Corporate Finance, Collateralized Borrowing, and Monetary Policy

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Abstract

We construct a monetary model in which entrepreneurs facing uncertainty in input costs and returns of projects may finance investment internally and with bank credit. Entrepreneurs using money as a down payment and bonds as collateral can reduce the default probability. Working through these key channels, lower nominal policy rates and open market sales can reduce the real lending rate. The central bank's private asset purchases improve availability of credit and compress risk spreads. Our model identifies the risk-reducing channel of private asset purchases—the policy functions as if the government had supplied more bonds, and the increased collateralizable bonds are allocated more to corporate borrowings with a higher lending risk. Risk-retention requirements associated with asset purchases are essential to welfare. As uncertainty with respect to input costs and investment returns intensifies, the central bank should lower the optimal risk-retention rate to encourage lending and reduce business failures.

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1 Introduction

We build a general equilibrium model with uncertainty in input costs and returns of investment to study channels through which conventional and unconventional monetary policies affect liquidity, risk spreads, business failures, and investment. Our model captures the decisions businesses face when financing risky projects. Risky projects, such as R&D, are difficult to finance for several reasons, including low rates of success, volatile returns, and uncertainty surrounding input costs, as pointed out by Hall and Lerner (2010). The inherent riskiness of investment projects in particular causes banks to be especially cautious in lending, and to require collateral and higher compensation than otherwise. In times of heightened uncertainty, financial risks increase and the impact on the economy can be severe. During the COVID-19 pandemic, for instance, economic activity was seriously disrupted, and defaults and risk spreads surged.[1] To counter the pandemic's effects on the financial system, central banks around the world made tremendous efforts to provide the market with sufficient liquidity and credit.

Our model features corporate financing choices under various types of uncertainty, which allows us to study the impacts of heightened risk on economic activity and the ways in which different monetary policies might mitigate those impacts. In our model, entrepreneurs may not find suitable suppliers offering the inputs required for investment projects, and access to bank credit is not ensured.[2] These frictions, together with uncertainty about the cost of the inputs that will be required for an investment project, create entrepreneurs' needs for cash holdings (liquid retained earnings) and bank credit. Moreover, the investment return is uncertain and is private information to the entrepreneur, and when borrowers declare default the lender needs to incur a monitoring cost (interpreted as the bankruptcy cost) to observe the return. This friction makes lending risky and external financing difficult; the difficulty, however, can be mitigated if borrowers hold safe assets such as money and government bonds. Money plays a dual role in this economy: An unbanked entrepreneur can finance investment internally, and this is the insurance role; a banked entrepreneur may use money as a down payment, which lowers the default probability to save the expected

[1] According to S&P report, "2020 Annual Global Corporate Default and Rating Transition Study" (April 7, 2021), 226 rated issuers defaulted in 2020, resulting in a rise of global corporate defaults of 91% from 2019. High-yield spreads in the U.S. climbed to almost 10% in March 2020 at the onset of the pandemic.

[2] According to the World Bank Enterprise Surveys, in 2019 33.7% of firms currently have a bank loan or line of credit, and 11.1% of loans are rejected

monitoring cost, and thus, helps the entrepreneur to obtain credit on more favorable terms, and this is the risk-reducing role. Risk-free and fully recognizable government bonds serving as collateral have a similar risk-reducing role as down payments. While money is universally acceptable in this economy, bonds relies on banks' asset transformation. Due to limited access to bank credit, money and bonds are not perfect substitutes, and this feature has crucial implications for open market operations.

Specifically, we introduce corporate finance into a monetary model, as in Rocheteau, Wright, and Zhang (2018), with an intermediary structure featuring costly state verification similar to Williamson (1986). Each entrepreneur chooses a portfolio of money and bonds, and is endowed with an indivisible risky investment project. After the idiosyncratic input cost is realized, entrepreneurs are heterogeneous with respect to whether the input cost is fully covered by their internal funds. Those whose input costs exceed their internal funds can resort to external financing if they have access to bank credit. Due to different input cost realizations, the loan amounts, interest payments, and default probabilities differ across borrowers. The marginal borrowers, those with the largest input cost among those receiving bank credit, bear the highest default risk. In financing investment projects, entrepreneurs exhaust their retained earnings and debt secured by safe assets before they resort to loans backed by risky investment returns that involve monitoring costs. This result is consistent with the pecking-order theory of capital structure in Myers (1984).

We derive an investment threshold such that if an entrepreneur's randomly drawn input cost is above this threshold, the investment project will not be implemented either because banks find it too risky to lend under the circumstances, or the entrepreneur finds it unprofitable to invest even if credit is available. The investment threshold is mainly determined by entrepreneurs' holdings of internal funds and bonds and the severity of credit market frictions. Monetary policy works mainly by affecting entrepreneurs' holdings of money and bonds and the investment threshold. Any policy that raises the investment threshold may result in more investment projects to obtain external financing; however, it does not necessarily increase net output (final output subtracting the aggregate bankruptcy cost) because marginal borrowers have the highest tendency to default.

It is shown that lower nominal policy rates get passed through to the real lending rate and risk spreads by increasing entrepreneurs' cash holdings, which they can use for down payments. This policy also raises aggregate investment and net output. For a given money growth rate, open market sales compress risk spreads and boost aggregate investment by increasing bonds as collateral and reducing lenders' risk exposure. Through this risk-reducing channel, an open market sale can reduce the average lending rate and increase aggregate lending.[3] The result that open market sales can boost the economy is similar to some studies wherein assets are used as means of payment or collateral for consumer credit,[4] while assets in our paper reduce the default risk of corporate collateralized borrowings.

Our model also allows us to study the unconventional monetary policies that were implemented during the past two economic downturns. During the Great Recession and 2019 coronavirus outbreak, several central banks, including the Federal Reserve and the European Central Bank, purchased a large amount of private assets such as bank loans, asset-backed securities, and corporate bonds. In the Main Street Lending Program, for example, the Fed announced it would buy 95 percent of the loans banks had made to qualified firms while the issuing bank would keep a 5 percent stake. Caballero, Farhi, and Gourinchas (2017) argue that if a shortage of safe assets is the main reason behind an economic downturn, then a central bank engaged in quantitative easing should purchase riskier assets. To investigate this issue, the type of unconventional monetary policy we consider is the purchase of private assets.

The private assets purchased by the central bank in the model are bank loans, which can be interpreted as corporate debt or securities backed by collateralized corporate loans. When conducting the policy, the central bank requires that the issuers of private assets retain a certain fraction of the assets. We show that the asset purchase program enlarges the portion of a loan collateralized by bonds for a given borrowing amount; that is, the policy transfers a fraction of the lending risk to the central bank, and thereby incentivizing private banks to lend more. The innovation of the policy is to enhance the efficacy of the risk-reducing role of bonds—it functions as if the government had supplied more safe assets to expand the aggregate stock of collateral; and more importantly, the increased stock of collateral is allocated more to borrowers with higher input

[3] Our result highlights the importance of bonds in their use as collateral in corporate financing and the importance of considering their role when evaluating the effects of monetary policy. Evidence shows that collateralized borrowing is widespread—77% of industrial loans from financial institutions require collateral. See Fan, Nguyen, and Qian (2020). They use the World Bank Enterprise Surveys (WBES) to investigate firms' collateralized borrowing in 131 countries between 2005–2017. They find that the average loan-to-value ratio is about 60%, but when bonds, equities, and other financial assets are pledged as collateral, the loan-to-value is 83%.

[4] See, e.g., Williamson (2012), Andolfatto and Williamson (2015), and Dong and Xiao (2019).

costs, which implies that the central bank gives a larger subsidy to corporate borrowings with a higher lending risk.

A novel result is that risk-retention requirements are essential for welfare under certain circumstances. The financial friction of costly state verification prevents some socially efficient investment projects from being implemented. Private asset purchases make credit available to entrepreneurs with high input costs who could not borrow without the policy, whereas those marginal borrowers bear the highest default cost. The optimal risk-retention rate trades off the benefit of more investment projects being implemented and higher bankruptcy costs. We find that the central bank's purchase of private assets helps compress risk spreads, increase aggregate lending, and reduce business failures. As uncertainty with respect to input costs and investment returns rises, the central bank should lower the risk-retention rate to encourage lending. Finally, we show that the central bank's asset purchases under the optimal risk-retention rate can dominate, in terms of welfare, some policies such as the fiscal authority's direct subsidies to bank lending; moreover, the efficacy of asset purchases does not hinge on whether the central bank foregoes collateral of the loans it purchases.

Related literature. Our model has features related to two strands of the literature. The first includes studies on financial frictions using costly state verification as proposed by Townsend (1979); see, e.g., Williamson (1987), Bernanke and Gertler (1989), and Christiano, Motto, and Rostagno (2014), which are contributions regarding the role of agency problems in business cycles. The second strand includes models with explicit roles for money and liquidity, which are based on Lagos and Wright (2005) and Rocheteau and Wright (2005) (see references in Lagos, Rocheteau, and Wright 2017). A model with explicit roles for money and fully flexible prices is suitable for studying monetary policy, and we generate pass-through without nominal rigidities.

The closely related papers include Rocheteau, Wright, and Zhang (2018), who impose exogenous pledgeability constraints on firms, Imhof, Monnet, and Zhang (2018), who introduce limited commitment by banks and risky loans due to moral hazard, and Bethune et al. (2022), who consider lending relationships to study optimal monetary policy in the aftermath of a crisis. By considering uncertainty in both the input cost and investment returns, we identify the risk-reducing channel through which monetary policy, especially private asset purchases, impacts the economy. Jackson and Madison (2022) study the entry and exit of firms with different ages in the framework of Rocheteau et al. (2018), while in our paper the ex post heterogeneity of firms with respect to loan amount generates different business failures across firms with different sizes of investment.

Our study on financing choices under uncertainty is motivated by the empirical findings which show that firms hold a larger position of safe assets such as cash and government bonds when facing risky investment decisions; see, for instance, Hall (2002), Bates, Kahle, and Stulz (2009), and Brown, Fazzari, and Petersen (2009). Moreover, the inherent riskiness of projects is likely to cause banks to require more collateral. Our paper is also related to research that studies the role (and shortage) of safe assets as collateral and the implications for aggregate liquidity; for example, Caballero, Farhi, and Gourinchas (2017) and Gorton (2017).[5]

Studies using search monetary models with costly state verification include an extension in Williamson (2012) and Chu and Li (2024). In Williamson (2012), entrepreneurs do not hold any assets; in our paper, however, corporate financing is affected by entrepreneurs' asset holdings, and we study how the response of corporate financing decisions and portfolio choices to various shocks and policy shifts impact the effectiveness of monetary policy. While Williamson (2012) considers private asset purchases of a given amount, the central bank in our model chooses the amount of purchases optimally, which allows us to study the design of the optimal risk-retention rate. Chu and Li (2024) study the implication of secured credit for monetary equilibrium wherein risky assets are used as collateral, and the focus is on how increased risk of the pledged asset affects consumers' borrowing costs and asset prices. Another related paper is Greenwood, Sanchez, and Wang (2010), who show that as the efficiency of the monitoring technology increases, the spread between the expected marginal product of capital and its user cost shrinks, a situation which leads to more capital accumulation and higher economic growth. In our model, however, a more efficient monitoring technology may decrease total output because entrepreneurs respond by holding lower real balances.

Studies that feature the complementarity of money and credit include, for example, Rocheteau, Wright, and Zhang (2018), who show that firms holding large amounts of cash rely less on credit, and they negotiate lower interest payments. In Lagos and Zhang (2022), accepting money as

[5] Our approach to risk-free government bonds as collateral to back loans when only a part of a firm's value (or, the expected return of investment) is pledgeable is related to Holmstrom and Tirole's (1998) corporate finance model, which generates endogenous pledgeability constraints to firms.

payment is always an option for producers, and such an outside option serves as a discipline device in negotiating a better terms with financial intermediaries. Our paper contributes a new mechanism to the literature on the complementarity of money and credit through the information channel of costly state verification—money used as a down payment lowers the bank's expected monitoring cost, which raises the availability of credit and also makes credit less expensive.[6]

There is a large literature on the transmission channel of unconventional monetary policy. For example, studies that consider limited enforcement of banks in a New Keynesian model include Gertler and Karadi (2011), who investigate how government credit policy such as direct lending mitigates the disruptions of financial crisis when bank balance sheet constraints are tightened, Gertler, Kiyotaki, and Queralto (2012), who show that the anticipation of the credit policy can make banks to adopt a riskier balance sheet, Correia et al. (2021), who illustrate that the policy of credit subsidies may dominate government direct lending, and Caballero and Simsek (2021), who study how asset purchase programs transfer aggregate risk to central bank's balance sheet and boost aggregate demand by raising asset prices. From a New Monetarist perspective, Williamson (2016) shows that the central bank's purchase of long-term bonds with reserves increases banks' collateral to back the deposit issuance, and Williamson (2018) and Kang (2019) study how the central bank's purchase of mortgages interacts with private incentives to falsify the quality of mortgages. Financial frictions in our model mainly stem from the costly state verification, and we identify the risk-reducing channel through which private asset purchases compress risk spreads, increase aggregate lending, and reduce business failures.

The rest of the paper is organized as follows. The next section presents the model. Section 3 derives properties of the debt contract. In Section 4 we study monetary equilibrium with bank credit. Section 5 considers the conventional money policy. Section 6 studies private asset purchases by the central bank, and we also show that our main findings are robust in various extensions. Section 7 concludes. The appendix contains omitted proofs and derivations of equations, and the online supplementary appendix presents analyses of additional results and extensions.

[6] Studies that consider money and credit as competing means of payment include, for instance, Gu, Mattesini and Wright (2016), who show that increased real balances offset a reduction in credit availability and maintain a certain level of liquidity; moreover, money is not essential when credit is ample.

2 The model

Time is indexed by $t = 0, 1, 2, ...$, and there are two stages within each period. In stage 1, there are two markets: a competitive market for capital and a competitive market for credit; in stage 2, agents settle debt and trade the general good (taken as the numéraire) and assets in a frictionless centralized market. Capital and the general good are perfectly divisible, both of which perished fully within a period.

There are three types of agents—entrepreneurs, suppliers, and banks. Entrepreneurs are potential investors and need capital as the input for implementing investment projects; suppliers can produce capital at unit cost in stage 1; and banks are credit providers. The population of entrepreneurs is one; the population of suppliers is irrelevant due to the constant returns technology to produce capital. The population of banks is small relative to the mass of entrepreneurs who need external financing so that banks can hold diversified portfolios. All agents can produce the general good at unit cost in stage 2. Agents of each type have the same linear utility function in stage 2:

$$u(C, H) = C - H, \tag{1}$$

where C and H are units of the numéraire consumed and produced, respectively. Agents discount across periods according to $\beta = \frac{1}{1+r}$, where $r > 0$ is the rate of time preference.

At the beginning of stage 1 in each period, an entrepreneur is endowed with an indivisible investment project, which requires random units of capital, $\tilde{k}$, to implement. The input, $\tilde{k}$, is distributed according to the distribution function, $F(k)$, with the continuously differentiable density function, $f(k)$, which is strictly positive on $[\underline{k}, \overline{k}] \subset [0, \infty)$. An entrepreneur's investment project becomes obsolete at the end of the period. A project, which requires k units of capital, produces random units of the general good, $\tilde{\omega}$, in stage 2 if k units of capital are used as input and produces zero units otherwise. The project return is drawn from the distribution function, $G(\omega)$. The associated density function, $g(\omega)$, is continuously differentiable and positive on $[\underline{\omega}, \overline{\omega}] \subset [0, \infty)$, with the mean $E(\tilde{\omega}) = \mu_{\omega}$. The random return, $\tilde{\omega}$, and random capital input, $\tilde{k}$, are independent, and both are independently and identically distributed. The realization of $\tilde{\omega}$, denoted by ω, is costlessly observable only to the entrepreneur who implements the project.

Once an entrepreneur's input cost k is realized, she enters the market for capital with probability $\alpha \in (0, 1]$, which captures the frictions involved in searching for the right supplier of the necessary input. The entrepreneur can trade in the credit market with probability $\phi \in (0,1)$, where she can borrow from a bank to finance the purchase of capital. Entrepreneurs with access to banking are called banked entrepreneurs, and those who do not have access to banking are called unbanked entrepreneurs.

In the competitive credit market, borrowers offer contracts that will be evaluated by banks in terms of the expected return they offer, and banks compete for the contract. There is ex post information asymmetry between lenders and borrowers: An entrepreneur's project return is private information, and anyone else needs to incur a monitoring cost to observe the project return. Specifically, if a borrower does not repay the debt, the lender needs to incur a cost (in terms of the numéraire), γ, to observe the realized return of the project. After that, the lender can seize the defaulter's project return and the pledged assets.

Banks make loans to entrepreneurs by issuing redeemable claims, called banknotes, which entrepreneurs use to purchase capital from suppliers. In so doing banks transform illiquid corporate loans into liquid liabilities. Assume that banknotes are perfectly recognizable, and we consider the case in which the issuers of banknotes commit to redeem notes at par in stage 2. In this environment, without loss of generality, we consider that bank loans are one-period contracts.

Suppliers produce capital and trade with entrepreneurs in a competitive market. They accept fiat money and banknotes as means of payment. We assume lack of commitment, enforcement, and record keeping between entrepreneurs and suppliers, and thus trade credit is infeasible. In online supplementary Appendix B we consider bank credit and trade credit, both of which are subject to costly state verification, and bonds are used as collateral in both types of credit. It is shown that monetary policy implications are similar to those in the baseline model.

A government provides two types of perfectly recognizable assets: fiat money and one-period real bonds. Let $M_{s,t}$ denote the nominal money supply per entrepreneur in period t. The fiat money supply grows at a fixed gross rate, σ, according to $M_{s,t} = \sigma M_{s,t-1}$ via lump-sum transfers if $\sigma > 1$, or taxes if $\sigma < 1$, in stage 2. Assume that $\sigma \geq \beta$. Let q_t denote the value of money in terms of the numéraire, and $m_{s,t} = q_t M_{s,t}$ the real balances in period t. The one-period bond is of supply, $z_{s,t}$, per entrepreneur. The price of newly issued bonds in terms of the numéraire is ψ_t. Each unit of bonds pays the bearer one unit of the numéraire in stage 2. The government fully commits to redeem bonds at par, so bonds are considered as risk-free. Assume that bonds cannot be used as a medium of exchange;[7] however, entrepreneurs can collateralize bonds in a debt contract. Assume that lenders incur no cost to seize government bonds if borrowers default.

We summarize the timing of events in Figure 1.

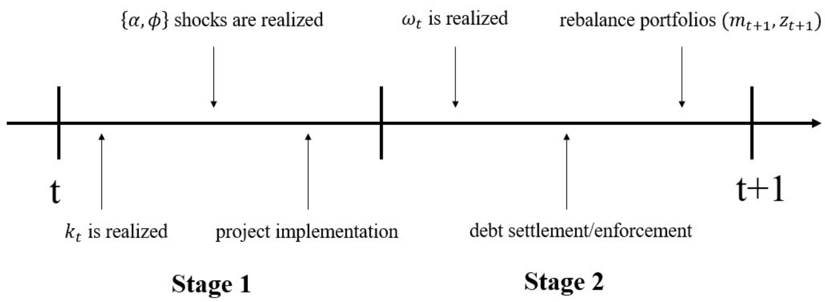


Figure 1: Timing of events

3 The optimal debt contract

We follow Williamson's (1986, 1987) approach to derive debt contracts as optimal arrangements between borrowers and lenders in the competitive credit market (see online supplementary Appendix M for the proof), while we emphasize how entrepreneurs' holdings of internal funds and bonds affect the terms of credit.

We study stationary equilibria in which real balances are constant: $q_t M_{s,t} = q_{t-1} M_{s,t-1} = m_s$, implying that $\sigma = \frac{q_{t-1}}{q_t}$; i.e., the money growth rate equals the inflation rate. Let $V(m, z)$ and $W(m, z)$ denote an entrepreneur's expected life-time utility from entering stages 1 and 2, respectively, where m and z denote an entrepreneur's holdings of real balances and government bonds. Because producing capital requires unit cost and the market is competitive, the equilibrium price of capital is 1 unit of the numéraire. For a project that requires k units of capital to implement, its input cost is k. We refer to the uncertainty concerning the amount of capital that will be required for investment as the input cost uncertainty. We consider $F(\mu_\omega) > 0$, so that potentially profitable projects exist and a risk-neutral entrepreneur would want to implement such projects.

[7] Some studies consider exogenous restrictions to prevent bonds from being used as a means of payment in certain trades, for example, Shi (2008) and Lagos (2010).

Investment with internal funds only. In this case an entrepreneur's randomly drawn input cost, k, is fully covered by her internal funds (real balances), m, and credit is not needed. The entrepreneur pays k with fiat money for capital in stage 1 and obtains the return, ω, in stage 2. Moreover, she receives a lump-sum transfer, T, produces H, consumes C units of the numéraire, and adjusts her holding of real balances, m', and bonds, z', for the next period.

The entrepreneur solves the following problem in stage 2:

$$\max_{\{C,H,m',z'\}} \quad W(m-k,z) = C - H + \beta\,\mathbb{E}\,V(m',z')$$
$$s.t. \quad C = H + m - k + z + T + \omega - \sigma m' - \psi z',$$

where expectations for $\mathbb{E}\,V(m',z')$ are with respect to the input cost and project return. Substituting C from the budget constraint into the objective, we derive the first-order conditions:

$$-\sigma + \beta\,\mathbb{E}\,V_m(m',z') \geq 0, \text{ “ = ” if } \quad m' > 0, \tag{2}$$

$$-\psi + \beta\,\mathbb{E}\,V_z(m',z') \geq 0, \text{ “ = ” if } \quad z' > 0, \tag{3}$$

where $\mathbb{E}\,V_m(m',z')$ and $\mathbb{E}\,V_z(m',z')$ are the marginal expected values of taking an additional unit of real balances and bonds, respectively, into the next period. Conditions (2) and (3) determine the optimal portfolio, (m',z'), independent of the initial holdings of m and z.

3.1 The debt contract

If the randomly drawn input cost is larger than the entrepreneur's internal funds, external financing is necessary for implementing the project. We solve for the debt contract problem and show that, by using more internal funds as a down payment and more bonds as collateral to reduce the lender's exposure to default risk, entrepreneurs can lower the interest payment.

An entrepreneur's default rule. Consider an entrepreneur holding (m,z), endowed with a project $k > m$, who has access to banking. The debt contract, (d,a,x), specifies the down payment $d \leq m$, the units of bonds required as collateral, $a \leq z$, and a non-contingent repayment, x, for the borrowing amount, $\ell = k - d$, which is set to maximize the entrepreneur's expected payoff while satisfying the bank's participation constraint (where d, a, x are in terms of the numéraire).

The entrepreneur, who borrows at stage 1 with the contract, (d,a,x), chooses whether or not to make repayment after the project return, ω, is realized in stage 2. Let $W^j(m-d,z,a,x)$, $j=R,D$, be the expected utility to an entrepreneur who repays or defaults, respectively, and $\mathbb{E}\,V^j(m'^j,z'^j)$, the associated expected utility from entering stage 1 of the next period with m'^j real balances and z'^j bonds. If an entrepreneur repays x, her continuation value is

$$\begin{aligned}\max_{\{m'^R,z'^R\}} \quad & W^R(m-d,z,a,x) = C - H + \beta\,\mathbb{E}\,V^R(m'^R,z'^R)\\ s.t. \quad & C = H + m - d + z + \omega - x + T - \sigma m'^R - \psi z'^R;\end{aligned}$$

if she does not repay, her return and collateral, $\omega + a$, is seized by the bank, and the continuation value is

$$\begin{aligned}\max_{\{m'^D,z'^D\}} \quad & W^D(m-d,z,a,x) = C - H + \beta\,\mathbb{E}\,V^D(m'^D,z'^D)\\ s.t. \quad & C = H + m - d + z - a + T - \sigma m'^D - \psi z'^D.\end{aligned}$$

The optimal portfolio choices satisfy the following first-order conditions, $j=R,D$:

$$-\sigma + \beta\,\mathbb{E}\,V^j_m(m'^j,z'^j) \;\geq\; 0, \text{ “ = ” if } \; m'^j > 0, \tag{4}$$

$$-\psi + \beta\,\mathbb{E}\,V^j_z(m'^j,z'^j) \;\geq\; 0, \text{ “ = ” if } \; z'^j > 0. \tag{5}$$

Because confiscation of project returns and collateral is the only punishment on defaulters, and entrepreneurs are *ex ante* identical before the input cost is realized, $\mathbb{E}\,V^D(\cdot) = \mathbb{E}\,V^R(\cdot)$. Therefore, from (4) and (5), entrepreneurs bring identical portfolios to the next period whether or not they have defaulted. This, together with (2) and (3), implies $m'^D = m'^R = m'$ and $z'^D = z'^R = z'$.

Given optimal asset holdings, (m',z'), we rewrite $W^j(m-d,z,a,x)$ as

$$\begin{aligned}W^R(m-d,z,a,x) &= m - d + z + T + (\omega - x) - \sigma m' - \psi z' + \beta\,\mathbb{E}\,V(m',z'),\\ W^D(m-d,z,a,x) &= m - d + z - a + T - \sigma m' - \psi z' + \beta\,\mathbb{E}\,V(m',z').\end{aligned}$$

An entrepreneur chooses to default if $W^D(m-d,z,a,x) > W^R(m-d,z,a,x)$; that is, default occurs if $\omega + a < x$, the realized project return plus the collateralized bonds is lower than the repayment. Thus, an entrepreneur defaults if

$$\omega < \underbrace{x - a}_{:=y}, \tag{6}$$

where we denote $y = x - a$ and refer it as the *effective repayment*. The default probability is

$$\Pr(\omega < x - a) = \Pr(\omega < y) = G(y).$$

The contract problem. With the default rule described by (6) and the linearity of $W(\cdot)$, the expected utility of an entrepreneur who has borrowed to implement a project is

$$\begin{aligned} W(m-d,z,a,x) &= \int_{x-a}^{\overline{\omega}} W^R(m-d,z,a,x)dG(x) + \int_{\underline{\omega}}^{x-a} W^D(m-d,z,a,x)dG(x) \\ &= -d - a + \int_{x-a}^{\overline{\omega}} \omega\, dG(\omega) - (x-a)[1-G(x-a)] + W(m,z,0,0), \end{aligned} \tag{7}$$

where $W(m,z,0,0)$ is the expected value of forgoing the investment project. An entrepreneur's net expected payoff from borrowing to implement the project k with the contract, (d,a,x), is

$$\pi_e = W(m-d,z,a,x) - W(m,z,0,0).$$

Substituting $y = x - a$, into (7), we rewrite an entrepreneur's net expected payoff as

$$\pi_e(y) = -d - a + \int_{y}^{\overline{\omega}} \omega\, dG(\omega) - y[1-G(y)]. \tag{8}$$

The contract, (d,a,x), gives the bank the net expected payoff from extending a loan, $\ell = k - d$,

$$\pi_b(x) = \int_{x-a}^{\overline{\omega}} x\, dG(\omega) + \int_{\underline{\omega}}^{x-a} (\omega + a - \gamma)dG(\omega) - \ell. \tag{9}$$

The right side of (9) shows that, if $\omega \geq x - a$, the bank receives the repayment x; if $\omega < x - a$, the borrower defaults, and the bank pays the cost, γ, to observe the project return and seize the return, ω, and collateral, a. The last term, $\ell = k - d$, is the amount of banknotes issued for the loan, which is the bank's cost of funds because it needs to redeem banknotes at par. The expected payoff must be non-negative for the bank to accept the contract.

With the notation of the effective repayment, $y = x - a$, we rewrite (9) as

$$\pi_b(y) = \underbrace{y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y)}_{:=B(y)} - \underbrace{(k-d-a)}_{:=\ell_y} \tag{10}$$

As shown in (10), the bank's expected payoff from the original contract, (d,a,x), can be expressed as a payoff from a contract, (d,a,y), with a down payment, $d + a$, the loan amount, $\ell_y = k - d - a$, which is solely backed by the risky project return, $B(y)$. The down payment, d, and collateral, a, play the same role in reducing the lender's risk exposure.[8]

With $\pi_e(y)$ and $\pi_b(y)$ expressed in (8) and (10), we rewrite the original contract problem as:

$$\{d^*, a^*, y^*\} = \arg \max_{\{d,a,y\}} -d - a + \int_y^{\overline{\omega}} \omega \, dG(\omega) - y[1 - G(y)]$$
$$\text{s.t.} \quad \pi_b(y) \geq 0; \; \pi_e(y) \geq 0; \; m \geq d \geq 0; \; z \geq a \geq 0. \tag{11}$$

The solution to the original contract is the down payment, d, collateral, a, and the noncontingent repayment, x, for the loan $\ell = k - d$. The original contract delivers the same outcome as the contract expressed in (11), where the entrepreneur uses the down payment, $d + a$, takes the risky loan, $\ell_y = k - d - a$, and promises to repay y if $\omega \geq y$; otherwise, she defaults, and the bank seizes ω. We will use the contract characterized by (d, a, y) for the following analysis.

Under the intermediary structure with costly state verification, there is an asymmetry in the payoff functions of borrowers and lenders—the entrepreneur's profit is decreasing in y; that is, $\pi_e'(y) = -[1 - G(y)] < 0$ for all $y \in [0, \overline{\omega})$, while the bank's expected payoff is not monotonically increasing in y. This situation creates endogenously credit constraints, which we elaborate below.

First note that since the entrepreneur's profit decreases in y, the bank's participation constraint in (11) must be binding. Substituting $\pi_b(y) = 0$ from (10) into the objective in (11), we have the entrepreneur's expected profit of using credit to invest in a project with input cost k as

$$\pi_e(y; k) = \mu_\omega - k - \gamma G(y), \tag{12}$$

where μ_ω is the mean return of the project.

Next we derive the credit limit, which is the *maximum risky loan amount* that banks are willing to lend, denoted as $\overline{\ell}_{yb}$. Assume that γ is small enough such that $B(y)$ and thereby $\pi_b(y)$ in (10) are strictly concave in y over $[0, \overline{\omega}]$.[9] Thus, there exists a repayment $y_b \in [0, \overline{\omega}]$, such that $B'(y_b) = 0$. The bank's binding participation constraint, $\pi_b(y) = 0$, thus implies that there is a maximum risky

[8] We assume no costs of collateralizing bonds, so real balances and bond collateral play the same role in the optimal contract (but in online supplementary Appendix H we consider costs of collateralizing bonds, and we find main results still hold). Moreover, collateralizing bonds in the model functions like selling bonds to the lender; however, this will not hold if selling bonds involves some costs. As we will show in Section 6, using bonds as collateral is the key to the effectiveness of the central bank's private asset purchases.

[9] The first-order condition is $B'(y) = 1 - G(y) - \gamma g(y)$, and we have $B'(\overline{\omega}) = -\gamma g(\overline{\omega}) < 0$ and $B(0) = 0$. Hence, if γ is small enough that $B'(0) = 1 - \gamma g(0) > 0$, then there exists y_b such that $B'(y_b) = 0$, and so y_b maximizes the bank's expected payoff. Moreover, $B(y_b) > 0$. See Williamson (1987) for a similar assumption.

loan amount, $\bar{\ell}_{yb}$, such that $B(y_b) = \bar{\ell}_{yb}$. That is, if an entrepreneur wishes to take a loan for which the risky loan amount $\ell_y > \bar{\ell}_{yb}$, she will find no repayment that satisfies the bank's participation constraint. Note that $\bar{\ell}_{yb}$ decreases as the monitoring cost rises.

The following proposition establishes the existence and properties of the effective repayment.

