$$I_{2n}=\int\frac{d^{2n}z}{SL(2,\mathbb{R})}\prod_{i<j}z_{ij}^{\alpha' s_{ij}}\frac{Pf' A_{2n\times 2n}}{z_{12}z_{34}\cdots z_{2n-1,2n}}\qquad A_{2n}=\frac{\alpha' s_{i,j}-\delta_{ij}}{z_{ij}}\quad\begin{cases}\delta_{12},\cdots,\delta_{2n-1,2n}=1\\ \text{else}\quad =0\end{cases}$$

For $n=3$

$$Pf' A_6^{56}=\begin{pmatrix}0&\frac{\alpha' s_{12}-\delta_{12}}{z_{12}}&\frac{\alpha' s_{13}}{z_{13}}&\frac{\alpha' s_{14}}{z_{14}}\\ &0&\frac{\alpha' s_{23}}{z_{23}}&\frac{\alpha' s_{24}}{z_{24}}\\ &&0&\frac{\alpha' s_{34}-\delta_{34}}{z_{34}}\\ &&&0\end{pmatrix}$$

$$=\frac{(-1)^{5+6}}{z_{56}}\left(\frac{\alpha' s_{12}-\delta_{12}}{z_{12}}\frac{\alpha' s_{34}-\delta_{34}}{z_{34}}-\frac{\alpha' s_{13}\alpha' s_{24}}{z_{13}z_{24}}+\frac{\alpha' s_{14}\alpha' s_{23}}{z_{14}z_{23}}\right)$$

$$=\frac{-1}{z_{12}z_{34}z_{56}}+\frac{-s_{14}s_{23}\alpha'^2}{z_{14}z_{23}z_{56}}+\frac{s_{13}s_{24}\alpha'^2}{z_{13}z_{24}z_{56}}$$

YM+F³

$$\Rightarrow I_{2n}=\int\frac{d^{2n}z}{SL}\prod_{i<j}z_{ij}^{\alpha' s_{ij}}\left(\underbrace{\frac{-1}{z_{12}^2z_{34}^2z_{56}^2}}_{1}+\underbrace{\frac{-s_{14}s_{23}\alpha'^2}{z_{12}z_{34}z_{56}^2\,z_{14}z_{23}}}_{2}+\underbrace{\frac{s_{13}s_{24}\alpha'^2}{z_{12}z_{34}z_{56}^2z_{13}z_{24}}}_{3}\right)$$

When pick Res on X_{13},X_{35},X_{51}

Res $X_{13}=0$ $\quad z_{12}^{\alpha' s_{12}}z_{13}^{\alpha' s_{13}}z_{14}^{\alpha' s_{14}}z_{15}^{\alpha' s_{15}}z_{16}^{\alpha' s_{16}}z_{23}^{\alpha' s_{23}}z_{24}^{\alpha' s_{24}}z_{25}^{\alpha' s_{25}}z_{26}^{\alpha' s_{26}}\prod_{2\le i<j}z_{i,j}^{\alpha' s_{ij}}=\text{koba.}$

Res $s_{12}=0$ $\quad$ Consider $\begin{matrix}u=z_2-z_1\\ v=z_2\end{matrix}\Rightarrow\begin{matrix}z_1=v-u\\ z_2=v\end{matrix}\qquad z_{1i}=z_i-z_1=u+\bigcirc$

$$\frac{\partial z_{1i}}{\partial u}=1$$

$$\Rightarrow\underset{\alpha' s_{12}=1}{\text{Res}}\int du\frac{u^{\alpha' s_{12}}}{u^2}\left(F(u)\right)=\frac{\partial F(u)}{\partial u}\Big|_{u=0}$$

$$\underset{\alpha' s_{12}}{\text{Res}}\int du\frac{u^{\alpha' s_{12}}}{u}\left(G(u)\right)=G(u)\Big|_{u=0}$$

$$-1\Rightarrow\left(\frac{\alpha' s_{13}}{z_{13}}+\frac{\alpha' s_{14}}{z_{14}}+\frac{\alpha' s_{15}}{z_{15}}+\frac{\alpha' s_{16}}{z_{16}}\right)\frac{1}{z_{34}^2z_{56}^2}\,z_{13}^{\alpha' s_{13}}z_{14}^{\alpha' s_{14}}\cdots z_{16}^{\alpha' s_{16}}\prod_{2<i<j}z_{ij}^{\alpha' s_{ij}}\Big|_{u=0\Leftrightarrow z_1=z_2}$$

$$z_{23}^{\alpha' s_{23}} z_{24}^{\alpha' s_{24}} \cdots z_{26}^{\alpha' s_{26}}$$

$$= \left(\frac{\alpha' s_{13}}{z_{23}} + \frac{\alpha' s_{14}}{z_{24}} + \frac{\alpha' s_{15}}{z_{25}} + \frac{\alpha' s_{16}}{z_{26}}\right) \frac{1}{z_{34}^2 z_{56}^2} \underline{z_{23}^{\alpha' s_{12,3}} z_{24}^{\alpha' s_{12,4}} z_{25}^{\alpha' s_{12,5}} z_{26}^{\alpha' s_{12,6}} \prod_{2<i<j} z_{i,j}^{\alpha' s_{i,j}}}.$$

koba 5 point

$$= \frac{z_{34}^{\alpha' s_{34}}}{z_{34}^2} \left(\frac{\alpha' s_{13}}{z_{23}} + \frac{\alpha' s_{14}}{z_{24}} + \frac{\alpha' s_{15}}{z_{25}} + \frac{\alpha' s_{16}}{z_{26}}\right) z_{23}^{\alpha' s_{12,3}} z_{35}^{\alpha' s_{35}} z_{36}^{\alpha' s_{36}} z_{24}^{\alpha' s_{12,4}} z_{25}^{\alpha' s_{12,5}} z_{26}^{\alpha' s_{12,6}} \prod_{3<i<j} z_{ij}^{\alpha' s_{ij}}.$$

$$2 \Rightarrow \left(\frac{-s_{14} s_{23} \alpha'^2}{z_{34} z_{56}^2 z_{14} z_{23}}\right)$$

$$3 \Rightarrow \left(\frac{\alpha'^2 s_{13} s_{24}}{z_{34} z_{56}^2 z_{13} z_{24}}\right)$$

Res $s_{34}=0$ $\quad u = z_4 - z_3 \quad v = z_4 \quad \frac{\partial z_{3i}}{\partial u} = 1 \quad \frac{\partial z_{i3}}{\partial u} = -1$

$$\Rightarrow \left[\frac{\alpha' s_{13}}{z_{23}^2} + \left(\frac{\alpha' s_{13}}{z_{23}} + \frac{\alpha' s_{14}}{z_{24}} + \frac{\alpha' s_{15}}{z_{25}} + \frac{\alpha' s_{16}}{z_{26}}\right)\left(-\frac{\alpha' s_{12,3}}{z_{2,3}} + \frac{\alpha' s_{35}}{z_{35}} + \frac{\alpha' s_{36}}{z_{36}}\right)\right] \frac{1}{z_{56}^2} z_{23}^{\alpha' s_{12,3}} z_{24}^{\alpha' s_{12,4}} z_{25}^{\alpha' s_{12,5}} z_{26}^{\alpha' s_{16}} z_{35}^{\alpha' s_{35}} z_{36}^{\alpha' s_{36}} z_{45}^{\alpha' s_{45}} z_{46}^{\alpha' s_{46}} z_{56}^{\alpha' s_{56}}$$

$u=0$ on $z_3 = z_4$

$$= \left[\frac{\alpha' s_{13}}{z_{24}^2} + \left(\frac{\alpha' s_{1,34}}{z_{2,4}} + \frac{\alpha' s_{15}}{z_{25}} + \frac{\alpha' s_{16}}{z_{26}}\right)\left(-\frac{\alpha' s_{12,3}}{z_{24}} + \frac{\alpha' s_{35}}{z_{45}} + \frac{\alpha' s_{36}}{z_{46}}\right)\right] \frac{1}{z_{56}^2} \underline{z_{24}^{\alpha' s_{12,34}} z_{25}^{\alpha' s_{12,5}} z_{26}^{\alpha' s_{12,6}} z_{45}^{\alpha' s_{34,5}} z_{46}^{\alpha' s_{34,6}} z_{56}^{\alpha' s_{56}}}$$

$$2 \Rightarrow \frac{-s_{14} s_{23} \alpha'^2}{z_{56}^2 z_{24}^2}$$ koba 4 point.

$$3 \Rightarrow \frac{\alpha'^2 s_{13} s_{24}}{z_{56}^2 z_{24}^2}$$ koba.

