Trim rule delete turn right conti from start
delete turn left conti from end.

Property of u equation

$$F_{ij} := F^{\text{trim}\ ++}_{W_{i+1,j+1}}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \overset{\text{Trim}}{\overset{\uparrow}{a}} & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ c & d \end{pmatrix}$$

$$(1\ 1) \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (a+c\ \ b+d) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = a+c$$

$\Rightarrow F^{\text{trim}\ ++}_{W_{i+1,j+1}} \Leftrightarrow F_{ij}$. equivalent definition.

We know that

$$u_{ij} = \frac{F^{+-}_{w_{ij}} F^{-+}_{w_{ij}}}{F^{++}_{W_{ij}} F^{--}_{u_{ij}}}$$

$$F^{+-}_{w_{ij}} = \Pi y^{L} F^{\text{Trim}\ +-}_{w_{ij}}$$

$$F^{-+}_{W_{ij}} = \Pi y^{R} F^{\text{Trm}\ -+}_{w_{ij}}$$

$$F^{++}_{w_{ij}} = \Pi y^{L} \Pi y^{R} F^{\text{Trm}\ ++}_{w_{ij}}$$

2 case $ij \in T$ or not

1. $u_{ij} = y_{ij} \dfrac{F^{\text{Trim}\ +-}_{u_{ij}} F^{\text{Trim}\ -+}_{u_{ij}}}{F^{\text{Trm}\ ++}_{u_{ij}} F^{\text{Trm}\ --}_{u_{ij}}}$ $\quad ij \in T$.

2. $u_{ij} = \dfrac{F^{\text{Trim}\ +-}_{u_{ij}} F^{\text{Trim}\ -+}_{u_{ij}}}{F^{\text{Trm}\ ++}_{u_{ij}} F^{\text{Trm}\ --}_{u_{ij}}}$ $\quad$ else

But study $F^{\text{Trm}\ -+}_{u_{ij}}$

Case 1, $F^{Trm-+}_{w_{i,j}}$ $\quad$ $i\ L\ L\ L\ y_1\ R\ y_2\ R \cdots j$

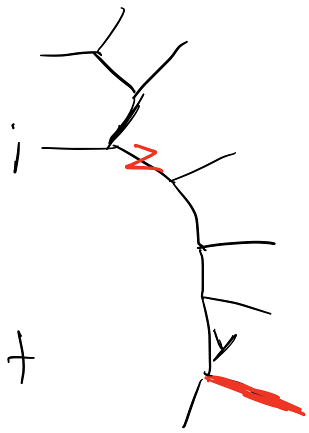
$\longrightarrow$ $R\ y_2\ R \cdots j$

But $W_{i+1,j} \Leftrightarrow$ $i{+}1\ R\ R\ R\ R\ L\ y_2 \cdots\cdots$

$$\Rightarrow \underline{F^{Trm++}_{w_{i+1,j}}} \Leftrightarrow \boxed{y_2 \cdots\cdots j} = \underline{F^{Trm-+}_{w_{i,j}}}$$

Case 2 $F^{Trm-+}_{w_{i,j}}$ $\quad$ $i\ R\ z\ R \cdots y \cdots L \cdots j$

$\longrightarrow$ $\boxed{z_R \cdots y \cdots j}$



But $W_{i+1,j} \Leftrightarrow$ $i{+}1\ R\ R\ R\ L\ z\ R \cdots y \cdots j$

$$\Rightarrow F^{Trm++}_{w_{i+1,j}} \Leftrightarrow \boxed{z_R \cdots y \cdots j} \Leftarrow F^{Trm-+}_{w_{i,j}}$$

$$\Rightarrow F^{Trm-+}_{w_{i,j}} = F^{Trm++}_{w_{i+1,j}} \overset{\text{by def}}{=} F_{i,j-1}$$

