$$A(z)B(w) = \sum_{n=-\infty}^{N} \frac{[AB]_n(w)}{(z-w)^n}$$

We recall that

$$[A[BC]_n]_m = (-1)^{ab}[B[AC]_m]_n + \sum_{j=0}^{m-1} \binom{m-1}{j} [[AB]_{m-j} C]_{n+j} \quad m \geq 1$$

Let $A = T$ $B = V_1$ $C = V_2$

$[TV_1]_2 = V_1$ $[TV_2]_2 = V_2$

$[TV_1]_1 = \partial V_1$ $[TV_2]_1 = \partial V_2$

We need to check

$m > 2 \Rightarrow [T[V_1V_2]_1]_m = 0$

$m = 2 \Rightarrow [T[V_1V_2]_1]_m = [V_1V_2]_1$

$m = 1 \Rightarrow [T[V_1V_2]]_m = \partial[V_1V_2]_1$

$n = 1$

$$\Rightarrow [T[V_1V_2]_1]_m = [V_1[TV_2]_m]_1 + \sum_{j=0}^{m-1} \binom{m-1}{j} [[TV_1]_{m-j} V_2]_{1+j}$$

when $m>2$

$$[T[V_1V_2]_1]_m = [V_1\ \underset{0}{\underset{\|}{0}}]_1 + (m-1)[[TV_1]_2 V_2]_{m-1} + [[TV_1]_1 V_2]_m$$

$$= 0 + (m-1)[V_1V_2]_{m-1} + [\partial V_1V_2]_m$$

$$= (m-1)[V_1V_2]_{m-1} + (1-m)[V_1V_2]_{m-1}$$

$$= 0$$

But $[\partial AB]_n = (1-n)[AB]_{n-1}$

When $m=2$

$$[T[V_1V_2]_1]_2 = [V_1[TV_2]_2]_1 + [[TV_1]_2V_2]_1 + [[TV_1]_1V_2]_2$$

$$= [V_1V_2]_1 + [V_1V_2]_1 + [\partial V_1V_2]_2$$

$$= 2[V_1V_2]_1 - [V_1V_2]_1 = [V_1V_2]_1$$

When $m=1$

$$[T[V_1V_2]_1]_1 = [V_1[TV_2]_1]_1 + [[TV_1]_1V_2]_1$$

$$= [V_1\,\partial V_2]_1 + [\partial V_1V_2]_1$$

$$= \partial[V_1V_2]_1 \quad \square$$

$\Rightarrow [V_1V_2]_1$ is Primary and conformal dimension 1.