Res $s_{56}=0$ $\quad u = z_6 - z_5 \quad v = z_6 \quad \frac{\partial z_{5,i}}{\partial u} = 1 \quad \frac{\partial z_{i,5}}{\partial u} = -1$

$$1 \Rightarrow \left\{\left[\frac{\alpha' s_{15}}{z_{25}^2}\left(-\frac{\alpha' s_{12,3}}{z_{24}} + \frac{\alpha' s_{35}}{z_{45}} + \frac{\alpha' s_{36}}{z_{46}}\right) + \frac{\alpha' s_{35}}{z_{45}^2}\left(\frac{\alpha' s_{1,34}}{z_{24}} + \frac{\alpha' s_{15}}{z_{25}} + \frac{\alpha' s_{16}}{z_{26}}\right)\right]\right.$$
$$\left. + \left[\frac{\alpha' s_{13}}{z_{24}^2} + \left(\frac{\alpha' s_{1,34}}{z_{24}} + \frac{\alpha' s_{15}}{z_{25}} + \frac{\alpha' s_{16}}{z_{26}}\right)\left(-\frac{\alpha' s_{12,3}}{z_{24}} + \frac{\alpha' s_{35}}{z_{45}} + \frac{\alpha' s_{36}}{z_{46}}\right)\right] \cdot \left(\frac{-\alpha' s_{12,5}}{z_{25}} - \frac{\alpha' s_{34,5}}{z_{45}}\right)\right\}$$
$$z_{24}^{\alpha' s_{12,34}} z_{25}^{\alpha' s_{12,5}} z_{26}^{\alpha' s_{12,6}} z_{45}^{\alpha' s_{34,5}} z_{4,6}^{\alpha' s_{34,6}} z_{56}^{\alpha' s_{56}} \Big|_{z_5 = z_6, u=0}$$

$$=\left[\frac{\alpha' S_{15}}{z_{26}^2}\left(-\frac{\alpha' S_{12,3}}{z_{24}}+\frac{\alpha' S_{3,56}}{z_{46}}\right)+\frac{\alpha' S_{35}}{z_{46}^2}\left(\frac{\alpha' S_{1,34}}{z_{24}}+\frac{\alpha' S_{1,56}}{z_{26}}\right)+\frac{\alpha' S_{13}}{z_{24}^2}\left(-\frac{\alpha' S_{12,5}}{z_{26}}-\frac{\alpha' S_{34,5}}{z_{46}}\right)\right.$$

$$\left.+\left(\frac{\alpha' S_{1,34}}{z_{24}}+\frac{\alpha' S_{1,56}}{z_{26}}\right)\left(\frac{-\alpha' S_{12,3}}{z_{24}}+\frac{\alpha' S_{3,56}}{z_{46}}\right)\left(\frac{-\alpha' S_{12,5}}{z_{26}}-\frac{\alpha' S_{34,5}}{z_{46}}\right)\right] z_{24}^{\alpha' S_{12,34}} z_{26}^{\alpha' S_{12,56}} z_{46}^{\alpha'_{34,56}}.$$

$$2\Rightarrow \frac{-\alpha'^2 S_{14}S_{23}}{z_{24}^2}\left(-\frac{\alpha' S_{12,5}}{z_{26}}-\frac{\alpha' S_{34,5}}{z_{46}}\right)$$

$$S_{14}=S_{1,34}-S_{1,3} \qquad 8$$

$$S_{23}=S_{12,3}-S_{1,3}$$

$$3\Rightarrow \frac{\alpha'^2 S_{13}S_{24}}{z_{24}^2}\left(\frac{-\alpha' S_{12,5}}{z_{26}}-\frac{\alpha' S_{34,5}}{z_{46}}\right)$$

$$S_{24}=S_{2,34}-S_{2,3}$$

$$=S_{12,34}-S_{1,34}-S_{12,3}+S_{1,3}$$

$$S_{13}S_{24}-S_{14}S_{23}$$

$$=S_{13}S_{12,34}-\cancel{S_{13}S_{1,34}}-\cancel{S_{13}S_{12,3}}+\cancel{S_{13}^2}$$

$$-S_{1,34}S_{12,3}+\cancel{S_{13}S_{1,34}}+\cancel{S_{13}S_{12,3}}-\cancel{S_{13}^2}$$

$$=(\varepsilon_1\cdot\varepsilon_2)(P_1\cdot P_2)-(\varepsilon_1,P_2)(\varepsilon_2\cdot P_1)$$

Identify $\varepsilon_1=P_1$ $\quad\varepsilon_2=P_3$ $\quad\varepsilon_3=P_5$ $\qquad z_2\to z_1$

$k_1=P_1+P_2$ $\quad k_2=P_3+P_4$ $\quad k_3=P_5+P_6$ $\qquad z_4\to z_2$, $\quad z_6\to z_3$

$$-1\Rightarrow 4\alpha'^2\left[\frac{\varepsilon_1\cdot\varepsilon_3}{z_{13}^2}\left(-\frac{\varepsilon_2\cdot k_1}{z_{12}}+\frac{\varepsilon_2\cdot k_3}{z_{23}}\right)+\frac{\varepsilon_2\cdot\varepsilon_3}{z_{23}^2}\left(\frac{\varepsilon_1\cdot k_2}{z_{12}}+\frac{\varepsilon_1\cdot k_3}{z_{13}}\right)+\frac{\varepsilon_1\cdot\varepsilon_2}{z_{12}^2}\left(-\frac{\varepsilon_3\cdot k_1}{z_{13}}-\frac{\varepsilon_3\cdot k_2}{z_{23}}\right)\right]$$

$$+8\alpha'^3\left[\left(\frac{\varepsilon_1\cdot k_2}{z_{12}}+\frac{\varepsilon_1\cdot k_3}{z_{13}}\right)\left(-\frac{\varepsilon_2\cdot k_1}{z_{12}}+\frac{\varepsilon_2\cdot k_3}{z_{23}}\right)\left(-\frac{\varepsilon_3 k_1}{z_{13}}-\frac{\varepsilon_3 k_2}{z_{23}}\right)\right]$$

$$2+3\ 8\alpha'^3\left[\frac{\underbrace{(\varepsilon_1\cdot\varepsilon_2)(k_1\cdot k_2)}_{=0}-(k_1\cdot\varepsilon_2)(k_2\cdot\varepsilon_1)}{z_{12}^2}\left(-\frac{\varepsilon_3 k_1}{z_{13}}-\frac{\varepsilon_3 k_2}{z_{23}}\right)\right]$$

$$=0 \ \because (k_1+k_2)^2=k_3^2=0.$$

$$=\underset{\substack{\shortparallel\\0}}{k_1^2}+\underset{\substack{\downarrow\\0}}{2k_1k_2}+\underset{\substack{\shortparallel\\0}}{k_2^2}$$

