The stringy δeformation.

The one-parymeter deformation of Tree level $Tr(\phi^3)$ for even number $2n$ of partisle, which would be basis of unification of $Tr(\phi^3)$ and Pion and gluons.

$$I_{2n}^{\delta} := \int_{\mathbb{R}^{2n-3}_{>0}} \prod_{I=1}^{2n-3} \frac{dy_I}{y_I} \prod_{(a,b)} u_{a,b}^{\alpha' X_{a,b}} \left(\frac{\prod_{(e,e)} u_{e,e}}{\prod_{(o,o)} u_{o,o}} \right)^{\alpha'\delta}$$

$$I_{2n}^{\delta} = I_{2n}^{Tr(\phi^3)} \left[\alpha' X_{e,e} \to \alpha'(X_{e,e}+\delta),\ \alpha' X_{o,o} \to \alpha'(X_{o,o}-\delta) \right].$$

$\alpha' X_{o,e} \to \alpha' X_{o,e}$

Claim.

1. $\alpha'\delta = 0 \Rightarrow$ usual stringy $Tr(\phi^3) \xrightarrow{\text{low energy}}$ field theory $Tr(\phi^3)$.

$$L_{Tr(\phi^3)} = Tr(\partial\phi)^2 + g\, Tr(\phi^3).$$

2. $\alpha'\delta \in (0,1) \Rightarrow$ at low energy $\longrightarrow$ field theory amplitude of $L_{NLSM} = \frac{1}{8\lambda^2}(\partial_\mu U^\dagger \partial^\mu U)$ $\quad U = (I+\lambda\phi)(I-\lambda\phi)^{-1}$

U(N) NLSM theory.

3. $\alpha'\delta = 1 \Rightarrow$ at low energy $\Rightarrow$ the amplitude of YMS.

gluons and adjoint scalars.

$$L_{YMS} = -Tr\left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}D^\mu\phi^I D_\mu\phi^I - \frac{g_{YM}^2}{4}\sum_{I\neq J}[\phi^I,\phi^J]^2\right)$$

We focus on the zero and factorization.

The arguement we can have the similiar factorization.

1, We prove that the δeformation is the unique way to preserve all $c_{i,j}$.

2. The shift only change the exponent of Y_i, but the zero's and factorization is independent of these exponents of Y_i.

Therefore the fact of $c_{i,j}$ is unchange under δeformation imply zero and factorization on I_n^{δ}.

Realization of the shift on momenta

Let x_i^{μ} is the vertices. $p_i^{\mu} = x_{i+1}^{\mu} - x_i^{\mu}$

To realize the shift for $2n$ particle process, we imagine adding $2n$ dimensions of spacetime, orthogonal to the original momentum polygon lives in.

t_a^{μ}, s_a^{μ} for $a = 1 \cdots n$.

$s_a \cdot x_j = t_a \cdot x_j = 0 \ \forall a, j \quad t_a \cdot t_b = t_a \cdot s_b = s_a \cdot s_b = 0$ for $a \neq b$.

And $t_a^2 = \frac{\delta}{2} \quad s_a^2 = -\frac{\delta}{2}$.

$x_{2k}^{\mu} \to x_{2k}^{\mu} + t_k^{\mu} \quad x_{2k+1}^{\mu} \to x_{2k+1}^{\mu} + s_k^{\mu}$.

$$X_{2k,2l} = (x_{2k} - x_{2l})^2 \to (x_{2k} - x_{2l} + t_k - t_l)^2 = X_{2k,2l} + \delta.$$

$$X_{2k+1,2l+1} = (x_{2k+1} - x_{2l+1})^2 \to (x_{2k+1} - x_{2l+1} + s_k - s_l)^2 = X_{2k+1,2l+1} - \delta.$$

$$X_{2k,2l+1} = (x_{2k} - x_{2l+1})^2 \to (x_{2k} - x_{2l+1} + t_k - s_l)^2 = X_{2k,2l+1}.$$

4 - Point Seformation stringy amplitude

$$I_{n=4}^{\delta} = \frac{\Gamma[\alpha'(X_{1,3}-\delta)]\,\Gamma[\alpha'(X_{2,4}+\delta)]}{\Gamma[\alpha'(X_{1,3}+X_{2,4})]}$$

1, $\delta=0$ and $\alpha' \ll 1$ field theory limit.

$$I_4^{\delta=0} = \frac{1}{\alpha' X_{1,3}} + \frac{1}{\alpha' X_{2,4}}$$

2, $\delta \neq 0$ $\alpha' X_{1,3}$, $\alpha' X_{2,4} \ll 1$

$$I_4^{\delta} \to \alpha' \, \Gamma[-\alpha'\delta]\,\Gamma[+\alpha'\delta] \times \underline{(X_{1,3}+X_{2,4})}$$

This is the NLSM amplitude with $\lambda^2 = \alpha' \Gamma[-\alpha'\delta]\,\Gamma(\alpha'\delta)$.

3, We pick the residue where $X_{1,3}=\delta$

We can see the residue $=1$. So this means we exchange a massive spin zero particle.