$$F^{Trm--}_{w_{i,j}} = F^{Trm++}_{w_{i+1,j+1}} = F_{i,j}$$

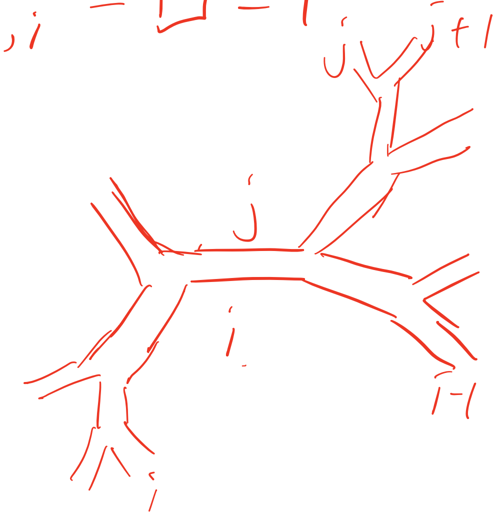
$$F^{Trm+-}_{w_{i,j}} = F^{Trm++}_{w_{i,j+1}} = F_{i-1,j}$$

$$\Rightarrow u_{i,j} = y_{ij}\,\frac{F^{Trm+-}_{w_{i,j}}\;F^{Trm-+}_{w_{i,j}}}{F^{Trm--}_{w_{i,j}}\;F^{Trm++}_{w_{i,j}}} = y_{i,j}\,\frac{F_{i-1,j}\;F_{i,j-1}}{F_{i,j}\;F_{i-1,j-1}} \quad (i,j)\in T$$

$$= \frac{F_{i-1,j}\;F_{i,j-1}}{F_{i,j}\;F_{i-1,j-1}} \quad \text{else}$$

$i-1$ L L L L L L L i $\Leftrightarrow w_{i-1,i}$

Notice ① $F_{i\,i+1} = 1$ $\because$ $F_{i\,i+1} := F^{Trm++}_{w_{i-1,i}} = \square = 1$



② $F_{i-1,j-1} = 1$ when $(i,j)\in T$

$\parallel$

$F^{Trm++}_{w_{i,j}}$

$\because$ $w_{i,j} = i\,RRRR\cdots R\,Y_{ij}\,LLLL\,j$

$\Rightarrow F^{Trm++}_{w_{ij}} = \square = 1$

Summary $F_{i,i+1} = 1$ and $F_{i-1,j-1} = 1$ when $(i,j)\in T$.

Super Tachyon.

$$\int \frac{d^{2n}z}{SL} \left\langle \prod_{j=1}^{2n} P_j \psi e^{iP_j X} \right\rangle$$

$$= \int \frac{d^{2n}z}{SL} \prod_{i<j} |z_{ij}|^{\alpha' S_{ij}} \, Pf(A_{2n\times 2n})$$

$A_{2n\times 2n} :=$ $a_{ij} = 0$ when $i=j$

$a_{ij} = \frac{\alpha' S_{ij}}{z_{ij}}$ when $i<j$

$$\Rightarrow \int \frac{d^{2n}z}{SL} \prod_{i<j} z_{ij}^{\alpha' S_{ij}} \, Pf(A_{2n\times 2n})$$

$$\Rightarrow \int \frac{d^{2n}z}{SL} \frac{\prod u_c^{\alpha' X_c} \prod z_{i\,i+1}^{\frac{1}{2}}}{\prod z_{i\,i+1}} Pf$$

$$\Rightarrow \int \frac{d^{2n}z}{SL\,PK} \prod u_c^{\alpha' X_c} \underline{\prod z_{i\,i+1}^{\frac{1}{2}} Pf}$$

$$= \int \frac{d^{2n}z}{SL\,PK} \prod u_c^{\alpha' X_c} \sum_{\text{partion}} \prod_{ij\in P} \alpha' S_{ij} \cdot \prod u_\alpha^{n^\alpha_{\text{partion}}}$$