Proposition 1. *There exists an effective repayment, y^*, such that $y^* = \min\{y|\pi_b(y) = 0\}$ solves the contract problem, (11), for a given risky loan amount, $\ell_y \in [0, \bar{\ell}_{yb}]$. Furthermore, (i) $y^* = 0$ if $\ell_y = 0$; $y^* > \ell_y$ if $\ell_y > 0$; (ii) $\frac{\partial y^*}{\partial \ell_y} > 0$; (iii) $\frac{\partial y^*}{\partial \gamma} > 0$.*

From Proposition 1, an entrepreneur's borrowing with a zero risky loan amount ($\ell_y = 0$) implies that her loan, $\ell = k - d$, is fully secured by bonds. In this case, the repayment is $x^* = a$, or, equivalently, $y^* = 0$. The result, $\frac{\partial y^*}{\partial \ell_y} > 0$, means that an entrepreneur can reduce the effective repayment and increase her expected payoff by using more internal funds and more collateral to lower the risky loan amount.[10] The reason is that, a lower repayment reduces the default probability and saves the expected bankruptcy cost. For a given ℓ_y, a larger monitoring cost implies a higher expected bankruptcy cost and, thus, the effective repayment, y^*, must rise to satisfy the bank's participation constraint. The following corollary considers a special case when $\gamma = 0$.

Corollary 1. *If there is no monitoring cost ($\gamma = 0$), then (i) $\bar{\ell}_{yb} = \mu_\omega$; (ii) $y^* > \ell_y$ for all $\ell_y \in (0, \bar{\ell}_{yb}]$;*

Corollary 1 shows that, if there is no monitoring cost, $\bar{\ell}_{yb} = \mu_\omega$, which implies all of banked entrepreneurs' potentially profitable projects ($k \leq \mu_\omega$) can obtain external finance.[11] Moreover, the net interest rate is strictly positive ($\frac{y^* - \ell_y}{\ell_y} > 0$) even when $\gamma = 0$.

The following proposition characterizes the optimal down payment and collateral decision for an entrepreneur who needs external finance.

Proposition 2. *Given the input cost, $k > m$, a banked entrepreneur is indifferent between using internal funds as a down payment and collateralized borrowing with bonds. Suppose a banked entrepreneur chooses the down payment, $d^* = m$. The amount of bonds collateralized is $a^* = \min\{z, k - m\}$.*

[10] This result implies that it does not matter whether banks can observe a borrower's asset holdings, as it is to an entrepreneur's best interest to offer as much money and bonds as needed in the debt contract.

[11] The case with $\gamma = 0$ wherein all profitable projects can be financed externally is similar to the unconstrained pledgeability case in Rocheteau, Wright, and Zhang (2018); however, the strictly positive net interest rate in their paper is due to the creditor' bargaining power, while in our paper it is to compensate the creditor's loss due to default.

The intuition underlying Proposition 2 is this. One unit of real balances or bonds can reduce one unit of the risky loan amount, while each asset is valued at one unit of the numéraire in stage 2. That is, both money and bonds have the same benefits in reducing the risky loan amount and identical opportunity costs. Therefore, entrepreneurs are indifferent between using internal funds as a down payment and collateralized borrowing with bonds. Without loss of generality, we consider that a banked entrepreneur uses internal funds before collateralizing bonds.[12] Proposition 2 shows the pecking order of financing investment: Entrepreneurs exhaust retained earnings and debt secured by safe assets before they resort to borrowing backed by risky project returns, which may involve a monitoring cost.

So far we have established the properties of the debt contract. A bank in our model lends to a large number of entrepreneurs and commits to redeem banknotes at par. In equilibrium the bank is perfectly diversified by making loans to a positive mass of entrepreneurs, and it receives a certain return from lending. Specifically, a bank holds a positive mass of loans for a given input cost k, and in equilibrium,

$$\int_{x-a}^{\overline{\omega}} x \, dG(\omega) + \int_{\underline{\omega}}^{x-a} (\omega + a - \gamma) dG(\omega) = k - d.$$

By lending to a large number of entrepreneurs, a bank receives a certain return ex post to redeem banknotes, $\ell = k - d$, at par.[13]

3.2 Credit constraints

By the properties of the contract described in Propositions 1 and 2, whether a banked entrepreneur will implement a project with the input cost $k > m$ depends on two factors: Banks must be willing to extend credit, and in those cases in which they are, the entrepreneur's expected payoff must also

[12] The other pecking order, entrepreneurs collateralizing bonds before using money, is payoff equivalent. Consider $k > \max\{m, z\}$. Proposition 2 implies that the entrepreneur pays the supplier with fiat money m plus banknotes $k - m$. If an entrepreneur collateralizes bonds first, $a^* = z$, then the down payment $d^* = \min\{m, k - z\}$, she pays the supplier with banknotes $\max\{k - m, z\}$ plus fiat money $\min\{m, k - z\}$. Given that banknotes and fiat money have the same value in stage 2, both payment schemes yield the same value, k, and are equally acceptable to the supplier. Moreover, both payment schemes require the same amount of safe assets from the entrepreneur, $d^* + a^* = \min\{m + z, k\}$, and they result in the identical risky loan amount, $\ell_y = k - (d^* + a^*)$, which leads to an identical repayment. Therefore, both pecking orders are payoff equivalent.

[13] In Williamson (1986), depositors collectively act as a perfectly diversified financial intermediary to economize on the monitoring cost. A profit-maximizing financial intermediary will lend to a large number of borrowers, which ensures a certain return to each depositor, so that depositors need not monitor the intermediary. A similar argument applies here.

be positive. First, given the maximum risky loan amount (credit limit), $\bar{\ell}_{yb}$, an entrepreneur with a project $k > m + z + \bar{\ell}_{yb}$, is not able to obtain credit; this threshold is denoted as $k_b = m + z + \bar{\ell}_{yb}$. Second, even if an entrepreneur can offer a contract which is acceptable to a bank, her expected payoff must be positive. We define the threshold, k_e, as $k_e \leq k_b$ (so that credit is feasible) such that it satisfies $\pi_e(y(k_e); k_e) = 0$.

Given the definition of k_e and the fact that $\pi_e(y)$ is decreasing in k, if k_e does not exist, a project with $k \leq k_b$ yields $\pi_e(y; k \leq k_b) \geq 0$. This implies that entrepreneurs with projects $k > k_b$ may find it profitable to invest and are willing to offer a higher repayment, but they cannot obtain credit because the implied risky loan amount is larger than $\bar{\ell}_{yb}$. The entrepreneurs are thus credit-constrained. If k_e exists, banked entrepreneurs can always finance profitable projects, a situation in which entrepreneurs are credit-unconstrained.

Definition 1. *In an equilibrium entrepreneurs are credit-unconstrained if there exists a $k_e \leq k_b$ satisfying $\pi_e(y(k_e); k_e) = 0$, and the investment threshold is $k^T = k_e$; otherwise, some entrepreneurs are credit-constrained, and $k^T = k_b = m + z + \bar{\ell}_{yb}$.*

4 Monetary equilibrium with bank credit

In this section we incorporate the solution to the contract problem into a general equilibrium framework and determine entrepreneurs' optimal portfolios and financing choices. We define some aggregate variables that are key to describing the performance of aggregate economy.

4.1 Optimal portfolios and financing choices

From Proposition 2 and Definition 1, the expected value of an entrepreneur with asset holdings, (m, z), is

$$\mathbb{E}V(m,z) = \alpha\Big[\underbrace{(1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k)}_{\text{(a)}} + \underbrace{\phi\int_{\underline{k}}^{k^T}(\mu_\omega - \gamma G(y^*(k)) - k)dF(k)}_{\text{(b)}}\Big] + W(m,z,0,0), \tag{13}$$

where $y^*(k)$ is the solution to the contract of an entrepreneur with the input cost k. Term (a) in (13) shows that if the entrepreneur is unbanked, her payoff is from financing a project with an input cost lower than internal funds; term (b) is the payoff from obtaining bank credit to finance the project up to k^T, where $k^T = k_b$ or k_e.

From (13), the expected marginal values of real balances and bonds are

$$\mathbb{E}\, V_m(m,z) = \alpha \left\{ (1-\phi)\underbrace{f(m)(\mu_\omega - m)}_{\text{(c)}} + \phi \left\{ \begin{array}{l} \overbrace{f(k^T)\dfrac{\partial k^T}{\partial m}\Big[\mu_\omega - \gamma G(y^\dagger) - k^T\Big]}^{\text{(d)}} \\ \underbrace{-\displaystyle\int_{m+z}^{k^T} \gamma g(y^*)\frac{\partial y^*}{\partial m} dF(k)}_{\text{(e)}} \end{array} \right\} \right\} + 1 \tag{14}$$

$$\mathbb{E}\, V_z(m,z) = \alpha\phi \Big\{ \underbrace{f(k^T)\frac{\partial k^T}{\partial z}\Big[\mu_\omega - \gamma G(y^\dagger) - k^T\Big]}_{\text{(f)}} - \underbrace{\int_{m+z}^{k^T} \gamma g(y^*)\frac{\partial y^*}{\partial z} dF(k)}_{\text{(g)}} \Big\} + 1, \tag{15}$$

where

$$\frac{\partial k^T}{\partial m} = \frac{\partial k^T}{\partial z} = \begin{cases} \frac{\gamma g(y^\dagger)}{1-G(y^\dagger)} < 1 & \text{when } k^T = k_e, \\ 1 & \text{when } k^T = k_b, \end{cases} \tag{16}$$

and $y^\dagger$ is the effective repayment for the marginal borrower with $k = k^T$. Term (c) in (14) depicts the benefit of an additional unit of real balances in expanding the chance to implement a project when the entrepreneur has to rely solely on internal financing. If the entrepreneur has access to bank credit, holding more real balances or government bonds expands the investment threshold, k^T, as depicted in (d) and (f), and reduces the risky loan amount for a given k, which, in turn, decreases the repayment, y^*, as depicted in (e) and (g).

Note that an extra unit of real balances and an extra unit of bonds have identical effects in raising the investment threshold; $\frac{\partial k^T}{\partial m} = \frac{\partial k^T}{\partial z}$, as shown in (16). Moreover, they have identical benefits in reducing the risky loan amount and, consequently, lead to lower repayment, $(\frac{\partial y^*}{\partial m} = \frac{\partial y^*}{\partial z} < 0)$.[14] These effects serve as an important transmission mechanism for monetary policy.

Using (4) and (5) lagged one period to eliminate $\mathbb{E}\, V_m(m,z)$ and $\mathbb{E}\, V_z(m,z)$ from (14) and (15), respectively, we have an entrepreneur's optimal asset holdings satisfying

$$\frac{\sigma}{\beta} = \underbrace{\alpha \Big\{ (1-\phi) f(m)(\mu_\omega - m) + \phi \Big[f(k^T)(\mu_\omega - k^T) - \gamma \int_{m+z}^{k^T} G(y^*) f'(k) dk \Big] \Big\}}_{\text{liquidity premium of money}} + 1, \tag{17}$$

[14] The effective repayment, y^*, is the solution to $\pi_b(y^*) = y^*[1-G(y^*)] + \int_{\underline{\omega}}^{y^*} \omega dG(\omega) - \gamma G(y^*) - (k-m-z) = 0$. Thus, $\frac{\partial y^*}{\partial m} = \frac{\partial y^*}{\partial z} = \frac{-1}{1-G(y^*)-\gamma g(y^*)} < 0$ for all $y^* < y_b$, where y_b satisfies $\pi_b'(y_b) = 0$.

$$\frac{\psi}{\beta} = \underbrace{\alpha\phi\Big\{f(k^T)(\mu_\omega - k^T) - \gamma\int_{m+z}^{k^T} G(y^*)f'(k)dk\Big\}}_{\text{liquidity premium of bond}} + 1. \tag{18}$$

Observing from (17) and (18) that, if $\phi = 1$, money and bonds are perfect substitutes. The (im)perfect substitution between money and bonds is captured as follows. Fiat money is universally accepted by suppliers as a means of payment, while bonds have to rely on banks to be transformed into banknotes; that is, entrepreneurs' access to bank credit affects the liquidity role of bonds. When $\phi < 1$, money enjoys a higher liquidity premium than bonds.

Definition 2. *Given $\{G(\cdot), F(\cdot), \gamma, \sigma, m_s, z_s\}$, a monetary equilibrium with bank credit is (m, ψ), satisfying (17) and (18), the market-clearing conditions for money and bonds, $m = m_s$ and $z = z_s$, and the effective repayment, $y^*(k)$, solves the contract problem (11) for $k \leq k^T$, where $k^T = k_e$ in an equilibrium with unconstrained credit, and $k^T = k_b$ in an equilibrium with constrained credit.*

The following proposition illustrates a monetary equilibrium with credit.[15]

Proposition 3. *Consider that the input cost and project return follow uniform distributions: $k \sim U[0, \epsilon_k]$, $\omega \sim U[0, \epsilon_\omega]$, where $\mu_k = \frac{\epsilon_k}{2}$ and $\mu_\omega = \frac{\epsilon_\omega}{2}$, and $\gamma < \epsilon_\omega$. Suppose $\sigma \geq \beta$. In a monetary equilibrium,*

$$\psi = \beta\Big[1 + \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\Big],\ \bar{\ell}_{yb} = \mu_\omega - \gamma + \frac{\gamma^2}{2\epsilon_\omega},\ y^* = (\overline{\omega} - \gamma) - \sqrt{(\overline{\omega} - \gamma)^2 - 2\epsilon_\omega \ell_y},$$

where $\ell_y = k - m - z$ is the risky loan amount for a given k. There are two cases:
(i) in an equilibrium with constrained credit,

$$\begin{aligned} k^T &= m + z + \bar{\ell}_{yb}, \\ m &= \mu_\omega - \phi(z + \bar{\ell}_{yb}) - \frac{(\sigma - \beta)\epsilon_k}{\alpha\beta}; \end{aligned}$$

(ii) in an equilibrium with unconstrained credit,

$$k^T = \begin{cases} \mu_\omega - \gamma\Big(1 - \sqrt{\frac{2(m+z)}{\epsilon_\omega}}\Big), & \text{if } \sigma > \beta, \\ \mu_\omega, & \text{if } \sigma = \beta, \end{cases}$$

[15] In equilibrium where assets bear strictly positive liquidity premia, $\sigma > \beta$ and $\psi > \beta$, suppliers do not have strictly positive gains from holding money and bonds as a store of value, and thus, they do not have demand for assets. If $\sigma = \beta$, money does not bear liquidity premium, and it does not matter whether suppliers hold money. A similar argument holds for bonds when $\psi = \beta$.

$$m \ solves \begin{cases} (1-\phi)(\mu_\omega - m) + \phi\gamma\left(1 - \sqrt{\frac{2(m+z)}{\epsilon_\omega}}\right) - \frac{(\sigma-\beta)\epsilon_k}{\alpha\beta} = 0, & if\ \sigma > \beta, \\ (1-\phi)(\mu_\omega - m) = 0, & if\ \sigma = \beta. \end{cases} \tag{19}$$

Proposition 3 implies that money and credit can coexist: an entrepreneur may have insufficient internal funds to cover the input cost so external financing is necessary, while monitoring is costly so internal funds are valuable. The inflation rate, σ, is the cost of internal financing, and the monitoring cost, γ, which affects repayment, is the main cost of external financing. The existence of an equilibrium depends on how entrepreneurs trade off the two costs to make financing choices. If the monitoring cost is too high, credit is not feasible. Even if monitoring involves no costs and all potentially profitable projects are eligible for bank credit, as indicated in Corollary 1, entrepreneurs still hold some internal funds since access to credit is not guaranteed ($\phi < 1$). Because inflation makes holding money costly, m decreases in the inflation rate σ. When inflation is low enough, entrepreneurs hold sufficient internal funds, which leads to the following proposition.

Proposition 4. *Under the Friedman rule ($\sigma = \beta$), entrepreneurs hold ample internal funds ($m = \mu_\omega$), and the economy achieves efficiency. External financing is not needed, and bonds are priced at the fundamental value.*

4.2 Aggregate variables

After the idiosyncratic input cost shock is realized, and financing choices are made, entrepreneurs are heterogeneous in terms of whether they invest in a project, the amount of loan taken, and the repayment. We define aggregate variables (the expected value per entrepreneur) that are key to studying the macroeconomic implications of monetary policy.

Firm entry and business failures. Entrepreneurs are regarded as entering the market if they implement a project. The firm entry rate is defined as $N = \alpha[(1-\phi)F(m) + \phi F(k^T)]$. The intermediary structure implies that an entrepreneur who has collateralized all bonds and defaults will lose all asset holdings and project returns, and is considered a business failure. The aggregate default is defined as $D = \alpha\phi \int_{m+z}^{k^T} G(y^*(k))dF(k)$. The business failure rate is the measure of defaulting firms divided by the measure of firms: $\frac{D}{N}$.

Aggregate lending and interest rates. The aggregate lending is defined as total credit, $TL = \alpha\phi \int_m^{k^T} (k-m) dF(k)$. The aggregate repayment is $X = \alpha\phi \int_m^{k^T} x^* \, dF(k)$, where x^* is the repayment of the original contract. The average real lending rate is defined as $R = \frac{X}{TL}$. The real yield of risk-free government bonds is $r_b = \frac{1}{\psi} - 1$, and the nominal bond yield is $\frac{\sigma}{\psi} - 1$. The risk spread is captured by the difference between the lending rate and the risk-free rate which is adjusted with the same time span as loans, $\frac{\beta}{\psi} - 1$; i.e., the risk spread is $S = R - \frac{\beta}{\psi}$.[16] Let υ_b denote the liquidity premium of bond defined in (18). Then, $S = R - (1 + \upsilon_b)$; the risk spread contains two components, a risk premium and a liquidity premium.

Aggregate investment and net output. The aggregate investment is defined as total capital used in implementing projects:

$$K = \alpha\Big[(1-\phi)\int_{\underline{k}}^{m} k \, dF(k) + \phi \int_{\underline{k}}^{k^T} k \, dF(k)\Big],$$

which also equals the suppliers' proceeds. We define net output, Y, as aggregate output subtracting the aggregate bankruptcy costs, $Y = \mu_\omega N - \gamma D$, where γD is total resources used in monitoring the project return of defaulting firms. An entrepreneur's expected profit is

$$P = \alpha\Big\{(1-\phi)\int_{\underline{k}}^{m} (\mu_\omega - k) dF(k) + \phi \int_{\underline{k}}^{k^T} [\mu_\omega - \gamma G(y^*) - k] dF(k)\Big\}. \tag{20}$$

The net output turns out to be equal to the amount of goods consumed by entrepreneurs and suppliers; i.e., $Y = P + K$.

5 Conventional monetary policy

The monetary authority commits to a constant money growth rate, and the fiscal authority issues real bonds. The consolidated government budget constraint is

$$(\sigma - 1) m_s + (\psi - 1) z_s = T + G, \tag{21}$$

where G is government expenditures, T is lump-sum transfers or taxes if $T < 0$, $(\sigma - 1) m_s$ is seigniorage from printing new money, and $(\psi - 1) z_s$ is net revenue from issuing debt. Assume that

[16] In this economy loans are taken in stage 1 and are repaid in stage 2. Each unit of bonds is sold at the price ψ and is redeemed in one unit of the numéraire next period. In defining the risk spread, we use the discounted price of bonds, ψ/β, instead of ψ, to calculate the risk-free rate.

the government employs an accommodating fiscal policy; that is, fiscal policy is passive, with T adjusting to satisfy (21).

The central bank can adjust the money growth rate (inflation), σ, or conduct open market operations as its monetary policy tools. Let i denote the return on illiquid nominal bonds, defined by the Fisher equation, $i = \sigma(1+r) - 1$. One can interpret i as the central bank policy rate. In a stationary monetary equilibrium an open market operation (OMO) that swaps money for bonds has the same real impact as changing only the bond supply (see, e.g., Rocheteau, Wright, and Xiao 2018). Thus, the type of OMOs we consider is that the monetary-fiscal authority changes the supply of government bonds by withdrawing or injecting money, and T is adjusted so that the government continues to issue the new level of bonds forever, and money still grows at the rate σ. Taking the policy rules as given, entrepreneurs determine holdings of real balances and bonds according to (17) and (18).

5.1 Adjusting the nominal policy rate

To gain clear policy implications and intuitions, we assume specific distributions for the input cost and project return in Proposition 5. We also illustrate the monetary policy implications from numerical examples with other specifications of distributions (see Figure 2).

Proposition 5. *Consider the specific distributions, $k \sim U[0, \epsilon_k]$, $\omega \sim U[0, \epsilon_\omega]$, where $\mu_k = \frac{\epsilon_k}{2}$, $\mu_\omega = \frac{\epsilon_\omega}{2}$ and $\gamma < \epsilon_\omega$. In an equilibrium with constrained credit, the effects of adjusting the nominal policy rate, i, are: $\frac{\partial m}{\partial i} < 0$, $\frac{\partial k^T}{\partial i} < 0$, $\frac{\partial N}{\partial i} < 0$, $\frac{\partial r_b}{\partial i} < 0$, $\frac{\partial S}{\partial i} > 0$, $\frac{\partial K}{\partial i} < 0$, $\frac{\partial Y}{\partial i} < 0$, $\frac{\partial P}{\partial i} < 0$, $\frac{\partial TL}{\partial i} = \frac{\partial R}{\partial i} = \frac{\partial D}{\partial i} = 0$.*

The key transmission channel of monetary policy is twofold. First, a higher nominal policy rate, i, raises the cost of holding real balances and reduces internal funds, which lowers unbanked entrepreneurs' liquidity for their purchase of capital, and banked entrepreneurs' down payment. Second, the policy affects the investment threshold—k^T falls because real balances, m, is reduced by higher i. Recall that for credit-constrained entrepreneurs, $k^T = m + z + \bar{\ell}_{yb}$, of which z and $\bar{\ell}_{yb}$ are not changed by the policy rate: $\bar{\ell}_{yb}$ is affected only by the monitoring cost and the distribution of project return ($\bar{\ell}_{yb} = \mu_\omega - \gamma + \frac{\gamma^2}{2\epsilon_\omega}$ by Proposition 3), and z is equal to the fiscal authority's supply of bonds. Consequently, lower internal funds with which unbanked entrepreneurs can finance investment, together with lower k^T for banked entrepreneurs, reduce firm entry, aggregate investment, and net output. As money holdings fall, the demand for bonds to facilitate external financing increases, which drives down the bond yield, r_b, and raises the risk spread.

To gain further insights on policy implications, we consider numerical exercises wherein the input cost, k, follows a lognormal distribution. From Figure 2 we have the following observations.[17] First, equilibria with unconstrained credit exist when the policy rate (or inflation) is low, while when i is above a threshold (indicated by the red line), equilibria feature constrained credit. Second, in both types of equilibria, as the central bank raises the policy rate, aggregate lending and net output fall, and the average lending rate and risk spread rise.

Complementarity of money and credit. Our model features the complementarity of money and credit through the information channel of costly state verification. To illustrate this point, in online supplementary Appendix L we consider a simplified model wherein fiat money is the only asset, and we provide a proposition to demonstrate a monetary equilibrium with credit (similar to Proposition 3). As the proposition clearly shows, entrepreneurs' real balances, m, decrease as the inflation rate σ (or, equivalently, the policy rate i) rises. Lower real balances raise the risky loan amount for a given input cost k, which leads to a higher repayment and default probability; that is, a higher nominal policy rate gets passed through to an individual entrepreneur's real lending rate. Moreover, for entrepreneurs whose input costs are slightly lower than the credit limit, lower real balances may tighten the credit constraint and exclude them from receiving credit. These two effects illustrate that money and credit could be complementary.

Though an individual's lending rate is raised by a higher policy rate, aggregate lending may not always fall. For entrepreneurs with an input cost slightly lower than their real balances, credit is not needed, but a higher policy rate may reduce real balances to such an extent that they need some credit for investment. For entrepreneurs who receive credit, reducing one unit of real balances increases one unit of credit needed. The above two effects imply that money and credit

[17] The followings are parameter settings for numerical examples for Figure 2: $\omega \sim U[0, 18]$, $log(k) \sim N$ with $E[log(k)] = 3$ and $Var[log(k)] = 1$, $\{\alpha, \beta, \gamma, \phi, \sigma, z_s\} = \{0.5, 1/1.02, 15, 0.7, 1.025, 0.75\}$. Figure 2 shows that in a credit-constrained equilibrium aggregate defaults decrease in the policy rate. With the assumed log-normal distribution for the input cost, as i increases, the probability that k occurs in the range, $[m + z, k_b]$, decreases, so that entrepreneurs have a lower chance to take risky loans and thus, the default probability falls. Using an exponential distribution of the input cost, for example, we observe that a raise in the policy rate increases aggregate defaults.

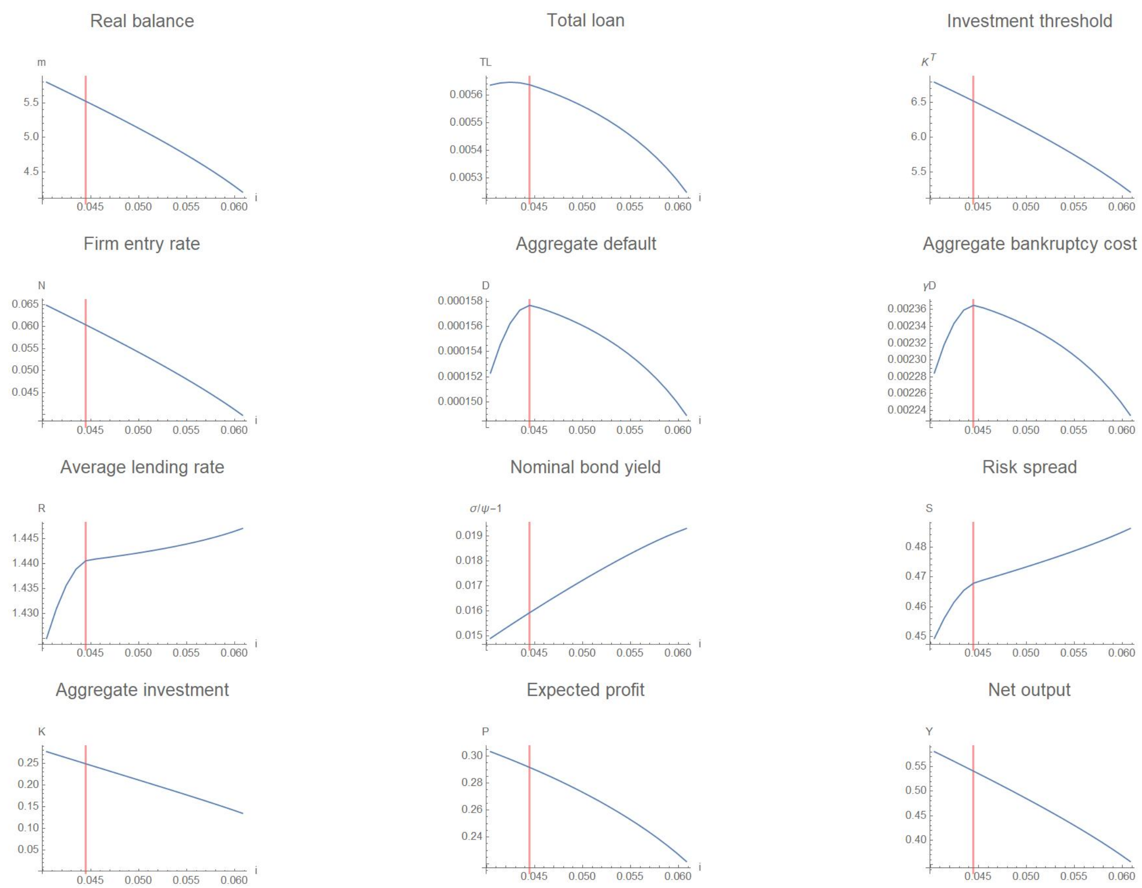


Figure 2: Effects of changes in the nominal policy rate

could be substitutes. Therefore, whether lower policy rates raise aggregate lending depends on the magnitudes of the complementarity effect and the substitutability effect. If the former effect dominates, a higher nominal policy rate reduces aggregate lending; otherwise, aggregate lending can even increase though the pass-through implies a rise in the individual real lending rate.

5.2 Open market operations

The following proposition illustrates the effects of OMOs by assuming a specific distribution, and we also consider numerical examples with other specifications of distributions (see Figure 3).

Proposition 6. *Consider the specific distributions, $k \sim U[0, \epsilon_k]$, $\omega \sim U[0, \epsilon_\omega]$, where $\mu_k = \frac{\epsilon_k}{2}$, $\mu_\omega = \frac{\epsilon_\omega}{2}$ and $\gamma < \epsilon_\omega$. In an equilibrium with constrained credit, the effects of open market operations* *are:* $\frac{\partial m}{\partial z_s} < 0$, $\frac{\partial k^T}{\partial z_s} > 0$, $\frac{\partial TL}{\partial z_s} > 0$, $\frac{\partial R}{\partial z_s} < 0$, $\frac{\partial r_b}{\partial z_s} > 0$, $\frac{\partial S}{\partial z_s} < 0$, $\frac{\partial K}{\partial z_s} > 0$, $\frac{\partial P}{\partial z_s} < 0$, $\frac{\partial N}{\partial z_s} = \frac{\partial D}{\partial z_s} = \frac{\partial Y}{\partial z_s} = 0$.

When the central bank conducts open market sales, entrepreneurs have more bonds pledged as collateral. Proposition 6 shows that as more bonds substitute for money as down payments, entrepreneurs tend to reduce their money holdings, consistent with the evidence that money demand decreases with pledgeability. Notice that $\bar{\ell}_{yb}$ is not affected by OMOs, and a larger z raises k^T even though money holdings decrease. The reason is that money enjoys a higher liquidity premium, and thus claims a higher price, than bonds. When the central bank sells one unit of bonds, it withdraws less than one unit of real balances from the market, and consequently, k^T increases. Proposition 6 shows that open market sales drive up the bond yield, reduce the aggregate lending rate, and compress the risk spread.[18]

Open market sales create two opposing forces for transmission to aggregate output: decreased m implies fewer investment projects can be financed internally, while a higher k^T implies that more entrepreneurs with relatively high input costs can receive bank loans. If the population of entrepreneurs who receive loans due to a higher k^T is larger than those who have to give up investment due to lower internal funds, firm entry and total output increase. The net output, $Y = \mu_\omega N - \gamma D$, however, is also affected by aggregate defaults. Notice that open market sales increase total asset holdings, $m + z$, and lenders do not need to monitor entrepreneurs with $k \leq m+z$ even if they do not repay; on the other hand, a rise in k^T implies that more entrepreneurs with high input costs can borrow and thus, high chances to incur the monitoring cost. If the former effect dominates the latter, aggregate defaults fall and net output rises from an open market sale. Figure 3 shows that when the bond supply is scarce (left of red line), entrepreneurs are credit-constrained, whereas when bonds are plentiful, the aggregate stock of collateral is so abundant that they are credit-unconstrained.[19] In both types of equilibria, aggregate defaults and risk spread decrease,

[18] In this section we conduct a stationary equilibrium analysis for OMOs, wherein the money growth rate is constant, and so is the policy rate. This situation is different from the conventional view that OMOs are conducted to adjust the policy rate. Online supplementary Appendix J studies temporary OMOs which allows the one-period money growth rate to change. We show that temporary open market purchases are expansionary and the policy rate falls.

[19] The followings are parameter settings for numerical examples for Figure 3: $\omega \sim U[0, 18]$, $log(k) \sim N$ with $E[log(k)] = 2.5$ and $Var[log(k)] = 1$, $\{\alpha, \beta, \gamma, \phi, \sigma, z_s\} = \{0.5, 1/1.02, 15, 0.7, 1.045, 0.1\}$.Under assumed uniform distribution, the credit-unconstrained equilibria show that first, higher i raises aggregate lending, the lending rate, and defaults; second, more bonds decrease risky loan amounts to such an extent that defaults and aggregate bankruptcy costs fall, and net output rises.

