2 ways to see the rule of curves.

$$L=\begin{pmatrix}1&1\\0&1\end{pmatrix}\qquad R=\begin{pmatrix}1&0\\1&1\end{pmatrix}$$

第一種 用階梯圖來看

Denote $w_{i,j} = i\,L\,y_1\,R\,y_2\,L\,y_3\,R\,j$

$i^{y_1}{}_{y_2}{}^{y_3}\ j$

Define $F^{++}_{w_{i,j}}$ = subset include beginning and end

$F^{+-}_{w_{i,j}}$ = subset include begin, exclude end

$F^{-+}_{w_{i,j}}$ = exclude begin, include end

$F^{--}_{w_{i,j}}$ = exclude both begin and end

$$F^{++}_{w_{i,j}} = \underset{i\quad j}{1} + \underset{i\ y_2\ j}{y_2} + \underset{i\ y_1 y_2\ j}{y_1y_2} + \underset{i\ y_2 y_3\ j}{y_2y_3} + \underset{i\ y_1 y_2 y_3\ j}{y_1y_2y_3}$$

F poly

all possible valid subset

$$F^{+-}_{w_{i,j}} = \underset{i}{1} + \underset{i\ y_2}{y_2} + \underset{i\ y_1 y_2}{y_1y_2}$$

$$F^{-+}_{w_{i,j}} = \underset{j}{1} + \underset{y_2\ j}{y_2} + \underset{y_2 y_3\ j}{y_2y_3}$$

$$F^{--}_{w_{i,j}} = \underset{\text{empty}}{1} + \underset{y_2}{y_2}$$

這邊跟 all loop chapter 5 的討論相似。

然後 $u_{i,j}$ definition 是 $u_{i,j} = \frac{F^{+-}_{w_{i,j}} \cdot F^{-+}_{w_{i,j}}}{F^{++}_{w_{i,j}} F^{--}_{w_{i,j}}}$.

答 = 乘 重

而我們可以用矩陣方式一步步構造 F^{++}、F^{+-}、F^{-+}、F^{--}

我們 Let $M_w = \begin{pmatrix} F^{++}_w & F^{+-}_w \\ F^{-+}_w & F^{--}_w \end{pmatrix}$

考慮 Word w

Case1 : $w = YRw'$ $\qquad w = Y_{w'}$

$\Rightarrow F^{++}_w = Y \cdot F^{++}_{w'}$

$F^{+-}_w = Y \cdot F^{+-}_{w'}$

$F^{-+}_w = F^{++}_{w'} + F^{-+}_{w'}$

$F^{--}_w = F^{+-}_{w'} + F^{--}_{w'}$

$\Rightarrow \begin{pmatrix} F^{++}_w & F^{+-}_w \\ F^{-+}_w & F^{--}_w \end{pmatrix} = \begin{pmatrix} Y & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} F^{++}_{w'} & F^{+-}_{w'} \\ F^{-+}_{w'} & F^{--}_{w'} \end{pmatrix}$

$= \begin{pmatrix} Y & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} F^{++}_{w'} & F^{+-}_{w'} \\ F^{-+}_{w'} & F^{--}_{w'} \end{pmatrix}$

$M_w = \begin{pmatrix} Y & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} M_{w'}$

case 2. $w = Y\ell w'$ $\qquad w = {}_Y{}^{w'}$

$\Rightarrow F^{++}_w = Y F^{++}_{w'} + Y F^{-+}_{w'}$

$F^{+-}_w = Y F^{+-}_{w'} + Y F^{--}_{w'}$

$F^{-+}_w = F^{-+}_{w'}$

$F^{--}_w = F^{--}_{w'}$

$\begin{pmatrix} F^{++}_w & F^{+-}_w \\ F^{-+}_w & F^{--}_w \end{pmatrix} = \begin{pmatrix} Y & Y \\ 0 & 1 \end{pmatrix} \begin{pmatrix} F^{++}_{w'} & F^{+-}_{w'} \\ F^{-+}_{w'} & F^{--}_{w'} \end{pmatrix}$