$$S_{14}S_{23}-S_{13}S_{24}=(k_1\cdot\varepsilon_2)(k_2\cdot\varepsilon_1)$$

$$S_{14}=\varepsilon_1(k_2-\varepsilon_2)(k_1-\varepsilon_1)\varepsilon_2$$

$$-\varepsilon_1\varepsilon_3(k_1-\varepsilon_1)(k_2-\varepsilon_2)$$

Check α'^{13} order cancel

$$\left(\frac{\varepsilon_1\cdot k_2}{z_{12}}+\frac{\varepsilon_1\cdot k_3}{z_{13}}\right)\left(\frac{-\varepsilon_2 k_1}{z_{12}}+\frac{\varepsilon_2 k_3}{z_{23}}\right)\left(\frac{-\varepsilon_3 k_1}{z_{13}}-\frac{\varepsilon_3 k_2}{z_{23}}\right)$$

$$+\frac{(\varepsilon_2 k_1)(\varepsilon_1 k_2)}{z_{12}^2}\left(-\frac{\varepsilon_3 k_1}{z_{13}}-\frac{\varepsilon_3 k_2}{z_{23}}\right)$$

$$=\left(\cancel{-\frac{\varepsilon_1 k_2}{z_{12}}\frac{\varepsilon_2 k_1}{z_{12}}}-\underline{\frac{\varepsilon_1 k_3}{z_{13}}\frac{\varepsilon_2 k_1}{z_{12}}+\frac{\varepsilon_1 k_2}{z_{12}}\frac{\varepsilon_2 k_3}{z_{23}}+\frac{\varepsilon_1 k_3}{z_{13}}\frac{\varepsilon_2 k_3}{z_{23}}}+\cancel{\frac{\varepsilon_2 k_1\,\varepsilon_1 k_2}{z_{12}^2}}\right)$$

$$\left(\frac{-\varepsilon_3 k_1}{z_{13}}-\frac{\varepsilon_3 k_2}{z_{23}}\right)$$

$$\Rightarrow \frac{\varepsilon_1 k_2}{z_{13}}\frac{\varepsilon_2 k_1}{z_{12}}+\frac{\varepsilon_1 k_2}{z_{12}}\left(-\frac{\varepsilon_2 k_1}{z_{23}}\right)+\left(\frac{-\varepsilon_1 k_2}{z_{13}}\frac{-\varepsilon_2 k_1}{z_{23}}\right)$$

$$=\varepsilon_1 k_2\,\varepsilon_2 k_1\left(\frac{1}{z_{12}z_{13}}-\frac{1}{z_{12}z_{23}}+\frac{1}{z_{13}z_{23}}\right)$$

$$=\varepsilon_1 k_2\,\varepsilon_2 k_1\left(\frac{z_{23}-z_{13}+z_{12}}{z_{12}z_{13}z_{23}}\right)=\frac{z_3-z_2-z_3+z_1+z_2-z_1}{z_{12}z_{23}z_{13}}=0$$

$= 0$. ∎

If reduce phaffian pick 46 $\Psi'=\begin{pmatrix} 0 & \frac{\alpha' s_{12}}{z_{12}}-\delta_{12} & \frac{\alpha' s_{13}}{z_{13}} & \frac{\alpha' s_{15}}{z_{15}} \\ & 0 & \frac{\alpha' s_{23}}{z_{23}} & \frac{\alpha' s_{25}}{z_{25}} \\ & & 0 & \frac{\alpha' s_{35}}{z_{35}} \\ & & & 0 \end{pmatrix}$

$$\Rightarrow \mathrm{Pf}'\Psi^{46}=\frac{(-1)^{(4+6)}}{z_{46}}\left(\frac{\alpha' s_{12}-\delta_{12}}{z_{12}}\frac{\alpha' s_{35}}{z_{35}}+\frac{\alpha' s_{15}}{z_{15}}\frac{\alpha' s_{23}}{z_{23}}-\frac{\alpha' s_{13}}{z_{13}}\frac{\alpha' s_{25}}{z_{25}}\right)$$

$$\Rightarrow \underset{x_{51}}{\mathrm{Res}}\ \underset{x_{35}}{\mathrm{Res}}=\underset{z_{56}}{\mathrm{Res}}\ \underset{z_{34}}{\mathrm{Res}}=\frac{\text{Koba}\cdot\text{tour point}}{z_{12}}\frac{1}{z_{46}}\Big(\underbrace{\frac{-1}{z_{12}}\frac{\alpha' s_{35}}{z_{46}}}_{①}+\underbrace{\frac{\alpha' s_{15}}{z_{16}}\frac{\alpha' s_{23}}{z_{24}}}_{②}-\underbrace{\frac{\alpha' s_{13}}{z_{14}}\frac{\alpha' s_{25}}{z_{26}}}_{③}\Big)$$

$$z_{12}^{\alpha' s_{12}} z_{14}^{\alpha' s_{1,34}} z_{16}^{\alpha' s_{1,56}} z_{24}^{\alpha' s_{2,34}} z_{26}^{\alpha' s_{2,56}} z_{46}^{\alpha' s_{34,56}} = \text{koba four point.}$$

$$\underset{x_{13}}{\mathrm{Res}} = \underset{z_{12}}{\mathrm{Res}} = \frac{-\alpha' s_{35}}{z_{46}^2}\left(\frac{\alpha' s_{1,34}}{z_{14}} + \frac{\alpha' s_{1,56}}{z_{16}}\right)$$

$$+ \frac{\alpha' s_{15}}{z_{16}}\frac{\alpha' s_{23}}{z_{24} z_{46}} - \frac{\alpha' s_{13}}{z_{14}}\frac{\alpha' s_{25}}{z_{26} z_{46}}$$

$$= \frac{-\alpha' s_{35}}{z_{46}^2}\left(\frac{\alpha' s_{1,34}}{z_{24}} + \frac{\alpha' s_{1,56}}{z_{26}}\right) + \frac{\alpha' s_{15}}{z_{26}}\frac{\alpha' s_{23}}{z_{24} z_{46}} - \frac{\alpha' s_{13}}{z_{24}}\frac{\alpha' s_{25}}{z_{26} z_{46}}$$

$$\Rightarrow \alpha'^2\left(-\frac{\varepsilon_2\varepsilon_3}{z_{23}^2}\left(\frac{\varepsilon_1 k_2}{z_{12}} + \frac{\varepsilon_1 k_3}{z_{13}}\right) + \frac{\varepsilon_1\varepsilon_3(k_1\varepsilon_2 - \varepsilon_1\varepsilon_2) - \varepsilon_1\varepsilon_2(k_1\varepsilon_3 - \varepsilon_1\varepsilon_3)}{z_{12} z_{23} z_{13}}\right)$$

$$= \alpha'^2\left[-\frac{\varepsilon_2\varepsilon_3}{z_{23}^2}\left(\frac{\varepsilon_1 k_2}{z_{12}} + \frac{\varepsilon_1 k_3}{z_{13}}\right) + \frac{\varepsilon_1\varepsilon_3(k_1\varepsilon_2) - \varepsilon_1\varepsilon_2(k_1\varepsilon_3)}{z_{12} z_{23} z_{13}}\right)$$

$$\langle V^0(z_1) V^{+}(z_2) V^{+}(z_3)\rangle$$

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$$\mathrm{Pf}^{36'}(\psi) = \begin{pmatrix} 0 & \frac{-1}{z_{12}} & \frac{\alpha' s_{14}}{z_{14}} & \frac{\alpha' s_{15}}{z_{15}} \\ & 0 & \frac{\alpha' s_{24}}{z_{24}} & \frac{\alpha' s_{25}}{z_{25}} \\ & & 0 & \frac{\alpha' s_{45}}{z_{45}} \\ & & & 0 \end{pmatrix}$$

$$\Rightarrow \frac{-1}{z_{36}}\left(\frac{-\alpha' s_{45}}{z_{12} z_{45}} + \frac{\alpha'^2 s_{15} s_{24}}{z_{15} z_{24}} - \frac{\alpha'^2 s_{14} s_{25}}{z_{14} z_{25}}\right)$$

$$\downarrow$$

$$\frac{\alpha' s_{45}}{z_{46}^2}\left(\frac{\alpha' s_{1,34}}{z_{24}} + \frac{\alpha' s_{1,56}}{z_{26}}\right) + \frac{\alpha'^2(s_{15}s_{24} - s_{14}s_{25})}{z_{46} z_{26} z_{24}}$$

