and, the policy paths including reserves growth* $\{\sigma_{H,t},\ \sigma_{X,t},\ \sigma_{F,t}\}_{t\geq 1}$*, interest rate* $\{R_{H,t},\ R_{F,t}\}_{t\geq 1}$*, and net nominal transfer* $\{T_{H,t},\ T_{F,t}\}_{t\geq 1}$*, the prices sequence* $\{e_t,\ P_{H,t},\ P_{F,t}\ P_{A,t},\ P_{B,t}.\ P_{A^*,t},\ P_{B^*,t}\}_{t\geq 1}$ *such that*

- *intermediate goods market clearing conditions (27) - (30) are satisfied;*
- *the reserves demand derived from (23) - (25) meets the supply and market clearing conditions (36) - (37) are satisfied;*
- *path of exchange rate follows (26);*
- *government flow budget constraints (34) - (35) are satisfied.*

3.4 Benchmark stationary equilibrium

For the rest of paper, I focus on stationary equilibrium and pick a particular stationary equilibrium without FXI from country H and without government transfers in both countries as the benchmark. Then I study the effect of FXI by comparing to the equilibrium result after FXI to the benchmark equilibrium in the next section. The following conditions characterize any stationary equilibrium in this model.

Proposition 3. *(Stationary equilibrium conditions) In any stationary equilibrium under fixed policy parameters* $\left\{\sigma_H,\ R_H,\ \frac{X}{P_F},\ \sigma_F,\ R_F\right\}$, *the equilibrium conditions include:*
Inflation rates, growth rate of reserve holdings X and the evolution of nominal exchange rate

$$\Pi_H = \sigma_H; \quad \Pi_F = \sigma_F = \sigma_X; \quad \Pi_e = \frac{\sigma_H}{\sigma_F}.$$

Relative prices

$$\frac{P_A}{P_H} = (\gamma_H)^{1/3}\left(\frac{\omega}{\frac{\sigma_H}{\beta R_H}-1}\right)^{2/3}; \qquad \frac{P_B}{P_F} = \left(\frac{\gamma_H}{\mu}\right)^{1/3}\left(\frac{1-\omega}{\frac{\sigma_F}{\beta R_F}-1}\right)^{2/3}$$

$$\frac{P_{A^*}}{P_F} = (\gamma_F)^{1/3}\left(\frac{\omega}{\frac{\sigma_F}{\beta R_F}-1}\right)^{2/3}; \qquad \frac{P_{B^*}}{P_F} = (\mu\gamma_F)^{1/3}\left(\frac{1-\omega}{\frac{\sigma_F}{\beta R_F}-1}\right)^{2/3}$$

Real reserves demand from final goods producers at stage 2

$$\frac{m_H}{P_H} = \left(\frac{1}{\gamma_H}\right)^{1/3}\left(\frac{\omega}{\frac{\sigma_H}{\beta R_H}-1}\right)^{4/3}; \quad \frac{m_F}{P_F} = \left(\frac{1}{\mu\gamma_F}\right)^{1/3}\left(\frac{1-\omega}{\frac{\sigma_F}{\beta R_F}-1}\right)^{4/3};$$

$$\frac{m_F^*}{P_F} = \left[\omega^{4/3}\left(\frac{1}{\gamma_F}\right)^{1/3} + (1-\omega)^{4/3}\left(\frac{\mu}{\gamma_H}\right)^{1/3}\right]\left(\frac{1}{\frac{\sigma_F}{\beta R_F}-1}\right)^{4/3}.$$

Production in intermediate goods market

$$A = \ell_A = \left(\frac{\omega/\gamma_H}{\frac{\sigma_H}{\beta R_H} - 1}\right)^{2/3}; \qquad B = \ell_B = \left(\frac{1}{\mu}\right)^{1/3}\left(\frac{(1-\omega)/\gamma_H}{\frac{\sigma_F}{\beta R_F} - 1}\right)^{2/3}$$

$$A^* = \ell_{A^*} = \left(\frac{\omega/\gamma_F}{\frac{\sigma_F}{\beta R_F} - 1}\right)^{2/3}; \qquad B^* = \ell_{B^*} = (\mu)^{1/3}\left(\frac{(1-\omega)/\gamma_F}{\frac{\sigma_F}{\beta R_F} - 1}\right)^{2/3}.$$

Notice that the equilibrium path of exchange rate follows, $\Pi_e = \frac{\sigma_H}{\sigma_F}$, which can be monotonically increasing, monotonically decreasing, or fixed, and has no effect on real production. Without loss of generality, we take one specific stationary equilibrium as our benchmark stationary equilibrium defined as below.

Definition 2. *(Benchmark equilibrium) In the benchmark stationary equilibrium, FXI is set as 0 and growth rate in reserves and interest rates are equal across countries, which implies $R_H = R_F = \sigma_H = \sigma_F = \sigma_X = \overline{\sigma}$. To accommodate such policy, the government transfers from both countries must be 0.*

4 Foreign Exchange Intervention

In this section, I analyze the effect of FXI conducted by country *H*. Particularly, I want to see whether a FXI changes production decision at stage 1 and redistributes wealth across agents at stage 2. I first consider the one-time intervention and then the persistent intervention.

4.1 One-time intervention

Assume the economy reaches the benchmark stationary equilibrium mentioned in Definition 2 at the end of $t = 0$ and there is a one-time FXI at stage 2 in $t = 1$, i.e. $\sigma_{X,2} > R_F$. It can be further divided into two cases: unanticipated intervention and anticipated intervention. One-time unanticipated intervention is an intervention implemented at stage 2 and agents are unaware of it when agents make production decision at stage 1. In contrast, agents are aware of one-time anticipated intervention stage 2 when they produce at stage 1.

Unanticipated intervention. In this case, agents are unaware of intervention until stage 2 begins. Therefore, the production at stage 1 is unaffected. As it has been demonstrated above, final producer's reserve holdings decisions at stage 2, i.e. equation (23) - (25), are functions of *future* inflation rate and interest rate which remain the same. Hence, final goods producers sell the same units of final goods in exchange of reserves. This leads to a one-time inflation deviation from the original equilibrium inflation rate and moves the exchange rate to a new level. Specifically, there will be a one-time inflation jump such that $\Pi_{H,1} > \overline{\sigma} > \Pi_{F,1}$ at stage 2 period 1 where $\overline{\sigma}$ is the inflation rate for both countries in the benchmark. The exchange rate will stay at a new higher level, i.e. $e_t = e_1 > e_0$ for all $t \geq 1$. However, the new level of exchange rate does not affect production decision because the relative prices across goods remain the same after $t \geq 2$. The changes in path of prices are depicted in Figure 3.

