In six point gluino amplitude, we consider our massless fermions are left-handed for simplicy.

$$I^6 = \int \frac{d^6 z}{SL(2,\mathbb{R})} \prod_{i<j} z_{ij}^{\alpha' s_{ij}} \, G(z_i, \dot\alpha_i)\, u^1_{\dot\alpha_1} u^2_{\dot\alpha_2} u^3_{\dot\alpha_3} u^4_{\dot\alpha_4} u^5_{\dot\alpha_5} u^6_{\dot\alpha_6}$$

$$= \int \frac{d^6 z}{SL(2,\mathbb{R}) \prod_{i=1}^{6} z_{i,i+1}} \prod_{i<j} z_{ij}^{\alpha' s_{ij}} \left(\prod_{i=1}^{6} z_{i,i+1}\right) G(z_i, \dot\alpha_i)\, u^1_{\dot\alpha_1} u^2_{\dot\alpha_2} u^3_{\dot\alpha_3} u^4_{\dot\alpha_4} u^5_{\dot\alpha_5} u^6_{\dot\alpha_6}$$

$$G(z_i, \dot\alpha_i) = \sum_{i=1}^{5} \frac{1}{z_{i6}} \left[k^{i\nu} - \frac{1}{8} k_\mu^6 M^{\mu\nu}_{(i)}\right] \left\{ \sum_{\pi \in S_5} \mathrm{sign}(\pi) \left[\frac{-1}{80} A^\pi_\nu(\dot\alpha_i) \frac{1}{z_{\pi(4),\pi(5)} z_{\pi(3)\pi(4)} z_{\pi(2)\pi(3)} z_{\pi(1)\pi(2)} z_{\pi(5)\pi(1)}} + \frac{1}{24} B^\pi_\nu(\dot\alpha_i) \frac{z_{\pi(4),6}}{z_{\pi(3),6} z_{\pi(4)\pi(5)} z_{\pi(3)\pi(4)} z_{\pi(2)\pi(3)} z_{\pi(1)\pi(2)} z_{\pi(5)\pi(1)}} \right] \right\}$$

where $A^\pi_\nu(\dot\alpha_i) := (\gamma_\nu \gamma^{\rho\lambda})^{\dot\alpha_6 \dot\alpha_{\pi(5)}} \gamma_\rho^{\dot\alpha_{\pi(4)} \dot\alpha_{\pi(3)}} \gamma_\lambda^{\dot\alpha_{\pi(2)} \dot\alpha_{\pi(1)}}$

$B^\pi_\nu(\dot\alpha_i) := (\gamma_\nu)^{\dot\alpha_6 \dot\alpha_{\pi(5)}} (\gamma^\rho)^{\dot\alpha_{\pi(4)} \dot\alpha_{\pi(3)}} (\gamma_\rho)^{\dot\alpha_{\pi(2)} \dot\alpha_{\pi(1)}}$.

$M^{\mu\nu}_{(i)}$ is a operator which acts on spinor index

$$M^{\mu\nu}_{(i)} f^{\dot\alpha_6 \dot\alpha_5 \cdots \dot\alpha_1} = (\gamma^{\mu\nu})^{\dot\alpha_i}{}_{\dot\beta} \, f^{\dot\alpha_6 \cdots \dot\alpha_{i+1} \dot\beta \dot\alpha_{i-1} \cdots \dot\alpha_1}.$$

But in order to use transform the Integrand into the product of u-variables, we recombine the summation into the following form.

$$G(z_i, \dot\alpha_i) = \sum_{i=1}^{5} \frac{1}{z_{i6}} \left[k^{i\nu} - \frac{1}{8} k_\mu^6 M^{\mu\nu}_{(i)}\right] \left\{ \sum_{\pi \in S_5} \mathrm{sign}(\pi) \left[\frac{-1}{80} A^\pi_\nu \frac{1}{z_{\pi(4),\pi(5)} z_{\pi(3)\pi(4)} z_{\pi(2)\pi(3)} z_{\pi(1)\pi(2)} z_{\pi(5)\pi(1)}} + \frac{1}{24} B^\pi_\nu \frac{z_{\pi(4),6}}{z_{\pi(3),6} z_{\pi(4)\pi(5)} z_{\pi(3)\pi(4)} z_{\pi(2)\pi(3)} z_{\pi(1)\pi(2)} z_{\pi(5)\pi(1)}} \right] \right\}$$