4, $\alpha'\delta \to 1$ $\alpha'\delta = 1-\varepsilon$

$\Rightarrow$ when $X_{1,3} = -\varepsilon$ the residue of this pole

$$\frac{\Gamma[\alpha' X_{2,4} + 1 - \varepsilon]}{\Gamma[\alpha' X_{2,4} - \varepsilon]} = \frac{\underline{X_{2,4}-\varepsilon}}{}$$

$\hookrightarrow$ we exchange a <u>spin one</u> and a <u>spin 0</u>

So as $\varepsilon \to 0$ massive spin 1 particle $\to$ massless

massive spin 0 particle $\to$ disappear.

The only consistent theory is YM.

$s, \alpha' \delta = 1 \Rightarrow I_4^{\delta=1} = \dfrac{\Gamma[\alpha' X_{1,3} - 1]\, \Gamma(\alpha' X_{2,4} + 1)}{\Gamma(\alpha'(X_{1,3} + X_{2,4}))}$

This shift keep the massless pole of $\alpha' X_{1,3} = 1$, but remove it at $X_{2,4} = 0$.

$\Rightarrow$ We must interpret this amplitude as two different colored Scalars A, B. $1^A 2^A 3^B 4^B$ So $X_{2,4}$ channel is forbidden.

$I_4^{\delta=1} = \dfrac{X_{2,4}}{X_{1,3}} + 1 \Leftrightarrow X_{1,3}$ chamel gluon exchange
+ four vertex contact diagram.

Summary

The shifted $2n$-particle amplitude.

$\delta = 0 \Rightarrow Tr(\phi^3)$

$\delta \neq 0 \Rightarrow$ NLSM But when $\alpha' \delta$ is small NLSM $\to Tr(\phi^3)$ at energy $\sim \delta$.

$\delta = 1 \Rightarrow M_{1,\dots,n}(1^{A_1} 2^{A_1} 3^{A_2} 4^{A_2} \dots (2n-1)^{A_n} (2n)^{A_n})$.
$\hookrightarrow$ n-gluons amplitude

NLSM $\alpha'\delta \in (0,1)$.

Some special properties in NLSM.

1. Odd-point NLSM Amplitude vanish.
2. Adler zero (when one particle's momentum $\to 0$ Amplitude $\to 0$).
3. The Factorization

Remark: In tree level, these three properties $\Rightarrow$ NLSM.

1. $$I_{2n}^{\delta} = \int_{\mathbb{R}^{2n-3}_{>0}} \prod_{I=1}^{2n-3} \frac{dy_I}{y_I} \prod_{(e,e)} u_{e,e}^{\alpha'(X_{e,e}+\delta)} \times \prod_{(o,o)} u_{o,o}^{\alpha'(X_{o,o}-\delta)} \times \prod_{(o,e)} u_{o,e}^{\alpha' X_{o,e}},$$

$\because \delta \neq 0$ we don't have the pole at $\underline{X_{o,o}, X_{e,e} = 0}$.
odd-point interaction propagator,

satisfy 1.

And $X_{o,e} \Leftrightarrow$ even-point propagator.

2. Because we don't have poles for $X_{e,e}, X_{o,o} = 0$, then we can have the following argument

skinny rectangle type 0 $\Rightarrow$ Adler zero.

$\Downarrow$

$C_{1,j} = 0 \quad \forall j = 3 \cdots n-1$

$\Downarrow$

$I_n^{\delta} = I_{up} \times I_{down} \times \underline{I_4(X_{1,3}^{\delta}, \alpha'(C_{km} - X_{1,3}^{\delta}))}$

$= 0$

$\Downarrow$

$P_1 \to 0$

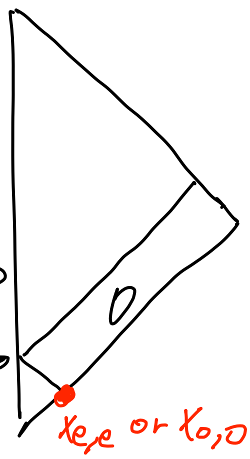
$\downarrow$

$C_{1,j} \to 0 \quad X_{1,3} \to 0$

$\Rightarrow \underline{I_4^{\delta}}$ is finite, as $X_{1,3} = 0$

there is no pole at $X_{1,3} = 0$

$\Rightarrow I_n^{\delta} \to 0$



NLSM field theory amplitude Assume $\alpha' \delta \ll 1$.

$$I^{\delta}_{2n} \to A^{\mathrm{Tr}(\phi^3)}_{2n}(X_{e,e} \to X_{e,e}+\delta,\ X_{o,o} \to X_{o,o}-\delta)$$

To get the real low energy behavior we need to expand

$X \ll \delta$, $\Leftrightarrow$ $\delta \to \infty$

$$A^{NLSM}_{2n} = \lim_{\delta\to\infty} \delta^{2n-2} A^{\mathrm{Tr}(\phi^3)}_{2n}(X_{e,e} \to X_{e,e}+\delta,\ X_{o,o} \to X_{o,o}-\delta).$$

Ex 4-point

$$A^{\mathrm{Tr}(\phi^3)}_{4}(X_{1,3} \to X_{1,3}-\delta,\ X_{2,4} \to X_{2,4}+\delta) = \frac{1}{X_{1,3}-\delta} + \frac{1}{X_{2,4}+\delta}$$

$$\delta \gg 1 \quad A^{\mathrm{Tr}(\phi^3)}_{4} \longrightarrow \frac{1}{\delta}(1-1) - \frac{1}{\delta^2}\underbrace{(X_{1,3}+X_{2,4})}_{A^{NLSM}_4} + O\left(\frac{1}{\delta^3}\right).$$

Factorization near zeros, for NLSM,

Because the $c_{i,j}$ are preserved, the factorization rule is the same.