$(k_1 - \varepsilon_1)(k_2 - \varepsilon_2)$

$$\frac{(-\varepsilon_3\varepsilon_2 + \varepsilon_3 k_2)}{z_{23}^2}\left(\frac{\varepsilon_1 k_2}{z_{12}} + \frac{\varepsilon_1 k_3}{z_{13}}\right) - \frac{\varepsilon_1\varepsilon_3(k_1\cdot k_2 - k_1\cdot\varepsilon_2 - k_2\cdot\varepsilon_1 + \varepsilon_1\varepsilon_2) - (\varepsilon_1 k_2 - \varepsilon_1\varepsilon_2)(k_1\varepsilon_3 - \varepsilon_1\varepsilon_3)}{z_{12} z_{23} z_{13}}$$

$$+ \varepsilon_3 k_1 \varepsilon_1 k_2 - \varepsilon_3 k_1 \varepsilon_1 \varepsilon_2 - \varepsilon_1\varepsilon_3 k_1 k_2 + k_1\varepsilon_2\,\varepsilon_1\varepsilon_3$$

$$\underline{\frac{\varepsilon_3 k_2}{z_{23}^2}\left(\frac{\varepsilon_1 k_2}{z_{12}}+\frac{\varepsilon_1 k_3}{z_{13}}\right)} = \frac{\varepsilon_1\varepsilon_3\, k_1 k_2 - \varepsilon_2 k_1 \varepsilon_1 k_2}{z_{12} z_{23} z_{13}}$$

$k_3 \to -k_1 - k_2$ $\qquad z_{12} - z_{13} = z_{32}$

$$\frac{\varepsilon_3 k_2\, \varepsilon_1 k_2\, z_{13} - \varepsilon_3 k_2 \varepsilon_1 k_2 z_{12}}{\underline{z_{23}^2\, z_{12} z_{13}}} = \frac{\varepsilon_3 k_2\, \varepsilon_1 k_2}{z_{12} z_{23} z_{13}}$$

$$\Rightarrow \frac{\varepsilon_3 k_2\, \varepsilon_1 k_2 - \varepsilon_1\varepsilon_3 k_1 k_2 + \varepsilon_2 k_1 \varepsilon_1 k_2}{z_{12} z_{23} z_{13}} = 0$$

pf 3 5

$$\mathrm{Pf}'\Psi^{35} = \begin{pmatrix} 0 & -\frac{1}{z_{12}} & \frac{\alpha' s_{14}}{z_{14}} & \frac{\alpha' s_{16}}{z_{16}} \\ & 0 & \frac{\alpha' s_{24}}{z_{24}} & \frac{\alpha' s_{26}}{z_{26}} \\ & & 0 & \frac{\alpha' s_{46}}{z_{46}} \\ & & & 0 \end{pmatrix}$$

$$\Rightarrow \mathrm{Pf}'\Psi^{35} = \frac{(-1)^{3+5}}{z_{35}}\left(-\frac{1}{z_{12}}\frac{\alpha' s_{46}}{z_{46}} + \frac{\alpha'^2 s_{16} s_{24}}{z_{24} z_{16}} - \frac{\alpha'^2 s_{14} s_{26}}{z_{14} z_{26}}\right)$$

$$\Rightarrow \underset{z_{56}}{\mathrm{Res}}\ \underset{z_{34}}{\mathrm{Res}} = \frac{-\alpha' s_{46}}{z_{12}^2 z_{46}^2} + \frac{1}{z_{12}}\left(\frac{\alpha'^2 s_{16} s_{24}}{z_{24} z_{16} z_{46}} - \frac{\alpha'^2 s_{14} s_{26}}{z_{14} z_{26} z_{46}}\right)$$

$$\underset{z_{12}}{\mathrm{Res}} = \frac{-\alpha' s_{46}}{z_{46}^2}\left(\frac{\alpha' s_{1,34}}{z_{2,4}} + \frac{\alpha' s_{1,56}}{z_{2,6}}\right) + \frac{\alpha'^2 (s_{16} s_{24} - s_{14} s_{26})}{z_{24} z_{26} z_{46}}$$

$$s_{46} = k_2 \cdot k_3 + \underline{\varepsilon_2 \varepsilon_3} - \varepsilon_2 k_3 - \varepsilon_3 k_2$$

$$s_{16} s_{24} = (\varepsilon_1 k_3 - \varepsilon_1 \varepsilon_3)(k_1 k_2 + \varepsilon_1 \varepsilon_2 - \varepsilon_1 k_2 - \varepsilon_2 k_1) = [\varepsilon_1 k_3 - \varepsilon_1 \varepsilon_3][(\varepsilon_1\varepsilon_2 - \varepsilon_1 k_2) + (k_1 k_2 - \varepsilon_2 k_1)]$$

$$s_{14} s_{26} = (\varepsilon_1 k_2 - \varepsilon_1 \varepsilon_2)(k_1 k_3 + \varepsilon_1 \varepsilon_3 - \varepsilon_1 k_3 - \varepsilon_3 k_1) = (\varepsilon_1 k_2 - \varepsilon_1 \varepsilon_2)[(\varepsilon_1 \varepsilon_3 - \varepsilon_1 k_3) + (k_1 k_3 - \varepsilon_3 k_1)]$$

$$s_{16} s_{24} - s_{14} s_{26} = (\varepsilon_1 k_3 - \varepsilon_1 \varepsilon_3)(k_1 k_2 - \varepsilon_2 k_1) - (\varepsilon_1 k_2 - \varepsilon_1 \varepsilon_2)(k_1 k_3 - \varepsilon_3 k_1)$$

$$= (-\varepsilon_1 k_3 + \varepsilon_1 \varepsilon_3)\varepsilon_2 k_1 + (\varepsilon_1 k_2 - \varepsilon_1 \varepsilon_2)(\varepsilon_3 k_1)$$

$$= \underline{(\varepsilon_1 \varepsilon_3)(\varepsilon_2 k_1)} - \underline{(\varepsilon_1 \varepsilon_2)(\varepsilon_3 k_1)} - (\varepsilon_1 k_3)(\varepsilon_2 k_1) + (\varepsilon_1 k_2)(\varepsilon_3 k_1)$$

$(\varepsilon_2 k_3 + \varepsilon_3 k_2)(\varepsilon_1 k_2) - (\varepsilon_1 k_3)(\varepsilon_2 k_1) + (\varepsilon_1 k_2)(\varepsilon_3 k_1)$

$= \underbrace{(\varepsilon_1 k_2)(\varepsilon_3 k_2 + \varepsilon_3 k_1)}_{\overset{\shortparallel}{0}} + \underbrace{(\varepsilon_2 k_3)(\varepsilon_1 k_2) - (\varepsilon_1 k_3)(\varepsilon_2 k_1)}$

$\underbrace{(\varepsilon_2 k_3)(\varepsilon_1 k_2) + (\varepsilon_1 k_2)(\varepsilon_2 k_1)}_{\overset{\shortparallel}{0}}$

$\Rightarrow$ each give the same answer.

$56 \leftrightarrow \quad i\varepsilon_1 \cdot \partial X e^{ik_1 X} \quad i\varepsilon_2 \cdot \partial X e^{ik_2 X} \quad i\varepsilon_3 \cdot \partial X e^{ik_3 X}$

$\quad - \varepsilon_1 \psi k_1 \psi e^{ik_1 X} \quad \varepsilon_2 \psi k_2 \psi e^{ik_2 X} \quad i\varepsilon_3 \partial X e^{ik_3 X}$

$46 \leftrightarrow \quad (i\varepsilon_1 \partial X + k_1 \psi \varepsilon_1 \psi) e^{ik_1 X} \quad e^{-\phi} \varepsilon_2 \psi e^{ik_2 X} \quad e^{-\phi} \varepsilon_3 \psi e^{ik_3 X}$

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might use condition like $k_1^2 = 0$ $\varepsilon_1^2 = 0$.