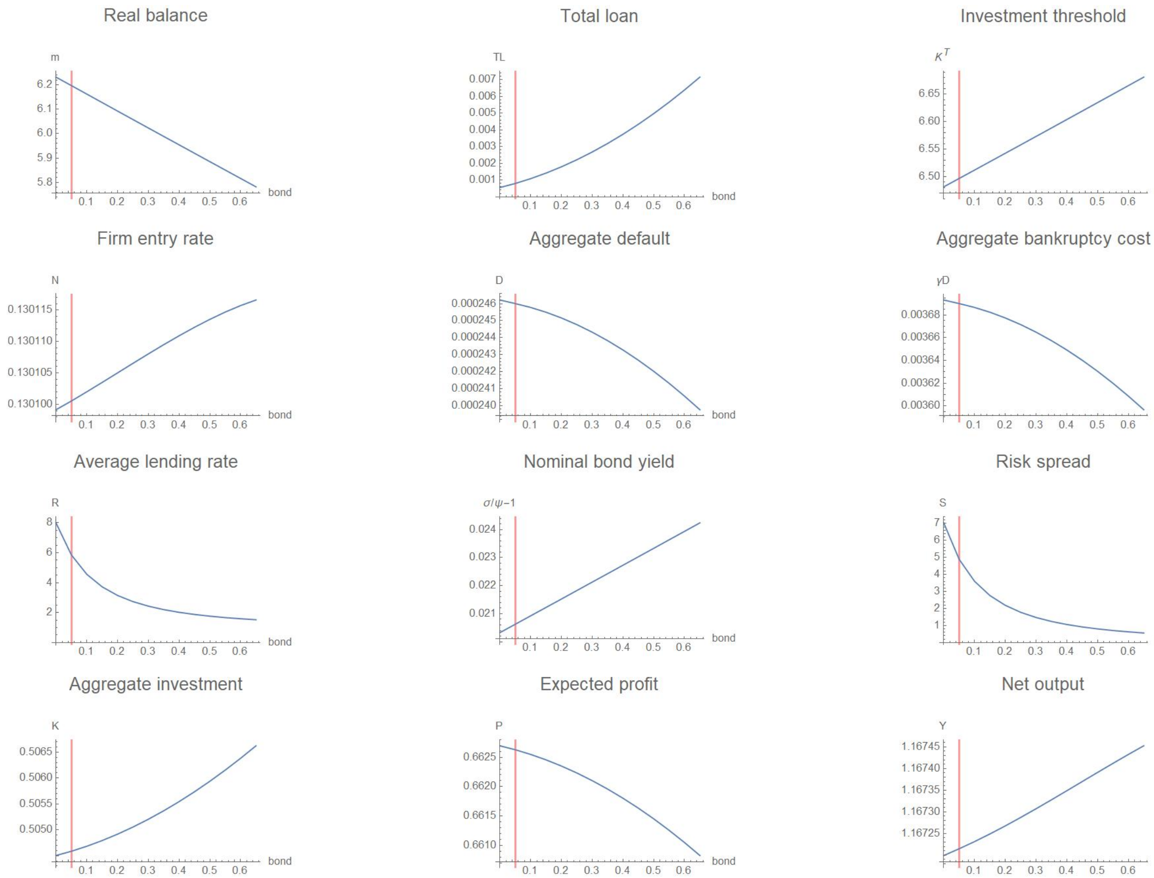


Figure 3: Effects of open market operations

firm entry and net output rise from open market sales.[20]

The effectiveness of OMOs in our model relies on the premise that money and bonds are imperfect substitutes. If entrepreneurs have full access to bank credit, bonds become perfect substitutes for money, and this neutralizes OMOs as indicated by Wallace (1981). The monetary policy effects in our model are in line with the conventional wisdom that ample liquidity can stimulate the economy. What is different from the conventional wisdom is that an open market sale can lower the lending rate and risk spread, and raise net output. Our result suggests that the role of bonds in

[20]The risk spread is defined as $S = R - (1 + \upsilon_b)$. Changes in the risk spread due to a change in some parameter, χ, can be expressed as changes in the risk premium and in the liquidity premium: $\frac{\partial S}{\partial \chi} = \frac{\partial R}{\partial \chi} + (1 + \upsilon_b)^{-2} \frac{\partial \upsilon_b}{\partial \chi}$. In Figure 2, when the policy rate increases from .04 to .06, the risk spread increase by .037, and the changes in the risk premium and in the liquidity premium is .022 and .015, respectively. In Figure 3, when bond supply increases from .6 to .8, the risk spread changes by $-.210556$, which is mainly due to the change in the risk premium, $-.209421$.

collateralized borrowing should be considered when evaluating the effects of OMOs.[21]

6 Private asset purchases by the central bank

This section studies how the central bank's private asset purchases work to improve liquidity and investment, and why this type of unconventional policy is beneficial to the aggregate economy. The private asset purchase policy is captured as the central bank buying loans from the issuing banks in our model. To encourage liquidity provision for risky investment while containing risk-taking, the private asset purchase program requires the issuer of private assets to retain a fraction, $\theta \in (0, 1)$, of the assets on its book. The central bank pays reserves to purchase $1 - \theta$ fraction of assets, and receives $1 - \theta$ fraction of expected returns backed solely by risky investment returns. We call θ the risk-retention rate. Assume that the central bank has the same monitoring technology as private banks; that is, it cannot do better than private banks in the credit market.

We first illustrate that the central bank's private asset purchase policy, by strengthening the role of bodns as collateral, effectively reduces banks' risk exposure, increases entrepreneurs' expected profits for a given input cost realization, and raises the investment threshold. Then we derive the optimal risk-retention rate. Section 6.3 provides further discussions on policy implications. We also show that our main findings are robust in various extensions that incorporate, for instance, ex ante asymmetric information regarding the quality of investment projects, private banks' potential misrepresentation of the quality of loans, costs of managing loan applications, and costs of collateralizing bonds.

6.1 Effects of the private asset purchase policy

The bank's net expected payoff from a contract with down payment d and collateral a, shown in (10), is re-expressed here:

$$\pi_b(y) = \left\{ y[1 - G(y)] + \int_{\underline{\omega}}^{y} \omega \, dG(\omega) - \gamma G(y) \right\} - (k - d - a).$$

[21] Technological improvements and financial developments can lower monitoring costs or enhance the access to banking. Proposition 10 in Appendix A shows macroeconomic impacts of changes in monitoring costs and access to bank credit. We find that, unlike Rocheteau, Wright, and Zhang (2018), improving access to bank loans is not always beneficial in our model, and the key lies in the bankruptcy costs associated with external financing.

Because the safe asset used as collateral works like a down payment, the actual funding cost for the lending bank is $k - d - a$, while the loan balance on the bank's balance sheet is $\ell = k - d$. Under the asset purchase policy the issuing bank receives reserves (in terms of the numéraire) from the central bank for a fraction of the loan on its book, $(1-\theta)(k-d)$, and retains θ fraction of the expected returns backed solely by risky investment returns. The bank's expected payoff under the policy thus becomes

$$\begin{aligned}\pi_b^p(y) &= (1-\theta)(k-d) + \theta\Big[y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y)\Big] - (k-d-a)\\ &= \theta\Big\{y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) - \underbrace{(k-d-\frac{a}{\theta})}_{\ell_y^p}\Big\}.\end{aligned} \tag{22}$$

The bank's binding participation constraint by using (22), $\pi_b^p(y) = 0$, implies that the *effective* quantity of bonds as collateral becomes $\frac{a}{\theta}$, and the risky loan amount under the policy, denoted by ℓ_y^p, is $k - d - \frac{a}{\theta}$. The policy makes bonds more effective than money in reducing risk—one unit of bonds can reduce $\frac{1}{\theta} > 1$ units of risky loan amounts.

The policy also affects the contract problem and entrepreneurs' expected profits. Let k^p denote the investment threshold under the policy. The contract problem becomes:

$$\begin{aligned}\{d^*,\ a^*,\ y^*\} &= \arg\max_{\{d,a,y\}} -d - a + \int_{y}^{\overline{\omega}} \omega\, dG(\omega) - y[1-G(y)]\\ \text{s.t.}\quad \pi_b^p(y) &= \theta\{y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) - (k-d-\frac{a}{\theta})\} \geq 0,\\ 0 &\leq d \leq m,\ 0 \leq a \leq z,\end{aligned} \tag{23}$$

where the solution satisfies $\pi_b^p(y^*) = 0$, and

$$a^* = \begin{cases}\theta k, & \text{if } \underline{k} \leq k \leq \frac{z}{\theta}\\ z, & \text{if } \frac{z}{\theta} < k \leq k^p\end{cases}, \qquad d^* = \begin{cases}0, & \text{if } \underline{k} \leq k \leq \frac{z}{\theta}\\ k - \frac{z}{\theta}, & \text{if } \frac{z}{\theta} < k \leq m + \frac{z}{\theta}\\ m, & \text{if } m + \frac{z}{\theta} < k \leq k^p.\end{cases} \tag{24}$$

From (24), if the entrepreneur's input cost is such that $\underline{k} \leq k \leq \frac{z}{\theta}$, bonds are collateralized, and no down payment is needed. Money will be used if $k > \frac{z}{\theta}$, under which all bond holdings are collateralized. Note that the risky loan amount is strictly positive and monitoring may occur only for borrowers with the input cost $k > m + \frac{z}{\theta}$.

Proposition 7. *The private asset purchase program enhances the efficacy of bonds as collateral and changes the pecking order of financing investment. Under the policy, entrepreneurs collateralize bonds before using internal funds.*

Given the solution to the contract problem, the expected profit of banked entrepreneurs is

$$\pi_e = \begin{cases} \mu_\omega - k + (1-\theta)k & \text{if } \underline{k} \le k \le \frac{z}{\theta} \\ \mu_\omega - k + \frac{1-\theta}{\theta} z, & \text{if } \frac{z}{\theta} < k < m + \frac{z}{\theta} \\ \mu_\omega - \gamma G(y^*) - k + \frac{1-\theta}{\theta} z, & \text{if } m + \frac{z}{\theta} \le k \le k^p. \end{cases} \tag{25}$$

Condition (25) shows that the asset purchase policy increases entrepreneurs' profits: For borrowers with $k \in [\underline{k}, \frac{z}{\theta}]$, the policy subsidizes $(1-\theta)k$, and that is, the subsidy increases in the borrower's input cost; for those with the input cost $k \in (\frac{z}{\theta}, k^p]$, the policy subsidizes $\frac{(1-\theta)z}{\theta}$.

The expected profit of unbanked entrepreneurs, who use internal funds only to purchase capital, is

$$\pi_e = \mu_\omega - k, \qquad \text{if } \underline{k} \le k \le m. \tag{26}$$

Lemma 1. *Given an entrepreneur's portfolio, (m, z), the private asset purchase program raises the investment threshold by $\frac{1-\theta}{\theta} z$.*

Private asset purchases increase banks' incentives to lend by transferring a fraction of the lending risk to the central bank. The policy effectively enlarges the quantity of collateralizable bonds from a to $\frac{a}{\theta}$, reducing the risky loan and repayment for a given loan amount (and a given input cost). Lemma 1 shows that banks extend loans to entrepreneurs with an input cost up to $k^p = k^T + \frac{1-\theta}{\theta} z$. Entrepreneurs with an input cost higher than the initial investment threshold, k^T, who could neither borrow nor find it profitable to invest without the policy will use credit to invest under the policy. The private asset purchase policy improves the availability of credit.

By (24) to (26), an entrepreneur's expected profit in the beginning of a period is

$$\pi_e^p = \alpha(1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k) + \alpha\phi \left\{ \begin{array}{c} \int_{\underline{k}}^{\frac{z}{\theta}}(\mu_\omega - k + (1-\theta)k)dF(k) + \int_{\frac{z}{\theta}}^{m+\frac{z}{\theta}}[\mu_\omega - k + \frac{1-\theta}{\theta}z]dF(k) \\ + \int_{m+\frac{z}{\theta}}^{k^p}[\mu_\omega - \gamma G(y^*) - k + \frac{1-\theta}{\theta}z]dF(k) \end{array} \right\}. \tag{27}$$

Notice that (27) incorporates the entrepreneur's financing decision of exhausting collateralized borrowing before using internal funds. An entrepreneur's lifetime expected utility is

$$\mathbb{E}\, V^p(m, z) = \pi_e^p + W(m, z, 0, 0). \tag{28}$$

Given (28), an entrepreneur's optimal portfolio choices are characterized by the following conditions (see Appendix A for the derivation):

$$\frac{\sigma}{\beta} = \alpha\Big\{(1-\phi)(\mu_\omega - m)f(m) + \phi\Big[(\mu_\omega - k^p + \frac{1-\theta}{\theta}z)f(k^p) - \gamma\int_{m+\frac{z}{\theta}}^{k^p} G(y^*)f'(k)dk\Big]\Big\} + 1, \quad (29)$$

$$\frac{\psi}{\beta} = \frac{\alpha\phi}{\theta}\Big[(\mu_\omega - k^p + \frac{1-\theta}{\theta}z)f(k^p) - \gamma\int_{m+\frac{z}{\theta}}^{k^p} G(y^*)f'(k)dk + (1-\theta)[F(k^p) - F(\frac{z}{\theta})]\Big] + 1. \quad (30)$$

Conditions (29) and (30) show that the liquidity premia of money and bonds are changed by the risk-retention rate, θ.

6.2 The optimal risk-retention rate

In the benchmark economy the Friedman rule achieves the first best allocation (Proposition 4). However, the Friedman rule may be infeasible because frictions may make it impossible to withdraw money from entrepreneurs.[22] In this section, we consider the situation in which the Friedman rule is infeasible, and we ask what the risk-retention rate should be to maximize welfare.[23]

When conducting the policy, the central bank knows the decision rules of entrepreneurs and banks. To choose the optimal risk-retention rate, θ^*, the central bank anticipates the equilibrium m and ψ given by (29) and (30). Entrepreneurs and banks take θ^* as given, and the optimal portfolio choices, (29) and (30), ensure that the equilibrium m and ψ are as anticipated.

The central bank, by giving reserves, $(1-\theta)(k-d^*)$, to the lending bank, is entitled to $1-\theta$ of the expected return of loans backed solely by the project return. Given the investment threshold under the policy, k^p, the central bank's expected payoff is:

$$\pi_c = \alpha\phi\int_{\underline{k}}^{k^p}\Big\{(1-\theta)\Big[y[1-G(y)] + \int_{\underline{\omega}}^{y}\omega\, dG(\omega) - \gamma G(y)\Big] - (1-\theta)(k-d^*)\Big\}dF(k).$$

Using the private bank's binding participation constraint from (23) with (d^*, a^*, y^*), we obtain

$$\pi_c = -\alpha\phi\Big[\int_{\underline{k}}^{k^p}(\frac{a^*}{\theta} - a^*)dF(k)\Big] < 0; \quad (31)$$

[22] As claimed by Andolfatto (2013), withdrawing money is like asking people to pay lump-sum taxes, and one can view paying taxes as a form of debt subject to default, and it must be incentive-compatible for agents to pay taxes. For instance, Berentsen, Camera, and Waller (2007) assume the government has limited enforcement so that the net inflation rate is positive.

[23] Assume that entrepreneurs need to pay taxes to be qualified for the asset purchase program; e.g., they need to show that they paid taxes in order to borrow from banks. As long as the benefit enjoyed by entrepreneurs from the asset purchase program can cover the tax burden, it is possible to conduct asset purchases while the Friedman rule is not feasible.

that is, the central bank runs a deficit from purchasing private assets. Condition (31) says that the cost of the policy is equal to the benefit of increased collateralizable bonds and is the subsidy to entrepreneurs who receive bank credit.

Substituting the solution to the contract, a^*, from (24) into (31), we obtain

$$\pi_c = -\alpha\phi\Big\{\int_{\underline{k}}^{\frac{z}{\theta}}(1-\theta)kdF(k) + \frac{(1-\theta)z}{\theta}\Big[F(k^p) - F(\frac{z}{\theta})\Big]\Big\}. \tag{32}$$

The right side of (32) equals the total subsidy to entrepreneurs through the central bank's purchase of private assets (see also (25)). A lower θ implies a larger subsidy, and more reserves (financed by lump-sum taxes on entrepreneurs) are needed.

The central bank maximizes society's welfare in choosing the optimal risk-retention rate. The representative entrepreneur's expected life-time utility before all shocks are realized is $(1-\beta)\Pi_e = \pi_e^p + C^* - H^*$, where π_e^p given by (27) is the consumption of expected profits of investment. Due to the constant return to scale technology to producing capital and competition in the capital market, suppliers' net expected utility from producing capital and consuming the payment is zero. Therefore, we use $\Omega = \pi_e^p + \pi_c$ as the welfare criterion, which is the expected utility of entrepreneurs subtracting the cost of policy, and π_c is given by (32). Using (25) and (26), we have the following welfare measure:[24]

$$\Omega = \alpha\Big\{(1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k) + \phi\Big[\int_{\underline{k}}^{k^p}(\mu_\omega - k)dF(k) - \int_{m+\frac{z}{\theta}}^{k^p}\gamma G(y^*)dF(k)\Big]\Big\}. \tag{33}$$

To find the optimal risk-retention rate, θ^*, the central bank maximizes Ω in (33), subject to $\pi_b^p(y) = 0$, and the government's budget constraint.

Proposition 8. *Consider the specific distributions, $k \sim U[\mu_k - \frac{\epsilon_k}{2}, \mu_k + \frac{\epsilon_k}{2}]$, $\omega \sim U[\mu_\omega - \frac{\epsilon_\omega}{2}, \mu_\omega + \frac{\epsilon_\omega}{2}]$, where $\{\mu_k - \frac{\epsilon_k}{2},\ \mu_\omega - \frac{\epsilon_\omega}{2}\}$ are close to 0, and $\gamma < \epsilon_\omega$. In a monetary equilibrium, there exists an optimal risk-retention rate, $\theta^* \in (0,1)$, such that*

(i) the investment threshold is raised and equals the mean return of projects; i.e., $k^p = \mu_\omega$;

(ii) the optimal risk-retention rate should be lower when the input cost and project return become more risky, when it is harder to meet with the supplier, and when the policy rate is lower; i.e., $\frac{\partial\theta^*}{\partial\epsilon_\omega} < 0$, $\frac{\partial\theta^*}{\partial\epsilon_k} < 0$, $\frac{\partial\theta^*}{\partial\alpha} > 0$;

[24] The welfare measure, Ω, is bounded because, when the investment threshold is raised by policy, so is the marginal borrower's input cost. The second term in (33), $\int_{\underline{k}}^{k^p}(\mu_\omega - k)dF(k)$, starts to decrease once k^p is above μ_ω.

(iii) the optimal risk-retention rate and social welfare decrease in inflation; i.e., $\frac{\partial \theta^*}{\partial \sigma} < 0$ *and* $\frac{\partial \Omega}{\partial \sigma}|_{\theta=\theta^*} < 0$.

By assuming specific distributions in Proposition 8, we gain clear policy implications and intuitions. The optimal risk-retention rate, $\theta^* \in (0, 1)$, is chosen to raise the investment threshold to such a level that all socially efficient projects ($k \leq \mu_\omega$) would be implemented. By sharing a fraction of the lending risk, through its purchase of private assets, the central bank can enhance welfare. As a negative shock increases uncertainty with respect to the input cost and investment returns facing businesses, or increases the difficulty of meeting the right supplier, the central bank should lower the risk-retention rate to encourage lending; moreover, the optimal risk-retention rate decreases in inflation.[25] Therefore, when higher inflation raises the cost of holding real balances, the central bank needs to lower the optimal risk-retention rate; that is, it should purchase more loans and share a larger fraction of lending risk to improve availability of credit.

To further investigate the transmission mechanisms of the private asset purchase program, we illustrate how it affects welfare. As the policy improves the efficacy of bonds as collateral, entrepreneurs may decrease their demand for money. The first term in (33), $\int_{\underline{k}}^{m} (\mu_\omega - k) dF(k)$, which is unbanked entrepreneurs' profit, may decrease due to lower real balances. The second term is the profit of banked entrepreneurs, $\int_{\underline{k}}^{k^p} (\mu_\omega - k) dF(k)$, which increases as the policy raises the investment threshold to k^p. The last term is the aggregate bankruptcy cost, $\int_{m+\frac{z}{\theta}}^{k^p} \gamma G(y^*) dF(k)$. The policy results in projects of relatively high input costs being funded by raising the investment threshold, and it also enlarges the quantity of collateralizable bonds. Thus, the input cost range for entrepreneurs whose borrowing involves strictly positive risky loan amounts (and so monitoring may occur) is changed from $(m + z, k^T)$ to $(m + \frac{z}{\theta}, k^p)$. If, for instance, $F(m + \frac{z}{\theta} < k < k^p)$ is small; that is, the number of borrowers with strictly positive risky loan amounts is small, aggregate bankruptcy costs may be reduced by the policy. Finally, though the policy raises welfare, it has a distributional effect: unbanked entrepreneurs are hurt and banked entrepreneurs are subsidized by the private asset purchase program.

We summarize the effects of private asset purchases on the aggregate economy in the following Proposition and Figure 4.

[25] If the central bank conducts asset purchases along with adjusting the policy rate, then under the circumstances that withdrawing money is infeasible, the optimal policy is to set $i = r$ (or equivalently $\sigma = 1$) and $\theta = \theta^* \in (0, 1)$.

Proposition 9. *Given the assumptions and θ^* in Proposition 8, the private asset purchase program raises firm entry, aggregate lending, investment, and net output, and it reduces the average lending rate and the business failure rate.*

In Figure 4, the input cost k follows a lognormal distribution. The variable $\widehat{\upsilon}$ is defined as $\frac{\upsilon_p}{\upsilon}$, where υ_p denotes the variable under private asset purchase policy and υ denotes the variable under no policy; if $\widehat{\upsilon} > 1$, the value of υ is higher under policy. We observe from Figure 4 that $\widehat{D} < 1$, $\widehat{S} < 1$, and $\widehat{\Omega} > 1$, which implies that the private asset purchase policy under the optimal risk-retention rate reduces aggregate bankruptcy costs, risk spreads, and raises welfare.

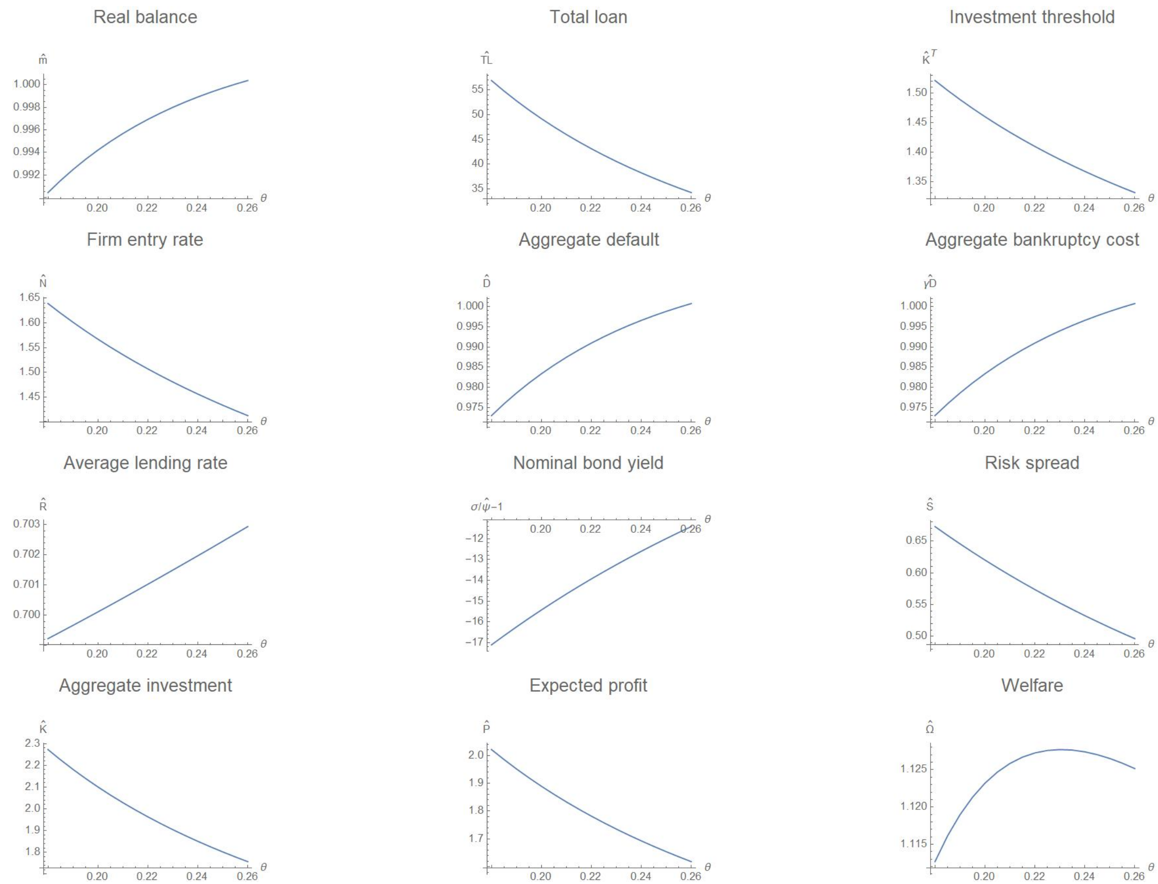
6.3 Discussion

This section further discusses policy implications of private asset purchases and several extensions.

Discussion on private asset purchases. The asset purchase program functions as enlarging the quantity of collateralizable bonds from a to $\frac{a}{\theta}$, but its transmission channels and macroeconomic consequences are different from open market sales, which directly increase the supply of bonds. By Proposition 7, the pecking order of financing investment is reversed by private asset purchases. Comparing (29) with (17), one sees that the liquidity premium of money is changed by the program. In numerical experiments we choose θ so that collateralizable bonds under private asset purchases rise by the same amount as the increase in the bond supply under an open market sale. We find that asset purchases result in higher net output. The reason is this. The increased bonds in an open market sale are evenly distributed to entrepreneurs, and after the input cost shocks are realized, increased bonds held by unbanked entrepreneurs and by banked entrepreneurs with input cost larger than the investment threshold are idle. However, the asset purchase program allocates the increased collateralizable bonds to only entrepreneurs who are eligible for bank credit; moreover, it allocates more bonds to borrowers with higher input costs, which implies that the central bank gives a larger subsidy to corporate borrowings with higher lending risk (see equation (25)).[26]

Note that by raising the efficacy of bonds as collateral, private asset purchases can increase the demand for bonds to such an extent that nominal bond yields may be negative. The policy rate

[26] This result hinges on the timing of events considered in this paper. For instance, if entrepreneurs learn whether they have access to banking before making portfolio choices, then increased bonds under open market sales can be allocated to only banked entrepreneurs.



Notes:
1. The aggregate bankruptcy cost is defined as γD. Because γ is exogenously given, the relative aggregate default rate, $\widehat{D} = \frac{D_p}{D}$, thus measures the relative aggregate bankruptcy cost.
2. We use the absolute value of the difference, $\triangle(\frac{\sigma}{\psi}) = (\frac{\sigma}{\psi_p} - 1) - (\frac{\sigma}{\psi} - 1)$, to measure the changes in the nominal bond yield, because the nominal bond yield under policy could be negative. The nominal bond yield falls, which implies a higher bond price, under the asset purchase program.
3. The relative welfare level, $\widehat{\Omega}$, reaches the highest value at the optimal risk-retention rate, $\theta^* = 0.2305$.
4. The parameter settings are the same as those for Figure 2.

Figure 4: Effects of private asset purchases

considered in Section 5 is the interest rate on totally illiquid bonds based on the Fisher equation, and the rate is constrained by the zero lower bound. Under the asset purchase policy, the central bank may consider the nominal yield on the short-term bond (which is liquid due to its role as collateral) as the monetary policy instrument (see, e.g., Geromichalos and Herrenbrueck, 2022). In so doing, the central bank chooses the optimal risk-retention rate and uses the resulting nominal bond yield as the policy interest rate, while keeping the money growth rate as such that i is still confined by the zero lower bound. Thus, aggregate liquidity is improved by the asset purchase program and the short-term target rate can be below the zero lower bound.

For ease of exposition, the asset purchase policy we have considered is the one wherein the central bank forgoes collateral backing the loans it purchases. In online supplementary Appendix C we show that the main results hold whether or not the central bank obtains a fraction of collateral. The asset purchase policy considered there requires the issuer of private assets to retain θ_1 fraction of the assets, and the issuer receives θ_1 fraction of the expected returns backed solely by risky investment returns and θ_2 fraction of collateral once defaults occur (and the central bank obtains $1-\theta_2$ fraction of collateral). We call θ_1 the *risk-retention rate*, and θ_2 the *collateral-retention rate*. It is shown that the optimal policy can be summarized by the ratio between the risk-retention rate and collateral-retention rate, $\theta_c^* = \frac{\theta_1}{\theta_2}$; moreover, the optimal ratio θ_c^* is equal to θ^*, which is the optimal risk-retention rate found in Section 6.2 wherein the central bank forgoes collateral backing private assets. Hence, all the results in Propositions 8 and 9 hold under this setup of policy. The intuitive reason is this. A lower risk-retention rate θ_1 means that banks bear a lower lending risk, while a higher collateral-retention rate θ_2 decreases banks' losses when defaults occur. The central bank thus can choose various combinations of θ_1 and θ_2 to achieve the optimal ratio, $\theta_c^* = \theta^*$.

We also compare the central bank's private asset purchases with the fiscal authority's direct subsidies to bank lending (see online supplementary Appendix D). The policy of direct subsidies considered here is as follows. Suppose that the fiscal authority aims to raise the investment threshold to $k^{pf} > k^{Tf}$, where k^{Tf} is the investment threshold without policy. The policy subsidizes a bank which extends loans to credit-constrained entrepreneurs who could not borrow without policy, i.e., borrowers with the input cost $k \in (k^{Tf}, k^{pf}]$, by the amount, $k - k^{Tf}$, and banks still earn zero profits. It is shown that private asset purchases with the optimal risk-retention rate yield higher social welfare than the policy of direct subsidies, even when k^{pf} is set optimally. The reason is twofold. Different from private asset purchases which increase the efficacy of bonds as collateral, direct subsidies work like a substitute for the down payment. Entrepreneurs thus hold much lower real balances, a situation which leads to a smaller expected profit for unbanked entrepreneurs. Moreover, entrepreneurs whose loans are eligible for the direct subsidy are marginal borrowers, who have the highest tendency to default; consequently, aggregate bankruptcy costs are higher under direct subsidies. The central bank's purchase of private assets is essential in the sense that it achieves an outcome that the policy of direct subsidies cannot.[27]

Extensions. In the spirit of Li, Rocheteau, and Weill (2012), we incorporate ex ante asymmetric information regarding the quality of investment projects into the benchmark model (see online supplementary Appendix E). If an entrepreneur with a project of input cost k fails to find the right supplier so that she cannot invest, by paying a fixed cost $f_c > 0$, she can make a fake project of input cost k. A fake project produces zero output, and banks cannot distinguish between a fake project and a productive project. Assume that an entrepreneur can exert an effort cost, $s(k)$, when offering a debt contract, and the effort cost is observable to banks. We show that an honest entrepreneur offers a debt contract, $\{d^*(k), a^*(k), s^*(k), y^*(k)\}$, that leaves no gains of making fake projects, and the bank will believe that the offer is provided by an entrepreneur holding a productive project; that is, the debt contract should satisfy an extra constraint, a *no-fraud condition.* In equilibrium entrepreneurs do not make fake projects (see, e.g., Li, Rocheteau, and Weill 2012). The macroeconomic effects of private asset purchases are qualitatively similar to those stated in Proposition 9. Compared to (33), the welfare measure now incorporates an extra term, aggregate entrepreneurs' effort costs, $\int_{\underline{k}}^{k^p} s^*(k)dF(k)$, which is decreasing in the risk-retention rate. To deter the entry of fake projects and restrain aggregate effort costs, the welfare-maximizing central bank chooses a higher optimal risk-retention rate but achieves lower welfare level than those in the basic model. Therefore, the threat of making fake projects to deceive banks results in an extra cost to the society.