$\Rightarrow M_w = \begin{pmatrix} Y & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} M_{w'}$

So given word W we can 用上面的方法一次構造 F_W^{++}、F_W^{+-}、F_W^{-+}、F_W^{--} By M_W 的方式

$$W = i \quad R \quad Y_1 \quad L \quad Y_2 \quad L \quad Y_3 \quad R \quad Y_4 \quad j$$

$$M_W = \begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}Y_1&0\\0&1\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}Y_2&0\\0&1\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}Y_3&0\\0&1\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}Y_4&0\\0&1\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

Summary：可以用數 subset 的方式看 或是用矩陣相乘 是等價的.

$F_{i,j}$ in new representation

1. Consider $W_{i+1,j+1}$ $\quad W_{i+1,j+1} = i{+}1\, R Y_1 L Y_2 R Y_3 L Y_4$
2. Trim $W_{i+1,j+1}$ $\Rightarrow W^{Trim}_{i+1,j+1} = L\underbrace{Y_2 R} = \underline{L}\,\underline{W'}$
3. $F_{i,j} = (1,1)\underline{\begin{pmatrix}Y_2&0\\0&1\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}}\begin{pmatrix}1\\0\end{pmatrix}$ 照定義 New 那篇

$$= (1,1)\, M_{W'}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= (1,0)\begin{pmatrix}1&1\\0&1\end{pmatrix} M_{W'}\begin{pmatrix}1\\0\end{pmatrix}$$

因為 Trim 完後第一個字一定是 L 所以一定可以這樣討論。

$$= (1,0)\, M_{w^{Trim}_{i+1,j+1}} \begin{pmatrix}1\\0\end{pmatrix}$$

$$= (1,0) \begin{pmatrix} F^{++}_{w^{Trim}_{i+1,j+1}} \\ F^{-+}_{w^{Trim}_{i+1,j+1}} \end{pmatrix}$$

$$= F^{++}_{w^{Trim}_{i+1,j+1}}$$

Summary $F_{i,j}$ follow New representation 的定義 可以推得 $F^{++}_{w^{Trim}_{i+1,j+1}}$.

而用圖像式來看那 $F^{++}_{w^{Trim}_{i+1,j+1}}$ 就是把路對應的valid subset 取和，而起終點交換只是把路對應的高低圖對最右邊做鏡像