$\Rightarrow$ where $n^\alpha_{\text{parton}} \in \pm\frac{1}{2}, \pm 1$

$$\prod u_c^{X_c} = \prod_{i<j} z_{ij}^{\alpha' S_{ij}} \prod z_{i\,i+1}^{2\alpha' p^2} \prod z_{i\,i+2}^{-\alpha' p_i^2}.$$

In this case $P_i^2 = \frac{1}{2\alpha'}$

$$\Rightarrow \prod u_c^{X_c} = \prod z_{ij}^{\alpha' S_{ij}} \prod z_{i\,i+1} \prod z_{i\,i+1}^{-\frac{1}{2}}$$

$$\Rightarrow \prod z_{ij}^{\alpha' S_{ij}} = \frac{\prod u_c^{X_c} \prod z_{i\,i+1}^{\frac{1}{2}}}{\prod z_{i\,i+1}}$$

$$u_{ij} = \frac{z_{i-1\,j}\, z_{i\,j-1}}{z_{ij}\, z_{i-1\,j-1}}$$

$$= \frac{F_{i-1,j}\, F_{i\,j-1}}{F_{i,j}\, F_{i-1,j-1}} \qquad ij \notin T$$

$$= y_{ij} \frac{F_{i-1,j}\, F_{i\,j-1}}{F_{i,j}\, F_{i-1,j-1}} \qquad ij \in T$$

$$\Rightarrow \prod u_c^{\alpha' X_c} = \prod y_T^{\alpha' X_T} \prod_{i<j} F_{ij}^{\alpha' S_{ij}} \prod F_{i\,i+1} \prod F_{i\,i+2}^{-\frac{1}{2}}$$

$$\Rightarrow \int \prod \frac{dy_T}{y_T} \prod y_T^{\alpha' X_T} \prod_{i<j} F_{ij}^{\alpha' S_{ij}} \prod F_{i\,i+1}^{1} \prod F_{i\,i+2}^{-\frac{1}{2}} \cdot \sum_{\text{partion}\,\alpha\in T} \prod y^{n^\alpha_{\text{parton}}} \prod_{ij\in P} \alpha' S_{ij} \cdot \frac{1}{F_{ij}} \cdot \prod F_{i\,i+2}^{\frac{1}{2}}$$

$$\Rightarrow \int \prod \frac{dy_T}{y_T} \prod y_T^{\alpha' X_T} \prod_{i<j} F_{ij}^{\alpha' S_{ij}} \cdot \left(\sum_{\text{partion}\,\alpha\in T} \prod y^{n^\alpha_{\text{parton}}} \prod \frac{\alpha' S_{ij}}{F_{ij}} \right)$$

Notice when we focus on the zero of Amplitude. we only care the $F_{i,j}$'s exponent, when all exponent is nonpositive interger then it's a zero.

Consider $\alpha' c_{ij}$ in a rectangle is nonpositive integer.

Then consider 2 case

1. (ij) in the partition also in rectangle.

when $\alpha' c_{ij} = 0$ then this term is zero

when $\alpha' c_{ij}$ is negative interger, $\Rightarrow F_{ij}^{-\alpha' c_{ij}} \cdot F_{ij}^{-1} = F^{\mathbb{Z}_{\geq 0}}$.

$\Rightarrow$ it is still scaleless integral $\Rightarrow$ this term goes to zero.

2. (ij) in the partion $\notin$ retangle.

Then it is zero, because it doesn't change the intergrand about scaleless intergal..

Summary: When $\alpha' c_{i,j}$ are nonpositive integer in the rectangle.

$\Rightarrow I^{2n\text{-}ST} = 0$.

The zero is the same as the Bosonic case.

After scaffolding $\Rightarrow$ SYM n point amplitude.

Notice If we can prove $y_\alpha^{n^\alpha_{parton}}$ $\exists$ $\alpha \in (13)(35)\cdots(2n-1)$ st

$n^\alpha_{partion} \in \mathbb{Z}_{\leq 0} \Rightarrow n^\alpha_{parton} = -1$ or 0.