Although the production at period $t = 1$ is unaffected, there exists a wealth redistribution from domestic intermediate goods sector in country H (sector A) to all the other sectors. When stage 2 begins, all sectors carry the predetermined revenue received at stage 1. However, only the revenue of domestic intermediate goods sector in country H (sector A) is in the form of reserves H. Therefore, when there is a one-time unanticipated FXI causing $\Pi_{H,1} > \overline{\sigma} > \Pi_{F,1}$, the purchase power of reserves H relative to reserves F decreases as the inflation rate is higher in terms of Home reserves.

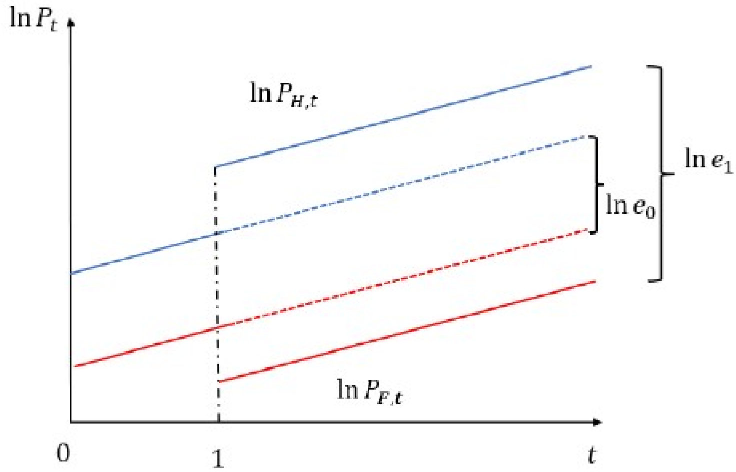

Figure 3: One-time FXI

Anticipated intervention. In this case, government in country H announces there will be a FXI at stage 2 when stage 1 begins. As the unanticipated intervention case, the FXI alters the paths of final goods prices in the same direction, which leads to the same exchange rate path. Therefore, according to optimal labor supply conditions (10) and (20), the production in intermediate good A reduces while the production in intermediate goods B, A^*, and B^* increases at a given price of intermediate good. Notice all agents expect the inflation deviation from the original equilibrium path, and thereby the changes in purchase power of reserves at stage 2. Hence, agents are more willing to produce to exchange for reserves if the purchase power of reserves rises. Vice versa. However, the terms-of-trade and expenditure share on foreign intermediate goods for final goods producer Z^* remain the same due to the general equilibrium effect as (32) and (33) suggest. Consequently, trade deficit (or surplus, if any) for country F in the bilateral trade with country H is unaffected by one-time FXI. The same redistributional effect at stage 2 as unanticipated case still remains. The effect of one-time FXI in the model is summarized in Proposition 4.

Proposition 4. *(One-time FXI) Suppose the economy reaches a stationary benchmark equilibrium defined in Definition 2 at stage 2 in $t = 0$. If there is an unanticipated one-time FXI occurs on stage 2 in $t = 1$, then it induces $\Pi_{H,1} > \overline{\sigma} > \Pi_{F,1}$ and a one-time surge in exchange rate, $e_t = e_1 > e_0$ for $t \geq 1$. Also, there is a redistribution of wealth from sector A to all the other sectors and all* *the production at stage 1 is unaffected. If there is an anticipated one-time FXI occurs on stage 2 in $t = 1$, then it reduces production in intermediate good A and raises production in intermediate goods B, A*, and B*. Meanwhile, the path of exchange rate and the redistribution of wealth are identical to the unanticipated case.*

In NOEM literature, FXI is purely conducted by inflation instead of open market operation. I provide the analysis when a one-time anticipated FXI is implemented by a pure inflation in the Appendix B. In such scenario, the production of domestic sector in country H still decreases while the production of other sectors remain the same. The differences might explain why open market operation instead of pure inflation is commonly used in FXI, which documented in Adler et al. (2021).

4.2 Persistent intervention

If agents are aware of inventions in the future and the interventions consecutively last more than 2 periods, then it is considered as a persistent intervention. Assume the economy starts at the *beginning* of stage 2 in $t = 0$ when centralized global market has not opened yet. Two types of persistent FXI are investigated in this section. The first one is *active* intervention, which means country H conducts a persistent FXI while ceteris paribus. The second one is *passive* intervention, which means country H needs to conduct FXI due to her policy goals such as maintaining a fixed level of real international reserves balance when the monetary policy in country F changes.

Active FXI. To keep the model manageable, I analyze a two-period FXI financed by the combination of Home reserves issuance and lump-sum transfers without changing interest rate. That is, only $\{\sigma_{H,t},\ \sigma_{X,t},\ T_{H,t}\}$ are different from benchmark equilibrium. The details are following.

The economy starts at stage 2 in $t = 0$ and Home government announces that there will be a two-period FXI at stage 2 in $t = 1, 2$. Monetary policy in country F is fixed, which means $\sigma_{F,t+1} = R_{F,t+1} = \overline{\sigma}$ and thereby $T_{F,t} = 0$ for all $t \geq 0$. The interest rate on reserves H is fixed, i.e. $R_{H,t+1} = \overline{\sigma}$ for all $t \geq 0$, and let $\sigma_{HH} = \sigma_{H,t+1} = \sigma_{X,t+1} > \overline{\sigma}$ so that $\triangle X_t > 0$ for $t = 1, 2$ to capture FXI. The lump-sum transfer (or tax if negative) in country H changes accordingly to satisfy the real budget government H constraint in $t = 1, 2$ such that

$$\frac{(\sigma_{HH}/R_F - 1)X_t}{P_{F,t}} + \frac{T_{H,t}}{P_{H,t}} = \left(\frac{\sigma_{HH}}{R_H} - 1\right)\frac{M_{H,t}}{P_{H,t}}.$$

After a two-period FXI, all the policy parameters return to the benchmark setting.

Higher inflation rates in final goods price measured in reserves H is caused by a larger issuance of Home reserves during the FXIs. Meanwhile, the FXIs reduce the growth rate of Foreign reserves, which leads to lower inflation rates in final goods price measured in reserves F. The price dynamic is summarized in Proposition 5.