My guess V

$k_1 \psi e^{ik_1 X} \quad k_2 \psi e^{ik_2 X}$

$\int d\theta_1 d\theta_2 d\theta_3 d\theta_4 \; \underline{e^{ik_1 X + \theta_1 k_1 \psi}} \; \underline{e^{ik_2 X + \theta_2 k_2 \psi}} \; \underline{e^{ik_3 X + \theta_3 k_3 \psi}} \; \underline{e^{ik_4 X + \theta_4 k_4 \psi}}$

$\therefore \; e^{ik_1 X} \theta_1 k_1 \psi \quad e^{ik_2 X} \theta_2 k_2 \psi \quad e^{ik_3 X} \theta_3 k_3 \psi \quad e^{ik_4 X} \theta_4 k_4 \psi$

koba $\left(\frac{k_1 k_2}{z_{12}} \frac{k_3 k_4}{z_{34}} - \frac{k_1 k_3}{z_{13}} \frac{k_2 k_4}{z_{24}} + \frac{k_1 k_4}{z_{14}} \frac{k_2 k_3}{z_{23}} \right)$. $\quad \frac{k_1 k_2}{z_{12}} \frac{-k_3 k_1 - k_3 k_2}{z_{34}} + \frac{k_1 k_3}{z_{13}} \frac{k_2 k_3 + k_2 k_1}{2}$

$\int d\theta_1 \; |z_{12} + \theta_1 \theta_2|^{\alpha' s_{12}} \; |z_{13} + \theta_1 \theta_3|^{\alpha' s_{13}} \; |z_{14} + \theta_1 \theta_4|^{\alpha' s_{14}} \; |z_{23} + \theta_2 \theta_3|^{\alpha' s_{23}} \; |z_{24} + \theta_2 \theta_4|^{\alpha' s_{24}} \; |z_{34} + \theta_3 \theta_4|^{\alpha' s_{34}}$

$$= z_{12}^{\alpha' s_{12}}\left(1+\frac{\alpha' s_{12}\theta_1\theta_2}{z_{12}}\right) z_{13}^{\alpha' s_{13}}\left(1+\frac{\alpha' s_{13}\theta_1\theta_3}{z_{13}}\right)$$
$$z_{14}^{\alpha' s_{14}}\left(1+\frac{\alpha' s_{14}\theta_1\theta_4}{z_{14}}\right) z_{23}^{\alpha' s_{23}}\left(1+\frac{\alpha' s_{23}\theta_2\theta_3}{z_{23}}\right)$$
$$z_{24}^{\alpha' s_{24}}\left(1+\frac{\alpha' s_{24}\theta_2\theta_4}{z_{24}}\right) z_{34}^{\alpha' s_{34}}\left(1+\frac{\alpha' s_{34}\theta_3\theta_4}{z_{34}}\right)$$

$$\frac{p_1\psi e^{ip_1X(z_1)}\, p_2\psi e^{ip_2X(z_2)}}{z_{12}}$$

$z_2 - z_1 = u$

$z_2 = v$

$e^{ik_1X} k_1\psi\, e^{ik_2X} k_2\psi$

$p_1 \cdot p_2 = 1$

$p_1^2 = -1$ $\quad p_1 \cdot p_2$ $\quad s_{12} = 2p_1p_2$

$p_2^2 = -1$ $\quad (p_1+p_2)^2 = 0$ $\quad s_{12} = 2p_1p_2$

$= \cancel{(p_1+p_2)^2 + 1}$

$= (p_1+p_2)^2 + 2$

$= X_{1,3} + 2$

$$\frac{z_1^{\alpha' s_{11}}}{z_1^2} = z_1^{-\alpha' s_{11} - 2}$$

$\alpha' s_{11} = 0$

$$\frac{z_1^{\alpha' s_{11}}}{z_1} = z^{-\alpha' s_{11} - 1}$$

$\alpha' s_{11} = 1$

$(p_1 + p_2) = k_1$

$p_1 = \varepsilon_1$

$p_1^2 + p_1 p_2 = k_1 \cdot \varepsilon_1$

$= 0$

ε_1

$\cancel{p_1 + p}$

$\cancel{p_1 + p}$

$2p_1p_2 + p_1^2 + p_2^2 = 0$

$\quad -1 \quad -1$

$p_1 p_2 = 1$

$(p_1 + p_2)^2 = 2$

s_{12}

$$\underbrace{p_1\psi e^{ip_1X}\, p_2\psi e^{ip_2X}}_{z_{12}}\ \underbrace{p_3\psi e^{ip_3X}}_{z_{34}}\ p_4\psi e^{ip_4X}\ \underbrace{p_5\psi e^{ip_5X}\, p_6\psi e^{ip_6X}}_{z_{56}}$$

$5 \cdot 6$

$\downarrow$

$$(-1)^{11} \frac{(p_5 p_6)}{z_{56}^2} \quad i p_5 \partial X e^{ik_3X}$$

Res $\alpha' s_{12} = -1 \Rightarrow \alpha' s_{12} + 1 = 0$

$(z_{11})^{\alpha' s_{12}}$

Consider $P_1\psi e^{iP_1X}\ P_2\psi e^{iP_2X}\ \cdots\ P_5\psi e^{iP_5X}\ P_6\psi e^{iP_6X}$ $\quad 6\to 3$

Consider $\alpha' s_{ii} = 2\alpha' P_iP_i = 1$

Fix two ψ to contract

If 5,6 $\Rightarrow P_1\psi e^{iP_1X} P_2\psi e^{iP_2X}\cdots \dfrac{(-1)^{5+6}e^{iP_5X}\, e^{iP_6X}}{z_{56}}$

Consider the residue on $\alpha' s_{2i-1\,2i}=-1\ \forall i=1\cdots n.$ $\quad \alpha' s_{12}-1=0$

$\Rightarrow$ Check in this condition whether $2i-1$ and $2i$ scalar fusing to a gluon. $\quad (z_{12})^{\alpha' s_{12}}$

(pf) $(P_{2i-1}+P_{2i})^2 = P_{2i-1}^2+P_{2i}^2+2P_{2i-1}P_{2i}$

$= \dfrac{1}{2\alpha'}+\dfrac{1}{2\alpha'}+s_{2i-1,2i}$

$= \dfrac{1+\alpha' s_{2i-1,2i}}{\alpha'}$ $\quad\Rightarrow$ when $\alpha' s_{2i-1,2i}=-1 \Rightarrow (P_{2i-1}+P_{2i})^2=0.$

Pick Residue on $\alpha' s_{2i-1,2i}=-1 \Leftrightarrow \mathrm{Res}\ z_{2i-1,2i}=0.$

But $:e^{iP_1X}e^{iP_2X}: \sim z_{12}^{\alpha' s_{12}} = \dfrac{z_{12}^{\alpha' s_{12}+1}}{z_{12}}$

$\Rightarrow \mathrm{Res}_{z_{12}} \Rightarrow P_1\psi P_2\psi e^{i(P_1+P_2)X(z_2)} + (P_1\cdot P_2)\, iP_1\cdot\partial X e^{i(P_1+P_2)X(z_2)}$

$\mathrm{Res}_{z_{34}} \Rightarrow P_3\psi P_4\psi e^{i(P_3+P_4)X(z_4)} + (P_3\cdot P_4)\, iP_3\cdot\partial X e^{i(P_3+P_4)X(z_4)}$

$\mathrm{Res}_{z_{56}} \Rightarrow\ iP_5\,\partial X e^{i(P_5+P_6)X(z_6)}$

$\Rightarrow \langle [P_1\psi P_2\psi + (P_1\cdot P_2)\, iP_1\cdot\partial X] e^{i(P_1+P_2)X(z_2)} [P_3\psi P_4\psi + (P_3\cdot P_4)\, iP_3\cdot\partial X] e^{i(P_3+P_4)X(z_4)}\ (P_5\cdot P_6)\, iP_5\partial X e^{i(P_5+P_6)X(z_6)}$

$\langle [\varepsilon_1\psi (k_1-\varepsilon_1)\psi - \frac{1}{2\alpha'} i\varepsilon_1\cdot\partial X e^{ik_1X(z_1)}][\varepsilon_2\psi(k_2-\varepsilon_2)\psi - \frac{1}{2\alpha'} i\varepsilon_2\partial X e^{ik_2X(z_2)}]$

$[-i\varepsilon_3\partial X e^{ik_3X(z_3)}]\rangle$

Notice $\varepsilon_1\psi(k_1-\varepsilon_1)\psi = \varepsilon_1\psi k_1\psi - \underbrace{\varepsilon_1\psi\varepsilon_1\psi}_{=0}$.