$u_{2i-1\,2i}$ X

$u_{2i\,2i}$ X

$u_{2i-1\,2i+1}$ V

$u_{2i\,2i+1}$ X

$[\because Z_{2i-1\,2i}$ can only be construct by

$\Rightarrow$ must corresponding to $u_{2i-1\,2i+1}$

$\Rightarrow \frac{1}{Z_{2i-1\,2i}} \Leftrightarrow \frac{1}{u_{2i-1\,2i+1}}$

Bosonic tachyon.

$$\prod u_c^{\alpha' X_c} = \prod z_{ij}^{\alpha' s_{ij}} \prod z_{i\,i+1}^{2} \prod z_{i\,i+2}^{-1}$$

$$\int \frac{d^n z}{SL} \prod z_{ij}^{\alpha' s_{ij}} = \int \frac{d^n z}{SL} \prod u_c^{\alpha' X_c} \cdot \frac{\prod z_{i\,i+2}}{\prod z_{i\,i+1}^{2}}$$

$$= \int \frac{d^n z}{SL \prod z_{i\,i+1}} \prod u_c^{\alpha' X_c} \cdot \frac{1}{\prod u_c}$$

$$= \int \prod_T \frac{dy_T}{y_T} \prod u_c^{\alpha' X_c - 1}$$

$$= \int \prod_T \frac{dy_T}{y_T} \prod y_T^{\alpha' X_T - 1} \cdot \prod_{i<j} F_{ij}^{\alpha' s_{ij}}$$

$$= \int \prod \frac{dy_T}{y_T^2} y_T^{\alpha' X_T} \prod_{i<j} F_{ij}^{-\alpha' c_{ij}}$$

$\Rightarrow$ Zero is the same as the $\mathrm{Tr}\,\phi^3$ case.

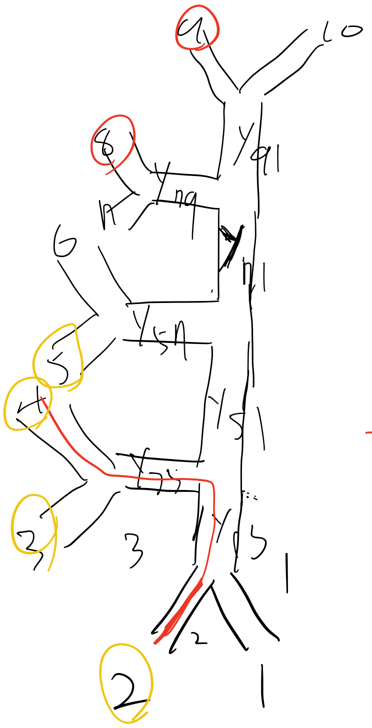
Scaffolding.

$$\int \frac{d^{2n} z}{SL} \prod z_{ij}^{\alpha' s_{ij}} \xrightarrow{\text{Residue on}} \begin{array}{l} s_{12} = \frac{-2}{\alpha'} \\ s_{34} = \frac{-2}{\alpha'} \\ \vdots \end{array} \quad n \text{ point YM.}$$

$$\|$$

$$\int \prod_S \frac{dy_S}{y_S^2} y_S^{\alpha' X_S} \prod^{n-3} \frac{dy_T}{y_T^2} y_T^{\alpha' X_T} \prod_{i<j} F_{ij}^{\alpha' s_{ij}} \xrightarrow[X_S = 0]{\text{Residue}} n \text{ point YM.}$$