Consider the $2n$ particle amplitude into (even point $\times$ even point)

We follow the same rule before and with the same shift for the lower point amplitude.

But when amplitude $\to$ odd $\times$ odd

We need some indentification of which particle is ϕ which is π.

We only give the final rule.

$$A^{NLSM}_{2n}(C_{\#} \neq 0) = \left(\frac{1}{X_B} + \frac{1}{X_T}\right) \times A^{down,NLSM} \times A^{up,NLSM}$$

$$A^{NLSM}_{2n}(C_{\#} \neq 0) = \left(X_B + X_T\right) \times A^{down,NLSM+Tr\phi^3} \times A^{up,NLSM+Tr\phi^3}$$

Before we look at the stringy amplitude, we first study how to use colored scalars to describe a gluon (which has polarization).

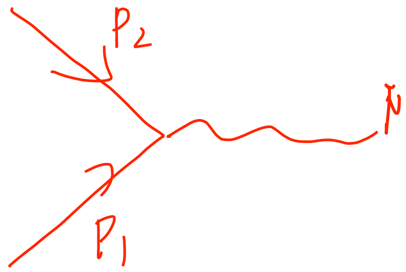
The 2n - scalar to n-gluons

We first check the degree of freedom in each side.

$$2n\text{-scalars}\quad \frac{2n(2n-1)}{2} - 2n - \underbrace{n}_{\text{Residue}} = \frac{2n(2n-4)}{2}$$

$$n\text{-gluons}\quad \underbrace{\frac{n(n-3)}{2}}_{(p_i\cdot p_j)} + \underbrace{\frac{n(n-1)}{2}}_{\varepsilon_i\cdot\varepsilon_j} + \underbrace{n(n-1)}_{\varepsilon_i\cdot p_j} - \underbrace{n}_{\varepsilon_i(p_1+p_2\cdots p_n)=0,\ i=1,2,3\cdots n} = \frac{2n(2n-4)}{2}$$

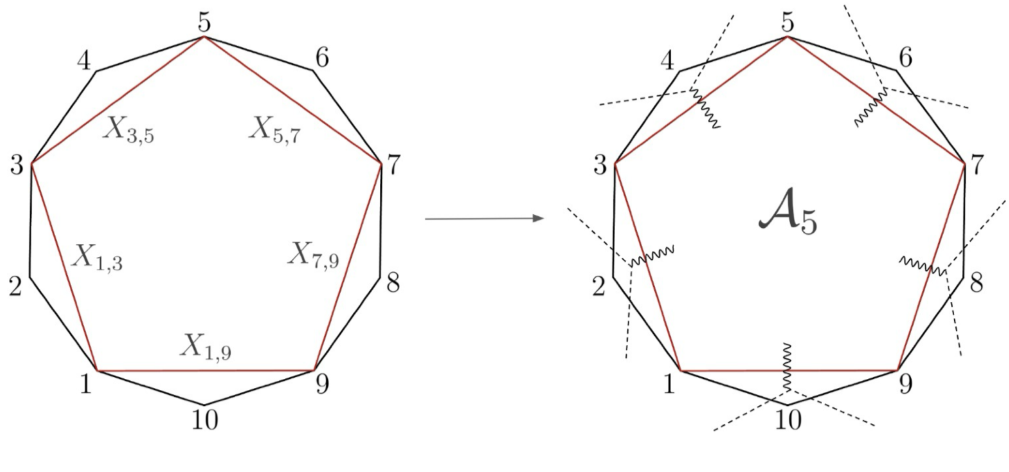
Secondly, we consider that the scalar can interact with gluon.



$\longrightarrow g_{YM}(p_2 - p_1)^\mu$.

So we take $(p_{2i-1} + p_{2i})$ as the ith gluon momentum (q_i) and require $(p_{2i-1}+p_{2i})^2 = X_{2i-1,2i+1} = 0$ on-shell gluon.

So we take n - residue $X_{2i-1,2i+1}=0$ to make n-gluons.



The polarization of each external gluon we can use $\varepsilon_i \propto (P_{2i} - P_{2i-1})$ to descibe them which comes from the feynman rule of two scalars-one gluon.

Remark: The guage invariance of $\varepsilon_i \to \varepsilon_i + \alpha q_i$ and the multilinearity in polarization give me the form of our Amplitude.