$\Rightarrow$ We can replace $\varepsilon_1\psi(k_1-\varepsilon_1)\psi$ as $\varepsilon_1\psi k_1\psi$

$$\Rightarrow \langle\left[\left(\varepsilon_1\psi k_1\psi - \frac{1}{2\alpha'}i\varepsilon_1\partial X\right)e^{ik_1X}\right]\left[\left(\varepsilon_2\psi k_2\psi - \frac{1}{2\alpha'}i\varepsilon_2\partial X\right)e^{ik_2X}\right]\left[-i\varepsilon_3\partial Xe^{ik_3X}\right]\rangle$$

$$= \frac{(-1)^{5+6}}{4\alpha'^2}\langle(2\alpha' k_1\psi\varepsilon_1\psi + i\varepsilon_1\partial X)e^{ik_1X}\,(2\alpha' k_2\psi\varepsilon_2\psi + i\varepsilon_2\partial X)e^{ik_2X}\;i\varepsilon_3\partial Xe^{ik_3X}\rangle$$

$$\mathrm{pf}\,A = \sum_{j=2}^{2n}(-1)^j a_{1j}\,\mathrm{Pf}(A_{\hat{1}\hat{j}}). \qquad a_{1j} = \frac{S_{1j}}{z_{1j}}$$

$$\int\frac{(-1)^{1+j}}{z_{1j}}\mathrm{Pf}(A_{ij}) = -SYM \qquad \int \mathrm{pf}A = SYM.$$

$$\sum_{1j}\alpha' S_{1j}\int\frac{(-1)^{1+j}}{z_{1j}}\mathrm{Pf}(A_{1j}) = \sum_{1j}\alpha' S_{1j}(-SYM)$$

$$= \sum_{1j}\int\frac{(-1)^{1+j}\alpha' S_{1j}}{z_{1j}}\mathrm{Pf}(A_{1j}) = \frac{-\alpha'}{1}S_{11} - SYM$$

$$= \int \mathrm{Pf}A = SYM$$

$$= \frac{1}{4\alpha'^2}\Big[4\alpha'^2\left[\varepsilon_1\psi k_1\psi e^{ik_1X}\,\varepsilon_2\psi k_2\psi e^{ik_2X}(-i\varepsilon_3\partial Xe^{ik_3X})\right]$$
$$-\,i\varepsilon_1\partial Xe^{ik_1X}\,i\varepsilon_2\partial Xe^{ik_2X}\,i\varepsilon_3\partial Xe^{ik_1X}\Big]$$

$$= \frac{1}{4\alpha'^2}\Big[4\alpha'^2\Big[\frac{\varepsilon_1k_2k_1\varepsilon_2 - \varepsilon_1\varepsilon_2k_1k_2}{z_{12}^2}\left(-\frac{\varepsilon_3k_1}{z_{13}} - \frac{\varepsilon_3k_2}{z_{23}}\right) - 2\alpha'\frac{\varepsilon_1\varepsilon_2}{z_{12}^2}\left(2\alpha'\frac{\varepsilon_3k_1}{z_{13}} + 2\alpha'\frac{\varepsilon_3k_2}{z_{23}}\right)$$

$$\varepsilon_3\partial X(z_3)e^{ik_1X(z_1)} = 2i\alpha'\frac{\varepsilon_3k_1}{z_{31}} = -2i\alpha'\frac{\varepsilon_3k_1}{z_{13}}$$

Let $\varepsilon_1\partial X(z_1)\,e^{ik_2X(z_2)} = 2i\alpha'\frac{\varepsilon_1k_2}{z_{12}}e^{ik_2X(z_2)}$

$$\varepsilon_1\partial X(z_1)\,\varepsilon_2\partial X(z_2) = -2\alpha'\frac{\varepsilon_1\varepsilon_2}{(z_{12})^2}$$

For 46

$$P_1\psi e^{iP_1X}\, P_2\psi e^{iP_2X}\, P_3\psi e^{iP_3X}\, \underbrace{P_4\psi e^{iP_4X}\, P_5\psi e^{iP_5X}\, P_6\psi e^{iP_6X}}_{\frac{1}{z_{46}}}$$

$\mathrm{Res}\ \alpha' s_{12}=-1$ $[P_1\psi P_2\psi-(P_1\cdot P_2)\, iP_1\partial X]e^{ik_1X}$

$\alpha' s_{34}=-1$ $P_3\psi e^{ik_2X}$

$\alpha' s_{56}=-1$ $P_5\psi e^{ik_3X}$

$$\Rightarrow \langle [P_1\psi P_2\psi-\frac{1}{2\alpha'}iP_1\partial X]e^{ik_1X}\, P_3\psi e^{ik_2X}\, P_5\psi e^{ik_3X}\cdot\frac{1}{z_{46}}]\rangle.$$

$$\Rightarrow \langle [\varepsilon_1\psi k_1\psi-\frac{1}{2\alpha'}i\varepsilon_1\partial X]e^{ik_1X}\, \varepsilon_2\psi e^{ik_2X}\, \varepsilon_3\psi e^{ik_3X}]\rangle\cdot\frac{1}{z_{23}}$$

$$\Rightarrow \langle [\varepsilon_1\psi k_1\psi-\frac{1}{2\alpha'}i\varepsilon_1\partial X]e^{ik_1X}\, \varepsilon_2\psi e^{-\phi}e^{ik_2X}\, \varepsilon_3\psi e^{-\phi}e^{ik_3X}\rangle.$$

$$=\frac{-1}{2\alpha'}\langle [2\alpha' k_1\psi\varepsilon_1\psi+i\varepsilon_1\partial X]e^{ik_1X}\, \varepsilon_2\psi e^{-\phi}e^{ik_2X}\, \varepsilon_3\psi e^{-\phi}e^{ik_3X}\rangle$$

For 36

$$(-1)^{a}\langle [k_1\psi\varepsilon_1\psi-\frac{1}{2\alpha'}i\varepsilon_1\partial X]e^{ik_1X}\, e^{-\phi}P_4\psi e^{ik_2X}\, e^{-\phi}P_5\psi e^{ik_3X}]\rangle$$

$$=\frac{-1}{2\alpha'}\langle [2\alpha' k_1\psi\varepsilon_1\psi+i\varepsilon_1\partial X]e^{ik_1X}\, e^{-\phi}(-1)^{a}(k_2-\varepsilon_2)\psi e^{ik_2X}\, e^{-\phi}\varepsilon_3\psi e^{ik_3X}]\rangle$$

$$=\frac{-1}{2\alpha'}\langle [2\alpha' k_1\psi\varepsilon_1\psi+i\varepsilon_1\partial X]e^{ikX}\, e^{-\phi}(\varepsilon_2-k_2)\psi e^{ik_2X}\, e^{-\phi}\varepsilon_3\psi e^{ik_3X}]\rangle$$