Proposition 5. *(Price dynamic of active FXI) Consider the economy starts at stage 2 in* $t = 0$ *and Home government announces that there will be a two-period FXI at stage 2 in* $t = 1, 2$*. The changes in policy parameters are* $\sigma_{HH} = \sigma_{H,t+1} = \sigma_{X,t+1} > \overline{\sigma}$ *and* $T_{H,t}$ *for* $t = 1,\ 2$*. The rest parameters remain the same as those in the benchmark equilibrium. In this case, the price dynamic is* $\Pi_{H,1} > \Pi_{H,2} > \overline{\sigma} > \Pi_{F,2} > \Pi_{F,1}$*, and* $\Pi_{H,t} = \Pi_{F,t} = \overline{\sigma}$ *for all* $t \geq 3$*. The intervention induces instant price level changes in country H (F) such that* $\Pi_{H,1} > \Pi_{H,0} > \overline{\sigma} > \Pi_{F,0} > \Pi_{F,1}$*.*

Comparing to the stationary benchmark equilibrium, an active FXI leads to different paths of price levels in both countries and hence different paths of production at stage 1 and nominal exchange rate which are summarized in Corollary 1.

Corollary 1. *Based on the price dynamic in Proposition 5, the active FXI leads to different paths of relative prices and production at stage 1 such that* $\frac{P_{A,1}}{P_{H,1}} < \frac{P_{A,2}}{P_{H,2}} < \frac{P_{A,t}}{P_{H,t}}$*,* $\frac{P_{j,1}}{P_{F,1}} > \frac{P_{j,2}}{P_{F,2}} > \frac{P_{j,t}}{P_{F,t}}$*,* $\ell_{A,1} < \ell_{A,2} < \ell_{A,t}$*, and* $\ell_{j,1} > \ell_{j,2} > \ell_{j,t},\ \ j \in \{B,\ A^*,\ B^*\}$ *where* $t \geq 3$ *and the economy returns to stationary equilibrium at* $t = 3$*. The path of nominal exchange rate alters accordingly, i.e.* $\Pi_{e,1} > \Pi_{e,k} > 1$ *for* $k = 0,\ 2$*.*

As Corollary 1 suggests, the effects at stage 1 production are qualitatively the same as Proposition 4. However, the effect lasts for two periods since the intervention continues for two periods. There are two things noteworthy. First, the policy effect is diminishing although the extent in FXI is the same, i.e. $\sigma_{X,2} = \sigma_{X,3}$. This implies that the extent of FXI needs to be ever-increasing to maintain the same level of policy effect for government H. Second, the price levels ($P_{H,0}$ and $P_{F,0}$) react immediately for the intervention in the future, which leads to an instant change in nominal exchange rate, e_0. Agents in this model are forward-looking. Since the path of price levels will alter due to FXI, final goods producers foresee the real rate of return on reserves changes accordingly and modify their real demand on reserves, causing instant price levels and nominal exchange rate movements even without FXI in $t = 0$.

Passive FXI. Passive intervention means Home central bank changes monetary policy to follow its policy goals in response to the changes in Foreign central bank monetary policy. In this section, I compare another steady state equilibrium with the benchmark equilibrium. Assume the policy goal is to maintain a fixed level of real Foreign reserve balance.[13] However, the Foreign central bank decides to adopt an expansionary monetary policy by raising the growth rate of Foreign reserves while fixing interest rate. Specifically, we have $\sigma_{HH} = \sigma_X = \sigma_F > R_F = \overline{\sigma}$ and thus the transfer in country F becomes positive, i.e. $\frac{T_F}{P_F} > 0$, from government budget constraint (22). Despite the rest policy parameters are unrestricted, I further assume $T_H = 0$, $\sigma_H = \sigma_{HH}$ and let R_H be a free parameter to satisfy government budget constraint (16), which yields additional insights.

In the stationary equilibrium under the new parameter setting, if it exists, real reserve demand from each agent is constant across time. According to Proposition 3, we can find all variables in the new equilibrium. However, we need to verify whether government H can finance such persistent FXI. Combining government budget (34) with Home reserves market clearing condition (36) and agent's reserve holdings decision in the stationary equilibrium (23), the government budget constraint in country H can be rewritten as

$$(\frac{\sigma_X}{R_F} - 1)\frac{X}{P_F} = \underbrace{(\frac{\sigma_H}{R_H} - 1)\left(\frac{1}{\gamma_H}\right)^{1/3}\left(\frac{\omega}{\frac{\sigma_H}{\beta R_H} - 1}\right)^{4/3}}_{\equiv \tau(\frac{\sigma_H}{R_H})} \tag{38}$$

where the left hand side is the expense on FXI and right hand side is seigniorage for government H. Notice that there exists a maximum level of seigniorage and hence the extent of persistent FXI

[13] Central banks might want to maintain a certain level of foreign reserves to lend to citizens for foreign debt settlement. Since central banks can only issue own currency, precautionary foreign reserves accumulation is essential for this purpose. Recently, the Federal Reserve Bank had established temporary swap lines for nine central banks to accommodate their liquidity needs. With these swap lines, precautionary foreign reserves accumulation, at least for US dollars, is unnecessary for these nine central banks. Nevertheless, these swap lines are not common and were ended in July 2021. For details, see Federal Reserve Board announcement.

is limited without the support from fiscal sector. The results are summarized in the Proposition 6 and Figure 4.

Proposition 6. *(Seigniorage Laffer curve) Let* $\delta = \frac{\sigma_H}{R_H}$*. There exists* $\delta^* \in (1, \infty)$ *such that* $\delta^* = \arg\max\ \tau(\delta)$*. In addition, a stationary equilibrium with a persistent FXI without the support from fiscal sector is feasible if only if* $(\frac{\sigma_X}{R_F} - 1)\frac{X}{P_F} \leq \tau(\delta^*)$*.*

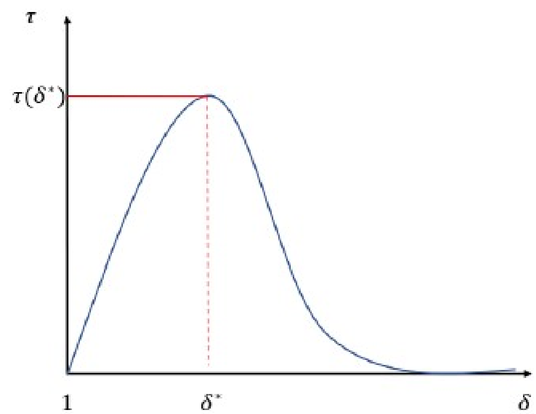

Figure 4: Seigniorage

Generally speaking, government H has two options to finance a persistent FXI, either $\delta < \delta^*$ or $\delta > \delta^*$, when $(\frac{\sigma_X}{R_F} - 1)\frac{X}{P_F} < \tau(\delta^*)$.. Nevertheless, the production is higher in the equilibrium with higher rate of return on reserve H, i.e. $\delta < \delta^*$, and thereby I focus on the case $\delta \in (1, \delta^*)$ for the rest analysis.