[27] Online supplementary Appendix I studies one-period private asset purchases and shows that results of Proposition 9 still hold. In the basic model entrepreneurs may choose lower real balances under the policy because bonds are more efficient than money in reducing the risky loan amount. However, if the central bank announces the one-period asset purchase policy *after* entrepreneurs have chosen their holdings of real balances, policy effects on macroeconomic conditions are more expansionary than otherwise. Therefore, compared to the basic model, the central bank sets a lower optimal risk-retention rate and achieves a higher welfare level under the short-run asset purchase policy.

If banks have information advantage on their loan portfolios than the central bank, one may wonder whether private banks may have incentives to retain loans with low default risk and sell those with high default risk to the central bank. To address this issue, in online supplementary Appendix F we apply Li, Rocheteau, and Weill's (2012) idea of counterfeiting assets to model banks' potential misrepresentation of the quality of loans (see also Williamson 2018). Assume that a bank, by paying a fixed cost, $f_{bc} > 0$, can misrepresent the quality of loans sold to the central bank, and the central bank cannot recognize the quality of individual loans it purchases. We derive a *no-misrepresentation constraint* under which a bank chooses not to misrepresent the quality of loans sold to the central bank, and Proposition 9 still holds in this economy. The central bank faces an additional constraint when choosing the optimal risk-retention rate. If the fraud cost f_{bc} is sufficiently low such that banks' no-misrepresentation constraint binds, then compared with the basic model, the central bank chooses a higher optimal risk-retention rate to contain the incentive of misrepresentation, and the policy achieves lower welfare.

Our next extension considers the situation wherein banks need to pay a fixed cost, $f_m > 0$, to manage a loan application (see online supplementary Appendix G). The fixed cost f_m tightens banks' participation constraint and, thereby, when offering the contract, entrepreneurs need to provide more safe assets or a higher repayment than otherwise to satisfy the bank's participation constraint. Under the private asset purchase policy, the bank's net expected payoff from a contract with input cost k becomes $\pi_b^p = \theta \left\{ B(y) - (k - d - \frac{a-f_m}{\theta}) \right\}$. Compared to (22), here the policy enlarges the quantity of collateralizable bonds to $\frac{a-f_m}{\theta}$, rather than $\frac{a}{\theta}$. We show that entrepreneurs pledge bonds as collateral before they use down payment, as stated in Proposition 7. If the bond supply is ample ($z > f_m$), policy implications stated in Proposition 9 still holds. However, the fixed cost, f_m, reduces the portion of collateral that would be enlarged by the policy, and the optimal risk retention rate should be set lower than otherwise. If the bond supply is scarce ($z < f_m$), the policy would bring a loss to banks, and this suggests no central bank interventions in the form of private asset purchases.

Finally, we extend our analysis to incorporate the cost of collateralizing bonds (see online supplementary Appendix H). In the basic model, each unit of bonds pays the bearer one unit of the numéraire, and if the borrower defaults, the lending bank receives one unit of the numéraire from each unit of bonds it forecloses. The cost of collateralizing bonds is modeled as follows. For each unit of bonds pledged as collateral, the bank receives $\rho \in (0, 1)$ units of the numéraire if default occurs; thus, $1-\rho$ units of the numéraire is considered as the cost of collateralizing one unit of bonds. We find that adding this cost into the model does not alter the qualitative effects of OMOs and the private asset purchase policy. A higher cost of collateralizing bonds makes external financing more expensive, and reduces the maximum risky loan amount that a bank is willing to extend, and lowers the investment threshold. Hence, to raise the investment threshold to the optimal level, the central bank should choose a lower risk-retention rate than that in the basic model.[28]

7 Conclusion

This paper considers corporate finance with respect to implementing risky investment projects to study the macroeconomic implications of monetary policy and in particular the central bank's purchase of private assets. We have established that the key transmission channels of monetary policy are money, when it's used for down payments, and government bonds, when they're used as collateral in external financing, both of which can reduce lenders' risk exposure and save the bankruptcy cost. A lower nominal policy rate and open market sales can compress risk spreads and increase investment. The central bank's private asset purchases improve the availability of credit, and appropriate risk-retention requirements are essential to welfare.

In times when uncertainty is heightened, financial risks increase and the impact on the economy can be severe. During the Great Recession and COVID-19 pandemic, for instance, the worldwide economy was severely impacted by rising uncertainty in investment and the disruption of supply chains. In response to these adverse economic events, several central banks purchased a large amount of private assets with the goal of restoring credit market activity. A policy suggestion we draw from our analysis for periods of an economic downturn or crisis is this. The private asset purchase program with an optimal risk-retention rate helps improve credit availability and reduce business failures. As the uncertainty and difficulties for businesses intensify, the central bank should bear a larger fraction of the lending risk by requiring a lower risk-retention rate.

[28] We compare the effectiveness of OMOs and private asset purchases under different levels of nominal interest rates in online supplementary Appendix K. In numerical examples we calculate the maximum welfare levels both policies can achieve given the nominal policy rate. We find that, for different levels of the policy rates, the private asset purchase policy attains higher maximum welfare (and thus is more effective) than OMOs. And the difference between the maximum welfare levels of both policies is larger under the situation when the policy rate is higher.

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Appendix A. Proofs and derivation of equations

Proof of Proposition 1. This proof contains four parts. We first show that the solution to a debt contract with $\ell_y \in [0,\ \overline{\ell}_{yb}]$ exists; next we show $y^* = 0$ if $\ell_y = 0$ and $y^* > \ell_y$ if $\ell_y > 0$; then we show that $\frac{\partial y^*}{\partial \ell_y} > 0$; and finally $\frac{\partial y^*}{\partial \gamma} > 0$.

First, given $(d^*,\ a^*)$, the input cost, k, and the corresponding risky loan amount, $\ell_y \in [0, \overline{\ell}_{yb}]$, where $\overline{\ell}_{yb} > 0$, the contract problem, (11), is rewritten as

$$y^* = \arg\max_{y} -d^* - a^* + \int_{y}^{\overline{\omega}} \omega \, dG(\omega) - y[1 - G(y)] \tag{34}$$

$$\text{s.t.} \quad \pi_b = \underbrace{y[1 - G(y)] + \int_{\underline{\omega}}^{y} \omega \, dG(\omega) - \gamma G(y)}_{\equiv B(y)} - \ell_y = 0.$$

By definition, we know that for all $\ell_y \in [0, \overline{\ell}_{yb}]$, $B(y_b) - \ell_y \geq 0$, where $\pi_b'(y_b) = B'(y_b) = 0$. From the bank's participation constraint we know that if $y = 0$, then $\pi_b(0) = -\ell_y \leq 0$. With the assumption on γ, we have a strict concave $B(y)$, and $B'(y) > 0$ for all $y \in [0, y_b)$. Hence, we conclude that there exists a $y^* \in [0, y_b] \subset [0, \overline{\omega}]$, which solves the contract problem.

It is straightforward to see $y = 0$ is a solution for a contract with $\ell_y = 0$. If $\ell_y > 0$, then a contract with $y = \ell_y$ does not satisfy bank's participation condition because

$$\pi_b(\ell_y) = B(\ell_y) - \ell_y = -\int_{\underline{\omega}}^{\ell_y} (\ell_y - \omega) \, dG(\omega) - \gamma G(\ell_y) < 0.$$

Since $B'(y) > 0$ for all $y \in [0, y_b)$, it must be that $y^* > \ell_y$ to satisfy the bank's participation condition. Moreover, $\frac{\partial y^*}{\partial \ell_y} > 0$ because $B'(y) > 0$.

Finally, the bank's participation condition must be satisfied after γ changes. Thus, for a given ℓ_y, the binding bank's participation condition implies

$$\frac{\partial y^*}{\partial \gamma} = \frac{G(y^*)}{1 - G(y^*) - \gamma g(y^*)} > 0$$

for all $y^* < y_b$.

Proof of Corollary 1. When $\gamma = 0$, $B(y) = y[1 - G(y)] + \int_{\underline{\omega}}^{y} \omega dG(\omega)$ and $B'(y) = 1 - G(y) \geq 0$ for all $y \in [0, \overline{\omega}]$. Therefore, $y_b = \overline{\omega}$, and $\overline{\ell}_{yb} = B(\overline{\omega}) = \mu_\omega$. Furthermore, since $B(y) = y[1 - G(y)] +$

$\int_{\underline{\omega}}^{y} \omega dG(\omega) < y \int_{\underline{\omega}}^{\overline{\omega}} dG(\omega) = y$, for all $\ell_y \in (0, \bar{\ell}_{yb}]$, $\pi_b = B(\ell_y) - \ell_y < 0$. Hence, it must be $y^* > \ell_y$ to satisfy bank's participation condition, $\pi_b = B(y^*) - \ell_y = 0$.

Proof of Proposition 2. According to (12), $\pi_e(y;k) = \mu_\omega - \gamma G(y) - k$, and so the entrepreneur's net expected payoff decreases in the repayment y^*, where y^* is the solution to the contract problem, (34). From Proposition 1, $\frac{\partial y^*}{\partial \ell_y} > 0$, where the risky loan amount $\ell_y = k - d - a$, so a banked entrepreneur with a given input cost, k, has an incentive to use more money and more bonds to reduce ℓ_y, which implies exhausting safe assets first is optimal for a banked entrepreneur. Because $\frac{\partial \ell_y}{\partial d} = \frac{\partial \ell_y}{\partial a}$, the effects of real balances and bonds in reducing the risky loan amount are identical. Moreover, $\frac{\partial y^*}{\partial \ell_y}\frac{\partial \ell_y}{\partial d} = \frac{\partial y^*}{\partial \ell_y}\frac{\partial \ell_y}{\partial a}$, which implies that the two options reduce the effective repayment y^* by the same amount. Thus, an entrepreneur is indifferent between using money as a down payment and bonds as collateral. Without loss of generality, we consider the case where entrepreneurs use real balances before bonds. Thus, when $k > m$, $d^* = m$ and $a^* = \min\{z, k - m\}$.

Derivation of (17) and (18). Using (4) and (5) lagged one period to eliminate $\mathbb{E}\, V_m(m,z)$ and $\mathbb{E}\, V_z(m,z)$ from (14) and (15), respectively, we derive (17) and (18) as follows:

$$
\begin{aligned}
\frac{\sigma}{\beta} &= \alpha \left\{ \begin{array}{l} (1-\phi)(\mu_\omega - m) f(m) + \\ \phi \left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T) \dfrac{\partial k^T}{\partial m} - \displaystyle\int_{m+z}^{k^T} \gamma g(y^*) \dfrac{\partial y^*}{\partial m} f(k) dk \right] \end{array} \right\} + 1 \\
&= \alpha \left\{ \begin{array}{l} (1-\phi)(\mu_\omega - m) f(m) + \\ \phi \left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T) \dfrac{\partial k^T}{\partial m} + \underbrace{\displaystyle\int_{m+z}^{k^T} \gamma g(y^*) \dfrac{\partial y^*}{\partial k} f(k) dk}_{\text{Integration by parts}} \right] \end{array} \right\} + 1 \\
&= \alpha \left\{ \begin{array}{l} (1-\phi)(\mu_\omega - m) f(m) + \\ \phi \left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T) \dfrac{\partial k^T}{\partial m} + \left[\gamma G(y^*) f(k)\right]_{k=m+z}^{k=k^T} - \displaystyle\int_{m+z}^{k^T} \gamma G(y^*) f'(k) dk \right] \end{array} \right\} + 1 \\
&= \alpha \left\{ (1-\phi)(\mu_\omega - m) f(m) + \phi \left[(\mu_\omega - k^T) f(k^T) - \int_{m+z}^{k^T} \gamma G(y^*) f'(k) dk \right] \right\} + 1;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\psi}{\beta} &= \alpha\phi\left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T)\frac{\partial k^T}{\partial z} - \int_{m+z}^{k^T} \gamma g(y^*)\frac{\partial y^*}{\partial z} f(k)dk\right] + 1 \\
&= \alpha\phi\left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T)\frac{\partial k^T}{\partial z} + \underbrace{\int_{m+z}^{k^T} \gamma g(y^*)\frac{\partial y^*}{\partial k} f(k)dk}_{\text{Integration by parts}}\right] + 1 \\
&= \alpha\phi\left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T)\frac{\partial k^T}{\partial z} + \Big[\gamma G(y^*) f(k)\Big]_{k=m+z}^{k=k^T} - \int_{m+z}^{k^T} \gamma G(y^*) f'(k)dk\right] + 1 \\
&= \alpha\phi\left[(\mu_\omega - k^T) f(k^T) - \int_{m+z}^{k^T} \gamma G(y^*) f'(k)dk\right] + 1,
\end{aligned}
$$

where $y^\dagger = y^*(k^T)$ and we have used $\frac{\partial y^*}{\partial m} = \frac{\partial y^*}{\partial z} = -\frac{\partial y^*}{\partial k}$ in the second step, $\frac{\partial k^T}{\partial m} = \frac{\partial k^T}{\partial z} = 1$ for the constrained equilibrium, and $\mu_\omega - \gamma G(y^\dagger) - k^T = 0$ for the unconstrained equilibrium.

Proof of Proposition 3. Consider that the input cost and project return follow uniform distributions: $k \sim U[0, \epsilon_k]$, $\omega \sim U[0, \epsilon_\omega]$, where $\mu_k = \frac{\epsilon_k}{2}$ and $\mu_\omega = \frac{\epsilon_\omega}{2}$, and $\gamma < \epsilon_\omega$. We first calculate the efficient repayment y^* by equation (10). Given the risky loan amount $\ell_y = k - m - z$,

$$
y^* = (\overline{\omega} - \gamma) - \sqrt{(\overline{\omega} - \gamma)^2 - 2\epsilon_\omega \ell_y},
$$

which is defined when $0 \leq \ell_y \leq \frac{(\overline{\omega}-\gamma)^2}{2\epsilon_\omega} = \mu_\omega - \gamma + \frac{\gamma^2}{2\epsilon_\omega} \equiv \overline{\ell}_{yb}$.

According to (17) and (18), and given the uniform distribution assumption on k and ω, the following conditions hold in a monetary equilibrium:

$$
\begin{aligned}
\frac{\sigma}{\beta} &= \frac{\alpha}{\epsilon_k}[(1-\phi)(\mu_\omega - m) + \phi(\mu_\omega - k^T)] + 1, \\
\frac{\psi}{\beta} &= \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T) + 1.
\end{aligned}
$$

Therefore,

$$
\psi^* = \beta\left[1 + \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right].
$$

In a credit-constrained equilibrium, $k^T = k_b$, and thereby,

$$
\begin{aligned}
k^T &= m + z + \overline{\ell}_{yb}, \\
m^* &= \mu_\omega - \phi(z + \overline{\ell}_{yb}) - \frac{(\sigma - \beta)\epsilon_k}{\alpha\beta}.
\end{aligned}
$$

In a credit-unconstrained equilibrium, $k^T = k_e$, where k_e satisfies

$$\pi_e = \mu_\omega - \gamma G(y^*) - k_e = 0$$

given an entrepreneur's asset holdings, $(m,\ z)$. This implies

$$k^T = k_e = \mu_\omega - \gamma\left(1 - \sqrt{\frac{2(m+z)}{\epsilon_\omega}}\right),$$

when $\sigma > \beta$. The real balances, m^*, solves

$$m^* = (1-\phi)(\mu_\omega - m) + \phi\gamma\left(1 - \sqrt{\frac{2(m+z)}{\epsilon_\omega}}\right) - \frac{\sigma-\beta}{\alpha\beta}\epsilon_k = 0,$$

based on (17). If $\sigma = \beta$, then $m^* = \mu_\omega = k^T$.

Proof of Proposition 4 Under the Friedman rule, $\sigma = \beta$, the liquidity premium of money is 0, from (17). This can be achieved only by $m = \mu_\omega$. Hence, by holding $m = \mu_\omega$, entrepreneurs can implement all potentially profitable project; i.e., $k \leq \mu_w$. This also implies that bank credit is not needed and the investment threshold is $k^T = \mu_\omega$. Because for all projects the input cost is fully financed by internal funds, bonds claim 0 liquidity premium, and thus, they are priced at the fundamental value.

Proof of Propositions 5 and 6. With the uniform distribution assumptions and the results of Proposition 3, we have following conditions in a credit-constrained equilibrium:

$$\begin{aligned}
m^* &= \mu_\omega - \phi(z + \bar{\ell}_{yb}) - \frac{(\sigma-\beta)\epsilon_k}{\alpha\beta}, \\
\psi^* &= \beta\left[1 + \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right], \\
k^T &= m + z + \bar{\ell}_{yb}, \\
\bar{\ell}_{yb} &= \mu_\omega - \gamma + \frac{\gamma^2}{2\epsilon_\omega}, \\
y^* &= (\overline{\omega} - \gamma) - \sqrt{(\overline{\omega}-\gamma)^2 - 2\epsilon_\omega \ell_y}.
\end{aligned}$$

With the aggregate variables defined in Section 4.2, we have

$$
\begin{aligned}
N &= \frac{\alpha}{\epsilon_k}\left(\mu_\omega - \frac{\sigma-\beta}{\alpha\beta}\epsilon_k\right), \\
D &= \frac{\alpha\phi}{\epsilon_k\epsilon_\omega}\left[\bar{\ell}_{yb}(\epsilon_\omega-\gamma) - \frac{(\epsilon_\omega-\gamma)^3}{3\epsilon_\omega}\right], \\
TL &= \frac{\alpha\phi}{\epsilon_k}\frac{(z+\bar{\ell}_{yb})^2}{2}, \\
X &= \frac{\alpha\phi}{\epsilon_k}\left[\frac{(z+\bar{\ell}_{yb})^2}{2} - \frac{\bar{\ell}_{yb}^2}{2} + \bar{\ell}_{yb}(\epsilon_\omega-\gamma) - \frac{(\epsilon_\omega-\gamma)^3}{3\epsilon_\omega}\right], \\
R &= \frac{X}{TL}, \\
r_b &= \left\{\beta\left[1+\frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right]\right\}^{-1} - 1, \\
S &= R - \left[1+\frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right]^{-1}, \\
K &= \frac{\alpha}{2\epsilon_k}\left[(1-\phi)m^2 + \phi(m+z+\bar{\ell}_{yb})^2\right], \\
Y &= \mu_\omega N - \gamma D, \\
P &= Y - K.
\end{aligned}
$$

The effects of changes in the policy rate, i.

Because i is defined by the Fisher equation, $i = \sigma(1+r)-1$, the effects of changes in i take the same signs as those of changes in σ. Hence, we show the comparative statics with respect to σ:

$$
\begin{aligned}
&\frac{\partial m}{\partial \sigma} = -\frac{\epsilon_k}{\alpha\beta} < 0, && \frac{\partial k^T}{\partial \sigma} = \frac{\partial m}{\partial \sigma} < 0, && \frac{\partial N}{\partial \sigma} = -\frac{1}{\beta} < 0, \\
&\frac{\partial r_b}{\partial \sigma} = -\left\{\beta\left[1+\frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right]\right\}^{-2}\phi < 0, && \frac{\partial S}{\partial \sigma} = -\beta\frac{\partial r_b}{\partial \sigma} > 0, && \frac{\partial K}{\partial \sigma} = \frac{-\left[m+\phi(z+\bar{\ell}_{yb})\right]}{\beta} < 0, \\
&\frac{\partial Y}{\partial \sigma} = -\frac{\mu_\omega}{\beta} < 0, && \frac{\partial P}{\partial \sigma} = -\frac{\sigma-\beta}{\alpha\beta^2}\epsilon_k < 0, && \frac{\partial TL}{\partial \sigma} = 0, \\
&\frac{\partial R}{\partial \sigma} = 0, && \frac{\partial D}{\partial \sigma} = 0.
\end{aligned}
$$

The effects of open market operations, z_s.

$$\frac{\partial m}{\partial z_s} = -\phi < 0, \qquad \frac{\partial k^T}{\partial z_s} = (1-\phi) > 0, \qquad \frac{\partial TL}{\partial z_s} = \frac{\alpha\phi(z+\overline{\ell}_{yb})}{\epsilon_k} > 0,$$

$$\frac{\partial R}{\partial z_s} = \frac{\alpha\phi(z+\overline{\ell}_{yb})}{\epsilon_k TL}(1-R) < 0, \qquad \frac{\partial r_b}{\partial z_s} = \frac{\alpha\phi(1-\phi)}{\beta\epsilon_k\left[1+\frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right]^2} > 0, \qquad \frac{\partial S}{\partial z_s} = \frac{\partial R}{\partial z_s} - \beta\frac{\partial r_b}{\partial z_s} < 0,$$

$$\frac{\partial K}{\partial z_s} = \frac{\alpha\phi(1-\phi)(m+k^T)}{\epsilon_k} > 0, \qquad \frac{\partial P}{\partial z_s} = -\frac{\partial K}{\partial z_s} < 0, \qquad \frac{\partial N}{\partial z_s} = 0,$$

$$\frac{\partial D}{\partial z_s} = 0, \qquad \frac{\partial Y}{\partial z_s} = 0.$$

Proposition 10 and the proof. We first state the proposition regarding the macroeconomic effects of changes in the monitoring cost and bank access, and then we offer some explanations and the proof.

Proposition 10. *Consider the specific distributions,* $k \sim U[0,\epsilon_k]$*,* $\omega \sim U[0,\epsilon_\omega]$ *where* $\mu_k = \frac{\epsilon_k}{2}$*,* $\mu_\omega = \frac{\epsilon_\omega}{2}$*, and* $\gamma < \epsilon_\omega$*. In an equilibrium with constrained credit,*
(i) the effects of changes in the monitoring cost are: $\frac{\partial m}{\partial \gamma} > 0$, $\frac{\partial k^T}{\partial \gamma} < 0$, $\frac{\partial D}{\partial \gamma} < 0$, $\frac{\partial TL}{\partial \gamma} < 0$, $\frac{\partial r_b}{\partial \gamma} < 0$, $\frac{\partial K}{\partial \gamma} < 0$, $\frac{\partial N}{\partial \gamma} = 0$, *and when* γ *is big,* $\frac{\partial R}{\partial \gamma} < 0$, $\frac{\partial S}{\partial \gamma} < 0$, $\frac{\partial Y}{\partial \gamma} > 0$, $\frac{\partial P}{\partial \gamma} > 0$;
(ii) the effects of changes in the access to banking are: $\frac{\partial m}{\partial \phi} < 0$, $\frac{\partial k^T}{\partial \phi} < 0$, $\frac{\partial D}{\partial \phi} > 0$, $\frac{\partial TL}{\partial \phi} > 0$, $\frac{\partial r_b}{\partial \phi} < 0$, $\frac{\partial S}{\partial \phi} > 0$, $\frac{\partial Y}{\partial \phi} < 0$, $\frac{\partial N}{\partial \phi} = \frac{\partial R}{\partial \phi} = 0$, *and when* ϕ *is small,* $\frac{\partial K}{\partial \phi} > 0$, $\frac{\partial P}{\partial \phi} < 0$.

A rise in the monitoring cost impacts the economy mainly through its effects on asset holdings and the cost of lending risk. Demands for money and bonds rise because both assets can reduce the probability of incurring the monitoring cost, and the bond yield falls. Higher monitoring costs reduce $\overline{\ell}_{yb}$, and aggregate lending falls. The investment threshold, $k^T = m + z + \overline{\ell}_{yb}$, is lower since the rise in m cannot compensate for the decrease in $\overline{\ell}_{yb}$, and aggregate investment falls as well. A lower k^T implies a smaller risky loan amount, which leads to lower interest payment. When the access to bank credit rises (ϕ increases), the usefulness of bonds increases. Entrepreneurs have a higher demand for bonds, which drives the bond yield lower, and they reduce their real balances, which results in lower k^T. When more loans are granted due to higher access to bank credit, defaults and the aggregate bankruptcy costs are more likely to rise, and net output falls.

We now offer the proof of Proposition 10 by using the results in the proof of Proposition 5.

The effects of changes in the monitoring cost, γ.

We assume $\epsilon_\omega > \gamma$ to ensure that bank credit is feasible.

$$\frac{\partial m}{\partial \gamma} = \frac{\phi(\epsilon_\omega - \gamma)}{\epsilon_\omega} > 0, \qquad \frac{\partial k^T}{\partial \gamma} = -\frac{(1-\phi)(\epsilon_\omega - \gamma)}{\epsilon_\omega} < 0,$$

$$\frac{\partial D}{\partial \gamma} = \frac{-\alpha\phi}{\epsilon_k \epsilon_\omega} \frac{\epsilon_\omega - \gamma}{\epsilon_\omega}[2(\epsilon_\omega - \gamma)^2 + \epsilon_\omega] < 0, \qquad \frac{\partial TL}{\partial \gamma} = \frac{-\alpha\phi(z + \bar{\ell}_{yb})}{\epsilon_k} \frac{\epsilon_\omega - \gamma}{\epsilon_\omega} < 0,$$

$$\frac{\partial r_b}{\partial \gamma} = \frac{-\alpha\phi\frac{\epsilon_\omega - \gamma}{\epsilon_\omega}}{\beta\epsilon_k\left[1 + \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right]^2} < 0, \qquad \frac{\partial K}{\partial \gamma} = -\frac{\alpha\phi(1-\phi)(\epsilon_\omega - \gamma)}{\epsilon_k\epsilon_\omega}(z + \bar{\ell}_{yb}) < 0,$$

$$\frac{\partial N}{\partial \gamma} = 0, \qquad \frac{\partial R}{\partial \gamma} = \frac{\bar{\ell}_{yb}}{\epsilon_\omega(z + \bar{\ell}_{yb})^3}\left[\frac{(\epsilon_\omega - \gamma)^2}{6} - \gamma z\right],$$

$$\frac{\partial S}{\partial \gamma} = \frac{\partial R}{\partial \gamma} + \frac{\alpha\phi(1-\phi)(\epsilon_\omega - \gamma)}{\epsilon_k\epsilon_\omega\left[1 + \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right]^2}, \qquad \frac{\partial Y}{\partial \gamma} = \frac{\alpha\phi\bar{\ell}_{yb}}{\epsilon_k\epsilon_\omega}\left[2\gamma + \frac{2}{3} - \epsilon_\omega\right],$$

$$\frac{\partial P}{\partial \gamma} = \frac{\partial Y}{\partial \gamma} - \frac{\partial K}{\partial \gamma}.$$

These conditions imply

$$\frac{\partial R}{\partial \gamma} < 0 \text{ iff } \frac{(\epsilon_\omega - \gamma)^2}{6} < \gamma z,$$

$$\frac{\partial S}{\partial \gamma} < 0 \text{ iff } \frac{(\epsilon_\omega - \gamma)^2}{6} + \frac{2\alpha\phi(1-\phi)\epsilon_\omega(z - \bar{\ell}_{yb})^3}{\epsilon_k\left[1 + \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right]^2} < \gamma z,$$

$$\frac{\partial Y}{\partial \gamma} < 0 \text{ iff } \epsilon_\omega - \frac{2}{3} < \gamma,$$

$$\frac{\partial Y}{\partial \gamma} > 0 \text{ implies } \frac{\partial P}{\partial \gamma} > 0.$$

The effects of changes in access to banking, ϕ.

$$\frac{\partial m}{\partial \phi} = -(z+\bar{\ell}_{yb}) < 0, \qquad \frac{\partial k^T}{\partial \phi} = -(z+\bar{\ell}_{yb}) < 0,$$

$$\frac{\partial D}{\partial \phi} = \frac{\alpha}{\epsilon_k \epsilon_\omega}\left[\bar{\ell}_{yb}(\epsilon_\omega - \gamma) - \frac{(\epsilon_\omega-\gamma)^3}{3\epsilon_\omega}\right] > 0, \qquad \frac{\partial TL}{\partial \phi} = \frac{\alpha}{\epsilon_k}\frac{(z+\bar{\ell}_{yb})^2}{2} > 0,$$

$$\frac{\partial r_b}{\partial \phi} = \frac{-\alpha[\mu_\omega - k^T + \phi(z+\bar{\ell}_{yb})]}{\epsilon_k \beta\left[1+\frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right]^2} < 0, \qquad \frac{\partial R}{\partial \phi} = 0,$$

$$\frac{\partial S}{\partial \phi} = \frac{\beta}{\psi^{*2}}\frac{\partial \psi^*}{\partial \phi} > 0, \qquad \frac{\partial N}{\partial \phi} = 0,$$

$$\frac{\partial Y}{\partial \phi} = -\frac{\partial D}{\partial \phi} < 0, \qquad \frac{\partial K}{\partial \phi} = \frac{\alpha(z+\bar{\ell}_{yb})^2}{2\epsilon_k}(1-2\phi),$$

$$\frac{\partial P}{\partial \gamma} = \frac{\alpha(z+\bar{\ell}_{yb})^2}{\epsilon_k}\left[\phi - \frac{1}{2} - \frac{\bar{\ell}_{yb}^2}{(z+\bar{\ell}_{yb})^2}\frac{2\epsilon_\omega}{3(\epsilon_\omega-\gamma)}\right].$$

These conditions imply

$$\frac{\partial K}{\partial \phi} > 0 \text{ iff } \phi < \frac{1}{2},$$

$$\frac{\partial P}{\partial \phi} < 0 \text{ iff } \phi < \frac{1}{2} + \frac{\bar{\ell}_{yb}^2}{(z+\bar{\ell}_{yb})^2}\frac{2\epsilon_\omega}{3(\epsilon_\omega-\gamma)}.$$

Proof of Proposition 7. Given the down payment, d^*, and collateralized bonds, a^*, entrepreneurs follow the default rule as described in (6) under the policy. Hence, entrepreneurs have the expected payoffs as described in (8). The bank's expected payoff is changed under the policy:

$$\pi_b^p(y) = \theta\left[y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega \, dG(\omega) - \gamma G(y) - \underbrace{(k - d - \frac{a}{\theta})}_{\ell_y^p}\right],$$

which is shown in (22). The contract problem under policy thus can be described as

$$\{d^*,\ a^*,\ y^*\} = \arg\max_{\{d,a,y\}} -d - a + \int_y^{\overline{\omega}} \omega\, dG(\omega) - y[1-G(y)]$$

$$\text{s.t.} \quad \pi_b^p(y) = \theta\left[y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) - (k - d - \frac{a}{\theta})\right] = 0,$$

$$0 \le d \le m,\ 0 \le a \le z.$$

Notice that under the policy the risky loan amount becomes $\ell_y^p = k - d - \frac{a}{\theta}$, and one unit of bonds collateralized can reduce $\frac{1}{\theta} > 1$ unit of the risky loan amount, while one unit of real balances as a down payment can reduce one unit of the risky loan amount.