$$= \sum_{\pi \in S_5} \mathrm{sign}(\pi) \Bigg\{ \sum_{i=1}^{5} \frac{1}{z_{i6}} \Bigg[-\frac{1}{80}\left(k^{i\nu} - \frac{1}{8} k^6_\mu M^{\mu\nu}_{(i)}\right) A^\pi_\nu \frac{1}{z_{\pi(4),\pi(5)} z_{\pi(3)\pi(4)} z_{\pi(2)\pi(3)} z_{\pi(1)\pi(2)} z_{\pi(5)\pi(1)}}$$

$$+ \frac{1}{24}\left(k^{i\nu} - \frac{1}{8} k^6_\mu M^{\mu\nu}_{(i)}\right) B^\pi_\nu \frac{z_{\pi(4),6}}{z_{\pi(3),6}\, z_{\pi(4)\pi(5)} z_{\pi(3)\pi(4)} z_{\pi(2)\pi(3)} z_{\pi(1)\pi(2)} z_{\pi(5)\pi(1)}} \Bigg] \Bigg\}$$

First we denote $\left(k^{i\nu} - \frac{1}{8} k^6_\mu M^{\mu\nu}_{(i)}\right) A^\pi_\nu u^6_{\dot\alpha_6} = C^{i,\pi}$

$\left(k^{i\nu} - \frac{1}{8} k^6_\mu M^{\mu\nu}_{(i)}\right) B^\pi_\nu u^6_{\dot\alpha_6} = D^{i,\pi}$

consider the following two sum.

$$\sum_{i=1}^{5} \left(k^{i\nu} - \frac{1}{8} k^6_\mu M^{\mu\nu}_{(i)}\right) A^\pi_\nu = -k^{6\nu} A^\pi_\nu - \frac{1}{8} \sum_{i=1}^{5} k^6_\mu M^{\mu\nu}_{(i)} A^\pi_\nu := C^\pi(\dot\alpha_i)$$

$$\sum_{i=1}^{5} \left(k^{i\nu} - \frac{1}{8} k^6_\mu M^{\mu\nu}_{(i)}\right) B^\pi_\nu = -k^{6\nu} B^\pi_\nu - \frac{1}{8} \sum_{i=1}^{5} k^6_\mu M^{\mu\nu}_{(i)} B^\pi_\nu := D^\pi(\dot\alpha_i)$$

6 4 3 2 5 1   6 5 4 3 2 1   6 5 4 2 2 1 $6\, u^4_{\pi^{-1}(4)}, u^3$

$\pi\uparrow$ 6 5 4 3 2 1   6 5 4 3 2 1

Keep track on the spinor index, we can find that the permutation act on C is equivalent to act on the u's ordering.

$$\sum_{k=1}^{5} C^{k,\pi}(\dot\alpha_i)\, u^5_{\dot\alpha_5} u^4_{\dot\alpha_4} u^3_{\dot\alpha_3} u^2_{\dot\alpha_2} u^1_{\dot\alpha_1} = \sum_{k=1}^{5} C^{k,\pi=1}(\dot\alpha_i)\, u^5_{\dot\alpha_{\pi^{-1}(5)}} u^4_{\dot\alpha_{\pi^{-1}(4)}} u^3_{\dot\alpha_{\pi^{-1}(3)}} u^2_{\dot\alpha_{\pi^{-1}(2)}} u^1_{\dot\alpha_{\pi^{-1}(1)}}$$

$$\sum_{k=1}^{5} D^{k,\pi}(\dot\alpha_i)\, u^5_{\dot\alpha_5} u^4_{\dot\alpha_4} u^3_{\dot\alpha_3} u^2_{\dot\alpha_2} u^1_{\dot\alpha_1} = \sum_{k=1}^{5} D^{k,\pi=1}(\alpha_i)\, u^5_{\dot\alpha_{\pi^{-1}(5)}} u^4_{\dot\alpha_{\pi^{-1}(4)}} u^3_{\dot\alpha_{\pi^{-1}(3)}} u^2_{\dot\alpha_{\pi^{-1}(2)}} u^1_{\dot\alpha_{\pi^{-1}(1)}}$$

And after the careful calculation, we can find the following properties.