Because $\sigma_{HH} = \sigma_H = \sigma_X = \sigma_F$, the nominal exchange rate is fixed. Under the new parameter setting, if the persistent FXI is feasible, then the real effect of a persistent FXI is summarized in the Proposition 7.

Proposition 7. *(Persistently passive FXI) The production in all sectors in the stationary equilibrium with a feasible persistently passive FXI in response to a higher growth rate in Foreign reserves is lower than the production in the benchmark stationary equilibrium defined in Definition 2. Furthermore, the production of domestic sector in country H* (ℓ_A) *declines more as the real Foreign reserve holdings target in government H,* $\frac{X}{P_F}$*, increases.*

Two things in Proposition 7 are noteworthy. First, although country F experiences a lower production at stage 1, agents in country F are subsidized at stage 2 because of positive transfers from government F. Second, central bank in country H might need to adjust interest rate on reserve H to finance a persistent FXI. When the interest rate on reserves, R_H, changes, UIP is generally not held in a stationary equilibrium with persistent FXI, which is presented in the Corollary 2.

Corollary 2. *(Failure of UIP) When the interest rate on reserves, R_H, has to be adjusted to finance a persistently passive FXI, the standard UIP, i.e. $\Pi_e \frac{R_F}{R_H} = 1$, is generally not satisfied.*

4.3 Calibration

In this section, I calibrate the model by using macro monthly data from Taiwan and the US. The data sample period starts from January 2000 to December 2021 unless otherwise stated. To calibrate the benchmark equilibrium defined in Definition 2, there are 9 parameter values need to be assigned. Table 1 summarizes the parameter value and the corresponding source or target. Here are the details of calibration. The time discount rate β targets 2% annual real rate of 3-month T-bils computed using the method in Hamilton et al. (2016). Economic relative scale μ is set to match the average of relative aggregate annual real GDP from 2000-2019. The relative production cost γ_F/γ_H targets the average term-of-trade and import-export value ratio $\frac{P_B^* B^*}{P_B(\mu B)}$ to target average import-export value ratio of Taiwan-US bilateral trade. In addition, I normalize $\gamma_H = 1$. The home bias parameter ω is calibrated by using the ratio of imported expenditure over domestic expenditure $\frac{\chi}{1-\chi}$ from (33) to target the average of imported expenditure from Taiwan to US. For the reserve growth rate $\sigma_H,\ \sigma_F$, I use the average annual growth rate in M2 in both countries. The growth rate of USD reserve holding in Taiwanese central bank σ_X equals σ_F as the Proposition 3 requires. For the rate of return on reserves $R_H,\ R_F$, I use average annualized overnight interbank rate in both countries. The real balance of USD reserve holding in Taiwan $\frac{X}{P_F}$ is calculated through (38) after substituting all the other calibrated parameters.

In this calibrated parameter setting, if Taiwanese central bank conducts a one-time anticipated intervention by increasing 1% USD reserves purchase and finance it by issuing home reserves, then the growth rate of home reserve needs to be raised from the final goods price in NTD (P_H) is raised by 0.34% and the wealth of domestic sector in Taiwan reduces by $(\frac{m_H/P_H(1-\frac{1}{}^{-1})}{m_H/P_H+m_F/P_F})\times 100\% = 0.2143\%$

Parameter	Value	Source
β	0.98	Hamilton et al. (2016)
μ	0.05	Average RGDP
γ_H	1	Normalization
γ_F	0.073	Average import-export ratio
ω	0.84	Average imported expenditure share
σ_H	5.1%	NTD M2 growth
σ_X	7.1%	equals σ_F as Proposition 3
σ_F	7.1%	USD M2 growth
R_H	1.007	Average Taiwanese interbank rate
R_F	1.016	Average US interbank rate
X/P_F	24.48	Calculated from (38)

Table 1: Calibrated parameters

measured by real GDP. Part of this wealth reduction would be transferred to exporter in Taiwan because they hold USD as well. As a result, the portion that transferred to foreign countries such as US would be even smaller. Nevertheless, the cost is still significant for domestic sector in Taiwan.

5 Conclusion

In this paper, I have reexamined the real effects of foreign exchange intervention (FXI) in the context of a two-country model. The intervention analyzed involves a non-dominant currency country depreciating her own currency against a dominant currency country through open market operations. The analysis is conducted under the assumptions that international trade follows the dominant currency paradigm (DCP) and prices are flexible.

In contrast to existing literature, my model predicts that FXI does not lead to an expenditure switching effect. Instead, I find that prices decrease and quantities increase for all goods priced in dominant currency, which include the export sector for both countries. Furthermore, the analysis reveals a wealth redistribution effect from the domestic sector in the non-dominant currency country to the rest of the world. This effect implies the domestic sector as the sole victim of FXI, while sectors in the dominant currency country do not experience negative consequences. In the calibration exercise, I find the domestic price increase by 0.34% and the wealth of domestic sector in Taiwan reduces by 0.2143% of real GDP if the central bank in Taiwan increases US reserves purchase by 1% and finances FXI by seigniorage.

In summary, this paper sheds new insights into the study of FXI. It provides a framework that rationalizes the common practices of FXI observed in the real world. Moreover, it highlights that the domestic sector in the non-dominant currency country, rather than sectors in the dominant currency country, bears the burden of FXI.

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Appendix A - Proof

Proof of Proposition 1

The marginal productivity of any intermediate goods at stage 1 approaches to positive infinity as the usage goes to 0. Therefore, if the prices of intermediate goods at stage 1 are finite, then the usage must be positive in final goods production. Moreover, final goods producers need to prepare reserves at previous stage 2 to facilitate the transactions at current stage 1. Consequently, final goods producers hold a positive amount of reserves at stage 2 and the corresponding first order conditions, (5), (6), are binding. □