The solution to the contract problem satisfies $\pi_b^p(y) = 0$, and

$$a^* = \begin{cases} \theta k, & \text{if } \underline{k} \le k \le \frac{z}{\theta} \\ z, & \text{if } \frac{z}{\theta} < k \le k^p \end{cases}, \qquad d^* = \begin{cases} 0, & \text{if } \underline{k} \le k \le \frac{z}{\theta} \\ k - \frac{z}{\theta}, & \text{if } \frac{z}{\theta} < k \le m + \frac{z}{\theta} \\ m, & \text{if } m + \frac{z}{\theta} < k \le k^p, \end{cases}$$

which is shown in (24). The solution shows that banked entrepreneurs collateralize bonds before they use real balances as a down payment, which is different from Proposition 2. For instance, a banked entrepreneur with the input cost, $k \in [\underline{k}, \frac{z}{\theta}]$, will propose a contract characterized by $\{d^*,\ a^*,\ y^*\} = \{0,\ \theta k,\ 0\}$, wherein a fraction of bonds are collateralized, and no down payment is needed. Money will be used if $k > \frac{z}{\theta}$, under which all bond holdings are collateralized. When $k \in (m + \frac{z}{\theta}, k^p]$, the risky loan amount is strictly positive, which implies $y^* > 0$. Note that the contract, $\{d^*,\ a^*,\ y^*\} = \{0,\ \theta k,\ 0\}$, implies that the gross repayment of the original contract is $x^* = y^* + a^* = \theta k$. This also implies that, without the subsidy from the private asset purchase program, the lending bank experiences a loss by extending the loan. Finally, by binding bank's participation constraint and the solution to the contract problem (24), we obtain an entrepreneur's expected payoff, (25).

Proof of Lemma 1. Let $B(y)$ denote the bank's expected payoff backed solely by the risky project return:

$$B(y) = y[1 - G(y)] + \int_{\underline{\omega}}^{y} \omega \, dG(\omega) - \gamma G(y).$$

The bank's expected payoff under the policy, (22), can be rewritten as

$$\pi_b^p(y) = \theta \left[B(y) - \ell_y^p \right], \tag{35}$$

where $\ell_y^p = k - d - \frac{a}{\theta}$.

First, consider the case with constrained credit. Let $y_b \in [0, \overline{\omega}]$ denote the repayment such that $B'(y_b) = 0$, and $\overline{\ell}_{yb}$ denote the associated maximum risky loan amount such that $B(y_b) - \overline{\ell}_{yb} = 0$. From Definition 1, the investment threshold is $k_b = m + z + \overline{\ell}_{yb}$ without policy. Under the policy, let $y_b^p \in [0, \overline{\omega}]$ denote the repayment such that $B'(y_b^p) = 0$, and $\overline{\ell}_{yb}^p$ is the associated maximum risky loan amount such that $B(y_b^p) - \overline{\ell}_{yb}^p = 0$. Therefore, $\pi_b^p(y_b^p) = \theta \left[B(y_b^p) - \overline{\ell}_{yb}^p \right] = 0$, and the investment threshold under the policy is $k_b^p = m + \frac{z}{\theta} + B(y_b^p)$. Note that the repayment associated with the maximum risky loan amount depends only on the monitoring cost and the distribution of the project return, and thus, $y_b^p = y_b$, and $B(y_b) = B(y_b^p)$. Thereby, $k_b^p - k_b = \frac{1-\theta}{\theta} z$.

Next, consider the case with unconstrained credit. Without policy, there exists $(y_1,\, k_e)$ that solves

$$\begin{cases} \mu_\omega - \gamma G(y_1) - k_e = 0 \\ B(y_1) - (k_e - m - z) = 0, \end{cases} \tag{$*$}$$

while under the policy, $(y_2,\, k_e^p)$ solves

$$\begin{cases} \mu_\omega - \gamma G(y_2) - k_e^p + \frac{1-\theta}{\theta} z = 0 \\ B(y_2) - (k_e^p - m - \frac{z}{\theta}) = 0. \end{cases} \tag{$*'$}$$

Given that $(y_1,\, k_e)$ solves the system, $(*)$, we let $(y_2,\, k_e^p) = (y_1,\, k_e + \frac{1-\theta}{\theta} z)$ and verify that this is the solution for the system, $(*')$. Therefore, $k_e^p - k_e = \frac{1-\theta}{\theta} z$.

Derivation of (29) and (30). From (28), we have

$$\begin{aligned}
\frac{\sigma}{\beta} &= \alpha \left\{ \begin{array}{l} (1-\phi)(\mu_\omega - m) f(m) + \\ \phi \left[\begin{array}{l} (\mu_\omega - m - z) f(m + \dfrac{z}{\theta}) + \\ (\mu_\omega - \gamma G(y^\dagger) - k^p + \dfrac{1-\theta}{\theta} z) f(k^p) \dfrac{\partial k^p}{\partial m} - (\mu_\omega - m - z) f(m + \dfrac{z}{\theta}) - \\ \displaystyle\int_{m+\frac{z}{\theta}}^{k^p} \gamma g(y^*) \frac{\partial y^*}{\partial m} f(k) dk \end{array} \right] \end{array} \right\} + 1 \\
&= \alpha \left\{ (1-\phi)(\mu_\omega - m) f(m) + \phi \left[\begin{array}{l} (\mu_\omega - \gamma G(y^\dagger) - k^p + \dfrac{1-\theta}{\theta} z) f(k^p) \dfrac{\partial k^p}{\partial m} + \\ \underbrace{\displaystyle\int_{m+\frac{z}{\theta}}^{k^p} \gamma g(y^*) \frac{\partial y^*}{\partial k} f(k) dk}_{\text{Integration by parts}} \end{array} \right] \right\} + 1 \\
&= \alpha \left\{ (1-\phi)(\mu_\omega - m) f(m) + \phi \left[\begin{array}{l} (\mu_\omega - \gamma G(y^\dagger) - k^p + \dfrac{1-\theta}{\theta} z) f(k^p) \dfrac{\partial k^p}{\partial m} + \\ \left[\gamma G(y^*) f(k) \right]_{k=m+\frac{z}{\theta}}^{k=k^p} - \displaystyle\int_{m+\frac{z}{\theta}}^{k^p} \gamma G(y^*) f'(k) dk \end{array} \right] \right\} + 1 \\
&= \alpha \left\{ (1-\phi)(\mu_\omega - m) f(m) + \phi \left[(\mu_\omega - k^p + \frac{1-\theta}{\theta} z) f(k^p) - \int_{m+\frac{z}{\theta}}^{k^p} \gamma G(y^*) f'(k) dk \right] \right\} + 1,
\end{aligned}$$

where $y^\dagger = y^*(k^p)$ and we have used $\frac{\partial y^*}{\partial m} = -\frac{\partial y^*}{\partial k}$ in the second step, $\frac{\partial k^p}{\partial m} = 1$ for the constrained equilibrium, and $\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta}z = 0$ for the unconstrained equilibrium, to obtain the last line.

Let $\hat{z} = \frac{z}{\theta}$. Then,

$$
\begin{aligned}
\frac{\partial \mathbb{E} V^p(m,z)}{\partial \hat{z}} &= \alpha\phi \begin{bmatrix} (\mu_\omega - z)f(\hat{z}) + \\ (\mu_\omega - m - z)f(m+\hat{z}) - (\mu_\omega - z)f(\hat{z}) + \int_{\hat{z}}^{m+\hat{z}} (1-\theta)dF(k) + \\ (\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta}z)f(k^p)\frac{\partial k^p}{\partial \hat{z}} - (\mu_\omega - m - z)f(m+\hat{z}) - \\ \int_{m+\hat{z}}^{k^p} \gamma g(y^*)\frac{\partial y^*}{\partial \hat{z}} f(k)dk + \int_{m+\hat{z}}^{k^p} (1-\theta)dF(k) \end{bmatrix} + \theta \\
&= \alpha\phi \begin{bmatrix} (\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta}z)f(k^p)\frac{\partial k^p}{\partial \hat{z}} + \\ \underbrace{\int_{m+\hat{z}}^{k^p} \gamma g(y^*)\frac{\partial y^*}{\partial k} f(k)dk}_{\text{Integration by parts}} + (1-\theta)[F(k^p) - F(\hat{z})] \end{bmatrix} + \theta \\
&= \alpha\phi \begin{bmatrix} (\mu_\omega - k^p + \frac{1-\theta}{\theta}z)f(k^p) - \int_{m+\hat{z}}^{k^p} \gamma G(y^*)f'(k)dk + \\ (1-\theta)[F(k^p) - F(\hat{z})] \end{bmatrix} + \theta,
\end{aligned}
$$

where $y^\dagger = y^*(k^p)$ and we have used $\frac{\partial y^*}{\partial \hat{z}} = -\frac{\partial y^*}{\partial k}$ in the second step, $\frac{\partial k^p}{\partial \hat{z}} = 1$ for the constrained equilibrium, and $\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta}z = 0$ for the unconstrained equilibrium. Then,

$$
\begin{aligned}
\frac{\psi}{\beta} &= \frac{\partial \mathbb{E} V^p}{\partial z} = \frac{\partial \mathbb{E} V^p}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial z} = \frac{1}{\theta}\frac{\partial \mathbb{E} V^p}{\partial \hat{z}} \\
&= \frac{\alpha\phi}{\theta} \begin{bmatrix} (\mu_\omega - k^p + \frac{1-\theta}{\theta}z)f(k^p) - \int_{m+\frac{z}{\theta}}^{k^p} \gamma G(y^*)f'(k)dk + \\ (1-\theta)[F(k^p) - F(\frac{z}{\theta})] \end{bmatrix} + 1.
\end{aligned}
$$

Proof of Proposition 8. This proof contains three parts. We first show that there is an optimal risk-retention rate for a given policy rate, $\theta^* \in (0,1)$, such that $k^p = \mu_\omega$; next we show $\frac{\partial \theta^*}{\partial \epsilon_\omega} < 0$, $\frac{\partial \theta^*}{\partial \epsilon_k} < 0$, $\frac{\partial \theta^*}{\partial \alpha} > 0$ and $\frac{\partial \theta^*}{\partial i} < 0$; and finally we prove that social welfare decreases in the policy rate ($\frac{\partial \Omega^*}{\partial i}|_{\theta=\theta^*} < 0$).

(i) The optimal risk-retention rate is derived from the first-order condition to the problem which maximizes the welfare measure, (33). Define $\hat{z} = \frac{z}{\theta}$. From Lemma 1 we have the following condition:

$$\frac{\partial k^p}{\partial \theta} = \frac{\partial (k^T - z + \hat{z})}{\partial \theta} = \frac{\partial k^T}{\partial m}\frac{\partial m}{\partial \theta} + \frac{\partial \hat{z}}{\partial \theta}. \tag{36}$$

From the bank's binding participation constraint under the policy, we obtain

$$\frac{\partial y^*}{\partial \theta} = \frac{\partial y^*}{\partial (m+\hat{z})}\frac{\partial (m+\hat{z})}{\partial \theta} = -\frac{\partial y^*}{\partial k}(\frac{\partial m}{\partial \theta} + \frac{\partial \hat{z}}{\partial \theta}). \tag{37}$$

With (36) and (37), the first-order condition is given by

$$\begin{aligned}
\frac{\partial \Omega}{\partial \theta} &= \alpha \left\{ \begin{array}{l} (1-\phi)(\mu_\omega - m)f(m)\dfrac{\partial m}{\partial \theta} + \\ \phi \left[(\mu_\omega - k^p)f(k^p)\dfrac{\partial k^p}{\partial \theta} - \gamma G(y^\dagger)f(k^p)\dfrac{\partial k^p}{\partial \theta} - \displaystyle\int_{m+\hat{z}}^{k^p} \gamma g(y^*)\dfrac{\partial y^*}{\partial \theta} f(k)dk \right] \end{array} \right\} \\
&= \alpha \left\{ \begin{array}{l} (1-\phi)(\mu_\omega - m)f(m)\dfrac{\partial m}{\partial \theta} + \\ \phi \left[(\mu_\omega - \gamma G(y^\dagger) - k^p)f(k^p)(\dfrac{\partial k^T}{\partial m}\dfrac{\partial m}{\partial \theta} + \dfrac{\partial \hat{z}}{\partial \theta}) + \displaystyle\int_{m+\hat{z}}^{k^p} \gamma g(y^*)\dfrac{\partial y^*}{\partial k}(\dfrac{\partial m}{\partial \theta} + \dfrac{\partial \hat{z}}{\partial \theta}) f(k)dk \right] \end{array} \right\} \\
&= \alpha \left\{ \begin{array}{l} \left\{ (1-\phi)(\mu_\omega - m)f(m) + \phi \left[(\mu_\omega - \gamma G(y^\dagger) - k^p)f(k^p)\dfrac{\partial k^T}{\partial m} + \underbrace{\displaystyle\int_{m+\hat{z}}^{k^p} \gamma g(y^*)\dfrac{\partial y^*}{\partial k} f(k)dk}_{G_A} \right] \right\} \dfrac{\partial m}{\partial \theta} + \\ \phi \left[(\mu_\omega - \gamma G(y^\dagger) - k^p)f(k^p) + \underbrace{\displaystyle\int_{m+\hat{z}}^{k^p} \gamma g(y^*)\dfrac{\partial y^*}{\partial k} f(k)dk}_{G_A} \right] \dfrac{\partial \hat{z}}{\partial \theta} \end{array} \right\} \\
&= \alpha \left\{ \begin{array}{l} \left\{ (1-\phi)(\mu_\omega - m)f(m) + \phi \left[\begin{array}{l} (\mu_\omega - k^p)f(k^p)\dfrac{\partial k^T}{\partial m} - \gamma G(y^\dagger)f(k^p)(\dfrac{\partial k^T}{\partial m} - 1) \\ - \displaystyle\int_{m+\hat{z}}^{k^p} \gamma G(y^*)f'(k)dk \end{array} \right] \right\} \dfrac{\partial m}{\partial \theta} + \\ \phi \left[(\mu_\omega - k^p)f(k^p) - \displaystyle\int_{m+\hat{z}}^{k^p} \gamma G(y^*)f'(k)dk \right] \dfrac{\partial \hat{z}}{\partial \theta} \end{array} \right\}
\end{aligned} \tag{38}$$

where $y^\dagger = y^*(k^p)$ and $G_A = \gamma G(y^\dagger)f(k^p) - \int_{m+\hat{z}}^{k^p} \gamma G(y^*)f'(k)dk$ is obtained by applying the integration by parts. Notice that k^T is the investment threshold without policy when entrepreneurs hold portfolio (m, z).

Given Lemma 1 and uniform distribution assumptions, we have $f'(k) = 0$ and $f(k^p) = f(k^T)$. We rewrite (29) as

$$\frac{\sigma}{\beta} = \alpha \left\{ (1-\phi)(\mu_\omega - m)f(m) + \phi(\mu_\omega - k^T)f(k^T) \right\} + 1,$$

which is the optimal choice of real balances under the policy. Therefore, if the input cost follows a uniform distribution, the decision of money holding is unaffected by the policy because the marginal benefit of money remains the same, which implies

$$\frac{\partial m}{\partial \theta} = 0.$$

Substituting $\frac{\partial m}{\partial \theta} = 0$ into (38), we have

$$\frac{\partial \Omega}{\partial \theta} = \alpha\phi(\mu_\omega - k^p) f(k^p)\frac{\partial \hat{z}}{\partial \theta} = -\frac{\alpha\phi z}{\theta^2}(\mu_\omega - k^p) f(k^p),$$

which suggests that the welfare measure, Ω, reaches the maximum when $k^p = \mu_\omega$ or $k^p > \overline{k}$. We focus on the non-trivial case with $k^p = \mu_\omega$ in the following analysis. Thus, the optimal risk-retention rate, θ^*, satisfies

$$\mu_\omega = k^p = k^T + \frac{1-\theta^*}{\theta^*} z,$$

which implies

$$\theta^* = \frac{z}{\mu_\omega - k^T + z} \in (0, 1) \tag{39}$$

when $k^T < \mu_\omega$. The second-order condition is

$$\frac{\partial^2 \Omega}{\partial^2 \theta} = -\frac{\partial \frac{\alpha\phi z}{\epsilon_k \theta^2}(\mu_\omega - k^p)}{\partial \theta} = 2\frac{\alpha\phi z}{\epsilon_k \theta^3}(\mu_\omega - k^p) + \frac{\alpha\phi z}{\epsilon_k \theta^2}\frac{\partial k^p}{\partial \theta}. \tag{40}$$

Substituting θ^* from (39) into equation (40), we have $\frac{\partial^2 \Omega}{\partial^2 \theta} \mid_{\theta=\theta^*} < 0$, so the sufficient condition holds.

(ii) We investigate how increased uncertainty, difficulties in meeting with a supplier, and a higher policy rate affect the optimal risk-retention rate. As the Fisher equation, $i = \sigma(1 + r) - 1$, implies that the policy rate (i) and inflation rate (σ) is one-to-one, in this proof, for convenience we use σ instead of i in the derivation. Condition (39) implies $\frac{\partial \theta^*}{\partial k^T} > 0$, and from Proposition 3, we have $\frac{\partial k^T}{\partial \alpha} > 0$, $\frac{\partial k^T}{\partial \epsilon_\omega} < 0$, $\frac{\partial k^T}{\partial \epsilon_k} < 0$, and $\frac{\partial k^T}{\partial \sigma} < 0$ (see below for detailed derivations). Hence, we conclude

$$\frac{\partial \theta^*}{\partial \alpha} = \frac{\partial \theta^*}{\partial k^T}\frac{\partial k^T}{\partial \alpha} > 0; \quad \frac{\partial \theta^*}{\partial \epsilon_\omega} = \frac{\partial \theta^*}{\partial k^T}\frac{\partial k^T}{\partial \epsilon_\omega} < 0;$$
$$\frac{\partial \theta^*}{\partial \epsilon_k} = \frac{\partial \theta^*}{\partial k^T}\frac{\partial k^T}{\partial \epsilon_k} < 0; \quad \frac{\partial \theta^*}{\partial \sigma} = \frac{\partial \theta^*}{\partial k^T}\frac{\partial k^T}{\partial \sigma} < 0.$$

Derivations.

Case 1. Equilibrium with constrained credit.

$$k^T = \mu_\omega + (1-\phi)(z + \overline{\ell}_{yb}) - \frac{\sigma - \beta}{\alpha\beta}\epsilon_k.$$

Thus,

$$\frac{\partial k^T}{\partial \alpha} = \frac{\sigma-\beta}{\alpha^2\beta}\epsilon_k > 0, \quad \frac{\partial k^T}{\partial \epsilon_\omega} = (1-\phi)\frac{\partial \bar{\ell}_{yb}}{\partial \epsilon_\omega} < 0,$$
$$\frac{\partial k^T}{\partial \epsilon_k} = -\frac{\sigma-\beta}{\alpha\beta} < 0, \quad \frac{\partial k^T}{\partial \sigma} = -\frac{1}{\alpha\beta} < 0.$$

Case 2. Equilibrium with unconstrained credit (assume $\sigma > \beta$).

$$m \quad \text{solves} \quad (1-\phi)(\mu_\omega - m) + \phi\gamma\left(1-\sqrt{\frac{2(m+z)}{\epsilon_\omega}}\right) - \frac{(\sigma-\beta)}{\alpha\beta}\epsilon_k = 0,$$
$$k^T \quad = \quad \mu_\omega - \gamma + \gamma\sqrt{\frac{2(m+z)}{\epsilon_\omega}}.$$

Thus,

$$\frac{\partial m}{\partial \alpha} > 0, \quad \frac{\partial m}{\partial \epsilon_k} < 0, \quad \frac{\partial m}{\partial \epsilon_\omega} > 0, \quad \frac{\partial m}{\partial \sigma} < 0.$$

The condition, $\frac{\partial m}{\partial \epsilon_\omega} > 0$, implies $\partial\sqrt{\frac{2(m+z)}{\epsilon_\omega}}/\partial\epsilon_\omega < 0$. Hence,

$$\frac{\partial k^T}{\partial \alpha} = \frac{\gamma}{\epsilon_\omega}\left(\frac{2(m+z)}{\epsilon_\omega}\right)^{-\frac{1}{2}}\frac{\partial m}{\partial \alpha} > 0,$$
$$\frac{\partial k^T}{\partial \epsilon_k} = \frac{\gamma}{\epsilon_\omega}\left(\frac{2(m+z)}{\epsilon_\omega}\right)^{-\frac{1}{2}}\frac{\partial m}{\partial \epsilon_k} < 0,$$
$$\frac{\partial k^T}{\partial \epsilon_\omega} = \frac{\gamma}{\epsilon_\omega}\left(\frac{2(m+z)}{\epsilon_\omega}\right)^{-\frac{1}{2}}\left[\partial\sqrt{\frac{2(m+z)}{\epsilon_\omega}}/\partial\epsilon_\omega\right] < 0,$$
$$\frac{\partial k^T}{\partial \sigma} = \frac{\gamma}{\epsilon_\omega}\left(\frac{2(m+z)}{\epsilon_\omega}\right)^{-\frac{1}{2}}\frac{\partial m}{\partial \sigma} < 0.$$

(iii) Given that $\frac{\partial \theta^*}{\partial i} < 0$, we now show that $\frac{\partial \Omega}{\partial i}|_{\theta=\theta^*} < 0$. Again, we use $\frac{\partial \Omega}{\partial \sigma}|_{\theta=\theta^*}$ to capture the effect of the policy rate on welfare. From part (i) of Proposition 8, we know θ^* such that $k^p = \mu_\omega$ for any given σ. Therefore, taking the derivative of welfare measure Ω in (33) with respect to σ when $\theta = \theta^*$, we have

$$\frac{\partial \Omega}{\partial \sigma}|_{\theta=\theta^*} = \alpha\left[(1-\phi)(\mu_\omega - m)f(m)\frac{\partial m}{\partial \sigma} - \phi\frac{\partial\int_{m+\frac{z}{\theta^*}}^{k^{p*}}\gamma G(y^*)dF(k)}{\partial \sigma}\right].$$

The first term in the brackets is negative because $\frac{\partial m}{\partial \sigma} < 0$. As for the second term, we observe that it is an integral from $m + \frac{z}{\theta^*}$ to k^{p*}. By Lemma 1,

$$k^{p*} - (m + \frac{z}{\theta^*}) = k^T - (m + z), \tag{41}$$

which equals the risky loans taken by the marginal borrowers. This amount, (41), remains unchanged in a credit-constrained equilibrium since $k^T - (m + z) = \bar{\ell}_{yb}$. However, in a credit-unconstrained equilibrium this risky loan amount will increase as σ increases, because m is decreased by higher inflation and entrepreneurs need to rely more on bank credit. Thus, the second term in the bracket is non-positive, and we conclude that welfare decreases in the policy rate.

Proof of Proposition 9. Here we show how the private asset purchase policy changes the aggregate variables, $\{N, TL, K, Y, R, \frac{D}{N}\}$. In the proof, we use the results from Proposition 8 that the real balances, m, is unchanged under the policy while the investment threshold is raised to $k^p = \mu_\omega > k^T$. Below the variables with a superscript p denote those under the policy.

N (firm entry)

$$N^p - N = \alpha\left[(1-\phi)F(m) + \phi F(k^p)\right] - \alpha\left[(1-\phi)F(m) + \phi F(k^T)\right] = \alpha\phi \int_{k^T}^{k^p} dF(k) > 0.$$

TL (total loans; aggregate lending)

$$\begin{aligned} TL^p - TL &= \alpha\phi \left[\int_0^{\frac{z}{\theta^*}} kdF(k) + \int_{\frac{z}{\theta^*}}^{m+\frac{z}{\theta^*}} \frac{z}{\theta^*} dF(k) + \int_{m+\frac{z}{\theta^*}}^{k^p} (k-m)dF(k)\right] - \alpha\phi \int_m^{k^T} (k-m)dF(k) \\ &= \frac{\alpha\phi}{\bar{k}} \left[\frac{m+z}{\theta^*} + \frac{(1-\theta^{*2})z^2}{2\theta^{*2}} + \frac{(1-\theta^*)\bar{\ell}_{yb} z}{\theta^*}\right] > 0. \end{aligned}$$

K (investment)

$$\begin{aligned} K^p - K &= \alpha\left[(1-\phi)\int_0^m kdF(k) + \phi \int_0^{k^p} kdF(k)\right] - \alpha\left[(1-\phi)\int_0^m kdF(k) + \phi\int_0^{k^T} kdF(k)\right] \\ &= \alpha\phi \int_{k^T}^{k^p} kdF(k) > 0. \end{aligned}$$

Y (net output)

To derive the changes in net output, Y, we need to investigate first the changes in aggregate defaults, D, which is

$$D^p - D = \alpha\phi \int_{m+\frac{z}{\theta^*}}^{k^p} G(y^*)dF(k) - \alpha\phi \int_{m+z}^{k^T} G(y^*)dF(k) = 0.$$

Hence,

$$Y^p - Y = (\mu_\omega N^p - \gamma D^p) - (\mu_\omega N - \gamma D) = \mu_\omega (N^p - N) > 0.$$

R (average lending rate)

First, notice that

$$R^p = \frac{X^p}{TL^p} = \frac{X + A}{TL + A}$$

where

$$A = \frac{\alpha\phi}{\overline{k}} \left[\frac{m+z}{\theta^*} + \frac{(1-\theta^{*2})z^2}{2\theta^{*2}} + \frac{(1-\theta^*)\overline{\ell}_{yb} z}{\theta^*} \right].$$

The increase in the total loan amounts, $TL^p - TL$, is due to that the policy effectively enlarges the quantity of collateralizable bonds from a to $\frac{a}{\theta}$, and the change of pecking order in financing investment. From (24) and Lemma 1, we infer that the risky loan amount is unaffected by the policy; i.e., $\int_{m+z}^{k^T} (k - m - z) dF(k) = \int_{m+\frac{z}{\theta}}^{k^p} (k - m - \frac{z}{\theta}) dF(k)$. This implies that the difference, $TL^p - TL$, is fully secured by bonds.

Meanwhile, the repayment, x^*, of a debt contract can be decomposed into $a^* + y^*$, with $y^* > 0$ when the risky loan amount is strictly positive. The contract with the same risky loan amount will have the same effective repayment. Because the risky loan amount is not affected by the policy, and uniform distribution of k assigns an equal probability to each risky loan amount, the aggregate effective repayment is not changed by the policy; i.e., $\int_{m+z}^{k^T} y^* dF(k) = \int_{m+\frac{z}{\theta}}^{k^p} y^* dF(k)$. This implies that the change in the aggregate repayment, $X^p - X$, comes from the collateralized bonds, a^*. When a loan is fully secured, the solution to the debt contract problem is $y^* = 0$, or equivalently, $x^* = k - d^*$. Hence the repayment just is equal to the loan amount. We also know that $R = \frac{X}{TL} \geq 1$, and the strict inequality holds if the aggregate risky loan amount is larger than

0. Therefore, $TL^p - TL = A$ as is shown above. We now verify $X^p - X$

$$
\begin{aligned}
X^p - X &= \alpha\phi \left\{ \begin{array}{l} \left[\int_0^{\frac{z}{\theta^*}} k dF(k) + \int_{\frac{z}{\theta^*}}^{m+\frac{z}{\theta^*}} \frac{z}{\theta^*} dF(k) + \int_{m+\frac{z}{\theta^*}}^{k^p} (y^* + \frac{z}{\theta^*}) dF(k) \right] - \\ \left[\int_m^{m+z} k dF(k) + \int_{m+z}^{k^T} (y^* + z) dF(k) \right] \end{array} \right\} \\
&= \alpha\phi \left\{ \begin{array}{l} \left[\int_0^{\frac{z}{\theta^*}} k dF(k) + \int_{\frac{z}{\theta^*}}^{m+\frac{z}{\theta^*}} \frac{z}{\theta^*} dF(k) + \int_{m+\frac{z}{\theta^*}}^{k^p} \frac{z}{\theta^*} dF(k) \right] - \\ \left[\int_m^{m+z} k dF(k) + \int_{m+z}^{k^T} z dF(k) \right] \end{array} \right\} \\
&= \frac{\alpha\phi}{\bar{k}} \left[\frac{m+z}{\theta^*} + \frac{(1-\theta^{*2}) z^2}{2\theta^{*2}} + \frac{(1-\theta^*)\bar{\ell}_{yb} z}{\theta^*} \right] > 0.
\end{aligned}
$$

Therefore,

$$
R^p - R = \frac{X + A}{TL + A} - \frac{X}{TL} < 0.
$$

D/N (business failure rate)

Because $N^p - N > 0$ and $D^p = D$, we have

$$
\frac{D^p}{N^p} - \frac{D}{N} < 0.
$$

Supplementary Appendix to "Corporate Finance, Collateralized Borrowing, and Monetary Policy"

This supplementary appendix establishes results to complement and extend the main analysis of the paper.

Appendix B. Bank credit and trade credit

We consider bank credit and trade credit, both of which are subject to costly state verification problems, and bonds are used as collateral in both types of credit. Assume that an entrepreneur has access to bank credit with probability $\phi_b \in (0,1)$. An unbanked entrepreneur has access to trade credit with probability $\phi_s \in (0,1)$. An entrepreneur's project return is private information, and a bank needs to incur a monitoring cost, γ_b, to observe the project return, and a supplier's monitoring cost is γ_s. To capture the special role of financial intermediaries, we assume $\gamma_b \leq \gamma_s$. We consider first the case wherein $\gamma_b < \gamma_s$, and then we show that the case with $\gamma_b = \gamma_s = \gamma$ is identical to the baseline model.

The solution to a debt contract derived in Section 3 can be applied to trade credit as well. However, due to the difference in monitoring costs, the maximum risky loan amounts and thus, the investment thresholds are different for bank credit and trade credit. Let y_j and $\bar{\ell}_{yj}$, $j = b, s$, denote the effective repayment and the maximum risky loan amount for bank credit and trade credit, respectively. Similar to the discussion in Section 3.2, a banked entrepreneur will implement a project with the input cost $k > m$ if banks are willing to extend credit (and the investment threshold is $k_{bb} = m + z + \bar{\ell}_{yb}$), and in those cases in which they are, the entrepreneur's expected payoff must also be positive (and the investment threshold is k_{eb}, where $k_{eb} \leq k_{bb}$ so that credit is feasible and it satisfies $\pi_e(y_b; k_{eb}) = 0$). Similarly, an unbanked entrepreneur with access to trade credit will implement a project with the input cost $k > m$ if suppliers are willing to extend credit (and the investment threshold is $k_{bs} = m + z + \bar{\ell}_{ys}$), and the entrepreneur's expected payoff must be positive (and the investment threshold is k_{es}, where $k_{es} \leq k_{bs}$ so that credit is feasible and it satisfies $\pi_e(y_s; k_{es}) = 0$).

We have the following definition analogous to Definition 1 in the main text.

Definition 3. *In an equilibrium entrepreneurs with (m, z) are credit-unconstrained if for both bank credit and trade credit, there exists a $k_{ej} \leq k_{bj}$ satisfying $\pi_e(y_j; k_{ej}) = 0$, and the investment threshold is $k_j^T = k_{ej}$; otherwise, some entrepreneurs are constrained in bank credit or trade credit, and $k_j^T = k_{bj} = m + z + \bar{\ell}_{yj}$, $j = b, s$.*

Optimal portfolios and financing choices

The expected value of an entrepreneur with asset holdings, (m, z), is

$$\mathbb{E}\,V(m,z) = \alpha \left\{ \begin{array}{l} \phi_b \displaystyle\int_{\underline{k}}^{k_b^T} (\mu_\omega - \gamma_b G(y_b^*(k)) - k) dF(k) + \\ (1-\phi_b)\Big[\underbrace{(1-\phi_s) \displaystyle\int_{\underline{k}}^{m} (\mu_\omega - k) dF(k)}_{\text{(a)}} + \underbrace{\phi_s \displaystyle\int_{\underline{k}}^{k_s^T} (\mu_\omega - \gamma_s G(y_s^*(k)) - k) dF(k)}_{\text{(b)}} \Big] \end{array} \right\} + W(m,z,0,0), \tag{42}$$

where $y_b^*(k)$ and $y_s^*(k)$ are the solutions to contracts in bank credit and trade credit, respectively. The first line in (42) is the expected payoffs of borrowing from a bank and financing a project with an input cost up to k_b^T, where $k_b^T = k_{eb}$ or k_{bb}. Term (a) in (42) shows that, with probability, $(1-\phi_b)(1-\phi_s)$, the entrepreneur has no access to any credit, and she can implement a project with an input cost lower than her internal funds. Term (b) shows that if the unbanked entrepreneur can borrow from a supplier, she can finance a project up to k_s^T where $k_s^T = k_{es}$ or k_{bs}.