$$W_{2,7} = 2\, L\, y_{13}\, R\, y_{14}\, R\, y_{16}\, L\, y_{68}\, R\, 7$$

$$= 2^{y_{13}}{}_{y_{14}}{}_{y_{16}}{}^{y_{68}}\, 7$$

$$W_{7,2} = 7\, L\, y_{68}\, R\, y_{16}\, L\, y_{14}\, L\, y_{13}\, R\, 2$$

$$= 7^{y_{68}}{}_{y_{16}}{}^{y_{14}}{}^{y_{13}}\, 2$$

鏡像

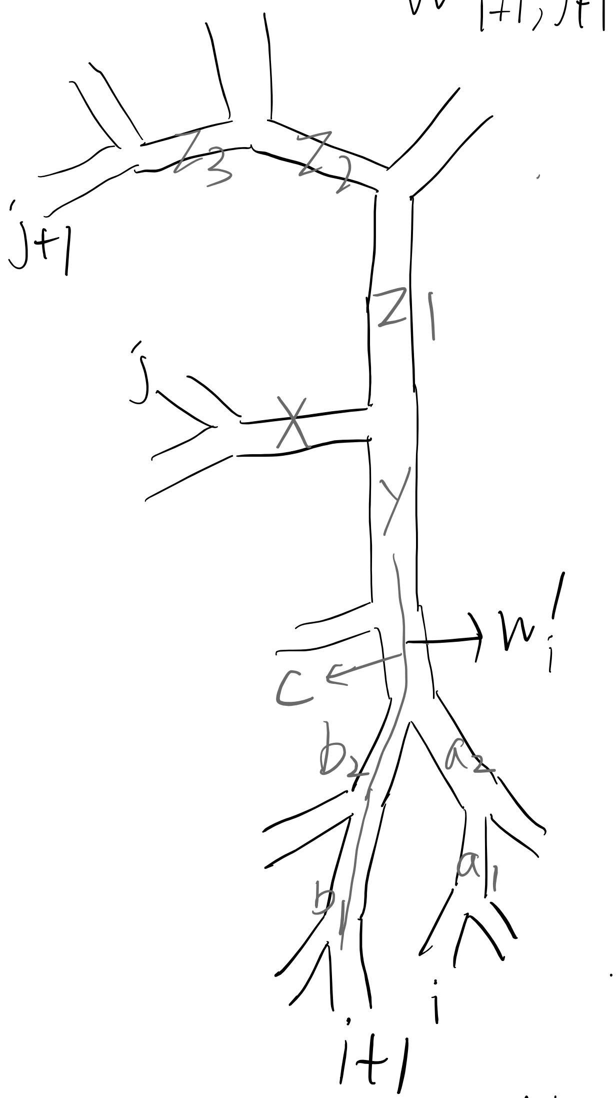
2、回到 $u_{i,j} = \dfrac{F^{+-}_{w_{i,j}} F^{-+}_{w_{i,j}}}{F^{++}_{w_{i,j}} F^{--}_{w_{i,j}}}$

先分析 $w_{i,j}$ 之間的關係

$w_{i,j+1}$

$w_{i+1,j}$

$w_{i+1,j+1}$



$w_{i,j} = w_i'\, YLXRj$

$w_{i,j+1} = w_i'\, YRz_1Lz_2Lz_3L(j+1)$

尾巴是j+1和j之間的關係是

把$w_{i,j}$中從j倒著走直到

遇到第一個L，然後把那個L改

成R後後面一路走L直到j+1

$w_{i,j} = w_i'\, y\, \boxed{L}\, x\, \underline{R}\, j$

$w_{i,j+1} = w_i'\, y\, R\, z_1 \underline{L}\, z_2 \underline{L}\, z_3 \underline{L}\, j+1$

而相同 尾不同 頭之間的關係

$W_{i,j} = i L a_1 L a_2 R C W_j$

$W_{i+1,j} = i{+}1 R b_1 R b_2 L C W_j$

i 到 i+1 的方法

先從 i 出發一直往後直到第一個 R 把他改成 L 以

後一直往前補 R 直到 i+1 R R R ⋯ L C

$W_{i,j} = i L a_1 L a_2 \boxed{R} C W_j$

$W_{i+1,j} = i{+}1 R b_1 R b_2 \boxed{L} C W_j$

分析 $W^{Trim}_{i+1,j}$、$W^{Trim}_{i,j+1}$、$W^{Trim}_{i+1,j+1}$ 跟 $W^{Trim}_{i,j}$ 的關係

1. $W_{i+1,j}$ 是把 $W_{i,j}$ 從左數來第一個 R 改成 L 前面補上一堆 R.

$W^{Trim}_{i+1,j}$ 的左邊會全刪掉直到那個做變更的 L.

$\Rightarrow W^{Trim}_{i+1,j}$ 會是 $W_{i,j}$ 右邊把連續 L 刪掉，而左邊是一路刪掉直到第一個 R 把他變 L.