By the ward identity of superstring amplitude

$$\Rightarrow \frac{-1}{2\alpha'}\langle [2\alpha' k_1\psi\varepsilon_1\psi+i\varepsilon_1\partial X]e^{ik_1X}\, e^{-\phi}\varepsilon_2\psi e^{ik_2X}\, e^{-\phi}\varepsilon_3\psi e^{ik_3X}]\rangle \quad \#$$

$\Rightarrow$ I can prove the invariant of choosing of different (i,j)

(i_1, j_1) (i_2, j_2) are equivalenc when $\underset{\substack{\uparrow \\ \{2k-1,2k\}}}{i_1, i_2}$ $\underset{\substack{\uparrow \\ \{2m-1,2m\}}}{j_1, j_2}$.

$$\text{Use } \underbrace{k_1\partial X(z_1)\; e^{ik_2X(z_2)}} = -2i\alpha'\frac{k_1\cdot k_2}{z_{12}}$$

$$\frac{(k_1-\varepsilon_1)\cdot\varepsilon_2}{z_{12}}\;\frac{\varepsilon_1\cdot(k_2-\varepsilon_2)}{z_{12}}\cdot -\frac{1}{2\alpha'}\, i\; -2i\alpha'\left(-\frac{\varepsilon_3 k_1}{z_{13}}-\frac{\varepsilon_3 k_2}{z_{23}}\right)$$

$$\underline{-\varepsilon_1\varepsilon_2\,\varepsilon_1 k_2 - k_1\varepsilon_2\,\varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_2\,\varepsilon_1\varepsilon_2}$$

2

$$\varepsilon_1\varepsilon_2(-\varepsilon_1 k_2 - k_1\varepsilon_2 + \varepsilon_1\varepsilon_2)$$

$$P_1P_3\big(-P_1(P_3+P_4) - (P_1+P_2)P_3 + P_1P_3\big)$$

$$-2\cancel{P_1P_3} - P_1P_4 - P_2P_3 + \cancel{P_1P_3}$$

$$-P_1P_3 - P_1P_4 - P_2P_3$$

$P_1^2 = \frac{1}{2\alpha}$ $\quad P_2^2 = \frac{1}{2\alpha}$

$P_1P_2 = -\frac{1}{2\alpha}$

$\varepsilon_1 = P_1 \quad k_1 = P_1 + P_2$

$\varepsilon_1\psi\, k_1\psi - \varepsilon_1\psi\,\varepsilon_1\psi$

$\varepsilon_1\psi(k_1-\varepsilon_1)\psi$

$\varepsilon_1 \to -\varepsilon_1 \quad -\varepsilon_1\psi(k_1+\varepsilon_1)\psi$

$$\varepsilon_1\psi\,(k_1-\varepsilon_1)\psi \qquad \varepsilon_2\psi\,(k_2-\varepsilon_2)\psi \qquad \varepsilon_3\psi\,(k_3-\varepsilon_3)\psi$$

$$\varepsilon_1(k_2-\varepsilon_2)\,(k_1-\varepsilon_1)\varepsilon_2 - \varepsilon_1\varepsilon_2(k_1-\varepsilon_1)\cdot(k_2-\varepsilon_2)$$

$$=(\varepsilon_1k_2-\varepsilon_1\varepsilon_2)(k_1\varepsilon_2-\varepsilon_1\varepsilon_2) - \varepsilon_1\varepsilon_2(k_1k_2-\varepsilon_1k_2-\varepsilon_2k_1+\varepsilon_1\varepsilon_2)$$

$$\varepsilon_1k_2\,k_1\varepsilon_2 - \varepsilon_1\varepsilon_2\,k_1\varepsilon_2 - \varepsilon_1\varepsilon_2\,\varepsilon_1k_2 + \varepsilon_1\varepsilon_2\,\varepsilon_1\varepsilon_2 - \varepsilon_1\varepsilon_2\,k_1k_2 + \varepsilon_1\varepsilon_2\,\varepsilon_1k_2 + \varepsilon_1\varepsilon_2\,\varepsilon_2k_1 - \varepsilon_1\varepsilon_2\,\varepsilon_1\varepsilon_2$$

$$\varepsilon_1(k_2-\varepsilon_2)\,(k_1-\varepsilon_1)\cdot(k_3-\varepsilon_3) - \varepsilon_1(k_3-\varepsilon_3)(k_1-\varepsilon_1)(k_2-\varepsilon_2)$$

$$=(\varepsilon_1k_2-\varepsilon_1\varepsilon_2)\,[k_1k_3-\varepsilon_1k_3-\varepsilon_3k_1+\varepsilon_1\varepsilon_3) - (\varepsilon_1k_3-\varepsilon_1\varepsilon_3)(k_1k_2-\varepsilon_1k_2-\varepsilon_2k_1+\varepsilon_1\varepsilon_2)$$

$$(\varepsilon_1k_2-\varepsilon_1\varepsilon_1)\,[\varepsilon_1\varepsilon_3-\varepsilon_1k_3+k_1k_3-\varepsilon_3k_1] - (\varepsilon_1k_3-\varepsilon_1\varepsilon_3)(-\varepsilon_1k_2+\varepsilon_1\varepsilon_2+k_1k_2-\varepsilon_2k_1)$$

$(-1)^{\cancel{A}+6}$

$\overline{z_{46}}$

1. $(i\varepsilon_1\partial X+\varepsilon_1\psi k_1\psi)e^{ik_1X}\,(i\varepsilon_2\partial X+\varepsilon_2\psi k_2\psi)e^{ik_2X}\,(i\varepsilon_3\partial X+\varepsilon_3\psi k_3\psi)e^{ik_3X}$

2. $(i\varepsilon_1\partial X+\varepsilon_1\psi_1 k_1\psi)e^{ik_1X}\,(e^{-\phi}\varepsilon_2\psi e^{ik_2X})(e^{-\phi}\varepsilon_3\psi e^{ik_3X})$ ✓

$(i\varepsilon_1\partial X+\varepsilon_1\psi k_1\psi)e^{ik_1X}(i\varepsilon\partial X+\varepsilon_2\psi k_2\psi)e^{ik_2X}\quad i\varepsilon_3\partial X\, e^{ik_3X}$

$=\mathrm{Spom}$

$\mathrm{Tr}\quad \begin{matrix} i\varepsilon_1\partial X & i\varepsilon_2\partial X & i\varepsilon_3\partial X \\ \varepsilon_1\psi k_1\psi & \varepsilon_2\psi k_2\psi & i\varepsilon_3\partial X \end{matrix}$

$$\left(\begin{matrix} 0 & \frac{-1}{z_{12}} & \frac{s_{13}}{z_{13}} & \frac{s_{14}}{z_{14}} & \frac{s_{15}}{z_{15}} & \frac{s_{16}}{z_{16}} \end{matrix}\right.$$

$$\mathrm{Pf}'\Psi^{ij}=\frac{(-1)^{i+j}}{z_{ij}}\mathrm{Pf}\,\Psi^{ij}$$

$$\mathrm{Pf}\Psi=\sum_{i<j}s^{ij}\text{-}s^{ij}\cdot\ \mathrm{Pf}'\Psi^{ij}=\frac{(-1)^{i+j}}{z_{ij}}\mathrm{Pf}\,\Psi^{ij}$$

$$\int\frac{\mathrm{Pf}\Psi\quad(\mathrm{Koba})}{z_{12}z_{23}\cdots z_{1n\text{-}1\,2n}z_{2n\,1}}=\sum_{j=2}^{2n}\frac{s^{1j}-s^{1j}}{}\int\frac{\mathrm{pf}\Psi^{1j}\,\mathrm{koba}}{z_{12}z_{23}\cdots z_{2n\,1}}$$

$$\ \ \|$$

$$=-\left(\sum s^{1j}-1\right)\mathrm{SYM}\quad -\ \mathrm{SYM}$$

$$=\left(1+s^{12}\right)\mathrm{SYM}$$

$\| \\ 0$

!