From (42), the expected marginal values of real balances and bonds are

$$\begin{aligned} \mathbb{E}\,V_m(m,z) &= \alpha(1-\phi_b)(1-\phi_s) f(m)(\mu_\omega - m) \\ &+ \alpha\phi_b \left\{ f(k_b^T)\frac{\partial k_b^T}{\partial m}\left[\mu_\omega - \gamma_b G(y_b^\dagger) - k_b^T\right] - \int_{m+z}^{k_b^T} \gamma_b g(y_b^*) \frac{\partial y_b^*}{\partial m} dF(k) \right\} \\ &+ \alpha(1-\phi_b)\phi_s \left\{ f(k_s^T)\frac{\partial k_s^T}{\partial m}\left[\mu_\omega - \gamma_s G(y_s^\dagger) - k_s^T\right] - \int_{m+z}^{k_s^T} \gamma_s g(y_s^*) \frac{\partial y_s^*}{\partial m} dF(k) \right\} + 1, \end{aligned} \tag{43}$$

$$\begin{aligned} \mathbb{E}\,V_z(m,z) &= \alpha\phi_b \Big\{ f(k_b^T)\frac{\partial k_b^T}{\partial z}\left[\mu_\omega - \gamma_b G(y_b^\dagger) - k_b^T\right] - \int_{m+z}^{k_b^T} \gamma_b g(y_b^*) \frac{\partial y_b^*}{\partial z} dF(k) \Big\} \\ &+ \alpha(1-\phi_b)\phi_s \Big\{ f(k_s^T)\frac{\partial k_s^T}{\partial z}\left[\mu_\omega - \gamma_s G(y_s^\dagger) - k_s^T\right] - \int_{m+z}^{k_s^T} \gamma_s g(y_s^*) \frac{\partial y_s^*}{\partial z} dF(k) \Big\} + 1, \end{aligned} \tag{44}$$

where

$$\frac{\partial k_j^T}{\partial m} = \frac{\partial k_j^T}{\partial z} = \begin{cases} \frac{\gamma_j g(y_{bj})}{1-G(y_{bj})} < 1 & \text{when } k_j^T = k_{ej}, \\ 1 & \text{when } k_j^T = k_{bj}, \end{cases} \quad j = b, s, \tag{45}$$

and $y_j^\dagger = y_j^*(k_j^T)$, $j = b, s$, when she borrows from a bank or a supplier. Interpretations of (43), (44), and (45) are the same as those of (14), (15), and (16), respectively.

Using (4) and (5) lagged one period to eliminate $\mathbb{E}\, V_m(m,z)$ and $\mathbb{E}\, V_z(m,z)$ from (43) and (44), respectively, we have an entrepreneur's optimal asset holdings satisfying

$$\begin{aligned}\frac{\sigma}{\beta} &= \alpha(1-\phi_b)(1-\phi_s)f(m)(\mu_\omega - m) + \alpha\phi_b\Big[f(k_b^T)(\mu_\omega - k_b^T) - \gamma_b\int_{m+z}^{k_b^T} G(y_b^*)f'(k)dk\Big] \quad (46)\\ &\quad +\alpha(1-\phi_b)\phi_s\Big[f(k_s^T)(\mu_\omega - k_s^T) - \gamma_s\int_{m+z}^{k_s^T} G(y_s^*)f'(k)dk\Big] + 1\\ \frac{\psi}{\beta} &= \alpha\phi_b\Big\{f(k_b^T)(\mu_\omega - k_b^T) - \gamma_b\int_{m+z}^{k_b^T} G(y_b^*)f'(k)dk\Big\} \quad (47)\\ &\quad +\alpha(1-\phi_b)\phi_s\Big\{f(k_s^T)(\mu_\omega - k_s^T) - \gamma_s\int_{m+z}^{k_s^T} G(y_s^*)f'(k)dk\Big\} + 1.\end{aligned}$$

Observing from equations (46) and (47) that, if $\phi_b = 1$ or $\phi_s = 1$, money and bonds are perfect substitutes. When entrepreneurs' access to banking or trade credit is not always ensured, $\phi_b < 1$ and $\phi_s < 1$, money enjoys a higher liquidity premium than bonds.

Definition 4. *Given $\{G(\cdot), F(\cdot), \gamma, \sigma, m_s, z_s\}$, a monetary equilibrium with bank credit and trade credit is (m, ψ), satisfying (46) and (47), the market-clearing conditions for money and bonds, $m = m_s$ and $z = z_s$, and the effective repayment, $y_j^*(k)$, solves the contract problem (11) with the monitoring cost, γ_j, for $k \leq k_j^T$, where $k_j^T = k_{ej}$ or $k_j^T = k_{b_j}$, $j = b, s$.*

There are three possible cases in which whether entrepreneurs are constrained in bank credit and/or trade credit. We first calculate the amount of internal funds m_{ej}, $j = b, s$, which is the amount an entrepreneur needs to hold to prevent being credit constrained when borrowing from a bank and a supplier, respectively. Comparing the real balances, m^*, held by entrepreneurs in equilibrium with the amount, m_{ej}, we know in which credit channel entrepreneurs are constrained. For example, if $m_{eb} < m_{es}$, then $m^* \in (m_{es}, \infty)$ implies entrepreneurs are not credit constrained in both channels; $m^* \in (m_{eb}, m_{es}]$ implies entrepreneurs are constrained in trade credit, but not in bank credit; $m^* \in [0, m_{eb}]$ implies entrepreneurs are constrained in both credit channels.

Aggregate variables and policy implications

We redefine the aggregate variables in Section 4.2, by considering both bank credit and trade credit.

The firm entry rate is defined as

$$N = \alpha[(1-\phi_b)(1-\phi_s)F(m) + \phi_b F(k_b^T) + (1-\phi_b)\phi_s F(k_s^T)].$$

The aggregate default is defined as $D = D_b + D_s$, where

$$D_b = \alpha\phi_b \int_{m+z}^{k_b^T} G(y_b^*(k))dF(k), \text{ and } D_s = \alpha(1-\phi_b)\phi_s \int_{m+z}^{k_s^T} G(y_s^*(k))dF(k).$$

The aggregate lending is defined as total bank loans plus trade credit, $TL = TL_b + TL_s$ where

$$TL_b = \alpha\phi_b \int_{m}^{k_b^T} (k-m)dF(k) \text{ and } TL_s = \alpha(1-\phi_b)\phi_s \int_{m}^{k_s^T} (k-m)dF(k).$$

The average interest payment of bank loans and trade credit are $X_b = \alpha\phi_b \int_m^{k_b^T} x_b^*\, dF(k)$ and $X_s = \alpha(1-\phi_b)\phi_s \int_m^{k_s^T} x_s^*\, dF(k)$ where x_j^* is the repayment of the original contract of bank credit and trade credit, $j = b, s$. The average real lending rates for bank credit and trade credit are defined as $R_j = \frac{X_j}{TL_j}$.

The aggregate investment is defined as

$$K = \alpha(1-\phi_b)(1-\phi_s) \int_{\underline{k}}^{m} k\, dF(k) + \alpha\Big[\phi_b \int_{\underline{k}}^{k_b^T} k\, dF(k) + (1-\phi_b)\phi_s \int_{\underline{k}}^{k_s^T} k\, dF(k)\Big].$$

We define net output, Y, as aggregate output subtracting the aggregate bankruptcy costs, $Y = \mu_\omega N - \sum \gamma_j D_j$. An entrepreneur's expected profit is

$$\begin{aligned} P &= \alpha(1-\phi_b)(1-\phi_s) \int_{\underline{k}}^{m} (\mu_\omega - k)dF(k) + \\ &\alpha\Big\{\phi_b \int_{\underline{k}}^{k_b^T} [\mu_\omega - \gamma_b G(y_b^*) - k]dF(k) + (1-\phi_b)\phi_s \int_{\underline{k}}^{k_s^T} [\mu_\omega - \gamma_s G(y_s^*) - k]dF(k)\Big\}. \end{aligned}$$

Using the aggregate variables defined above, we study the policy implications of adjusting the policy rate and OMOs with numerical simulations.[29] In the numerical exercises of Figures 5 and 6, real balances $m^* < m_{eb} < m_{es}$; that is, entrepreneurs are constrained in both credit channels, and the constraint in bank credit is less stringent than in trade credit because $\gamma_b < \gamma_s$. Figure 5 shows that as the central bank raises the policy rate, investment thresholds in both credit channels, k_b^T and k_s^T, fall, so do firm entry, aggregate investment, aggregate lending and net output, while the average lending rates of bank credit (R_b) and trade credit (R_s), and the respective risk spreads rise. These results are similar to those shown in Figure 2, which considers bank credit only. Figure 6 shows that aggregate defaults, average lending rates of bank credit and trade credit and respective

[29] In the numerical examples of Figures 5 and 6, we set $\phi_b = \phi_s = 0.7$ and $15 = \gamma_b < \gamma_s = 15.5$; other parameters are the same as those in Figures 2 and 3.

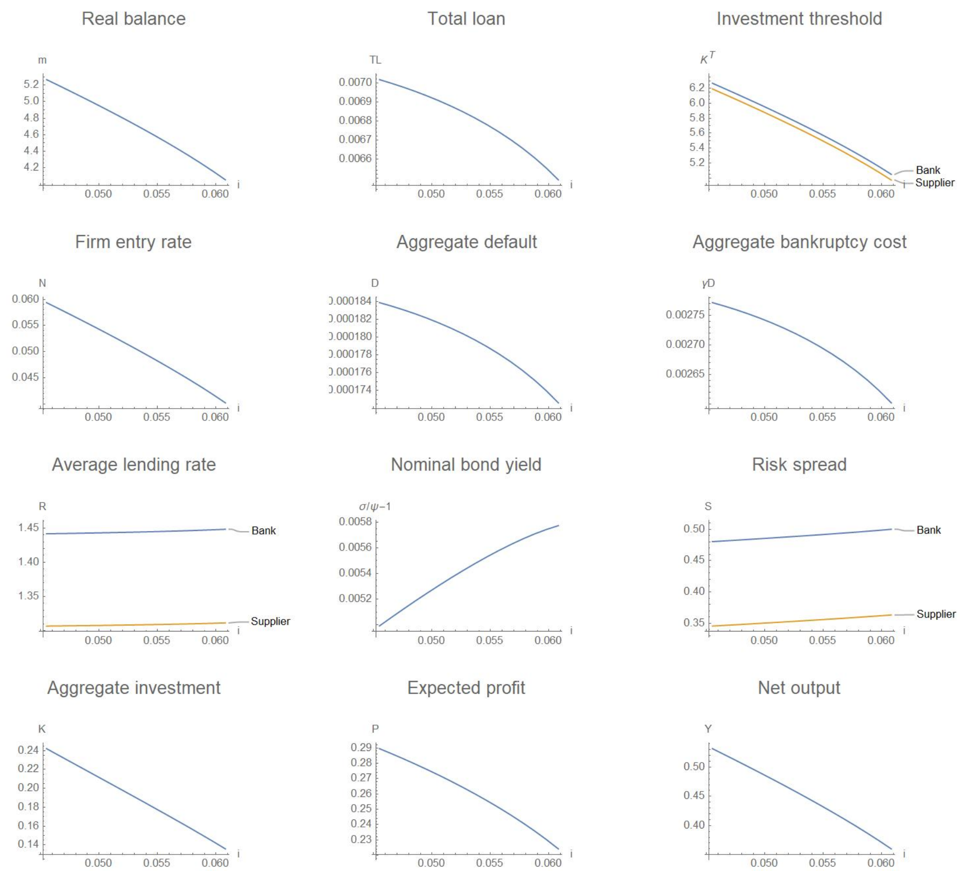

Figure 5: Effects of changes in the policy rate in the model with bank credit and trade credit

risk spreads decrease, and firm entry, investment thresholds, aggregate investment, and net output rise from open market sales. These results are consistent with what we have seen in Figure 3, wherein we consider bank credit only. Thus, we conclude that the policy implications in the model with two credit channels are similar to those in the benchmark model with bank credit only.

The case with $\gamma_b = \gamma_s = \gamma$

The expected value of an entrepreneur with asset holdings, (m, z), is

$$\begin{aligned}\mathbb{E}V(m,z) &= \alpha(1-\phi_b)\left[(1-\phi_s)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k) + \phi_s\int_{\underline{k}}^{k^T}(\mu_\omega - \gamma G(y^*(k)) - k)dF(k)\right] \\ &\quad + \alpha\phi_b\int_{\underline{k}}^{k^T}(\mu_\omega - \gamma G(y^*(k)) - k)dF(k) + W(m,z,0,0), \qquad (48)\end{aligned}$$

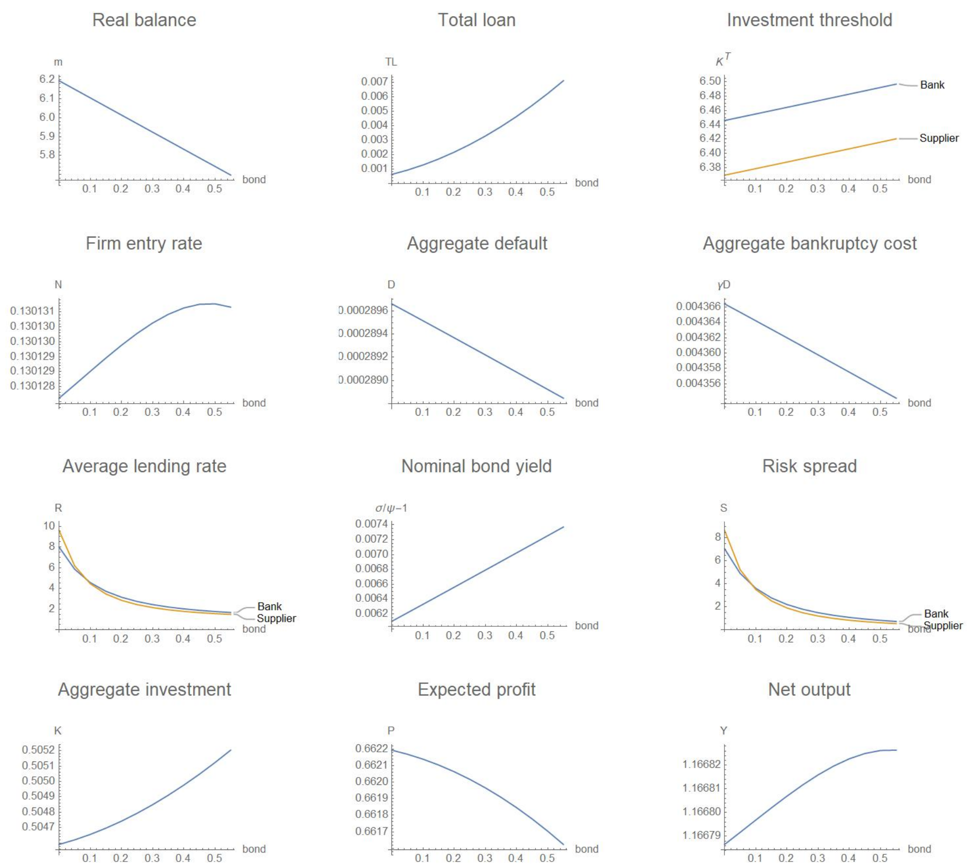

Figure 6: Effects of open market operations in the model with bank credit and trade credit

where the first line in (48) shows that if the entrepreneur has no access to bank credit, her payoff is from financing the project with internal funds only or with trade credit; the second line is the payoff from financing the project with bank loans. Let $\phi \equiv (1-\phi_b)\phi_s + \phi_b$ denote the probability that an entrepreneur has access to credit from banks or suppliers, and so $1-\phi = (1-\phi_b)(1-\phi_s)$ denote the probability that access to credit is not available. The expected value of an entrepreneur with asset holdings, (m, z), becomes

$$\mathbb{E}\, V(m,z) = \alpha\Big[(1-\phi)\int_{\underline{k}}^{m} (\mu_\omega - k)dF(k) + \phi\int_{\underline{k}}^{k^T} (\mu_\omega - \gamma G(y^*(k)) - k)dF(k)\Big] + W(m,z,0,0),$$

which is identical to (13). Therefore, the model is identical to that presented in the main text.

Appendix C. The central bank claiming collateral in asset purchases

This appendix considers the asset purchase policy wherein the central bank obtains a fraction of collateral backing the loans that it purchases. The policy requires the issuer of private assets to retain a fraction, $\theta_1 \in (0,1)$, of the assets on its book. The issuing bank retains θ_1 fraction of the expected returns backed solely by risky investment returns, and $\theta_2 \in [0,1]$ fraction of collateral once defaults occur. We call θ_1 the *risk-retention rate*, and θ_2 the *collateral-retention rate*. We show that the optimal policy can be summarized by the ratio between the risk-retention rate and collateral-retention rate; moreover, this optimal ratio is equal to θ^*, which is the optimal risk-retention rate found in Section 6.2 wherein the central bank forgoes collateral backing private assets ($\theta_2 = 1$).

The bank's expected payoff under the policy becomes

$$\begin{aligned}\pi_b^p(y) &= (1-\theta_1)(k-d) + \theta_1\Big[y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y)\Big] - (k-d-\theta_2 a) \\ &= \theta_1\Big\{y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) - \underbrace{(k-d-\frac{\theta_2 a}{\theta_1})}_{\ell_y^p}\Big\}. \end{aligned} \tag{49}$$

Condition (49) shows that, from the bank's viewpoint, the "effective" down payment is $d + \frac{\theta_2 a}{\theta_1}$, and the risky loan amount under the policy, denoted by ℓ_y^p, is $k - d - \frac{\theta_2 a}{\theta_1}$. From (49) it is obvious that if $\theta_2 = \theta_1$, the lending bank's profit is isomorphic to the one without policy and so the policy is ineffective; if $\theta_2 < \theta_1$, the policy makes bonds even less useful than without policy. Therefore, we consider $\theta_2 > \theta_1$ in this appendix. If the central bank forgoes collateral backing private assets, then $\theta_2 = 1$, which is the policy considered in the main text.

Let $\theta_c \equiv \frac{\theta_1}{\theta_2}$; then $\theta_c < 1$ given that we consider $1 \geq \theta_2 > \theta_1 > 0$. The contract problem becomes:

$$\begin{aligned} \{d^*,\ a^*,\ y^*\} &= \arg\max_{\{d,a,y\}} -d - a + \int_{y}^{\overline{\omega}} \omega\, dG(\omega) - y[1-G(y)] \\ \text{s.t.}\quad & \pi_b^p(y) = \theta_1\{y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) - (k-d-\frac{a}{\theta_c})\} \geq 0, \\ & 0 \leq d \leq m,\ 0 \leq a \leq z, \end{aligned} \tag{50}$$

where the solution satisfies $\pi_b^p(y) = 0$, and

$$a^* = \begin{cases} \theta_c k, & \text{if } \underline{k} \leq k \leq \frac{z}{\theta_c} \\ z, & \text{if } \frac{z}{\theta_c} < k \leq k^p \end{cases}, \qquad d^* = \begin{cases} 0, & \text{if } \underline{k} \leq k \leq \frac{z}{\theta_c} \\ k - \theta_c z, & \text{if } \frac{z}{\theta_c} < k \leq m + \frac{z}{\theta_c} \\ m, & \text{if } m + \frac{z}{\theta_c} < k \leq k^p. \end{cases} \tag{51}$$

The interpretation of (51) is the same as (24) in the main text.

Proposition 11. *Given $\theta_c < 1$, the private asset purchase program enhances the efficacy of bonds as collateral and changes the pecking order of financing investment. Under the policy, entrepreneurs collateralize bonds before using internal funds.*

Given the solution to the contract problem, the expected profit of entrepreneurs with access to bank credit is

$$\pi_e = \begin{cases} \mu_\omega - k + (1-\theta_c)k & \text{if } \underline{k} \le k \le \frac{z}{\theta_c} \\ \mu_\omega - k + \frac{1-\theta_c}{\theta_c}z, & \text{if } \frac{z}{\theta_c} < k < m + \frac{z}{\theta_c} \\ \mu_\omega - \gamma G(y^*) - k + \frac{1-\theta_c}{\theta_c}z, & \text{if } m + \frac{z}{\theta_c} \le k \le k^p. \end{cases} \tag{52}$$

The expected profit of unbanked entrepreneurs, who use internal funds only to purchase capital, is

$$\pi_e = \mu_\omega - k, \qquad \text{if } \underline{k} \le k \le m. \tag{53}$$

Lemma 2. *Given an entrepreneur's portfolio, (m, z), the private asset purchase program raises the investment threshold by $\frac{1-\theta_c}{\theta_c}z$.*

According to Lemma 2, banks extend loans to entrepreneurs with an input cost up to $k^p = k^T + \frac{1-\theta_c}{\theta_c}z$, and so the policy improves the availability of credit.

By (51)-(53), an entrepreneur's expected profit is

$$\pi_e^p = \alpha(1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k) + \alpha\phi \left\{ \begin{array}{c} \int_{\underline{k}}^{\frac{z}{\theta_c}}(\mu_\omega - k + (1-\theta_c)k)dF(k) + \int_{\frac{z}{\theta_c}}^{m+\frac{z}{\theta_c}}[\mu_\omega - k + \frac{1-\theta_c}{\theta_c}z]dF(k) \\ + \int_{m+\frac{z}{\theta_c}}^{k^p}[\mu_\omega - \gamma G(y^*) - k + \frac{1-\theta_c}{\theta_c}z]dF(k) \end{array} \right\}. \tag{54}$$

An entrepreneurs's lifetime expected utility is

$$\mathbb{E}V^p(m, z) = \pi_e^p + W(m, z, 0, 0). \tag{55}$$

Given (55), an entrepreneur's optimal portfolio choices are characterized by the following conditions:

$$\frac{\sigma}{\beta} = \alpha\Big\{(1-\phi)(\mu_\omega - m)f(m) + \phi\Big[(\mu_\omega - k^p + \frac{1-\theta_c}{\theta_c}z)f(k^p) - \gamma\int_{m+\frac{z}{\theta_c}}^{k^p} G(y^*)f'(k)dk\Big]\Big\} + 1, \tag{56}$$

$$\frac{\psi}{\beta} = \frac{\alpha\phi}{\theta_c}\Big[(\mu_\omega - k^p + \frac{1-\theta_c}{\theta_c}z)f(k^p) - \gamma\int_{m+\frac{z}{\theta_c}}^{k^p} G(y^*)f'(k)dk + (1-\theta_c)[F(k^p) - F(\frac{z}{\theta_c})]\Big] + 1. \tag{57}$$

Equations (51)-(57) have similar interpretations as those of their counterparts, (24)-(30), in the main text.

If we replace the notation θ_c with θ in (51)-(57), then those equations are exactly the same as (24)-(30). The expression of bank's expected profits, (50), is slightly different from (23) in the main text. Nevertheless, banks earn zero profit and thus it will not affect the equilibrium. Therefore, the proofs of Proposition 11 and Lemma 2 are the same as the proofs of Proposition7 and Lemma 1, and are omitted.

Optimal θ^c When conducting the policy, the central bank knows the decision rules of entrepreneurs and banks. To choose the optimal θ_c^*, the central bank anticipates the equilibrium m and ψ given by (56) and (57). Entrepreneurs and banks take θ_c^* as given, and the optimal portfolio choices, (56) and (57), ensure that the equilibrium m and ψ are as anticipated.

The central bank pays reserves to purchase $1-\theta_1$ fraction of assets, and receives $1-\theta_1$ fraction of expected returns backed solely by risky investment returns, and $1-\theta_2$ fraction of collateral. Given the investment threshold, k^p, the central bank's expected payoff is:

$$\pi_c = \alpha\phi \int_{\underline{k}}^{k^p} \Big\{(1-\theta_1)\Big[y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y)\Big] - (1-\theta_1)(k-d^*) + (1-\theta_2)a^*\Big\} dF(k).$$

Using the private bank's binding participation constraint from (50) with (d^*, a^*, y^*), we obtain

$$\pi_c = -\alpha\phi\Big[\int_{\underline{k}}^{k^p} \frac{1-\theta_c}{\theta_c} a^* dF(k)\Big] < 0. \tag{58}$$

Substituting the solution to the contract, a^*, from (51) into (58), we obtain

$$\pi_c = -\alpha\phi\Big\{\int_{\underline{k}}^{\frac{z}{\theta_c}} (1-\theta_c)k dF(k) + \frac{1-\theta_c}{\theta_c} z \Big[F(k^p) - F(\frac{z}{\theta_c})\Big]\Big\}. \tag{59}$$

The right side of (59) equals the total subsidy to entrepreneurs through the central bank's purchase of private assets. For borrowers with $k < \frac{z}{\theta_c}$, the central bank subsidizes $(1-\theta_c)k$; for those with $\frac{z}{\theta_c} < k < k^p$, the central bank subsidizes $\frac{1-\theta_c}{\theta_c} z$.

Equation (59) suggests that, given other parameters and $1 \geq \theta_2 > \theta_1 > 0$, the central bank's deficit is determined by the ratio, θ_c, regardless of the values of θ_1 and θ_2. That is, the central bank has the same deficit no matter what the combinations of θ_1 and θ_2 are, as long as they yield the same θ_c.

The central bank maximizes society's welfare in choosing the optimal θ_c. We use $\Omega = \pi_e^p + \pi_c$ as the welfare criterion, which is the expected utility of entrepreneurs subtracting the cost of policy, and π_c is given by (59). Using (52) and (53), we have the following welfare measure:

$$\Omega = \alpha\Big\{(1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k) + \phi\Big[\int_{\underline{k}}^{k^p}(\mu_\omega - k)dF(k) - \int_{m+\frac{z}{\theta_c}}^{k^p}\gamma G(y^*)dF(k)\Big]\Big\}. \tag{60}$$

To find the optimal θ_c, denoted by θ_c^*, the central bank maximizes Ω in (60), subject to $\pi_b^p(y) = 0$ in (49), and the government's budget constraint.

Proposition 12. *Given other parameters, $\theta_c^* = \theta^*$, where θ^* is the optimal risk-retention rate in Section 6.2, wherein the central bank forgoes collateral backing private assets.*

If we replace the notation θ_c with θ in the above equations, then they are the same as their counterparts in the main text, and thus, $\theta_c^* = \theta^*$. Given that we have solved and characterized the optimal risk-retention rate θ^* in Proposition 8, Proposition 12 implies that all the results in Propositions 8 and 9 hold under this policy. Proposition 12 also implies that for an optimal θ_c^*, there are infinite combinations of θ_1 and θ_2 that can achieve the goal. The intuition is this. A lower risk-retention rate (lower θ_1) means that the lending bank bears a lower lending risk, while a higher collateral-retention rate (higher θ_2) decreases the bank's loss when defaults occur. Therefore, the central bank can choose any combination of θ_1 and θ_2 to achieve θ_c^*, and to induce banks to lend to achieve the targeted investment threshold.[30]

We conclude that given that θ^* is the optimal risk-retention rate under the policy wherein the central bank forgoes collateral backing private assets, $\theta_c^* = \theta^*$ is also optimal for the policy wherein the central bank requires a part of collateral when it purchases private assets.

[30] In this model money and bonds function as down payment in a debt contract, and thus, the amounts of reserves and bonds that the lending bank receives do not matter as long as the bank still earns zero expected profits (see the first line of (49)).

Appendix D. Comparison of asset purchases with direct subsidies

This appendix considers the policy wherein the fiscal authority gives subsidies to bank lending. We show that the private asset purchase policy with the optimal risk-retention rate yields higher welfare than the fiscal policy of direct subsidies; that is, direct subsidies cannot achieve the same outcomes as the central bank's asset purchases in this economy.

We consider the following simple setup of direct subsidies. The fiscal authority targets credit-constrained entrepreneurs; that is, it gives a subsidy to banks that extend credit to those with input cost $k > k^{Tf}$, where k^{Tf} is the investment threshold without policy. When conducting the policy, the fiscal authority knows the decision rules of entrepreneurs and banks, such as how entrepreneurs' asset holding decisions and bank's participation condition react to the policy.

Specifically, the fiscal authority sets up the targeted investment threshold, k^{pf}, such that $k^{pf} > k^{Tf}$, and it subsidizes a bank with a transfer $s(k)$ so that the bank can lend to entrepreneurs with the input cost up to k^{pf}. The bank's profit under the policy becomes

$$\pi_b^f(y_f) = \underbrace{y_f[1 - G(y_f)] + \int_{\underline{\omega}}^{y_f} \omega \, dG(\omega) - \gamma G(y_f)}_{=B(y_f)} - (k - d - a) + s(k),$$
$$\text{where } s(k) = \begin{cases} 0 & \text{if} \quad k \leq k^{Tf}, \\ k - k^{Tf} & \text{if} \quad k \in (k^{Tf},\ k^{pf}]. \end{cases} \tag{61}$$

In (61), $B(y_f)$ is the bank's expected profit backed solely by the risky project return, and as shown in Section 3.1, there exists a y_b such that $y_b = \arg\max_{y_f}\{B(y_f)|y_f \in [0, \overline{\omega}]\}$. Notice from (61) that $B(y_f)$ is independent of policy. Therefore, the maximum risky loan amount is $\overline{\ell}_{yb} = B(y_b)$, and the investment threshold is $k^{Tf} = m^f + z + \overline{\ell}_{yb}$ when there is no direct subsidy. That is, $B(y_b) - (k - m^f - z) < 0$ if $k > k^{Tf}$, and a bank expects a loss if such loans are extended without a subsidy. Thus, a subsidy is necessary for banks to extend loans to entrepreneurs with an input cost $k > k^{Tf}$. The targeted investment threshold of the fiscal policy, k^{pf}, satisfies $k^{pf} \leq \mu_\omega$; that is, the policy does not subsidize the socially inefficient projects. Note that we focus on credit-constrained case and thereby, $k^{Tf} = m^f + z + \overline{\ell}_{yb}$ is the investment threshold without policy.

The contract problem under the direct subsidy policy becomes

$$
\begin{aligned}
&\{d^*,\ a^*,\ y_f^*\} = \arg\max_{\{d,a,y_f\}} -d-a+\int_{y_f}^{\overline{\omega}} \omega\, dG(\omega) - y_f[1-G(y_f)] \\
&\text{s.t.} \quad \pi_b^f(y_f) = B(y_f) - (k-d-a) + s(k) \geq 0, \\
&\qquad\qquad 0 \leq d \leq m^f,\ 0 \leq a \leq z.
\end{aligned} \tag{62}
$$

The debt contract problem is similar to (11) except for the subsidy, $s(k)$, which is exogenously given. The subsidy doesn't alter the efficacy of assets used as collateral as in the asset purchase program. Hence, the pecking order is the same as in Section 3.1, and Proposition 2 applies here. Also, entrepreneurs' default rule is the same as (6). Consequently, for entrepreneurs with $k \leq k^{Tf}$, because there is no subsidy to the lending bank, the effective repayment, y_f^*, is the same as the solution to (11). Note that for $k \leq k^{Tf}$, the effective repayment y_f^* depends on the borrower's input cost, k. For all $k \in (k^{Tf},\ k^{pf}]$, however, the bank's profit becomes

$$
\pi_b^f(y_f) = B(y_f) - (k^{Tf} - d - a),
$$

due to the subsidy, $s(k)$. The only debt contract acceptable to banks is $(d^*, a^*, y_f^*) = (m^f, z, y_b)$ for the borrower's input cost $k \in (k^{Tf},\ k^{pf}]$, where $y_b = \arg\max_{y_f}\{B(y_f) | y_f \in [0, \overline{\omega}]\}$. Because the effective repayment for entrepreneurs with $k \in (k^{Tf},\ k^{pf}]$ is y_b, they have the same default probability, $G(y_b)$.