Proof of Proposition 2

If both countries implement Friedman Rule, i.e. $\frac{\widehat{R}_H}{\widehat{\Pi}_H} = \frac{\widehat{R}_F}{\widehat{\Pi}_F} = \frac{1}{\beta}$, then the liquidity premium is 0 on both reserves for country H agents. This implies there is no cost to carry one additional unit of reserves to the next period even the additional unit of reserves is idle in the end. In such scenario, final goods producers can buy intermediate goods up to first-best level. On the other hand, the real rates of return on both reserves are the same implies UIP holds

$$\frac{\widehat{R}_F}{\widehat{\Pi}_F} = \frac{\widehat{R}_H}{\widehat{\Pi}_H} \longleftrightarrow 1 + \widehat{\lambda}_F = 1 + \widehat{\lambda}_H \longleftrightarrow \widehat{\Pi}_e \frac{\widehat{R}_F}{\widehat{R}_H} = 1$$

according to (13). □

Proof of Proposition 3

First, in any stationary equilibrium, the real balance holding for final goods producer at the end of stage 2 is constant across time. Therefore, given the fixed Foreign reserve holdings in government H, $\frac{X}{P_F}$, and the asset market clearing conditions (36)-(37), the inflation rates follow

$$\frac{M_H}{P_H} = \frac{\widehat{M}_H}{\widehat{P}_H} \to \sigma_H = \frac{\widehat{M}_H}{M_H} = \frac{\widehat{P}_H}{P_H} = \widehat{\Pi}_H; \qquad \frac{M_F}{P_F} = \frac{\widehat{M}_F}{\widehat{P}_F} \to \sigma_F = \frac{\widehat{M}_F}{M_F} = \frac{\widehat{P}_F}{P_F} = \widehat{\Pi}_F;$$

$$\frac{X}{P_F} = \frac{\widehat{X}}{\widehat{P}_F} \to \frac{\widehat{X}}{X_F} = \frac{\widehat{P}_F}{P_F} = \widehat{\Pi}_F = \sigma_F; \qquad \widehat{\Pi}_e = \frac{\widehat{P}_H/\widehat{P}_F}{P_H/P_F} = \frac{\widehat{\Pi}_H}{\widehat{\Pi}_F} = \frac{\sigma_H}{\sigma_F}.$$

Replace the steady state inflation rates with reserve growth rates and rewrite equilibrium conditions (23)-(25) as

$$\left(\frac{\omega}{\frac{\sigma_H}{\beta R_H}-1}\right)^2=\frac{m_H}{P_H}\frac{P_A}{P_H};\quad \left(\frac{1-\omega}{\frac{\sigma_F}{\beta R_F}-1}\right)^2=\frac{m_F}{P_F}\frac{P_{B^*}}{P_F};\quad \frac{\left(\frac{\omega^2}{P_{A^*}}+\frac{(1-\omega)^2}{P_B}\right)P_F}{\left(\frac{\sigma_F}{\beta R_F}-1\right)^2}=\frac{m_F^*}{P_F}. \tag{39}$$

With these three conditions, we can determine price ratios $\{\frac{P_A}{P_H},\ \frac{P_B}{P_F},\ \frac{P_{A^*}}{P_F},\ \frac{P_{B^*}}{P_F}\}$ through intermediate goods market clearing conditions (27)-(30). With the determined price ratios, we can find real reserve balances at stage 2 through rewritten equilibrium condition (39). Finally, we use price ratios to determine labor supply at stage 1 through intermediate goods producer's optimal conditions (10) and (20). □

Proof of Proposition 4

This proof contains two parts, unanticipated and anticipated interventions. We first prove the unanticipated intervention part.

Unanticipated part. Suppose we reach the stationary benchmark equilibrium defined in Definition 2 at stage 2 in $t=0$. The production decision at stage 1 in $t=1$ follows the original equilibrium path because the intervention at stage 2 in $t=1$ is unanticipated. When the intervention is implemented at stage 2 in $t=1$, the real demand of each reserve at stage 2 in $t=1$ remains unchanged comparing to the benchmark equilibrium because the intervention leaves future policy parameters unchanged. Hence, how many goods final goods producers are willing to sell to exchange for reserves remain unchanged at stationary equilibrium level. However, the quantity of reserves changes in $t=1$ and this will change the equilibrium path of prices. Take Home final goods producers as an example. They sell the same units of final goods at stage 2 in $t=1$ for reserve H and the revenue for them is

$$P_{H,t}Y_{H,t}=\frac{m_{H,t+1}}{R_{H,t+1}}$$

where $Y_{H,t}$ is the final goods sold for reserve H. Apparently, if interest rate is fixed, then the price increases in the quantity of reserve. The same logic can directly apply to find the relationship between $P_{F,t}$ and $\mu m_{F,t+1}+m^*_{F,t+1}$, which is the total demand for reserve F. The price of final goods increases (decreases) in terms of of reserves H (F) because the intervention increases (decreases) reserves H (F) when the interest rate remains unchanged. This leads to a higher (lower) inflation in country H (F) at stage 2 in $t = 1$. Due to law of one price, the exchange rate e_1 has a one-time surge. As for the redistribution effect, we measure their purchase power in reserve F when stage 2 in $t = 1$ begins, which are

$$\frac{m_{H,1}}{e_1}, \text{ producer } A; \qquad \chi m^*_{F,1}/\mu, \text{ producer } B$$
$$(1-\chi)m^*_{F,1}, \text{ producer } A^*; \qquad \mu m_{F,1}, \text{ producer } B^*$$

where $\chi = \frac{(1-\omega)^2 P_{A^*,1}}{(1-\omega)^2 P_{A^*,1} + \omega^2 P^*_{B,1}} \in [0,1]$ is the fixed proportion of reserves that final goods producers in country F spend on imported intermediate goods B. Since there is a one-time surge in exchange rate e_1, the equivalent Foreign reserves level of producers A decreases while there is no change for the rest producers. Therefore, the wealth is redistributed from domestic sector in country H (producer A) to the rest of the world. $\square$

Anticipated part. Follow the argument in unanticipated case, final goods producer's decision is unaffected. Hence, the effect of FXI on final goods price stay the same. Nevertheless, intermediate good producers at stage 1 in $t = 1$ now can anticipate the price change at stage 2. Therefore, according to optimal labor supply conditions (10) and (20), the production in goods A reduces while the production in goods B, A^* and B^* increases for a given price level of each intermediate good. Given the reserve holdings are fixed and DCP constraints are binding for all agents at stage 1, the result is a higher price and a less production in good A market while the opposite is true for good B, A^* and B^* markets comparing to the benchmark. Equation (33) means Foreign final goods producers expenditure shares on goods A^* and B are unchanged. This implies the revenue for intermediate goods producers in stage 1 are the same as unanticipated case. Consequently, the redistributional effect at stage 2 is the same as unanticipated case. $\square$

Proof of Proposition 5

Notice that $R_{H,t+1} = R_{F,t+1} = \overline{\sigma}$ for all $t \geq 0$ and $\sigma_{H,t+1} = \sigma_{X,t+1} = \sigma_{HH} > \overline{\sigma}$ for $t = 1, 2$ in Proposition 5. We first prove the price dynamic in country H.