2. $W^{Trim}_{i,j+1}$ 把 $W_{i,j}$ 左邊連續 R 刪掉，而右邊倒著一路刪掉直到第一個 L 變成 R

3. $W^{Trim}_{i+1,j+1}$ 把左邊一路刪掉到第一個 R 改成 L 再把右邊一路倒著刪 直到第一個 L 然後將其改成 R

Consider $W_{i,j} =$

$$= i\,R\,Y_1\,R \cdots R\,Y_\ell\,\underline{L}\,Z_1 \cdots\cdots Z_m\,\underline{R}\,W_r\,L\,W_{r-1} \cdots L\,W_1\,L\,j$$

在此 case $i+1$ 開頭的就是

$$i+1\,\underline{L}\,Y_1\,R\,Y_2\,R\,Y_3 \cdots\cdots$$

而 $j+1$ 結尾的就是

$$\cdots\cdots\cdots\ W_r\,L\,W_{r-1}\,L \cdots L\,W_1\,\underline{R}\,j+1$$

Denote $W^m = L\,Z_1 \cdots\cdots Z_m\,R$ 把所有連續R開頭去掉 L結尾去掉

$W^\ell = \underline{L}\,Y_1\,R \cdots R\,Y_\ell\,W^m$ 把所有連續L結尾去掉 把第一個遇到的開頭R改成L

$W^r = W^m\,W_r\,L\,W_{r-1} \cdots W_1\,\underline{R}$ 把所有連續R開頭去掉 把倒著數第一個遇到的L改成R.

$W^{\ell,r} = \underline{L}\,Y_1\,R \cdots R\,Y_\ell\,W^m\,W_r\,L\,W_{r-1} \cdots W_1\,\underline{R}$ 把第一個遇到的開頭R改成L 把倒著數第一個遇到的L改成R.

By all loop F^{++} F^{+-} F^{-+} F^{--} definition

$$\Rightarrow F^{++}_{W_{i,j}} = \prod_{\alpha=1}^{\ell} Y_\alpha \prod_{\beta=1}^{r} W_\beta\, F^{++}_{W^m} \qquad F^{+-}_{W_{i,j}} = \prod_{\alpha=1}^{\ell} Y_\alpha\, F^{++}_{W^r}$$

$$F^{-+}_{W_{i,j}} = \prod_{\beta=1}^{r} W_\beta\, F^{++}_{W^\ell} \qquad F^{--}_{W_{i,j}} = F^{++}_{W^{\ell,r}}$$

$$W^m = L z_1 \cdots\cdots z_m R$$

$$W^\ell = \underline{L} y_1 R \cdots\cdots R y_\ell W^m$$

$$W^r = W^m w_r L w_{r-1} \cdots w_1 \underline{R}$$

$$W^{\ell,r} = \underline{L} y_1 R \cdots R y_\ell W^m w_r L w_{r-1} \cdots w_1 \underline{R}$$

但注意

$$W^{Trim}_{i,j} = W^m \qquad F^{++}_{W_{i,j}} = \prod_{\alpha=1}^{\ell} y_\alpha \prod_{\beta=1}^{r} w_\beta F^{++}_{W^{Trim}_{i,j}} = \prod_{\alpha=1}^{\ell} y_\alpha \prod_{\beta=1}^{r} w_\beta F_{i-1,j-1}$$

$$W^{Trim}_{i+1,j} = W^\ell \quad \Rightarrow \quad F^{+-}_{W_{i,j}} = \prod_{\alpha=1}^{\ell} y_\alpha F^{+-}_{W^{Trim}_{i,j+1}} = \prod_{\alpha=1}^{\ell} y_\alpha F_{i-1,j}$$