Given the solution to the contract problem and the transfer scheme, $s(k)$, the expected profit of entrepreneurs with access to bank credit is

$$
\pi_e = \begin{cases} \mu_\omega - k & \text{if } \underline{k} \leq k < m^f + z \\ \mu_\omega - \gamma G(y_f^*) - k, & \text{if } m^f + z \leq k \leq k^{Tf} \\ \mu_\omega - \gamma G(y_b) - k^{Tf}, & \text{if } k^{Tf} < k \leq k^{pf}. \end{cases} \tag{63}
$$

The expected profit of unbanked entrepreneurs, who use internal funds only to purchase capital, is

$$
\pi_e = \mu_\omega - k, \qquad \text{if } \underline{k} \leq k \leq m^f. \tag{64}
$$

By (63) and (64), an entrepreneur's expected profit is

$$
\pi_e^f = \alpha(1-\phi)\int_{\underline{k}}^{m^f} (\mu_\omega - k) dF(k) + \alpha\phi \left\{ \begin{array}{c} \int_{\underline{k}}^{m^f+z} (\mu_\omega - k) dF(k) + \int_{m^f+z}^{k^{Tf}} [\mu_\omega - \gamma G(y_f^*) - k] dF(k) \\ + \int_{k^{Tf}}^{k^{pf}} [\mu_\omega - \gamma G(y_b) - k^{Tf}] dF(k) \end{array} \right\}. \tag{65}
$$

An entrepreneur's lifetime expected utility is

$$\mathbb{E}\, V^f(m^f, z) = \pi_e^f + W(m^f, z, 0, 0). \tag{66}$$

Given (66), an entrepreneur's optimal portfolio choices are characterized by the following conditions:

$$\frac{\sigma}{\beta} = \alpha\Big\{(1-\phi)(\mu_\omega - m^f)f(m^f) + \phi\Big[\gamma G(y_b)f(k^{Tf}) - \gamma\int_{m^f+z}^{k^{Tf}} G(y_f^*)f'(k)dk - [F(k^{pf}) - F(k^{Tf})]\Big]\Big\} + 1, \tag{67}$$

$$\frac{\psi}{\beta} = \alpha\phi\Big[\gamma G(y_b)f(k^{Tf}) - \gamma\int_{m^f+z}^{k^{Tf}} G(y_f^*)f'(k)dk - [F(k^{pf}) - F(k^{Tf})]\Big] + 1. \tag{68}$$

The fiscal authority knows how entrepreneurs' asset holding decisions (67) and (68) respond to the targeted investment threshold, k^{pf}. Thus, given k^{pf}, the total amount of subsidies (fiscal deficit) is

$$\pi_f = -\alpha\phi\Big[\int_{k^{Tf}}^{k^{pf}} (k - k^{Tf})dF(k)\Big] < 0. \tag{69}$$

The measure of social welfare is $\Omega^f = \pi_e^f + \pi_f$, which is the expected utility of entrepreneurs subtracting the cost of policy, and π_f is given by (69). Substituting (63), (64), and (69) into the welfare measure, Ω^f, we have the following expression:

$$\Omega^f = \alpha\left\{(1-\phi)\int_{\underline{k}}^{m^f}(\mu_\omega - k)dF(k) + \phi\left[\begin{array}{c}\displaystyle\int_{\underline{k}}^{k^{pf}}(\mu_\omega - k)dF(k) - \int_{m^f+z}^{k^{Tf}}\gamma G(y_f^*)dF(k) \\ \displaystyle -\int_{k^{Tf}}^{k^{pf}}\gamma G(y_b)dF(k)\end{array}\right]\right\}. \tag{70}$$

Next we show that the private asset purchase policy with the optimal risk-retention rate yields higher welfare than the policy of direct subsidy does. To see clearly differences in the mechanisms underlying both policies, we assume that the input cost k follows uniform distribution, which allows us to obtain analytical results by ignoring the probability of input cost realization. We also consider that the access to bank credit, ϕ, is not sufficiently high so that the chance for entrepreneurs to get subsidy is limited and internal and external financing are still needed for investment.

Let Ω and Ω^f denote the welfare measures under asset purchases and direct subsidies, respectively. Then,

$$
\begin{aligned}
&\Omega - \Omega^f \\
= \;& \alpha\Big\{(1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k) + \phi\Big[\int_{\underline{k}}^{k^p}(\mu_\omega - k)dF(k) - \int_{m+\frac{z}{\theta}}^{k^p}\gamma G(y_p^*)dF(k)\Big]\Big\} \\
& -\alpha\left\{(1-\phi)\int_{\underline{k}}^{m^f}(\mu_\omega - k)dF(k) + \phi\left[\begin{array}{l}\int_{\underline{k}}^{k^{pf}}(\mu_\omega - k)dF(k) - \int_{m^f+z}^{k^{Tf}}\gamma G(y_f^*)dF(k) \\ -\int_{k^{Tf}}^{k^{pf}}\gamma G(y_b)dF(k)\end{array}\right]\right\} \\
> \;& 0 \quad \forall\, k^{pf} \in (k^{Tf},\ \mu_\omega] \text{ when } \theta = \theta^*.
\end{aligned}
$$

Notice that y_p^* and y_f^* are the effective repayments under asset purchases and direct subsidies, respectively. There are two reasons for the result of welfare comparison. The fiscal policy gives a subsidy to the lending bank to cover the difference, $k - k^{Tf}$, which induces banks to lend to borrowers with relatively high input costs. Different from asset purchases which increase the efficacy of bonds as collateral, the direct subsidy works like a substitute for the down payment. Hence, entrepreneurs hold lower real balances, which leads to a smaller expected profit for unbanked entrepreneurs. Moreover, entrepreneurs whose loans are eligible for the direct subsidy have the highest repayment, and thus the highest tendency to default. For those borrowers, their risky loan amount is even larger than that under asset purchases, and consequently, aggregate bankruptcy cost is higher under direct subsidies. We summarize the main result in the following proposition.

Proposition 13. *Consider that the input cost follows uniform distribution over $k \in [0,\ \overline{k}]$. Given other parameters, the central bank's purchase of private assets with the optimal risk-retention rate yields higher social welfare than the fiscal policy of direct subsidies, even when k^{pf} is set optimally.*

Proof. We have shown in Proposition 8 that under the optimal risk-retention rate θ^*, the investment threshold $k^p = \mu_\omega$. Therefore,

$$
\begin{aligned}
\Omega - \Omega^f \;=\;& \alpha\phi\Big[\int_{k^{pf}}^{k^p}(\mu_\omega - k)dF(k) + \int_{k^{Tf}}^{k^{pf}}\gamma G(y_b)dF(k)\Big] \\
& +\alpha\left[(1-\phi)\int_{m^f}^{m}(\mu_\omega - k)dF(k) - \phi\Big(\int_{m+\frac{z}{\theta^*}}^{k^p}\gamma G(y_p^*)dF(k) - \int_{m^f+z}^{k^{Tf}}\gamma G(y_f^*)dF(k)\Big)\right].
\end{aligned}
\tag{71}
$$

The first line is positive because $k^{pf} \in (k^{Tf},\ \mu_\omega]$. Hence, it suffices to prove the statement by showing the second line is non-negative.

First, we show $\int_{m+\frac{z}{\theta^*}}^{k^p} \gamma G(y_p^*)dF(k) = \int_{m^f+z}^{k^{Tf}} \gamma G(y_f^*)dF(k)$, which are aggregate bankruptcy cost for banked entrepreneurs with certain input cost under two policies. Under asset purchases, each banked entrepreneur with input cost $k \in [m + \frac{z}{\theta^*},\ k^p]$ faces a non-negative repayment, y_p^*, such that

$$y_p^* = \arg\min_{y_p} \{B(y_p) - \underbrace{(k - m - \frac{z}{\theta^*})}_{\equiv \ell_y^p} = 0\}.$$

While under direct subsidies, each banked entrepreneur with input cost $k \in [m^f + z,\ k^{Tf}]$ faces a non-negative repayment, y_f^*, such that

$$y_f^* = \arg\min_{y_f} \{B(y_f) - \underbrace{(k - m^f - z)}_{\equiv \ell_y^f} = 0\}.$$

Since $B(\cdot)$ is independent of policy, we conclude that the effective repayments are equal ($y_p^* = y_f^*$) if and only if the risky loan amounts are the same ($\ell_y^p = \ell_y^f$). Furthermore, given an entrepreneur's portfolio, (m, z), and a policy, the risky loan amount is 1-to-1 function of the input cost. Based on these observations, we rewrite the expected bankruptcy cost as follows: $\int_{m+\frac{z}{\theta^*}}^{k^p} \gamma G(y_p^*)dF(k) = \int_0^{\bar{\ell}_{yb}} \gamma G(y_p^*)f(\ell_y^p)d\ell_y^p$ and $\int_{m^f+z}^{k^{Tf}} \gamma G(y_f^*)dF(k) = \int_0^{\bar{\ell}_{yb}} \gamma G(y_f^*)f(\ell_y^f)d\ell_y^f$. By the uniform distribution of k, $f(\ell_y^p) = f(\ell_y^f) = 1/\overline{k}$ is given and is independent of the risky loan amount. Moreover, under the two policies, the repayment $y_p^* = y_f^*$ when the risky loan amount is the same, we conclude that $\int_0^{\bar{\ell}_{yb}} \gamma G(y_p^*)f(\ell_y^p)d\ell_y^p = \int_0^{\bar{\ell}_{yb}} \gamma G(y_f^*)f(\ell_y^f)d\ell_y^f$; which implies $\int_{m+\frac{z}{\theta^*}}^{k^p} \gamma G(y_p^*)dF(k) = \int_{m^f+z}^{k^{Tf}} \gamma G(y_f^*)dF(k)$.

Next, we show that $m \geq m^f$. The real balances m and m^f need to satisfy equilibrium conditions (29) and (67) respectively. Using the input cost distribution, $f(k) = 1/\overline{k}$ for all $k \in [0,\ \overline{k}]$ and Lemma 1, we rewrite (29) as

$$\begin{aligned} \frac{\sigma}{\beta} &= \alpha\Big\{(1-\phi)(\mu_\omega - m)f(m) + \phi(\mu_\omega - k^T)f(k^p)\Big\} + 1 \\ &= \frac{\alpha}{\overline{k}}\Big\{(1-\phi)(\mu_\omega - m) + \phi(\mu_\omega - k^T)\Big\} + 1. \end{aligned} \tag{72}$$

Note that $k^T = m + z + \bar{\ell}_{yb}$ is the investment threshold for credit-constrained entrepreneurs without any policy. Given $f(k) = 1/\overline{k}$ for all $k \in [0,\ \overline{k}]$, we rewrite (67) as

$$\begin{aligned} \frac{\sigma}{\beta} &= \alpha\Big\{(1-\phi)(\mu_\omega - m^f)f(m^f) + \phi\Big[\gamma G(y_b)f(k^{pf}) - [F(k^{pf}) - F(k^{Tf})]\Big]\Big\} + 1 \\ &= \frac{\alpha}{\overline{k}}\Big\{(1-\phi)(\mu_\omega - m^f) + \phi\Big[\gamma G(y_b) - (k^{pf} - k^{Tf})\Big]\Big\} + 1. \end{aligned} \tag{73}$$

Because m and m^f need to satisfy equilibrium conditions (72) and (73), we can compare the relative size of real balances, $m - m^f$, by subtracting (72) from (73), and we obtain

$$\begin{aligned} m - m^f &= \frac{\phi}{1-\phi}[\mu_\omega - k^T - \gamma G(y_b) + k^{pf} - k^{Tf}] \\ &\geq \frac{\phi}{1-\phi}[\mu_\omega - k^T - (\mu_\omega - k^{Tf}) + k^{pf} - k^{Tf}] \\ &= \frac{\phi}{1-\phi}(k^{pf} - k^T) \geq 0, \end{aligned}$$

where we have used $\mu_\omega - \gamma G(y_b) - k^{Tf} \geq 0$ for credit-constrained entrepreneurs to get the inequality in the second line. Since the goal of direct subsidies is to raise the investment threshold for credit-constrained entrepreneurs, we have $k^{pf} - k^T \geq 0$, and thus, $m \geq m^f$. Consequently, with $\int_{m+\frac{z}{\theta^*}}^{k^p} \gamma G(y_p^*)dF(k) = \int_{m^f+z}^{k^{Tf}} \gamma G(y_f^*)dF(k)$ and $m \geq m^f$, the second line of (71) is non-negative and we prove the welfare level is higher under the asset purchase program

Finally, we briefly discuss the asset purchase policy with $\theta = 0$, which means that private banks sell all loans to the central bank. This case can be viewed as that the central bank replaces private banks in lending. Under this circumstance, if the central bank acts exactly as private banks do, the policy has no effects. If the central bank wishes to raise the investment threshold above the one without policy, it needs to issue reserves to credit-constrained entrepreneurs, and this runs a deficit. The mechanism underlying this policy is exactly the same as that underlying the policy of direct subsides. Therefore, the asset purchase policy with $\theta = 0$ is inferior to that with the optimal risk-retention rate, $\theta^* \in (0, 1)$, specified in Proposition 8.

Appendix E. Ex ante asymmetric information

Here we incorporate ex ante asymmetric information regarding the quality of investment projects into our benchmark model. We show that the main results are robust under the case with ex ante asymmetric information.

In the spirit of Li, Rocheteau, and Weill (2012), we consider ex ante asymmetric information as follows. If an entrepreneur who draws an investment project with input cost k does not find a right supplier for the input (with probability $1-\alpha$), her project will produce zero output, and it is called an *unproductive project* with input cost k. An entrepreneur with an unproductive project has the following counterfeiting technology: by paying a fixed cost (in terms of disutility), $f_c > 0$, the entrepreneur can turn her unproductive project into a *fake project*; a fake project produces zero output, and it is indistinguishable from a productive project with the same input cost. If an entrepreneur with an unproductive project has access to banking, she can decide whether to use the counterfeiting technology to make a fake project to apply for a loan.[31] Though banks cannot distinguish between a fake project and a productive project, they can observe the input cost of a project. Moreover, we assume that an entrepreneur can exert an effort cost (in terms of disutility), $s(k)$, when offering a debt contract, and the effort cost is observable to banks. Notice that the effort cost chosen by an entrepreneur depends on the input cost of her investment project, k. A banked entrepreneur with a productive project is called an honest entrepreneur, and a banked entrepreneur with a fake project is called a dishonest entrepreneur.

An honest entrepreneur with a project of input cost k, offers a debt contract with a down payment $d^*(k)$, collateral, $a^*(k)$, and chooses the effort cost, $s^*(k)$. To deceive a bank, an entrepreneur with a fake project pretends she has a productive project by offering the same contract as an honest entrepreneur. A dishonest entrepreneur, if acquiring a loan, will use the loan, $k - d^*(k)$, to buy general good in stage 2 for her own consumption, without producing any output, and she will certainly default the debt and lose collateral. Let $\pi_f(k)$ denote the net payoff from making a fake project with input cost k to borrow from a bank. Then,

$$\pi_f(k) = -f_c + k - d^*(k) - a^*(k) - s^*(k). \tag{74}$$

[31] Williamson (2018) uses a similar setup to consider the ex ante asymmetric information between borrowers of consumer credit and banks.

If $\pi_f(k) > 0$, an entrepreneur with an unproductive project chooses to make a fake project; if $\pi_f(k) \leq 0$, she chooses not to counterfeit.

Notice that an entrepreneur with an unproductive project $k < m + z + f_c$, has no incentive to counterfeit even with $s^*(k) = 0$. This is so because the optimal contract satisfies $k - (d^* + a^*) = \max\{0, k - m - z\}$, which implies $\pi_f < 0$. Thus, honest entrepreneurs with $k \in [\underline{k}, m + z + f_c]$ choose $s^*(k) = 0$, and entrepreneurs with unproductive projects $k \in [\underline{k}, m + z + f_c]$ does not counterfeit. Therefore, if the counterfeiting cost f_c is sufficiently high such that $m + z + f_c \geq k^T$, where k^T is the investment threshold, there is no fraud in equilibrium, and all results in the basic model hold.

Next we consider the situation wherein the counterfeiting cost f_c is low; i.e., $f_c < k^T - m - z$, and entrepreneurs with unproductive project $k \in [m + z + f_c, k^T]$ may have incentives to counterfeit. An honest entrepreneur will offer a debt contract that leaves no gains for making fake projects, and the bank will believe that the entrepreneur is holding a productive project; that is, the debt contract satisfies

$$\pi_f(k) = -f_c + k - m - z - s^*(k) \leq 0.$$

which is called *no-fraud condition.* The no-fraud condition implies that an honest entrepreneur's effort cost, $s^*(k)$, must be high enough, i.e., $s^*(k) \geq -f_c + k - m - z$, to eliminate the incentive to counterfeit. Under this condition, therefore, in equilibrium entrepreneurs will not make fake projects (e.g., see Li, Rocheteau, and Weill 2012). Below we suppress the notation k from the functions π_f and s^* when there is no confusion.

Given that there are no fake projects in equilibrium, banks' expected payoff remains the same as in the benchmark model; however, the optimal contract need to satisfy an extra constraint, the no-fraud condition, $\pi_f \leq 0$:

$$\begin{aligned} \{d^*, a^*, s^*, y^*\} = \arg \max_{\{d,a,s,y\}} & -d - a - s + \int_y^{\overline{\omega}} \omega \, dG(\omega) - y[1 - G(y)] = \pi_e(y) \\ \text{s.t.} \quad & \pi_b(y) \geq 0; \ \pi_e(y) \geq 0; \ \pi_f(s) \leq 0; \ m \geq d \geq 0; \ z \geq a \geq 0. \end{aligned} \tag{75}$$

From the previous discussion, entrepreneurs with unproductive project $k \in [m + z + f_c, k^T]$ may have incentives to counterfeit. And from equation (74), we observe that by increasing down payment d^* and collateral a^*, an honest entrepreneur can lower the effort cost s^* as well as repayment y^*.

Therefore, the solutions to the contract problem (75) are $d^* = m$ and $a^* = z$, and

$$s^* = k - m - z - f_c, \tag{76}$$

which makes $\pi_f = 0$. From the binding constraint $\pi_b(y^*) = 0$ and $\pi_f(s^*) = 0$, and (76), the expected profit for an honest entrepreneur with input cost k is

$$\pi_e = \mu_\omega - \gamma G(y^*) - k - s^* = \mu_\omega - \gamma G(y^*) - 2k + m + z + f_c,\ k \in [m + z + f_c, k^T]$$

Given the solution to the contract problem under the threat of fraud, (75), the expected payoff of an honest entrepreneur is

$$\pi_e = \begin{cases} \mu_\omega - k & \text{if } \underline{k} \le k \le m + z \\ \mu_\omega - \gamma G(y^*) - k, & \text{if } m + z \le k \le m + z + f_c \\ \mu_\omega - \gamma G(y^*) - 2k + m + z + f_c, & \text{if } m + z + f_c \le k \le k^T. \end{cases} \tag{77}$$

The expected value of an entrepreneur with asset holdings, (m, z), is

$$\begin{aligned} &\mathbb{E}\, V^f(m, z) - W(m, z, 0, 0) \\ =&\alpha \left\{ (1-\phi) \int_{\underline{k}}^{m} (\mu_\omega - k) dF(k) + \phi \left[\begin{array}{l} \int_{\underline{k}}^{m+z} (\mu_\omega - k) dF(k) + \int_{m+z}^{k^T} (\mu_\omega - \gamma G(y^*) - k) dF(k) \\ - \int_{m+z+f_c}^{k^T} [k - (m + z + f_c)] dF(k) \end{array} \right] \right\} \end{aligned} \tag{78}$$

Notice that banks' expected payoff and the maximum risky loan amount $\bar{\ell}_{yb}$ are the same as in the basic model; therefore, the threshold $k_b = m + z + \bar{\ell}_{yb}$ is not affected by the ex ante asymmetric information problem. However, the threshold, k_e (if it exists), is lowered by the effort cost s^*.

We conclude that main results of our basic model hold even if we incorporate an ex ante information problem, though the additional no-fraud condition, $\pi_f(k) \le 0$, implies that an entrepreneur's expected payoffs may be lowered than otherwise (see (77)).

Implications of the private asset purchase policy. Now we study the effects of central bank's private asset purchases. The bank's expected payoff is given by (22) in the main text. The contract problem under policy is

$$\begin{aligned} \{d^*,\ a^*,\ s^*,\ y^*\} = \arg \max_{\{d,a,s,y\}} & -d - a - s + \int_y^{\overline{\omega}} \omega\, dG(\omega) - y[1 - G(y)] \\ \text{s.t.} \quad & \pi_b^p(y) \ge 0, \pi_f(s) \le 0,\ 0 \le d \le m,\ 0 \le a \le z, \end{aligned} \tag{79}$$

which incorporates the no-fraud condition, $\pi_f(s) \leq 0$. Note that under this environment, Proposition 7 still holds, and entrepreneurs collateralize bonds before using internal funds. Moreover, because the cost of using one unit of internal funds and the cost of exerting one unit of effort are identical, and the benefits of using internal funds and exerting efforts to satisfy the no-fraud condition are identical as well, an entrepreneur is indifferent between using money and efforts in the contract. Hence, the solution to the contract problem is

$$a^* = \begin{cases} \theta k, & \text{if } \underline{k} \leq k \leq \frac{z}{\theta} \\ z, & \text{if } \frac{z}{\theta} < k \leq k^p \end{cases}, \qquad d^* = \begin{cases} 0, & \text{if } \underline{k} \leq k \leq \frac{z}{\theta} \\ k - \frac{z}{\theta}, & \text{if } \frac{z}{\theta} < k \leq m + \frac{z}{\theta} \\ m, & \text{if } m + \frac{z}{\theta} < k \leq k^p \end{cases}$$
$$s^* = \begin{cases} \max\{0, (1-\theta)k - f_c\}, & \text{if } \underline{k} \leq k \leq \frac{z}{\theta} \\ \max\{0, \frac{(1-\theta)z}{\theta} - f_c\}, & \text{if } \frac{z}{\theta} < k \leq m + \frac{z}{\theta} \\ \max\{0, k - m - z - f_c\}, & \text{if } m + \frac{z}{\theta} < k \leq k^p. \end{cases} \tag{80}$$

The solution a^* and d^* are the same as (24) in the main text, but here we need to incorporate the effort cost s^*.

Given the solution (d^*, a^*, s^*, y^*) to the contract problem, the expected profit of an honest banked entrepreneur is

$$\pi_e = \begin{cases} \mu_\omega - k + (1-\theta)k - s^*(k) & \text{if } \underline{k} \leq k \leq \frac{z}{\theta} \\ \mu_\omega - k + \frac{1-\theta}{\theta} z - s^*(k), & \text{if } \frac{z}{\theta} < k < m + \frac{z}{\theta} \\ \mu_\omega - \gamma G(y^*) - k + \frac{1-\theta}{\theta} z - s^*(k), & \text{if } m + \frac{z}{\theta} \leq k \leq k^p. \end{cases} \tag{81}$$

Compared with (25), the expected profit π_e described in (80), is lowered by the effort cost, $s^*(k)$.

Following the proof of Lemma 1, we verify that $k^p - k^T = \frac{1-\theta}{\theta} z$ for $k^T = k_b$ and $k^p - k^T < \frac{1-\theta}{\theta} z$ for $k^T = k_e$. Therefore, an entrepreneur's net expected payoffs at the beginning of a period is

$$\begin{aligned} &\mathbb{E}\, V^{pf}(m,z) - W(m,z,0,0) \\ =&\alpha(1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k) + \alpha\phi \left\{ \begin{array}{c} \int_{\underline{k}}^{\frac{z}{\theta}}(\mu_\omega - k + (1-\theta)k)dF(k) + \int_{\frac{z}{\theta}}^{m+\frac{z}{\theta}}[\mu_\omega - k + \frac{1-\theta}{\theta}z]dF(k) \\ + \int_{m+\frac{z}{\theta}}^{k^p}[\mu_\omega - \gamma G(y^*) - k + \frac{1-\theta}{\theta}z]dF(k) - \int_{\underline{k}}^{k^p} s^* dF(k) \end{array} \right\} \\ =&\alpha \left\{ (1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k) + \phi \left[\begin{array}{l} \int_{\underline{k}}^{k^p}(\mu_\omega - k)dF(k) - \gamma \int_{m+\frac{z}{\theta}}^{k^p} G(y^*)dF(k) \\ + \int_{\underline{k}}^{\frac{z}{\theta}}(1-\theta)k dF(k) + \int_{\frac{z}{\theta}}^{k^p}\frac{(1-\theta)z}{\theta}dF(k) - \int_{\underline{k}}^{k^p} s^* dF(k) \end{array} \right] \right\}. \end{aligned} \tag{82}$$

Following the analysis in Section 6.2, it is easy to show that, given the investment threshold under the policy, k^p, the central bank's expected payoff is

$$\pi_c = \alpha\phi \int_{\underline{k}}^{k^p} \Big\{(1-\theta)\Big[y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y)\Big] - (1-\theta)(k-d^*)\Big\} dF(k).$$

Using the bank's binding participation constraint from (79) with (d^*, a^*, s^*, y^*), we obtain

$$\begin{aligned}\pi_c &= \alpha\phi(1-\theta) \int_{\underline{k}}^{k^p} \Big[y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) - (k-m)\Big] dF(k) \\ &= \alpha\phi(1-\theta) \int_{\underline{k}}^{k^p} -\frac{a^*}{\theta} dF(k) = -\alpha\phi \int_{\underline{k}}^{k^p} \frac{1-\theta}{\theta} a^* dF(k),\end{aligned} \tag{83}$$

which is the same as (31), and can be described as (32):

$$\pi_c = -\alpha\phi\Big\{ \int_{\underline{k}}^{\frac{z}{\theta}} (1-\theta) k dF(k) + \frac{(1-\theta)z}{\theta} \Big[F(k^p) - F(\frac{z}{\theta})\Big] \Big\}. \tag{84}$$

Using $\Omega = \pi_e^p + \pi_c$ as the welfare criterion, where π_c is given by (84) and π_e^p by (82), we have the following welfare measure:

$$\Omega = \alpha \left\{ (1-\phi) \int_{\underline{k}}^{m} (\mu_\omega - k) dF(k) + \phi \left[\begin{array}{l} \int_{\underline{k}}^{k^p} (\mu_\omega - k) dF(k) - \gamma \int_{m+\frac{z}{\theta}}^{k^p} G(y^*) dF(k) \\ - \int_{\underline{k}}^{k^p} s^* dF(k) \end{array} \right] \right\}. \tag{85}$$

Compared to the welfare measure (33) in the main text, (85) includes an extra term, $-\int_{\underline{k}}^{k^p} s^* dF(k)$, which is the aggregate entrepreneurs' effort costs to eliminate the incentive to make fake projects.

From numerical examples we found that, implications of the private asset purchase policy still hold. The policy raises firm entry, aggregate lending, investment, and net output, and it reduces the average lending rate and the business failure rate, as described in Proposition 9. The optimal risk-retention rate is higher than that in the economy without the ex ante information problem. The intuitive reason is this. The aggregate effort costs, $\int_{\underline{k}}^{k^p} s^* dF(k)$, is decreasing in θ (see (80)); therefore, other things being equal, a welfare-maximizing central bank chooses a higher risk-retention rate than otherwise to restrain the aggregate effort costs, $\int_{\underline{k}}^{k^p} s^* dF(k)$. Moreover, under the respective optimal risk-retention rates, the welfare level obtained in this economy is lower than that in the economy without ex ante information. Therefore, the threat of making fake projects to deceive banks results in an extra cost to the society.

Appendix F. Asymmetric information between private banks and the central bank

In our benchmark model, a bank retains a fraction, $\theta \in (0, 1)$, of each loan in its portfolio, and sells the rest of loans to the central bank. Thus, the default risk of bank's retained loans and the loans purchased by the central bank are identical. If banks have information advantage on their loan portfolios than the central bank, one may wonder whether private banks may have incentives to retain loans with low default risk and sell those with high default risk to the central bank. In this Appendix, we study how this type of potential misrepresentation of the quality of bank loans affect the implications of the private asset purchase policy.

We apply Li, Rocheteau, and Weill's (2012) idea of counterfeiting assets to model banks' potential misrepresentation of the quality of loans under the private asset purchase policy (see also Williamson 2018). Assume that a bank, by paying a fixed cost, $f_{bc} > 0$, can misrepresent the quality of loans sold to the central bank. The central bank cannot recognize the quality of individual loans it purchases.

In the basic model, the total value of loans on the bank's book is $TL = \int_{\underline{k}}^{k^T} (k - d^*) dF(k)$. The asset purchase policy requires banks retain a fraction θ of each loan, which implies that the total value of loans that a bank retains is θTL, and the central bank pays reserves, $(1 - \theta)TL$, to purchase the rest of loans. Now if a bank chooses to pay the cost, f_{bc}, to misrepresent the quality of loans, then it can sell loans with the input cost above the (endogenously determined) threshold, k^f, such that the total value of loans sold to the central bank, $\int_{k^f}^{k^T} (k - d^*) dF(k)$, equals the amount of reserves, $(1 - \theta)TL$. This implies that the bank sells loans with the highest default risk in its portfolio, because loans with a higher input cost involves a higher default probability. One can show that k^f is increasing in the risk-retention rate, θ, and k^f approaches to k^T as θ is close to 1. That is, the severity of misrepresentation of loan quality decreases as banks are required to retain a higher fraction of loans.

Now we check whether a bank has an incentive to fake the quality of loans under the asset purchase policy, by comparing the gains of an honest bank and a dishonest bank (which misrepresents the default risk of loans sold to the central bank). Without the policy a bank's expected payoff of loan contracts with the input cost k is

$$\pi_b(k) = \underbrace{\left[y^*[1 - G(y^*)] + \int_{\underline{\omega}}^{y^*} \omega \, dG(\omega) - \gamma G(y^*) \right]}_{:=B(y^*)} - (k - d^* - a^*) = 0,$$

where $B(y^*)$ denotes the risky project return. Let Π_b denote the expected payoff from the entire portfolio of loans. Then,

$$\Pi_b = \int_{\underline{k}}^{k^T} \pi_b(k) dF(k).$$

Under the private asset purchase policy, an honest bank's expected payoff of its portfolio is

$$\Pi_b^p = \Pi_b - (1 - \theta) \int_{\underline{k}}^{k^T} B(y^*) dF(k) + (1 - \theta) TL,$$

where the term, $(1 - \theta) \int_{\underline{k}}^{k^T} B(y^*) dF(k)$, means that the risky project return of loans sold to the central bank is a $1 - \theta$ fraction of the risky project return of the bank's entire portfolio, $\int_{\underline{k}}^{k^T} B(y^*) dF(k)$, and the bank receives total reserves, $(1 - \theta) TL$. An honest bank's net expected payoff under policy is

$$\triangle \Pi_b^p = \Pi_b^p - \Pi_b = (1 - \theta) \int_{\underline{k}}^{k^T} [(k - d^*) - B(y^*)] dF(k) = (1 - \theta) \int_{\underline{k}}^{k^T} a^* dF(k), \tag{86}$$

where the last equality comes from the zero profit condition, $\pi_b(k) = 0$.