Country *H*. Combining (23) and (27), we can rewrite the Home reserves demand function as

$$\frac{\Pi_{H,t+1}}{R_H} = \beta\left[1 + \omega\left(\gamma_H\right)^{-1/4}\left(\frac{m_{H,t+1}}{P_{H,t+1}}\right)^{-3/4}\right]. \tag{40}$$

Let the economy restore back to stationary benchmark equilibrium at an arbitrary period $T > 2$, then $\Pi_{H,T+1} = \frac{P_{H,T+1}}{P_{H,T}} = \overline{\sigma}$ and $\frac{m_{H,T+1}}{P_{H,T+1}} = \frac{m_H}{p_H}$, where $\frac{m_H}{p_H}$ is the real remand for reserve H in stationary benchmark equilibrium. Next, we claim that $\Pi_{H,t+1} = \overline{\sigma}$ for all $t \geq 2$. We use contradiction to prove $\Pi_{H,T} = \overline{\sigma}$ first.

Suppose $\Pi_{H,T} \lesseqgtr \Pi_{H,T+1} = \overline{\sigma}$, then $\frac{m_{H,T}}{P_{H,T}} \gtreqless \frac{m_{H,T+1}}{P_{H,T+1}} = \frac{m_H}{p_H}$ which can be seen from (40). It further implies that $\Pi_{H,T+1} = \frac{P_{H,T+1}}{P_{H,T}} \gtreqless \frac{m_{H,T+1}}{m_{H,T}} = \overline{\sigma}$, which contradicts to the fact $\Pi_{H,T+1} = \overline{\sigma}$. Consequently, it must be $\Pi_{H,T} = \overline{\sigma}$. Notice we use reserve H market clearing condition $M_H = m_H$ to find the condition $\frac{m_{H,T+1}}{m_{H,T}} = \overline{\sigma}$. For all $t \in [2, T-1]$, the same argument is applied and hence $\Pi_{H,t+1} = \overline{\sigma}$ for all $t \geq 2$.

Based on the claim above, we now prove that $\Pi_{H,1} > \Pi_{H,2} > \Pi_{H,3} = \overline{\sigma}$. We use contradiction to prove it period by period. Suppose $\Pi_{H,2} \leqslant \Pi_{H,3}$, then real demand function of reserve suggests that

$$\frac{m_{H,2}}{P_{H,2}} \geqslant \frac{m_{H,3}}{P_{H,3}} \to \overline{\sigma} = \Pi_{H,3} = \frac{P_{H,3}}{P_{H,2}} \geqslant \frac{m_{H,3}}{m_{H,2}} = \sigma_{HH},$$

which contradicts the fact $\sigma_{HH} > \overline{\sigma}$. Thus, $\Pi_{H,2} > \Pi_{H,3}$. The revenue of Home reserves for final goods producers in country H at stage 2 is

$$P_{H,t}Y_{H,t} = \frac{m_{H,t+1}}{R_H} \to Y_{H,t} = \frac{\Pi_{H,t+1}}{R_H}\frac{m_{H,t+1}}{P_{H,t+1}} = \frac{\omega^{4/3}\gamma_H^{-1/3}}{(\frac{\Pi_{H,t+1}}{R_H})^{1/3}[1/\beta - \frac{R_H}{\Pi_{H,t+1}}]^{4/3}} \tag{41}$$

where $Y_{H,t}$ is the total final goods supply in exchange for Home reserves and (40) is substituted in the last equality. (41) suggests that $Y_{H,t}$ is decreasing in $\Pi_{H,t+1}/R_H$. Next, we rewrite the inflation in $t = 2$ as

$$\Pi_{H,2} = \frac{P_{H,2}}{P_{H,1}} = \frac{m_{H,3}}{m_{H,2}}\frac{Y_{H,1}}{Y_{H,2}} = \sigma_{HH}\frac{Y_{H,1}}{Y_{H,2}}.$$

From (41), we know $Y_{H,1} < Y_{H,2}$ because $\Pi_{H,2} > \Pi_{H,3}$. Therefore, $\Pi_{H,2} = \sigma_{HH}\frac{Y_{H,1}}{Y_{H,2}} < \sigma_{HH}$. Follow the same procedure, we prove that $\Pi_{H,1} > \Pi_{H,2}$ and $\Pi_{H,1} < \sigma_{HH}$. In conclusion, we have shown $\sigma_{HH} > \Pi_{H,1} > \Pi_{H,2} > \Pi_{H,3} = \overline{\sigma}$.

Finally, we prove that $\Pi_{H,1} > \Pi_{H,0} > \overline{\sigma}$. Suppose $\Pi_{H,0} \geqslant \Pi_{H,1}$, then real demand function of reserve suggests that

$$\frac{m_{H,0}}{P_{H,0}} \leqslant \frac{m_{H,1}}{P_{H,1}} \to \Pi_{H,1} = \frac{P_{H,1}}{P_{H,0}} \leqslant \frac{m_{H,1}}{m_{H,0}} = \overline{\sigma},$$

which contradicts the fact $\Pi_{H,1} > \overline{\sigma}$. Thus, $\Pi_{H,1} > \Pi_{H,0}$. Notice that $\Pi_{H,0} = \frac{m_{H,1}}{m_{H,0}} \frac{Y_{H,-1}}{Y_{H,0}} = \overline{\sigma}\frac{Y_{H,-1}}{Y_{H,0}} > \overline{\sigma}$ because of $\Pi_{H,1} > \Pi_{H,0}$, which implies $\frac{Y_{H,-1}}{Y_{H,0}} > 1$. $\square$

Country *F*. First, we notice that the growth rate in supply of *circulated* Foreign reserves follows that

$$\sigma^{\dagger}_{F,t+1} = \frac{1 - \mu\frac{\sigma_{X,t+1}}{\sigma_F}\frac{X_t}{M_{F,t}}}{1 - \mu\frac{X_t}{M_{F,t}}}\sigma_F$$

where $\sigma_{X,t+1} = \sigma_{HH}$ for $t = 1, 2$ and $\sigma_{X,t+1} = \overline{\sigma} = \sigma_F$ for $t > 2$. Thus, $\overline{\sigma} > \sigma^{\dagger}_{F,2} > \sigma^{\dagger}_{F,3}$ because $\frac{X_2}{M_{F,2}} > \frac{X_1}{M_{F,1}}$ and $\sigma_{HH} > \overline{\sigma}$. Meanwhile, $\sigma^{\dagger}_{F,t+1} = \overline{\sigma}$ for all t except for $t = 1,\ 2$. Following the same argument in country H, we combine (24) - (25) and (28) - (30) and rewrite the Foreign reserves demand function as

$$\frac{\Pi_{F,t+1}}{R_F} = \beta\left[1 + (1-\omega)\left(\mu\gamma_F\right)^{-1/4}\left(\frac{m_{F,t+1}}{P_{F,t+1}}\right)^{-3/4}\right] \tag{42}$$