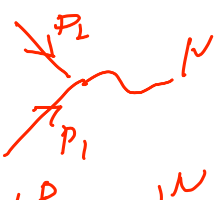
$$
\begin{aligned}
\mathcal{A}_n^{\text{gluon}} &= \sum_{j \neq \{2i-1, 2i, 2i+1\}} (X_{2i,j} - X_{2i-1,j}) \times \mathcal{Q}_{2i,j} \\
&= \sum_{j \neq \{2i-1, 2i, 2i+1\}} (X_{2i,j} - X_{2i+1,j}) \times \mathcal{Q}_{2i,j}.
\end{aligned}
$$

$Q_{2i,j}$ depend on $X_{i,j}$

But X $X_{2i,j}$.

This form combines 2 constraints. (multilinear, guage invariance)

Which is very useful to check the stringy amplitude satisfy the two constraints.

The polarization coming from the 

$\propto (P_2 - P_1)^\nu$

The momentum of gluons $= (P_{2i} + P_{2i-1})^\nu$

$\alpha' \delta = 1$ Gluons YMS

$$I_{2n}^{\alpha'\delta} = \int_{\mathbb{R}^{2n-3}_{>0}} \underbrace{\prod_{I=1}^{2n-3} \frac{dy_I}{y_I}}_{\Omega_{2n}} \prod_{(a,b)} u_{a,b}^{\alpha' X_{a,b}} \left(\frac{\prod_{(e,e)} u_{e,e}}{\prod_{(o,o)} u_{o,o}} \right)$$

We claim we can use this deformation stringy amplitude to describe the n-gluons amplitude in YMS.

First, we claim that the open bosonic string amplitude for 2n guage bosons with polarization ε_i

$$A_{2n}^{\text{tree}} = \int \frac{d^{2n} z_i}{SL(2,\mathbb{R})} \left(\prod_{i<j} z_{i,j}^{2\alpha' p_i \cdot p_j} \right) \exp\left(\sum_{i\neq j} 2 \frac{\varepsilon_i \cdot \varepsilon_j}{z_{i,j}^2} - \frac{\sqrt{\alpha'}\, \varepsilon_i \cdot p_j}{z_{i,j}} \right) \Big|_{\text{multi linear in } \varepsilon_i}$$

Under some special choices of our polarization in higher dim space,

$p_i \cdot \varepsilon_j = 0$ $\quad \varepsilon_i \cdot \varepsilon_j = 1 \quad (i,j) \in \{(1,2), (3,4), \dots (2n-1, 2n)\}$

$\quad = 0$ otherwise

Then $A_{2n}^{\text{tree}} \to I_{2n}^{\alpha'\delta = 1}$

Therefore, we can guess that $I_{2n}^{\alpha'\delta=1}$ is describing the gluons in higher dim.

And in the above choice, we can see that only they only interact with the neighbors ($\varepsilon_{2i} \cdot \varepsilon_{2i-1} \neq 0$), which can be interpreted as there being n different species of such scalars that do not mix.

$\underbrace{1^{A_1} 2^{A_1}}\ \underbrace{3^{A_2} 4^{A_2}} \cdots \underbrace{(2n-1)^{A_n} (2n)^{A_n}}$.

Secondly, the way we really get YMS amplitude is by the following procedure.

We take the residue of $X_{1,3} = X_{3,5} = \cdots = X_{1,2n-1} = 0$

n residue which are put all gluons on-shell.

We think of each gluon is like a pair of scalars,

$$I_n^{\text{gluon}} = \mathrm{Res}_{X_{1,3}=0}\left(\mathrm{Res}_{X_{3,5}=0} \cdots \left(\mathrm{Res}_{X_{1,2n-1}=0}\left(\Omega_{2n}^{\delta}\right)\right)\right)$$

If we want to check whether this is the amplitude of gluons, we need the following checks showing that the I_n^{gluon} satisfies all physical properties expected of gluons.

1. On shell invariance
2. Linearity of polarization
3. Consistent factorization on massless gluon pole.

We now focus on how to pick the residue.

We choose the triangulation T which contain each $X_{2i-1,2i+1} = (p_{2i-1}+p_{2i})^2$

Surprisingly, for such triangulation, we have

$$\frac{\prod_{(e,e)} u_{e,e}}{\prod_{(o,o)} u_{o,o}} \longrightarrow \frac{1}{y_{1,3}y_{3,5}\cdots y_{1,2n-1}} \times \frac{1}{\prod_{(k,m)\in\mathcal{T}'} y_{k,m}},$$

$$\mathcal{I}_{2n}^{\delta} = \int_{\mathbb{R}_{>0}^{2n-3}} \underbrace{\prod_{i=1}^{n} \frac{\mathrm{d}y_{2i-1,2i+1}}{y_{2i-1,2i+1}^2} \prod_{I\in\mathcal{T}'} \frac{\mathrm{d}y_I}{y_I^2} \prod_{(a,b)} u_{a,b}^{\alpha' X_{a,b}}}_{\Omega_{2n}},$$

In order to see the residue of $X_{1,3} = X_{3,5} = X_{5,7} \cdots = 0$.

We consider we take residue of $X_{1,3} = 0$ as example.

$$I_{2n}^{\delta} = \int_0^{\infty} \left(\frac{dY_{1,3}}{Y_{1,3}^2} Y_{1,3}^{\alpha' X_{1,3}} \right) \times \underbrace{F(Y_{I\in T}, X_{i,j})}$$

This part is regular as Y_s and $X_s \to 0$.

