

1. (10%) What is the inverse of the following real matrix?

$$\begin{pmatrix} r^2 - 2 & -r & 1 \\ -r & 1 & 0 \\ 3 - r^2 & r & -1 \end{pmatrix}$$

2. (10%) What is the smallest positive integer k such that

$$A^k = A$$

holds for any n -by- n diagonalizable complex matrix A whose eigenvalues are 0 and 1.

3. (10%) Let (x, y) be a point in \mathbb{R}^2 that is not the origin $(0, 0)$. Let L be the line of \mathbb{R}^2 passing (x, y) and $(0, 0)$. What is the matrix representation of the orthogonal projection of \mathbb{R}^2 on L with respect to the standard basis of \mathbb{R}^2 ?

4. (10%) Define function $f : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}$ as

$$f(x, y) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2,$$

for any vector x (respectively, y) with standard coordinate (x_1, x_2) (respectively, (y_1, y_2)). Let

$$u = (1, 0).$$

Find a vector v such that

$$f(x, y) = s_1t_1 + s_2t_2,$$

where (s_1, s_2) (respectively, (t_1, t_2)) is the coordinate of x (respectively, y) with respect to the ordered basis of \mathbb{R}^2 consisting of u and v .

5. (10%) Find the pseudo-inverse of

$$\frac{1}{6} \begin{pmatrix} 1 & 1 & 1 \\ 3 & 0 & -3 \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

見背面

題號： 414
科目：數學
節次： 4

國立臺灣大學 106 學年度碩士班招生考試試題

題號： 414
共 3 頁之第 2 頁

6. (10%) For $|A| = m$, how many relations on A are neither reflexive nor irreflexive.

7. (10%) Please fill in the blanks:

$$[(p \vee q) \wedge (\neg p \vee r)] \Rightarrow (\underline{\quad} \vee \underline{\quad}).$$

8. (10%) The solution to the recurrence equation

$$a_{n+2} = a_{n+1} + a_n$$

is of the form:

$$a_n = \left(\frac{A}{2\sqrt{5}}\right) \left(\frac{B}{2}\right)^n + \left(\frac{C}{2\sqrt{5}}\right) \left(\frac{D}{2}\right)^n.$$

Derive

$$A = \underline{\quad}$$

$$B = \underline{\quad}$$

$$C = \underline{\quad}$$

$$D = \underline{\quad}$$

in terms of (arbitrary initial conditions) a_0 and a_1 .

9. (5%) Prove that $n-1$ and n are relatively prime for $n \geq 2$.

10. (5%) A graph is _____ if and only if all its cycles have an even length.

11. (5%) For odd n , simplify

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{\frac{n-1}{2}}.$$

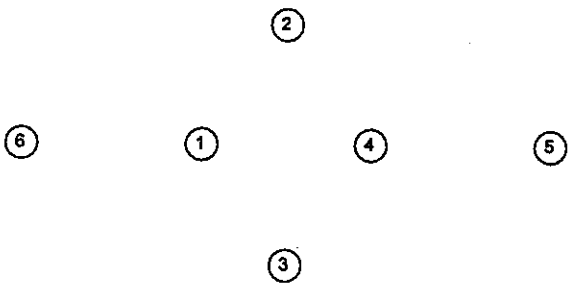
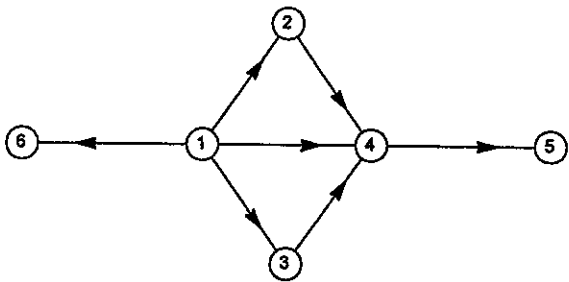
12. (5%) Consider the following directed graph (first plot). Draw its transitive closure in the second plot.

接次頁

題號： 414
科目：數學
節次： 4

國立臺灣大學 106 學年度碩士班招生考試試題

題號： 414
共 3 頁之第 3 頁



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