$$\frac{\Pi_{F,t+1}}{R_F} = \beta\left[1 + \psi_F\left(\frac{m^{*}_{F,t+1}}{P_{F,t+1}}\right)^{-3/4}\right] \tag{43}$$

where $\psi_F \equiv [\frac{\omega^2}{(\gamma_F(1-\chi))^{1/2}} + \frac{(1-\omega)^2}{(\frac{\gamma_H}{\mu}\chi)^{1/2}}]^{1/2}$ and $\chi = \frac{(1-\omega)^2 P_{A*}}{(1-\omega)^2 P_{A*} + \omega^2 P_B} = \frac{(1-\omega)^{4/3}\gamma_F^{1/3}}{(1-\omega)^{4/3}\gamma_F^{1/3} + \omega^{4/3}(\gamma_H/\mu)^{1/3}}$ is constant according to (33). Let the economy restore back to stationary benchmark equilibrium at an arbitrary period $T > 2$, then $\Pi_{F,T+1} = \frac{P_{F,T+1}}{P_{F,T}} = \overline{\sigma}$ and $\frac{m^{\dagger}_{F,T+1}}{P_{F,T+1}} = \frac{m^{\dagger}_F}{P_F}$, where $m^{\dagger}_{F,t} \equiv m^{*}_{F,t} + \mu m_{F,t}$ and $\frac{m^{\dagger}_F}{P_F}$ is the real remand for reserve F in stationary benchmark equilibrium. Next, we claim that for all $t \geq 2$, $\Pi_{F,t+1} = \overline{\sigma}$. We use contradiction to prove $\Pi_{F,T} = \overline{\sigma}$.

Suppose $\Pi_{F,T} \gtrless \overline{\sigma}$, then $\frac{m^{\dagger}_{F,T}}{P_{F,T}} \lessgtr \frac{m^{\dagger}_{F,T+1}}{P_{F,T+1}} = \frac{m^{\dagger}_F}{P_F}$ which can be seen from (42) and (43). Therefore, $\Pi_{F,T+1} = \frac{P_{H,T+1}}{P_{H,T}} \lessgtr \frac{m^{\dagger}_{F,T+1}}{m^{\dagger}_{F,T}} = \overline{\sigma}$, which contradicts to the fact $\Pi_{F,T+1} = \overline{\sigma}$. Consequently, it must be $\Pi_{F,T} = \overline{\sigma}$. Notice we use reserve F market clearing condition $M_F - \mu X = m^{\dagger}_F$ to find the condition $\frac{m^{\dagger}_{F,T+1}}{m^{\dagger}_{F,T}} = \overline{\sigma}$. For all $t \in [2, T-1]$, the same argument is applied and hence $\Pi_{F,t+1} = \overline{\sigma}$ for all $t \geq 2$.

Based on the claim above, we now prove that $\overline{\sigma} = \Pi_{F,3} > \Pi_{F,2} > \Pi_{F,1}$. We use contradiction to prove it period by period. Suppose $\Pi_{F,2} \geqslant \Pi_{F,3}$, then real demand function of reserve suggests that

$$\frac{m^{\dagger}_{F,2}}{P_{F,2}} \leqslant \frac{m^{\dagger}_{F,3}}{P_{F,3}} \to \overline{\sigma} = \Pi_{F,3} = \frac{P_{F,3}}{P_{F,2}} \leqslant \frac{m^{\dagger}_{F,3}}{m^{\dagger}_{F,2}} = \sigma^{\dagger}_{F,3},$$

which is contradicted because $\overline{\sigma} > \sigma^{\dagger}_{F,3}$ due to FXI. Thus, $\Pi_{F,2} < \Pi_{F,3}$. The revenue of Foreign reserves for final goods producers in both countries at stage 2 is

$$P_{F,t}Y_{F,t} = \frac{m^{\dagger}_{F,t+1}}{R_F} \to Y_{F,t} = \frac{\Pi_{F,t+1}}{R_F}\frac{m^{\dagger}_{F,t+1}}{P_{F,t+1}} = \frac{\mu^{2/3}(1-\omega)^{4/3}\gamma_F^{-1/3} + \psi^{4/3}}{(\frac{\Pi_{F,t+1}}{R_F})^{1/3}[1/\beta - \frac{R_F}{\Pi_{F,t+1}}]^{4/3}} \tag{44}$$

where $Y_{F,t}$ is the total final goods supply in exchange for Foreign reserves and (42) and (43) are substituted in the last equality. Next, we rewrite the inflation in $t = 2$ by using (44)

$$\Pi_{F,2} = \frac{P_{F,2}}{P_{F,1}} = \frac{m^{\dagger}_{F,3}}{m^{\dagger}_{F,2}}\frac{Y_{F,1}}{Y_{F,2}} = \sigma^{\dagger}_{F,3}\frac{Y_{F,1}}{Y_{F,2}} > \sigma^{\dagger}_{F,3}$$

because $\Pi_{F,2} < \Pi_{F,3}$ implies $Y_{F,1} > Y_{F,2}$. Next, suppose $\Pi_{F,1} \geqslant \Pi_{F,2}$, then real demand function of reserve suggests that

$$\frac{m^{\dagger}_{F,1}}{P_{F,1}} \leqslant \frac{m^{\dagger}_{F,2}}{P_{F,2}} \to \Pi_{F,2} = \frac{P_{F,2}}{P_{F,1}} \leqslant \frac{m^{\dagger}_{F,2}}{m^{\dagger}_{F,1}} = \sigma^{\dagger}_{F,2},$$

which contradicts the fact $\Pi_{F,2} > \sigma^{\dagger}_{F,3} > \sigma^{\dagger}_{F,2}$. Thus, $\Pi_{F,1} < \Pi_{F,2}$. Follow the same argument, we prove that $\Pi_{F,1} = \sigma^{\dagger}_{F,2}\frac{Y_{F,0}}{Y_{F,1}} > \sigma^{\dagger}_{F,2}$ because of $\Pi_{F,1} > \Pi_{F,2}$. In conclusion, we show $\overline{\sigma} = \Pi_{F,3} > \Pi_{F,2} > \Pi_{F,1}$.