F_{24}^{tt} F_{25}^{tt} $\cdots$ F_{210}

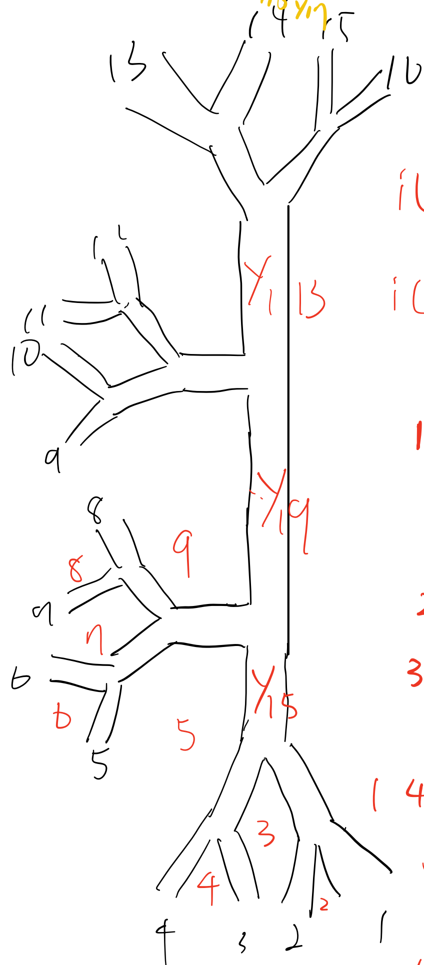


1. $C_{i7}=C_{i8}=0$ $i=1\cdots4$

$F_{w_{i+1}8}$ $F_{w_{i+1}9}$

6 L Y_{57} L Y_{17}	$6^{Y_{57}^{Y_{17}}}$
5 R Y_{57} L Y_{17}	$5_{Y_{57}}{}^{Y_{17}}$
4 L Y_{35} L Y_{15} R Y_{17}	$4^{Y_{35}^{Y_{15}}}{}_{Y_{17}}$
3 R Y_{35} L Y_{15} R Y_{17}	$3_{Y_{35}}{}^{Y_{15}}{}_{Y_{17}}$
2 L Y_{13} R Y_{15} R Y_{17}	$2^{Y_{13}}{}_{Y_{15}}{}_{Y_{17}}$
1 R Y_{13} R Y_{15} R Y_{17}	$1_{Y_{13}}{}_{Y_{15}}{}_{Y_{17}}$

Y_{17} L Y_{79} L 7	$Y_{17}{}^{Y_{79}^{7}}$
Y_{17} L Y_{79} R 8	$Y_{17}{}^{Y_{79}}{}_{8}$
Y_{17} R Y_{19} L 9	$Y_{17}{}_{Y_{19}}{}^{9}$
Y_{17} R Y_{19} R 10	$Y_{17}{}_{Y_{19}}{}_{10}$



$i(i+2)$ 13 35 57   8 27

$i(i+4)$ 15 59   4 17

	8.	12
$1_{Y_{13}}\ Y_{15}\ Y_{19}\ Y_{13}$	X	X
$2^{Y_{15}}\ Y_{15}\ Y_{19}\ Y_{13}$	✓	✓
$3\ Y_{35}{}^{Y_{15}}\ Y_{19}\ Y_{13}$	✓	✓
$4^{Y_{35}}\ Y_{15}\ Y_{19}\ Y_{13}$	✓	✓
$5\ Y_{57}\ Y_{57}\ Y_{19}\ Y_{13}$	✓	✓
$6^{Y_{57}}\ Y_{59}{}^{Y_{19}}\ Y_{13}$	X	✓
$7\ Y_{79}\ Y_{59}{}^{Y_{19}}\ Y_{13}$	X	✓
$8\ Y_{79}\ Y_{59}{}^{Y_{19}}\ Y_{13}$	X	✓

$Y_{19}{}^{Y_{913}{}^{Y_{91}{}^{9}}}$	X
$Y_{19}{}^{Y_{915}{}^{Y_{911}}}{}_{10}$	✓
$Y_{19}{}^{Y_{915}}\ Y_{1113}{}^{11}$	✓
$Y_{19}{}^{Y_{915}}\ Y_{1115}{}_{12}$	✓
$Y_{19}\ Y_{113}\ Y_{1515}{}^{13}$	✓
$Y_{19}\ Y_{113}\ Y_{1513}\ 14$	X