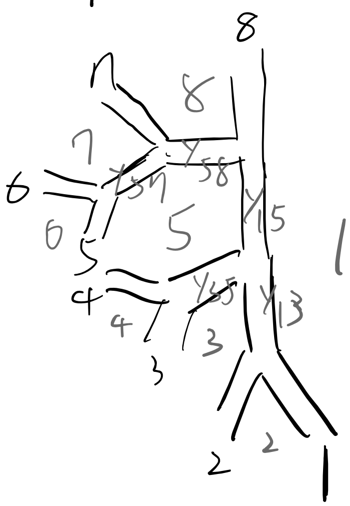
$$W^{Trim}_{i,j+1} = W^r \qquad F^{-+}_{W_{i,j}} = \prod_{\beta=1}^{r} y_\beta F^{++}_{W^{Trim}_{i+1,j}} = \prod_{\beta=1}^{r} w_\beta F_{i,j-1}$$

$$W^{Trim}_{i+1,j+1} = W^{\ell,r} \qquad F^{--}_{W_{i,j}} = F^{++}_{W^{Trim}_{i+1,j+1}} = F_{i,j}$$

$$\Rightarrow u_{i,j} = \frac{F^{+-}_{W_{i,j}} F^{-+}_{u_{i,j}}}{F^{++}_{W_{i,j}} F^{--}_{u_{i,j}}} = \frac{\prod_{\alpha=1}^{\ell} y_\alpha F_{i-1,j} \prod_{\beta=1}^{r} w_\beta F_{i,j-1}}{\prod_{\alpha=1}^{\ell} y_\alpha \prod_{\beta=1}^{r} w_\beta F_{i-1,j-1} \cdot F_{i,j}}$$

$$= \frac{F_{i-1,j} F_{i,j-1}}{F_{i-1,j-1} F_{i,j}}.$$

Special case when $(i,j) \in T$.



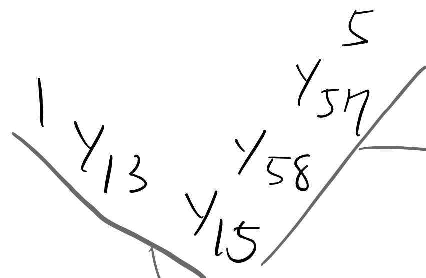
$\Rightarrow (1,3)、(1,5)、(3,5)、(5,8)、(5,7) \in T$

Consider $U_{1,5}$ $\quad (1,5) \in T$.

$\Rightarrow W_{1,5} = 1 R Y_{13} R Y_{15} L Y_{58} L Y_{57} L 5.$

特色是 $W_{1,5}$ 在 $Y_{1,5}$ 前是連續 R

後是連續 L



$$F^{++}_{w_{1,5}} = Y_{13} Y_{15} Y_{58} Y_{57} = \frac{(Y_{13} Y_{15})(Y_{57} Y_{58} Y_{15})}{Y_{15}} \quad F_{8,4} = 1$$

$$F^{+-}_{w_{1,5}} = (Y_{13} Y_{15}) F_{8,5} \quad (\because F_{1-1,5} = F_{8,5})$$

$$F^{-+}_{w_{1,5}} = (Y_{57} Y_{58} Y_{15}) F_{1,4}$$

$$F^{--}_{w_{1,5}} = F_{1,5}$$

$$\Rightarrow U_{1,5} = \frac{(Y_{13} Y_{15}) F_{8,5} \cdot (Y_{57} Y_{58} Y_{15}) F_{14}}{\frac{(Y_{13} Y_{15})(Y_{57} Y_{58} Y_{15})}{Y_{15}} F_{8,4} \cdot F_{1,5}}$$

$$= Y_{1,5} \frac{F_{8,5}\ F_{1,4}}{F\ \ F} = Y_{1,5} \frac{F_{1-1,5}\ F_{1,5-1}}{F\ \ F} \quad \square$$

$$\Rightarrow W^{Trim}_{i,j+1} = W_i^{\prime Trim,l} \, Y R$$

$$W^{Trim}_{i,j} = W_i^{\prime Trim,l} \, Y L X R$$

$$W^{Trim}_{i+1,j+1} = W_{i+1}^{\prime Trim,l} \, Y R$$

$$W^{Trim}_{i+1,j+1} = W_{i+1}^{\prime Trim,l} \, Y L X R$$