Finally, we prove that $\overline{\sigma} > \Pi_{F,0} > \Pi_{F,1}$. Suppose $\Pi_{F,0} \leqslant \Pi_{F,1}$, then real demand function of reserve suggests that

$$\frac{m^{\dagger}_{F,0}}{P_{F,0}} \leqslant \frac{m^{\dagger}_{F,1}}{P_{F,1}} \to \Pi_{F,1} = \frac{P_{F,1}}{P_{F,0}} \leqslant \frac{m^{\dagger}_{F,1}}{m^{\dagger}_{F,0}} = \overline{\sigma},$$

which contradicts the fact $\overline{\sigma} > \Pi_{F,1}$. Thus, $\Pi_{F,0} > \Pi_{F,1}$. Follow the same argument, we prove that $\Pi_{F,0} = \overline{\sigma}\frac{Y_{F,-1}}{Y_{F,0}} < \overline{\sigma}$ because $\Pi_{F,0} > \Pi_{F,1}$ implies $Y_{F,-1} < Y_{F,0}$. $\square$

Proof of Corollary 1

We combine optimal labor supply conditions at stage 1 (10) and (20) with the demand for intermediate goods (27) - (29) and get

$$\ell_{A,t} = \left(\frac{1}{\gamma_H}\frac{m_{H,t}}{P_{H,t}}\right)^{\frac{1}{2}}; \qquad \ell_{B,t} = \left(\frac{\chi}{\mu\gamma_H}\frac{m^*_{F,t}}{P_{F,t}}\right)^{\frac{1}{2}}$$

$$\ell_{A^*,t} = \left(\frac{1-\chi}{\gamma_F}\frac{m^*_{F,t}}{P_{F,t}}\right)^{\frac{1}{2}}; \qquad \ell_{B^*,t} = \left(\frac{\mu}{\gamma_F}\frac{m_{F,t}}{P_{F,t}}\right)^{\frac{1}{2}}$$

As we showed in the proof of Proposition 5, $\frac{m_{H,t}}{P_{H,t}}$ is negatively correlated with $\frac{\Pi_{H,t}}{R_{H,t}}$ and $\{\frac{m_{F,t}}{P_{F,t}},\ \frac{m^*_{F,t}}{P_{F,t}}\}$ are negatively correlated with $\frac{\Pi_{F,t}}{R_{F,t}}$. Hence, given the interest rates are fixed at $R_{F,t} = R_{H,t} = \overline{\sigma}$ for $t \geq 0$, the result $\Pi_{H,1} > \Pi_{H,2} > \overline{\sigma} > \Pi_{F,2} > \Pi_{F,1}$ from Proposition 5 suggests that $\ell_{A,1} < \ell_{A,2} < \ell_{A,3}$ and $\ell_{j,1} > \ell_{j,2} > \ell_{j,3},\ \ j \in \{B,\ A^*,\ B^*\}$. The production at stage 1 implies $\frac{P_{A,1}}{P_{H,1}} < \frac{P_{A,2}}{P_{H,2}} < \frac{P_A}{P_H}$, $\frac{P_{j,1}}{P_{F,1}} < \frac{P_{j,2}}{P_{F,2}} < \frac{P_j}{P_F}$, $j \in \{B,\ A^*,\ B^*\}$ according to optimal labor supply conditions at stage 1 (10) and (20).

The nominal exchange rate is defined as $e_t = \frac{P_{H,t}}{P_{F,t}}$ and thereby $\Pi_{e,t} = \frac{\Pi_{H,t}}{\Pi_{F,t}}$. The path of inflation rate, i.e. $\Pi_{H,1} > \Pi_{H,k} > \overline{\sigma} > \Pi_{F,k} > \Pi_{F,1}$, implies $\Pi_{e,1} > \Pi_{e,k} > 1$ for $k = 0, 2$. □

Proof of Proposition 6

The right hand side of equation (38) is the seigniorage, which is

$$\tau(\delta) = (\delta - 1)\left(\frac{1}{\gamma_H}\right)^{1/3}\left(\frac{\omega}{\frac{\delta}{\beta} - 1}\right)^{4/3}.$$

The maximum seigniorage can be achieved by setting $\delta^* = \arg\max \tau(\delta) = 4 - 3\beta$ from first order condition. The second order condition, which is

$$\frac{\partial^2 ln\ \tau(\delta)}{\partial^2 \delta}|_{\delta=\delta^*} = -[6(1-\beta)]^{-2} < 0,$$

is also satisfied. $\delta^* > 1$ because $1 > \beta$, which implies $\delta^* \in (1,\ \infty)$.

The left hand side of equation (38) is the expense of a persistent FXI without the support from fiscal sector. Therefore, a persistent FXI without the support from fiscal sector is feasible if and only if $(\frac{\sigma_X}{R_F} - 1)\frac{X}{P_F} \leq \tau(\delta^*)$. □

Proof of Proposition 7

The stationary equilibrium conditions suggest that all real reserve holdings and labor supply (hence production) at stage 1 drop because a lower real rate of return, $\frac{R_j}{\sigma_j}$, $j \in \{H, F\}$, on reserves. However, when the real Foreign reserve holdings, $\frac{X}{P_F}$, in government H is larger, the cost of FXI is higher. To finance a larger cost, central bank in country H needs to reduce R_H to increase seigniorage $\tau(\delta)$, which decreases the real rate of return on reserves H further. Therefore, the domestic sector in country H is less willing to produce intermediate goods ℓ_A in exchange of Home reserve m_H and the production decreases. $\square$

Proof of Corollary 2

In the benchmark stationary equilibrium, the policy parameters follow $R_H = R_F = \sigma_H = \sigma_X = \sigma_F = \overline{\sigma}$ and thus UIP holds, i.e. $\Pi_e \frac{R_F}{R_H} = \frac{\sigma_H / R_H}{\sigma_F / R_F} = 1$. However, in the stationary equilibrium with a persistent FXI, the policy parameters follow $\sigma_{HH} = \sigma_H = \sigma_X = \sigma_F > R_F = \overline{\sigma}$ while R_H needs to be adjusted to satisfy (38). Therefore, R_H generally not equals R_F. If that is the case, UIP does not hold, i.e. $\Pi_e \frac{R_F}{R_H} = \frac{\sigma_H / R_H}{\sigma_F / R_F} = \frac{R_F}{R_H} \neq 1$. $\square$

Appendix B - One-time pure inflation FXI

As the argument in the one-time intervention in section 4.1, the final goods producers sell the same equilibrium quantity of final goods at stage 2 at $t = 1$ because the future inflation rate and interest rate on reserves are the same. Therefore, there will be one-time price level jump if there is a one-time FXI at $t = 1$. However, in the case of pure inflation FXI, country H conducts FXI via pure inflation and hence the path of $P_{F,t}$ remains unaffected while there is a one-time surge in $P_{H,t}$ at $t = 1$. Based on this observation and optimal labor supply at stage 1, i.e. (10) and (20), the production in goods B, A^*, B^* remains the same since the path of $P_{F,t}$ is unchanged while the production in goods A decreases because of higher $P_{H,1}$. The result suggests that open market operation might be a better option comparing to pure inflation when it comes to FXI because there is no production increase in the traded sector in country H while the production decline in the domestic sector in country H remains.