$\langle V^{-1} V^{-1} V^{0} \rangle$

$$= \frac{\varepsilon_1 \varepsilon_2}{z_{12}^2}\left[-\frac{k_1\varepsilon_3}{z_{13}} - \frac{k_2\varepsilon_3}{z_{23}}\right] + \frac{1}{z_{12}}\left(\frac{\varepsilon_1\cdot\varepsilon_3}{z_{13}}\frac{\varepsilon_2\cdot k_3}{z_{23}} - \frac{\varepsilon_2\varepsilon_3}{z_{23}}\frac{\varepsilon_1 k_3}{z_{13}}\right)$$

$$(i\varepsilon_1 \dot{X} - \varepsilon_1\psi\, k_1\psi)e^{ik_1X}$$

$$\frac{p_1\psi e^{ip_1X(v-u)}\, p_2\psi e^{ip_2X(v)}}{u = z_{12}} \qquad \psi\cdot\psi = \qquad u = z_2 - z_1,\ v = z_2$$

$$\frac{p_1\psi\, p_2\psi\, e^{ik_1X(v)}}{\varepsilon_1\psi\,(k_1-\varepsilon_1)\psi\, e^{ik_1X}} + \frac{(p_1\cdot p_2)(-ip_1)\partial X e^{ik_1X}}{\frac{s_{12}}{2}(-i\varepsilon_1)\partial X e^{ik_1X}}$$

$1 = -2-3-4$

$$(i\varepsilon_1 \dot{X} + 2\alpha' k_1\psi\, \varepsilon_1\psi)e^{ik_1X}\ \varepsilon_2\psi e^{ik_2X}\ \varepsilon_3\psi e^{ik_3X}$$

Picture 12

$$[\varepsilon_1 \psi e^{ik_1X} e^{-\phi})(\varepsilon_2 \psi e^{ik_2X} e^{-\phi})[(i\varepsilon_3 \dot{X} + 2\alpha' k_3 \psi \varepsilon_3 \psi) e^{ik_3X}]$$

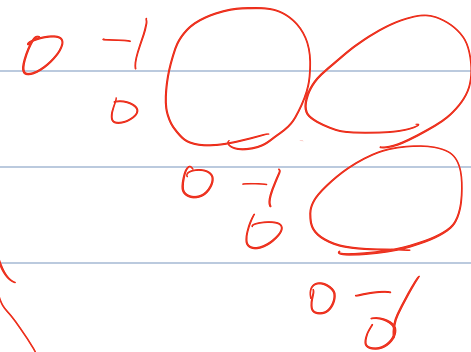
GW

$$(i\varepsilon_1 \dot{X} - \varepsilon_1 \psi k_1 \psi) e^{ik_1X} (i\varepsilon_2 \dot{X} - \varepsilon_2 \psi k_2 \psi) e^{ik_2X} (i\varepsilon_3 \dot{X} - \varepsilon_3 \psi k_3 \psi) e^{ikX}$$

$$i\varepsilon_1 \dot{X} e^{ik_1X}\ i\varepsilon_2 \dot{X} e^{ik_2X}\ i\varepsilon_3 \dot{X} e^{ik_3X}$$

$$+\ \varepsilon_1 \psi k_1 \psi e^{ik_1X}\ \varepsilon_2 \psi k_2 \psi e^{ik_2X}\ i\varepsilon_3 \dot{X} e^{ik_3X}$$

+ Permutation



$$|Z_{ij} - \theta_i \theta_j|^{\alpha' s_{ij}}$$

$$|Z_{ij} - \theta_i \theta_j|^{\alpha' s_{ij}+1} \quad |Z_{ij} - \theta_i \theta_j|^{-1}$$

$$\frac{1}{Z_{ij}}\left(1 + \frac{\theta_{ij}}{Z_{ij}}\right)$$

$P_2 - P_1 = \varepsilon_1$

$P_1 + P_2 = k_1$

$$S_{14} = \varepsilon_1 \cdot k_2 - \varepsilon_1 \cdot \varepsilon_2$$

$$S_{24} = k_1 \cdot k_2 - \varepsilon_1 \cdot k_2 - \varepsilon_2 \cdot k_1 + \varepsilon_1 \varepsilon_2$$

$\cancel{(\varepsilon_1 + k_1)}$

$(k_1 - \varepsilon_1) \cdot (k_2 + \varepsilon_2)$

$(k_1 \cdot \varepsilon_2)(k_2 \cdot \varepsilon_1)$ $\quad i\varepsilon_3 \partial X e^{ik_3X}$

$e^{ik_1X} \quad e^{ik_2X} \quad i\varepsilon_2 \partial X e^{ik_1X}$

$(\theta k_1 \psi + \phi \varepsilon \psi) \quad (\theta k_2 \psi +$

$S_{13} S_{24} - S_{14} S_{23}$

$$\left(\theta k \psi \, \phi \varepsilon \psi + \theta \phi \varepsilon \dot{X}\right) e^{ikX}$$

$$k \psi_1 \varepsilon_1 \psi_1 e^{ik_1X_1} \varepsilon_2 \dot{X}_2 e^{ik_2X_2} \varepsilon_3 \dot{X}_3 e^{ik_3X_3}$$

$$\left(k_1\psi \, \varepsilon_1\psi + i\varepsilon_1 \dot{X} e^{ik_1X}\right)\left(k_2\psi \, \varepsilon_2\psi + i\varepsilon_2 \dot{X} e^{ik_2X}\right)$$

$$\left(k_3\psi\varepsilon_3\psi + i\varepsilon_3\dot{X}\right) e^{ik_3X}$$

$$V(\varepsilon,k)=\int d\phi\, d\theta\; e^{ikX+\theta\phi\,\varepsilon\cdot\dot X+\theta k\cdot\psi-\phi\varepsilon\cdot\psi}$$

$$=\int d\phi\, d\theta\; e^{ikX}(1+\theta\phi\,\varepsilon\cdot\dot X)(1+\theta k\cdot\psi)(1-\phi\,\varepsilon\cdot\psi)$$

$$(-(\theta k\psi)(\phi\,\varepsilon\cdot\psi)+\theta\phi\,\varepsilon\cdot\dot X)e^{ikX}$$

?

$\varepsilon=k_1$

$k=k_1+k_2$

Res $$\frac{e^{ik_1X+\theta_1k_1\psi}\,e^{ik_2X+\theta_2k_2\psi}}{\cancel{e^{ik_1X}e^{ik_2X}(1+\theta k_1\psi)z_{12}}}$$

$$\frac{e^{ik_1X}(1+\theta_1k_1\psi)\,e^{ik_2X}(1+\theta_2k_2\psi)}{z_{12}}$$

$$(k_1\cdot k_2)-ik_1\partial X\,e^{i(k_1+k_2)X}$$

$u=z_2-z_1$

$v=z_2$

$$e^{i(k_1+k_2)X(z_2)}+$$

$z_1=v-u,$

$z_2=v$

$$\frac{e^{ik_1X(v-u)}(1+\theta_1k_1\psi(v-u))e^{ik_2X(v)}(1+\theta_2k_2\psi(v))}{u}$$

$k_1\cdot k_2$

$$\left(e^{ik_1X(v)}-ik_1\partial X(v)\,e^{ik_1X(v)}u\right)$$

$$\left(1+\theta_1k_1\psi(v)-u\theta_1k_1\partial\psi(v)\right)$$

1, Bosonic 我们能 tachyon 出发 ?

2. $\underbrace{p^4 e^{ikx} \quad p^4 e^{ikx}}_{\text{Intergrand}}$ $\quad \underline{\alpha' \to 0 ?}$?

3, zero structure

是否需要 Associahedron 才能 when zero structure