$Y_{19}\, Y_{113}\, Y_{151}\ 15$ ✗

$Y_{19}\, Y_{13}\, Y_{115}\ 16$ ✗

$9\ \ Y_{911}\, Y_{913}\, Y_{113}$ ✓

$10\ \ Y_{911}\, Y_{913}\, Y_{113}$ ✗

$11\ \ Y_{1113}\, Y_{915}\, Y_{113}$ ✗

$12\ \ Y_{1113}\, Y_{913}\, Y_{113}$ ✗

$\Downarrow$

$Y_{19} \Leftrightarrow C_{ij} \quad i \in 1 \cdots 8$
$\qquad\qquad\quad j \in 9 \cdots 12$

Tachyon $\{P\}$, $P^2 = \frac{1}{\alpha'}$

gluon $\{k\}$, $k^2 = 0$

level 2 $\{q\}$, $q^2 = -\frac{1}{\alpha'}$

Process $4n$ - tachyon $\quad (P_{2i-1} + P_{2i})^2 \to 0$

$$I^{4n,T} = \int \prod_{i=1}^{2n} \frac{dY_{2i-1,2i+1}}{Y_{2i-1,2i+1}^2} Y^{\alpha' X_{2i-1,2i+1}} \prod_{i=1}^{n} \frac{dY_{4i-3,4i+1}}{Y_{4i-3,4i+1}^2} Y^{\alpha' X_{4i-3,4i+1}} \prod_{i=1}^{n-3} \frac{dY_{1,5+4i}}{Y_{1,5+4i}^2} Y^{\alpha' X_{1,5+4i}} \prod_{i,j} F_{ij}^{-\alpha' C_{ij}}$$

Residue on $X_{2i-1,2i+1} = 0 \Leftrightarrow (P_{2i-1} + P_{2i}) = k_i \quad i$ from 1 to $2n$.

$P_{2i} = \varepsilon_i$

$I^{2n \cdot \text{gluon}}$

Residue on $X_{4i-3,4i+1} = (k_{2i-1} + k_{2i})^2 = -\frac{1}{\alpha'}$

$I^{n \cdot \text{level}\, 2}$ with vertex operator

$$E^{\mu\nu} = (\alpha' \varepsilon_1 \varepsilon_2 - 2\alpha' (\varepsilon_2 k_1)(\varepsilon_1 k_2)) k_1^\mu k_1^\nu + \alpha' (\varepsilon_1 k_2)(k_1^\mu \varepsilon_2^\nu + \varepsilon_2^\mu k_1^\nu) - \alpha' \varepsilon_2 k_1 (\varepsilon_1^\mu k^\nu + k_1^\mu \varepsilon_1^\nu)$$

$$V^{(2)} = B \cdot i\partial X + E \cdot i\partial X\, i\partial X$$

$$+ \frac{1}{2}(\varepsilon_1^{\mu}\varepsilon_2^{\nu} + \varepsilon_2^{\mu}\varepsilon_1^{\nu})$$

$$B^{\mu} = \left[\alpha(\varepsilon_1\varepsilon_2) - 2\alpha'^2(\varepsilon_2 k_1)(\varepsilon_1 k_2)\right] k_1^{\mu} - 2\alpha'(\varepsilon_2 k_1)\varepsilon_1^{\mu}.$$

Then we focus on the zero in level 2.

The zero from the intergral of the final triangulation $Y_{1,5+4k}$. So we focus on the F_{ij} which contain $[\# Y_{1,5+4k}(-)]$ after take the residue only some of $F_{i,j}$ has the above form. F_{ij} $i \in \{1,2,3 \cdots 4k\}$

$j \in \{5+4k \cdots 4n-4\}$

So if $C_{ij} = 0 \Rightarrow$ we have new form of zero.