We can see that as $X_{1,3} \to 0$ the integral over $Y_{1,3}$ diverge near 0.

So as $X_{1,3} \to 0$ the integral is dominated by the region $Y_{1,3}$ near 0.

$$\Rightarrow I^{\delta}_{2n} = \int_0^\infty \prod_{i=2}^{n} \frac{dY_{2i-1,2i+1}}{Y^2_{2i-1,2i+1}} Y_{2i-1,2i+1}^{\alpha' X_{2i-1,2i+1}} \times \int_0^{\varepsilon} \frac{dY_{1,3}}{Y^2_{1,3}} Y_{1,3}^{\alpha' X_{1,3}}$$

$$\times \left(F(Y_{1,3}=0, Y_I, X_{i,j}) + Y_{1,3} \frac{dF}{dY_{1,3}}\Big|_{Y_{1,3}=0} \cdots \right)$$

$\left(\int_0^{\varepsilon} \frac{dy}{y} y^A = \frac{\varepsilon^A}{A} \right.$ using analytic continuation. $\Big)$

$$\Rightarrow I^{\delta}_{2n} = \int_0^\infty \prod_{i=2}^{n} \frac{dY_{2i-1,2i+1}}{Y^2_{2i-1,2i+1}} Y_{2i-1,2i+1}^{\alpha' X_{2i-1,2i+1}} \times \left(\frac{\varepsilon^{X_{1,3}-1}}{X_{1,3}-1} F(Y_{1,3}=0) + \frac{\varepsilon^{X_{1,3}}}{X_{1,3}} \frac{dF}{dY_{1,3}}\Big|_{Y_{1,3}=0} \right)$$

$$\Rightarrow \mathrm{Res}_{X_{1,3}=0} I^{\delta}_{2n} = \int_0^\infty \prod_{i=2}^{n} \frac{dY_{2i-1,2i+1}}{Y^2_{2i-1,2i+1}} Y_{2i-1,2i+1}^{\alpha' X_{2i-1,2i+1}} \int_0^\infty \prod_{\substack{I \in T \\ I \neq Y_{2i-1,2i+1}}} \frac{dY_I}{Y_I^2} \times \left(\frac{dF}{dY_{1,3}}\Big|_{Y_{1,3}=0} \right)$$

$$= \int_0^\infty \mathrm{Res}_{Y_{1,3}=0} \left(\Omega_{2n} \right)$$

$$\Rightarrow I^{\text{gluon}}_n = \mathrm{Res}_{X_{1,3}=0} \left(\mathrm{Res}_{X_{3,5}=0} \left(\cdots \cdots \left(\mathrm{Res}_{X_{1,2n-1}=0} \left(I^{\delta}_{2n} \right) \right) \right) \right)$$

$$= \int_{\mathbb{R}^{n-3}_{>0}} \mathrm{Res}_{Y_{1,3}=0} \left(\mathrm{Res}_{Y_{3,5}=0} \left(\cdots \left(\mathrm{Res}_{Y_{1,2n-1}=0} \Omega_{2n} \right) \right) \right) \quad \square$$

We focus on the zero and factorization.

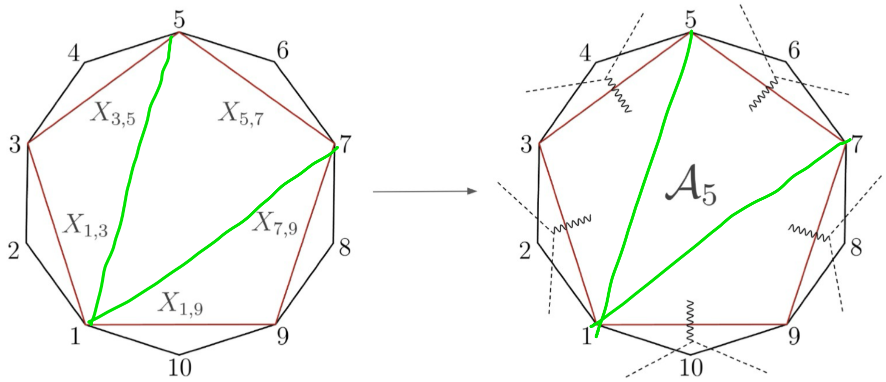
The deformation I_{2n}^{δ} has the same zero and factorization as $I_{2n}^{\mathrm{Tr}\,\phi^3}$.