Similarly, a dishonest bank's expected payoff is

$$\Pi_b^{pf} = \Pi_b - \int_{k^f}^{k^T} B(y^*) dF(k) + (1 - \theta) TL - f_{bc},$$

where the term, $\int_{k^f}^{k^T} B(y^*) dF(k)$, is the risky project return of loans sold to the central bank, which bear the highest default risk. The net gain is

$$\triangle \Pi_b^{pf} = \Pi_b^{pf} - \Pi_b = \int_{k^f}^{k^T} [(k - d^*) - B(y)] dF(k) - f_{bc} = \int_{k^f}^{k^T} a^* dF(k) - f_{bc}, \tag{87}$$

where we have used the condition that the total value of loans sold equals total reserves received, $\int_{k^f}^{k^T} (k - d^*) dF(k) = (1 - \theta) TL$. From (86) and (87), a bank chooses not to misrepresent the quality of loans sold to the central bank if it gains less than being honest; that is, if

$$\begin{aligned} &\triangle \Pi_b^{pf} - \triangle \Pi_b^p \\ &= \int_{k^f}^{k^T} a^* dF(k) - f_{bc} - (1 - \theta) \int_{\underline{k}}^{k^T} a^* dF(k) = \theta \int_{k^f}^{k^T} a^* dF(k) - (1 - \theta) \int_{\underline{k}}^{k^f} a^* dF(k) - f_{bc} \leq 0, \end{aligned} \tag{88}$$

which is called a *banks' no-misrepresentation condition.*

Different from the basic model, the central bank here faces an additional banks' no-misrepresentation condition, (88), when choosing the optimal risk-retention rate, θ^*. Note that $\triangle\Pi_b^{pf}$ decreases in k^f, and k^f increases in θ. From the binding no-misrepresentation condition, $\triangle\Pi_b^{pf} - \triangle\Pi_b^{p} = 0$, one can solve for the lower bound of the risk-retention rate, $\underline{\theta}$. Therefore, the central bank solves the following question to choose θ^*:

$$\theta^* = \arg\max \alpha\Big\{(1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k) + \phi\Big[\int_{\underline{k}}^{k^p}(\mu_\omega - k)dF(k) - \int_{m+\frac{z}{\theta}}^{k^p}\gamma G(y^*)dF(k)\Big]\Big\} \tag{89}$$
$$s.t. \quad \underline{\theta} \le \theta < 1,$$

In equilibrium, banks do not misrepresent the quality of loans sold to the central bank, and the optimal debt contract problem remains as in the basic model. The solution to the contract problem is thus described by (24). The asset demand functions are given by (29) and (30) for a given risk-retention rate θ.

We summarize some results from numerical examples. We use the same parameter setting as in Figure 4, and the fraud cost is set at $f_{bc} = 10^{-3}$. It is found that the optimal risk-retention rate is $\theta^* = 0.2305$, as in Figure 4, and $\theta^* > \underline{\theta}$, so that the bank's no-misrepresentation condition does not bind. Under this situation, the policy effects are not changed by the possibility of banks' misrepresentation of quality of loans. Notice that $\underline{\theta}$ decreases in f_{bc}. If f_{bc} becomes lower, $\underline{\theta}$ would be higher, and it is more likely that the banks' no-misrepresentation condition, (88), binds. Under this circumstance, the best the central bank can do is to set $\theta^* = \underline{\theta}$, and the policy is less effective in boosting the economy and it achieves lower welfare, compared to the basic model.

Appendix G. Bank's cost of managing loan applications

This appendix considers the situation wherein banks need to pay a fixed cost, $f_m > 0$, to manage a loan application. Our focus is on how this extra cost of lending affects the implications of the private asset purchase policy.

The bank's participation constraint in the contract problem is modified to incorporate the cost

f_m:

$$\pi_b = \underbrace{y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega \, dG(\omega) - \gamma G(y)}_{:=B(y)} - (k-d-a) - f_m \geq 0$$

Compared to the basic model, here an entrepreneur with input cost k need propose more down payment, more collateral, or a higher repayment, to satisfy the bank's participation constraint. Offering more down payment or collateral will be optimal as long as the entrepreneur has enough safe assets, because a higher repayment makes the lending bank more likely to incur the monitoring cost.

Under the private asset purchase policy, the bank's net expected payoff from a contract with input cost k becomes

$$\begin{aligned} \pi_b^p &= \theta B(y) - (k-d-a) + (1-\theta)(k-d) - f_m \\ &= \theta \left\{ B(y) - (k - d - \frac{a-f_m}{\theta}) \right\} \geq 0, \end{aligned} \tag{90}$$

where f is the cost of managing a loan application. Compared to bank's payoff under policy in the basic model, (22), (90) shows that here the policy enlarges the quantity of collateralizable bonds to $\frac{a-f_m}{\theta}$, rather than $\frac{a}{\theta}$. The contract problem is as (23), with the bank's participation constraint replaced by (90).

If the supply of bonds is ample ($z > f_m$), the optimal solution to the contract problem is

$$a^* = \begin{cases} f_m + \theta k, & \text{if } \underline{k} \leq k \leq \frac{z-f_m}{\theta} \\ z, & \text{if } \frac{z-f_m}{\theta} < k \leq k^p \end{cases}, \qquad d^* = \begin{cases} 0, & \text{if } \underline{k} \leq k \leq \frac{z-f_m}{\theta} \\ k - \frac{z-f_m}{\theta}, & \text{if } \frac{z-f_m}{\theta} < k \leq m + \frac{z-f_m}{\theta} \\ m, & \text{if } m + \frac{z-f_m}{\theta} < k \leq k^p. \end{cases}$$

(We consider the case wherein $\underline{k} < \frac{z-f_m}{\theta} < k^p$.) Entrepreneurs pledge bonds as collateral before they use down payment, as stated in Proposition 7. Thus, the policy implications in the basic model still hold. The difference is that, the fixed cost, f_m, reduces the portion of collateral that would be enlarged by the policy, and the optimal risk-retention rate should be lower than that in the basic model.

If the supply of bonds is scarce ($z < f_m$), condition (90) shows that the policy in fact brings a loss to the lending bank. Under this circumstance, the optimal risk-retention rate should be set at $\theta = 1$, which suggests no central bank interventions in the form of private asset purchases.

Appendix H. Costs of collateralizing bonds

In our benchmark model, each unit of bonds pays the bearer one unit of the numéraire, and if the borrower defaults, the lending bank receives one unit of the numéraire from each unit of bonds it forecloses. Moreover, there is no cost of collateralizing bonds. This appendix aims to study how incorporating the cost of collateralizing bonds into the basic model affects implications of monetary policy. We assume that for each unit of bonds pledged as collateral, the lending bank receives $\rho \in (0, 1)$ unit of the numéraire in the case of default; thus, $1 - \rho$ unit of the numéraire is considered as the cost of collateralizing one unit of bonds (we also call it the foreclosure cost).

The default rule is as described in (6), and the optimal debt contract is the same as in the basic model, except that the bank's expected payoff is reduced by the foreclosure cost as follows:

$$\begin{aligned}
\pi_b &= \int_{x^*-a^*}^{\overline{\omega}} x dG(\omega) + \int_{\underline{\omega}}^{x^*-a^*} (\omega + \rho a^* - \gamma) dG(\omega) - (k - d^*) \qquad (91)\\
&= y^*[1 - G(y^*)] + \int_{\underline{\omega}}^{y^*} \omega \, dG(\omega) - [\gamma + (1 - \rho)a^*]G(y^*) - (k - d^* - a^*)
\end{aligned}$$

where $y^* = x^* - a^*$. Equation (91) shows that if the borrower defaults, the bank's payoff is $\omega + \rho a^* - \gamma$; that is, the bank receives the project returns, ω, and ρa^* from the collateralized bonds, and it pays the monitoring cost, γ. One can rewrite the payoff as $\omega + a^* - [\gamma + (1 - \rho)a^*]$. Obviously, the foreclosure cost, $(1 - \rho)a^*$, plays the same role as the monitoring cost, γ.

Proposition 2 implies that, an entrepreneur with $k < m + z$ offers a contract with $d^* = m$, $a^* = k - m$, $y^* = 0$, so default will not occur, and the cost of collateralizing bonds does not matter. For $k > m + z$, default may occur, and $d^* = m$ and $a^* = z$. The bank's expected payoff can be rewritten as

$$\pi_b = y^*[1 - G(y^*)] + \int_{\underline{\omega}}^{y^*} \omega \, dG(\omega) - [\underbrace{\gamma + (1 - \rho)z}_{\equiv \gamma^\dagger}]G(y^*) - (k - m - z),$$

where $\gamma^\dagger = \gamma + (1 - \rho)z$ is called the *total enforcement cost.*

From numerical examples we have the following observations (the basic parameter setting is as in Figure 2). First, in the economy with $\rho \in (0, 1)$, the effects of OMOs and changes in the policy rates are qualitatively the same as in the basic model. Second, the cost of collateralizing bonds does not alter qualitatively the effects of the private asset purchase policy. A higher total enforcement cost makes external financing more expensive, and entrepreneurs hold more internal funds in response. We thus observe that aggregate defaults, the average lending rate, and the risk spread fall. Holding more internal funds reduces the expected bankruptcy cost, which increases entrepreneurs' expected profits. Nevertheless, a higher total enforcement cost reduces the maximum risky loan amount that a bank is willing to extend, and lowers the investment threshold. Hence, to raise the investment threshold to the optimal level, the central bank should choose a lower risk-retention rate than that in the basic model.

Appendix I. Effects of short-run private asset purchases

The policy implications in the main text are based on a steady state analysis. Here we consider a short-run private asset purchase policy as follows. At the beginning of period t, the central bank announces that it will purchase θ fraction of bank loans, and the policy will be implemented for only one period. The policy is announced at the beginning of period t, while entrepreneurs choose the holdings of real balances in stage 2 of period $t-1$. Therefore, unlike the basic model, here entrepreneurs cannot adjust holdings of real balances in response to policy.

Figure 4 in the main text shows policy effects for each period in a stationary equilibrium, so we can compare the one-period policy results with those shown in Figure 4. From numerical examples (with the same parameter setting as in Figure 4), we find that the short-run policy results are qualitatively the same as in the basic model. The policy raises firm entry, aggregate lending, investment, and net output, and it reduces the average lending rate and the business failure rate, as described in Proposition 9. For a given θ, compared to the results shown in Figure 4, the short-run policy results in higher investment threshold, firm entry, and aggregate investment, and lower defaults, aggregate bankruptcy costs, and average lending rate. Moreover, under the short-run policy, the optimal risk-retention rate is lower than that in the basic model. The main reason is that entrepreneurs cannot adjust holdings of real balances in response to the policy, while in the basic model entrepreneurs choose to hold lower real balances than without policy.

Note that if the central bank announces in stage 2 of period $t-1$ the short-run policy of purchasing loans at period t, entrepreneurs can adjust holdings of real balances in response to policy, and the short-run policy results will be the same as shown in Figure 4.

Appendix J. Temporary OMOs

In the main text we study OMOs in a stationary equilibrium, wherein the money growth rate (and the inflation rate), σ, is constant, so is the policy rate, $i = \sigma(1+r) - 1$. This is a type of permanent OMOs, which have the same real impact as changing only the bond supply. In the real world, central banks adopt OMOs in order to achieve the policy rate target. It is not easy to find evidences on the effects of OMOs which are consistent with the situation depicted in our steady state analysis wherein the policy rate does not change when OMOs are implemented. In this appendix we study temporary OMOs, whereby the central bank swaps money for bonds in period t, allowing for the one-period money growth rate and the inflation rate to change in response to OMOs, but it does not renew the operation at $t+1$. We show that temporary open market purchases are expansionary and the policy rate falls, a situation which is more consistent with the conventional view of the effects of OMOs.

To begin with, we define some notations. In period t, the money supply is M_t, the value of money is q_t, real balances are $m_t = q_t M_t$, the inflation rate is Π_t, where $1 + \Pi_t = \frac{q_{t-1}}{q_t}$, and the money growth rate is $\sigma_t = \frac{M_t}{M_{t-1}}$. The quantity of money is adjusted in stage 2 of each period, and so M_t denotes the quantity of money at stage 1 of period t, and at the end of stage 2 of period $t-1$. Suppose the money growth rate is σ in each period except in period t when temporary OMOs are implemented.

Following Rocheteau, Wright, and Xiao (2018), we consider the type of temporary OMOs as follows. Suppose that the central bank conducts a one-period open market purchase in stage 2 of period t by injecting $\triangle M_{t+1} > 0$ to purchase bonds, $\triangle z_{t+1} < 0$. This intervention implies that the quantity of money in period $t+1$ is $M_{t+1} = \sigma M_t + \triangle M_{t+1}$, and the real bond supply is $z_{t+1} = z + \triangle z_{t+1}$. The intervention will not be renewed in period $t+1$, and the nominal money supply will be adjusted in stage 2 of $t+1$ so that it reverses to the original path; i.e., the quantity of money in $t+2$ (and at the end of stage 2 of $t+1$) is $M_{t+2} = \sigma^2 M_t$.

For the purpose of exposition, we first consider $\sigma = 1$ as the benchmark case. Because the open market purchases will not be renewed in period $t+1$ and afterward, the economy goes back to the stationary equilibrium (with the money growth rate σ) after period $t+1$, and $q_s = q$ and $m_s = qM$ for all $s \geq t+1$. To understand the effects of OMOs, we need to know entrepreneurs' asset holdings in period t when the open market purchase is implemented. To solve for entrepreneurs' decisions in t, we need to know their decisions at stage 2 of $t+1$, at which time and afterwards the economy is in the stationary equilibrium. Entrepreneurs' asset holdings at stage 2 of $t+1$, $(m_{t+2},\ z_{t+2})$, satisfy the following two conditions:

$$\begin{aligned}
\frac{1+\Pi_{t+2}}{\beta} &= \alpha\Big\{(1-\phi)f(m_{t+2})(\mu_\omega - m_{t+2}) + \phi\Big[f(k_{t+2}^T)(\mu_\omega - k_{t+2}^T) - \gamma\int_{m_{t+2}+z_{t+2}}^{k_{t+2}^T} G(y^*)f'(k)dk\Big]\Big\} \\
&\quad +1, \qquad (92)\\
\frac{\psi_{t+1}}{\beta} &= \alpha\phi\Big\{f(k_{t+2}^T)(\mu_\omega - k_{t+2}^T) - \gamma\int_{m_{t+2}+z_{t+2}}^{k_{t+2}^T} G(y^*)f'(k)dk\Big\}+1, \qquad (93)
\end{aligned}$$

where $1+\Pi_{t+2} = \frac{q_{t+1}}{q_{t+2}}$. The equilibrium asset holding $(m_{t+2},\ z_{t+2}) = (m,\ z)$ because the economy is in the stationary equilibrium in $t+1$ and afterward. Substitute $m_{t+2} = m$, $z_{t+2} = z$ (both of which imply $k_{t+2}^T = k^T$), and $q_{t+2} = q$ into (92) and (93), we find that the right side of (92) and (93) are the marginal benefit of real balances and bond holdings, respectively, in the stationary equilibrium. This implies that $\psi_{t+1} = \psi$ and $1+\Pi_{t+2} = \frac{q_{t+1}}{q_{t+2}} = \frac{q_{t+1}}{q}$, where $q_{t+2} = q$ because the economy returns to stationary equilibrium after period $t+1$. The inflation rate equals original money growth rate, which means $1+\Pi_{t+2} = \sigma = 1$. Therefore, $1+\Pi_{t+2} = \frac{q_{t+1}}{q} = \sigma = 1$, and therefore, $q_{t+1} = q$.

With $q_{t+1} = q$, we have $m_{t+1} = q_{t+1}M_{t+1} = q(M+\triangle M_{t+1}) = m + q\triangle M_{t+1} = m+\triangle m_{t+1} > m$, because the period-$t$ open market purchase injects money, $\triangle M_{t+1} > 0$. Entrepreneurs hold more real balances at $t+1$ than in the stationary equilibrium; that is, $m_{t+1} > m$, which is the driving force behind the expansionary effect of temporary open market purchases.

We now consider entrepreneurs' asset holdings of period t. The optimal conditions are

$$\begin{aligned}
\frac{1+\Pi_{t+1}}{\beta} &= \alpha\Big\{(1-\phi)f(m_{t+1})(\mu_\omega - m_{t+1}) + \phi\Big[f(k_{t+1}^T)(\mu_\omega - k_{t+1}^T) - \gamma\int_{m_{t+1}+z_{t+1}}^{k_{t+1}^T} G(y^*)f'(k)dk\Big]\Big\}+1, \qquad (94)\\
\frac{\psi_t}{\beta} &= \alpha\phi\Big\{f(k_{t+1}^T)(\mu_\omega - k_{t+1}^T) - \gamma\int_{m_{t+1}+z_{t+1}}^{k_{t+1}^T} G(y^*)f'(k)dk\Big\}+1. \qquad (95)
\end{aligned}$$

Using (94), (95), and central bank's balanced budget of implementing OMOs,

$$0 = q_t\triangle M_{t+1} + \psi_t\triangle z_{t+1} = (1+\Pi_{t+1})\triangle m_{t+1} + \psi_t\triangle z_{t+1}, \qquad (96)$$

one can solve for $\{1+\Pi_{t+1},\ \triangle z_{t+1},\ \psi_t\}$ for a given $\triangle m_{t+1}$.

We summarize the results of temporary open market purchases when $\sigma = 1$ and real balances injection $\triangle m_{t+1} = 0.1$ as follows (the parameter setting is the same as in Figure 2). The solutions are $\{1 + \Pi_{t+1},\ \triangle z_{t+1},\ \psi_t\} = \{.9995,\ -.1008,\ .9911\}$. An increase in real balances due to the temporary open market purchase allows entrepreneurs to rely more on internal finance than otherwise. It allows unbanked entrepreneurs to invest because they have more real balances to cover the input cost. Therefore, we observe an increase in the firm entry rate, aggregate investment, entrepreneurs' expected profits, and net output; however, total loans are reduced, and the average lending rate and risk spread increase.[32] The policy rate calculated from Fisher equation, $i = (1 + \Pi_{t+1})/\beta - 1$ falls as $1 + \Pi_{t+1} = .9995 < \sigma$. Note that for the aggregate investment, the effect is $\frac{\partial K_{t+1}}{\partial \triangle m_{t+1}} = \alpha[(1 - \phi)m_{t+1}f(m_{t+1}) + \phi k^T_{t+1}f(k^T_{t+1})\frac{\partial k^T_{t+1}}{\partial \triangle m_{t+1}}]$ which is positive if $(1 - \phi)m_{t+1}f(m_{t+1}) > \phi k^T_{t+1}f(k^T_{t+1})(\frac{1+\Pi_{t+1}}{\psi_t} - 1)$. This condition is more likely to hold and the increase in aggregate investment is large when the policy rate is low (which implies that the liquidity premium differential between money and bonds is small). For instance, given $\triangle m_{t+1} = .1$, $\frac{\partial K_{t+1}}{\partial \triangle m_{t+1}} = .034$ and $.027$ when $\sigma = 1$ and $\sigma = 1.02$, respectively.

Appendix K. Effectiveness of policies under different nominal policy rates

This appendix compares the effectiveness of conventional and unconventional monetary policies under different levels of nominal policy rate, $i = \sigma(1 + r) - 1$. Proposition 4 shows that the Friedman rule (which implies $i = 0$) achieves the first best allocation. Therefore, when $i = 0$, unconventional monetary policy cannot improve welfare. However, as mentioned in Section 6.2, the Friedman rule may be infeasible because frictions makes it impossible to withdraw money from entrepreneurs. If withdrawing money is infeasible, the lowest feasible money growth rate is $\sigma = 1$, which implies $i = r$, the real interest rate. We have shown in Section 6.2 that, under this circumstance, unconventional monetary policy such as private asset purchases can improve welfare.

We now compare the effectiveness of OMOs and private asset purchases under different levels of nominal interest rate. We consider numerical examples wherein parameters follow the setting

[32] Using (94) - (96), we infer that $\triangle m_{t+1} + \triangle z_{t+1} < 1$. This implies that the risky loan amount increases after OMO ($\ell_y = k - (m + \triangle m_{t+1}) - (z + \triangle z_{t+1}) < k - m - z$) for a given input cost k. Proposition 1 shows that the effective repayment is increasing in risky loan amount and hence the real lending rate is increasing as well.

in Figure 2, except σ and z_s.[33] In the first case, we set $\sigma = 1$, which implies $i = r = .02$. In the second case we set $\sigma = 1.02$, which gives $i = .0404$. We compare the welfare levels under OMOs and the private asset purchase policy for $i = .02$ and $i = .0404$. Recall that under OMOs, the welfare measure is entrepreneurs' expected profits, (20), whereas under private asset purchases the welfare measure equals entrepreneurs' expected profits subtracting the central bank's cost of implementing the policy, (33).

We first calculate the welfare level in the benchmark setting wherein the supply of bonds is set at $z_s = .01$, and no private asset purchase policy ($\theta = 1$) is implemented; this benchmark welfare level is denoted as Ω_n. Next, we set $\theta = 1$ and compute the maximum welfare level that can be achieved by OMOs (which amounts to changing bond supply, z_s, in the basic model), denoting this welfare level as Ω_z. Then, we set bond supply at the benchmark level, $z_s = .01$, and compute the welfare level under private asset purchases with optimal risk-retention rate; denote this level as Ω_θ. Finally, we compare effectiveness between two policies by two measures: $\triangle\Omega \equiv \Omega_\theta - \Omega_z$, which captures the welfare difference between asset purchases and OMOs, and $\triangle\Omega(\%) \equiv \frac{\triangle\Omega}{\Omega_n} \times 100\%$, which captures the percentage change in the welfare level of a policy relative to the benchmark.

In the two cases, $i = .02$ and $i = .0404$, the optimal bond supply is 0, provided central bank cannot short sell bonds. The intuitive reason is this. As bond supply rises, the price of bonds fall, and entrepreneurs substitute away from real balances to bond holdings. This leads to lower welfare because entrepreneurs' expected profits increase in real balances. The optimal risk-retention rate is $\theta^* = .0806$ under $i = .02$, and $\theta^* = .0405$ under $i = .0404$. The welfare levels of the two cases are listed in the following table. From the numerical results, we draw the following conclusions. First, in terms of the maximum welfare level they can achieve, both policies are more efficient under low nominal policy rate (when $i = .02$). Second, the private asset purchase policy is more efficient than OMOs because $\triangle\Omega$ is positive under both policy rates. Finally, both effectiveness measures, $\triangle\Omega$ and $\triangle\Omega(\%)$, are larger in the case with $i = .0404$ than in the case with $i = .02$, a result suggests that effectiveness of the asset purchase policy is even higher under the situation when the nominal policy rate is high.

[33] The followings are parameter settings for numerical examples: $\omega \sim U[0, 18]$, $log(k) \sim N$ with $E[log(k)] = 3$ and $Var[log(k)] = 1$, $\{\alpha, \beta, \gamma, \phi, z_s, \theta\} = \{0.5, 1/1.02, 15, 0.7, 0.01, 1\}$.

	Ω_n	Ω_z	Ω_θ	$\triangle\Omega$	$\triangle\Omega(\%)$
$i = 0.0200$	0.3451	0.3451	0.3466	0.0015	0.43%
$i = 0.0404$	0.3053	0.3053	0.3118	0.0065	2.13%

Table 1: Effectiveness of policies under different nominal policy rates

Appendix L. The benchmark model without bonds

We consider the benchmark model with fiat money as the only safe asset. The optimal contract problem for a banked entrepreneur is the same with $a = z = 0$. The effective repayment y^* depends on the risky loan amount and so Proposition 1 still holds; however, here a banked entrepreneur can only use money as a down payment to reduce the risky loan amount. The definition of k_e is the same while $k_b = m + \bar{\ell}_{yb}$. Following the same approach as in the benchmark model, we obtain the first order condition for money demand,

$$\frac{\sigma}{\beta} = \alpha\Big\{(1-\phi)f(m)(\mu_\omega - m) + \phi\Big[f(k^T)(\mu_\omega - k^T) - \gamma\int_m^{k^T} G(y^*)f'(k)dk\Big]\Big\} + 1. \tag{97}$$

Proposition 14. *Consider that the input cost and project return follow uniform distributions: $k \sim U[0, \epsilon_k]$, $\omega \sim U[0, \epsilon_\omega]$, where $\mu_k = \frac{\epsilon_k}{2}$ and $\mu_\omega = \frac{\epsilon_\omega}{2}$, and $\gamma < \epsilon_\omega$. Suppose $\sigma \geq \beta$. In a monetary equilibrium,*

$$\bar{\ell}_{yb} = \mu_\omega - \gamma + \frac{\gamma^2}{2\epsilon_\omega},\ y^* = (\overline{\omega} - \gamma) - \sqrt{(\overline{\omega} - \gamma)^2 - 2\epsilon_\omega \ell_y},$$

where $\ell_y = k - m$ is the risky loan amount for a given k. There are two cases:

(i) in an equilibrium with constrained credit,

$$k^T = m + \bar{\ell}_{yb}, \quad m = \mu_\omega - \phi\bar{\ell}_{yb} - \frac{(\sigma-\beta)\epsilon_k}{\alpha\beta};$$

(ii) in an equilibrium with unconstrained credit,

$$k^T = \begin{cases} \mu_\omega - \gamma\Big(1 - \sqrt{\frac{2m}{\epsilon_\omega}}\Big), & \text{if } \sigma > \beta, \\ \mu_\omega, & \text{if } \sigma = \beta, \end{cases}$$

$$m \text{ solves} \begin{cases} (1-\phi)(\mu_\omega - m) + \phi\gamma\Big(1 - \sqrt{\frac{2m}{\epsilon_\omega}}\Big) - \frac{(\sigma-\beta)\epsilon_k}{\alpha\beta} = 0, & \text{if } \sigma > \beta, \\ (1-\phi)(\mu_\omega - m) = 0, & \text{if } \sigma = \beta. \end{cases} \tag{98}$$

As Proposition 14 shows, the real balances m is decreasing in the money growth rate σ. From Proposition 1, lower real balances imply a higher risky loan amount for a given input cost k. Consequently, the effective repayment increases and so does the default probability.

Appendix M. Proof of the optimal contract

For this proof, we focus on a non-trivial case that a banked entrepreneur wants to finance a project with input cost $k > m+z$ and hence, she needs to borrow the risky loan amount, $\ell_y = k-m-z > 0$, from a bank.

We adapt the proof from Williamson (1986, 1987). When a banked entrepreneur observes her project return ω, she sends a signal $\omega^d \in [\underline{\omega},\ \overline{\omega}]$ to the bank, which is a function of return realization. The contract specifies that, if $\omega^d \in D \subset [\underline{\omega},\ \overline{\omega}]$, then monitoring occurs, and if $\omega^d \in D^c$ (the complement set of D), monitoring does not occur. The repayment from the entrepreneur to the bank is $P(\omega)$ if $\omega^d \in D$, and $Q(\omega^d)$ if $\omega^d \in D^c$, where $P(\cdot)$ and $Q(\cdot)$ are functions on $[\underline{\omega},\ \overline{\omega}]$. If the entrepreneur wants to send a signal that $\omega^d \in D^c$, then she will always choose $\omega^* = \arg\min_{\omega^d \in D^c}\{Q(\omega^d)\}$. We denote the repayment $Q(\omega^*)$ as y. It remains to determine the repayment schedule $P(\omega)$, which satisfies the feasibility condition $0 \le P(\omega) \le \omega$. Whether or not an entrepreneur sends a signal to incur monitoring depends on the realization of project return, and she has no incentive to deviate from the contract in equilibrium, which implies that $\omega^d \in D$ if $P(\omega) < y$ and $\omega^d = \omega^*$ if $P(\omega) \ge y$. Let $A = \{\omega : P(\omega) < y\}$ and $B = \{\omega : P(\omega) \ge y\}$. Then, the optimal contract is a repayment schedule $\{P(\omega), y\}$, which maximizes the entrepreneur's expected profit while satisfying the bank's participation constraint:

$$\max_{\{P(\omega),\ y\}} \left\{ \int_A [\omega - P(\omega)]g(\omega)d\omega + \int_B [\omega - y]g(\omega)d\omega \right\}$$
$$s.t. \quad \int_A [P(\omega) - \gamma]g(\omega)d\omega + \int_B yg(\omega)d\omega \ge \ell_y.$$

If $\omega \in A$, the entrepreneur sends a signal $\omega^d \in D$ and monitoring occurs. Hence, the bank spends the monitoring cost γ and collects repayment $P(\omega)$. On the other hand, If $\omega \in B$, the entrepreneur sends a signal $\omega^d = \omega^*$ and repays y. The expected profit from the contract must be larger or equal to bank's funding cost, ℓ_y.

The following Proposition proves that the optimal repayment schedule is $P(\omega) = \omega$, independent of y. The debt contract in the basic model is an optimal contract under costly state verification.

Proposition 15. *The optimal repayment schedule is* $P(\omega) = \omega$, *independent of* y.

Proof. Suppose not, and there exists an optimal contract $\{P'(\omega), y'\}$, where $P'(\omega) < \omega$ for some $\omega \in A'$. First, note that the bank's participation constraint must hold with equality, since otherwise we could reduce $P'(\omega)$ for some ω such that the constraint would still hold, and increase the entrepreneur's expected profit. Letting $A' = \{\omega : P'(\omega) < y'\}$ and $B' = \{\omega : P'(\omega) \geq y'\}$, we have

$$\int_{A'} [P'(\omega) - \gamma] g(\omega) d\omega + \int_{B'} y' g(\omega) d\omega = \ell_y. \tag{99}$$

Since $P'(\omega) < \omega$ for some $\omega \in A'$, there exists another repayment schedule $P''(\omega)$ with $P''(\omega) \geq P'(\omega)$ for all ω and $P''(\omega) > P'(\omega)$ for some $\omega \in A'$, with $P''(\omega)$ continuous and monotone increasing on $[\underline{\omega},\ \overline{\omega}]$. There is then some y'', where $0 < y'' < y'$, such that, with $A'' = \{\omega : P''(w) < y''\}$ and $B'' = \{\omega : P''(w) \geq y''\}$,

$$\int_{A''} [P''(\omega) - \gamma] g(\omega) d\omega + \int_{B''} y'' g(\omega) d\omega = \ell_y. \tag{100}$$

From the two binding bank's participating constraints, (99) and (100), we obtain

$$\begin{aligned}
&\Big[\int_{A'} P'(\omega) g(\omega) d\omega + \int_{B'} y' g(\omega) d\omega\Big] - \Big[\int_{A''} P''(\omega) g(\omega) d\omega + \int_{B''} y'' g(\omega) d\omega\Big] \\
&= \gamma \Big[\int_{A'} g(\omega) d\omega - \int_{A''} g(\omega) d\omega\Big].
\end{aligned}$$

Now we show that the entrepreneur can attain a higher expected profit from the new repayment schedule $\{P''(\omega), y''\}$. The change in the objective function in changing the contract from $\{P'(\omega),\ y'\}$ to $\{P''(\omega),\ y''\}$ is then

$$\begin{aligned}
&\Big[\int_{A''} [\omega - P''(\omega)] g(\omega) d\omega + \int_{B''} [\omega - y''] g(\omega) d\omega\Big] - \Big[\int_{A'} [\omega - P'(\omega)] g(\omega) d\omega + \int_{B'} [\omega - y'] g(\omega) d\omega\Big] \\
&= \gamma \left[\int_{A'} g(\omega) d\omega - \int_{A''} g(\omega) d\omega\right].
\end{aligned}$$

The change is positive because for all $\omega \in A''$, it implies $P'(\omega) \leq P''(\omega) < y'' < y'$ and hence $\omega \in A'$ and $A'' \subset A'$. This implies that the initial repayment schedule, $\{P'(\omega),\ y'\}$, is not optimal, a contradiction. Since we can apply this argument to all repayment schedules $P(\omega) < \omega$ for all $\omega \in A$, and it must satisfy $0 \leq P(\omega) \leq \omega$, the only possibility is $P(\omega) = \omega$. $\square$