$$\sum_{k=1}^{5} C^{k,\pi=1}(\dot\alpha_i) = 0 \quad \Rightarrow \quad C^{5,\pi=1} = -C^{1,\pi=1} - C^{2,\pi=1} - C^{3,\pi=1} - C^{4,\pi=1}$$

$$\sum_{k=1}^{5} D^{k,\pi=1}(\dot\alpha_i) = 0 \quad \Rightarrow \quad D^{5,\pi=1} = -C^{1,\pi=1} - C^{2,\pi=1} - C^{3,\pi=1} - C^{4,\pi=1}$$

Then return to the integrand, we focus on the same permutation π.

$$\sum_{k=1}^{5} \frac{C^{k,\pi}}{z_{k6}} u^5_{\dot\alpha_5} u^4_{\dot\alpha_4} u^3_{\dot\alpha_3} u^2_{\dot\alpha_2} u^1_{\dot\alpha_1} = \sum_{k=1}^{5} \frac{C^{k,\pi=1}}{z_{k6}} u^5_{\dot\alpha_{\pi^{-1}(5)}} u^4_{\dot\alpha_{\pi^{-1}(4)}} u^3_{\dot\alpha_{\pi^{-1}(3)}} u^2_{\dot\alpha_{\pi^{-1}(2)}} u^1_{\dot\alpha_{\pi^{-1}(1)}}$$

For simplicy $C^{k,\pi=1} u^5_{\dot\alpha_{\pi^{-1}(5)}} u^4_{\dot\alpha_{\pi^{-1}(4)}} \cdots u^1_{\dot\alpha_{\pi^{-1}(1)}} := E^k$

$$\Rightarrow \sum_{k=1}^{5} \frac{E_k}{z_{k6}} = \sum_{k=1}^{4} \frac{E_k}{z_{k6}} + \left(-\frac{E_1}{z_{56}} - \frac{E_2}{z_{56}} - \frac{E_3}{z_{56}} - \frac{E_4}{z_{56}}\right)$$
$$= E_1\left(\frac{1}{z_{16}} - \frac{1}{z_{56}}\right) + E_2\left(\frac{1}{z_{26}} - \frac{1}{z_{56}}\right) + E_3\left(\frac{1}{z_{36}} - \frac{1}{z_{56}}\right) + E_4\left(\frac{1}{z_{46}} - \frac{1}{z_{56}}\right)$$
$$= E_1 \frac{z_{15}}{z_{16} z_{56}} + E_2 \frac{z_{25}}{z_{26} z_{56}} + E_3 \frac{z_{35}}{z_{36} z_{56}} + E_4 \frac{z_{45}}{z_{46} z_{56}}$$

Now we can see that the weight of each z_i in each term is zero.

Take $E_1 \frac{z_{15}}{z_{16} z_{56}}$ as example, the full intergrand of this term is

$E_1 \frac{z_{15}}{z_{16} z_{56}} \cdot \prod_{i=1}^{6} z_{i,i+1} \cdot \frac{1}{z_{\pi(4),\pi(6)} z_{\pi(3)\pi(4)} z_{\pi(2)\pi(3)} z_{\pi(1)\pi(2)} z_{\pi(5)\pi(1)}}$. the weight of each z_i is zero.

So we can transform them into the product of u-variable

Then, under this recombination we can analyze the zero of this amplitude. We change the variable to y and F by equation (3.7).

Although it is really complicated to change each term into u and then into $y \cdot F$, we can see the following pattern.

$$\prod_{i<j} z_{i,j}^{n_{i,j}} = \prod_{i<j} u_{i,j}^{\alpha_{i,j}} = \prod_{t\in T} y_t^{\alpha_t} \prod_{i<j} F_{i,j}^{n_{i,j}}$$

The only thing we care is the exponent of each $F_{i,j}$ when analyze the zero, so we can find that $n_{i,j} \in \{\pm 1, 0\}$ in each term.

Then under the ray-like triangulation, when we set,

$C_{1,3}, C_{1,4}, C_{1,5} \in -\mathbb{N} \Rightarrow y_{1,3}$ is scaleless

$C_{1,4}, C_{2,4}, C_{1,5}, C_{2,5} \in -\mathbb{N} \Rightarrow y_{1,4}$ is scaleless

$C_{1,5}, C_{2,5}, C_{3,5} \in -\mathbb{N} \Rightarrow y_{1,5}$ is scaleless