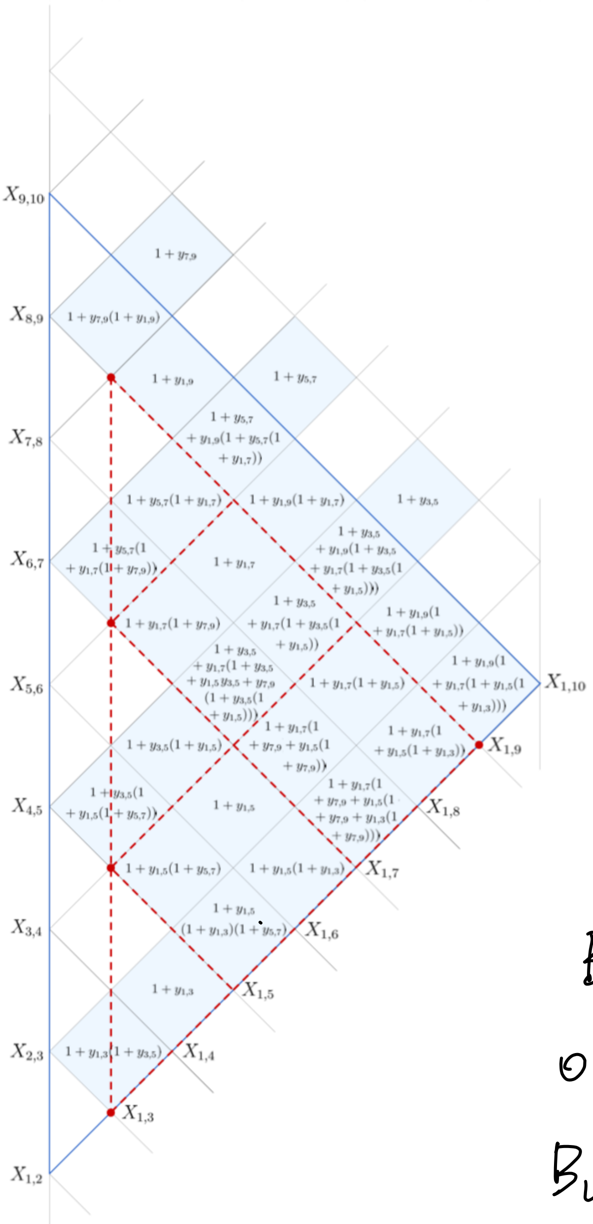
But whether these zero and factorization would survive after we take the nth residue.

Another difference is that in I_n^{gluon} case, we have different triangulization, so our previous trick of F might need some adjustment.

For example, we use $2n=10$ as example

$T=\{X_{1,3}, X_{3,5}, X_{5,7}, X_{7,9}, X_{1,9}, \underline{X_{1,5}}, \underline{X_{1,7}}\}$





If we turn off $c_{i,7} = c_{i,8} = 0$

$i = 1, 2, 3, 4$

We expect that we would get the zero.

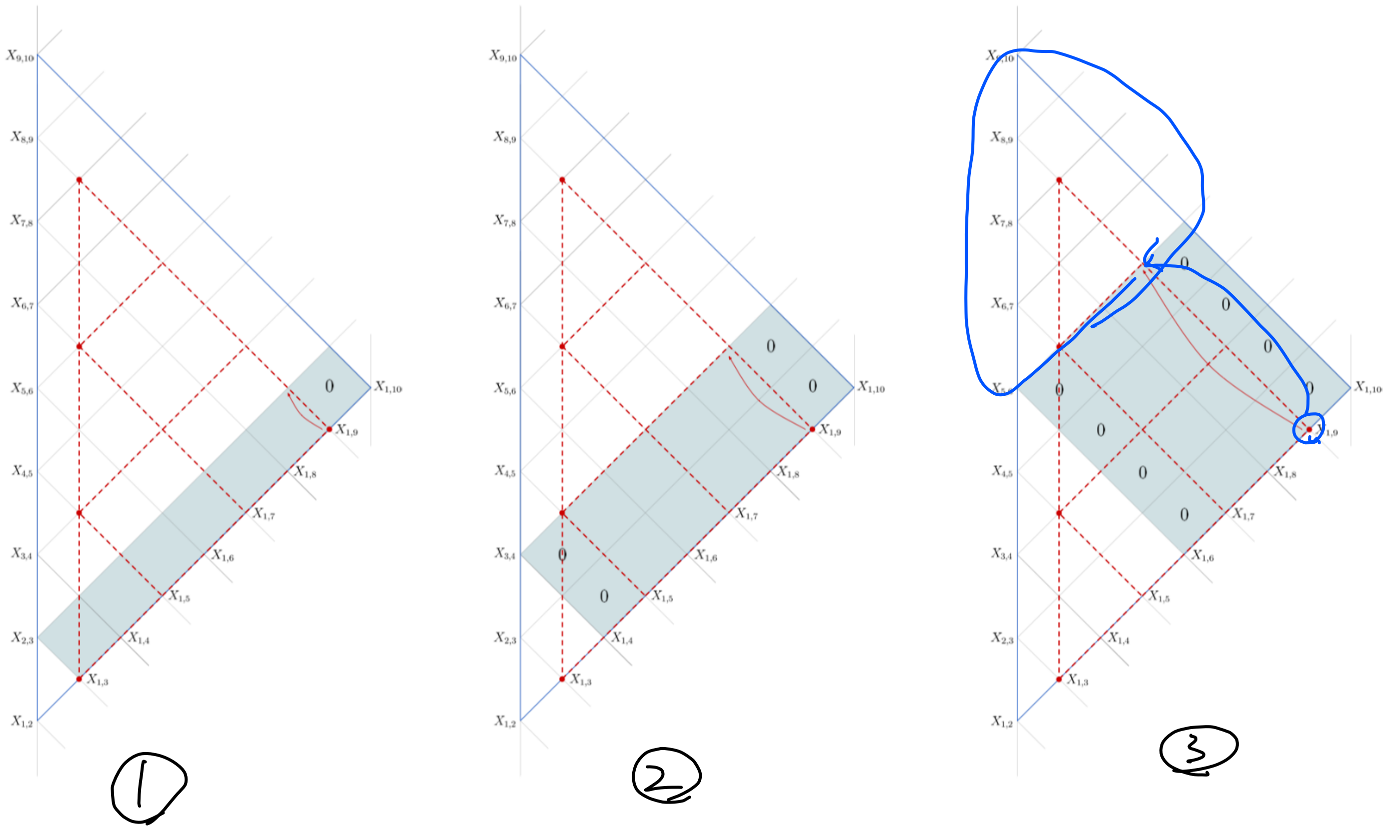
$$\Rightarrow \int_0^\infty \frac{dY_{1,7}}{Y_{1,7}} Y_{1,7}^{X_{1,7}} \times (\text{remaning integration indep of } Y_{1,7})$$

$$= 0$$

Because the F depend on $Y_{1,3}, Y_{1,5}, Y_{1,7}$ only in the casual diamond.

But those term outside the red triangle these dependence would disappear after we take the residue of each gluon.

Factorization.



① ② ③

③ $A_{10}(c_* \neq 0) \rightarrow \dfrac{\Gamma(\alpha' X_{1,6})\,\Gamma(\alpha' X_{5,10})}{\Gamma(\alpha'(X_{1,6}+X_{5,10}))} \times A_6^{\text{down}} \times A_6^{\text{up}}$

o,e unshifted.

From previous $Tr(\phi^3)$ factorization.

For A_6^{up} it depend on the varibles in upper triangle

but we need to replace the lower boundary with

$X_{1,i}$ (depend on the position of c_*).

But at least $X_{5,9}$ is replaced by $X_{1,9}$.

$\Rightarrow A_6^{\text{up}}$ after taking residue is a $A_3^{\text{gluon, up}}$

But in A_6^{down} there are two gluons because we

done take residue of $X_{1,5}=0$.

$\Rightarrow A_6^{down} \rightarrow A_4^{gluons+\phi}$ (2ϕ & 2 gluon).

Because we set $C_{i,6}=C_{i,9}=0$ $i\in 1\sim 2\sim 3\sim 4$, so after taking residue

$\Rightarrow X_{1,6}=X_{1,7}$ $X_{5,10}=X_{5,9}$

$$\Rightarrow A_{10}(C_*\neq 0) \xrightarrow{\text{low energy}} \left(\frac{1}{X_{1,6}}+\frac{1}{X_{5,10}}\right)\times A_6^{down}\times A_6^{up}$$

take 5 residue.

$$A_5^{gluons}(C_*\neq 0) \longrightarrow \left(\frac{1}{X_{1,7}}+\frac{1}{X_{5,9}}\right)\times A_3^{up,\,gluons}\times A_4^{down,\,gluon+\phi}.$$

Outlook

1. In $Tr(\phi^3)$ and NLSM theory, they find we can guess amplitude in the following rule.

1. $\dfrac{\text{Polynomial}(X_{i,j})}{\prod\limits_{1\le i<j-1\le n-1} X_{i,j}}$ → 1. polynomial with correct degree
2. At most linear in any $X_{i,j}$
3. Put the all skinny rectangle zero condition.

Then we can get the amplitude.

Lack the proof.

2. Consider the collapsing zero in $N=4$ SYM amplituhedron.

3. $A = \dfrac{N(X)}{D(X)}$ The locus of zeros of amplitude
$\Rightarrow N(X) = 0$.

But the hidden zero, tell us the $N(X)=0$ variety contains a large number of linear subspaces of different dimensions.

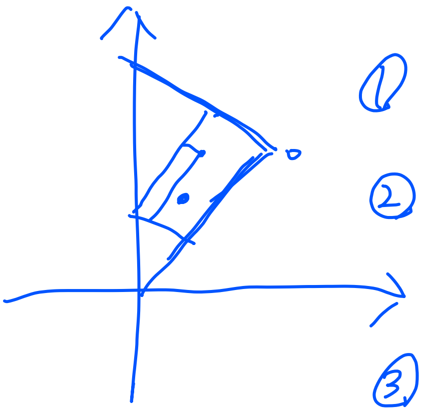
Guess $N(X)$ might be determinant of predictable matrix.

But if want to really check whether the scaffold amplitude is the YM amplitude

1. On-shell guage invariance.

2. linearity in polarizations

3. Consistent factorization on massless gluon poles.



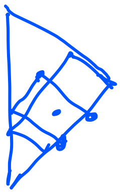
① $x_B = C^*$

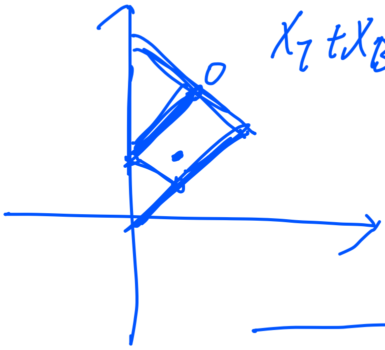
$x_B = 0$

② X in 

$X=0$ is redudant

③ The condition not redudant 

④ The constraint in $X_B = C^*$ is 



$X_T + X_B = C^*$

$X_B = C^*$

$X_B = 0$

The condition in A^{up} only relate to the later basis

The condition in $(A^{down}, X=0)$ relate to the former basis.

The vertex of polytope can be thought as any constraint in A^{up} and any constant in A^{down} could happen. So vertex

$$A^{up}\begin{pmatrix} 1,2,3=0 \\ 2,4,6=0 \\ 1,3,6=0 \\ \vdots \end{pmatrix} \times A^{down}\begin{pmatrix} 8,9=0 \\ 11,9=0 \\ 9,10=0 \\ \vdots \end{pmatrix}$$

$$\times A\begin{pmatrix} X_B=0 \\ X_T=0 \end{pmatrix}$$

$\alpha'\delta = 1$ and scaffold gluons.

$$I^{\delta}_{2n} = \int_{\mathbb{R}^{2n-3}_{\geq 0}} \prod_{I=1}^{2n-3} \frac{dy_I}{y_I} \prod_{(a,b)} u_{a,b}^{\alpha' X_{a,b}} \frac{\prod_{(e,e)} u_{e,e}}{\prod_{(o,o)} u_{o,o}}$$

1, We can check the open bosonic string amplitude for 2n guage boson (in the high dimension space) can be identified as $I_{2n}^{\delta=1, \mathrm{Tr}\phi^3}$.

$\Rightarrow$ The scalar scattering in $I_{2n}^{\alpha'\delta=1\ \mathrm{Tr}\phi^3}$ are gluons in higher dimensions.

2, But when we fix our polarization, only $\varepsilon_{2i} \cdot \varepsilon_{2i-1}$ are not zero, So they can be interpreted as there being n different species of scalars that do not mix.

3, To access the n-point gluons, we need to take n residues, to put each gluons on-shell. ($X_{1,3} = 0 = X_{3,5} \cdots = X_{1,2n-1}$)

Then we get the YMS amplitude in field-theory.

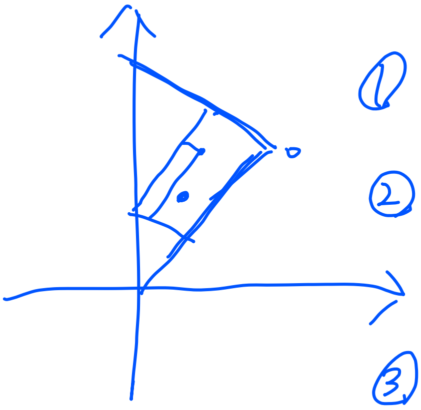
We can also check the DOF

$$2n \quad \mathrm{Tr}(\phi^3) \quad \frac{2n(2n-1)}{2} - 2n - \underset{\text{Residue}}{\underline{n}} = \frac{2n(2n-4)}{2}$$

$$n \text{ gluon} \quad \underset{(p_i \cdot p_j)}{\frac{n(n-3)}{2}} + \underset{\varepsilon_i \cdot \varepsilon_j}{\frac{n(n-1)}{2}} + \underset{\varepsilon_i \cdot p_j}{n(n-1)} - n = \frac{2n(2n-4)}{2}$$

$\varepsilon_i (p_1 + p_2 \cdots p_n) = 0$

$i = 1 \cdot 2 \cdot 3 \cdots n$



① ———— $x_B = c^*$

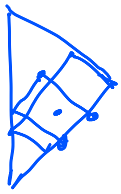
———— $x_B = 0$

② X in

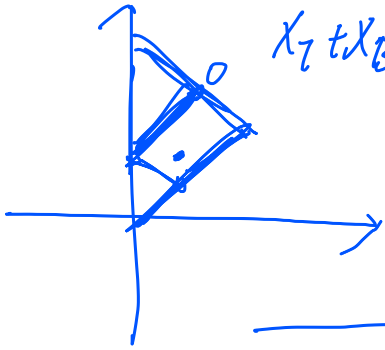
$X=0$ is redudant

③ The condition not redudant

④ The constraint in $X_B = C^*$ is



$X_T + X_B = C^*$



———— $X_B = C^*$

———— $X_B = 0$

The condition in A^{up} only relate to the later basis

The condition in $(A^{down}, X=0)$ relate to the former basis.

The vertex of polytope can be thought as any constraint in A^{up} and any constant in A^{down} could happen. So vertex

$$A^{up}\begin{pmatrix} 1,2,3=0 \\ 2,4,6=0 \\ 1,3,6=0 \\ \vdots \end{pmatrix} \times A^{down}\begin{pmatrix} 8,9=0 \\ 7,9=0 \\ 9,10=0 \\ \vdots \end{pmatrix}$$

$$\times A \begin{pmatrix} X_B=0 \\ X_T=0 \end{pmatrix}$$