

# 國立中興大學

110 學年度

碩士班考試入學招生

## 試 題

學系：資訊科學與工程學系  
甲組

科目名稱：基礎數學 A



本科目 **不可以** 使用計算機

本科目試題共 2 頁

## PART I

## 一. 選擇題 (單選或多重選擇, 每題二分)

- Let  $A=\{a,b,c\}$   $B=P(A)$ , the power set of  $A$ . Which of the following properties is correct?  
(a)  $\phi \subseteq B$  (b)  $\phi \in B$  (c)  $A \in B$  (d)  $A \subseteq B$  (note:  $\phi$  is the empty set)
- Let  $R$  be an equivalence relation on a set  $A$ . Both  $a$  and  $b$  are elements of  $A$  and are equivalent.  $[a]$  stands for the equivalence class of  $a$ . Which of the followings is correct?  
(a)  $aRb$  (b)  $bRa$  (c)  $[a] = [b]$  (d)  $[a] \cap [b] \neq \phi$
- Which of the following propositions is equivalent to " $p \rightarrow q$ "?  
(a)  $\neg q \rightarrow \neg p$  (b)  $\neg p \vee \neg q$  (c)  $\neg(q \wedge p)$  (d)  $\neg p \vee q$
- Which of the following numbers is a primitive root in  $Z_{13}$ ?  
(a) 2 (b) 3 (c) 5 (d) 7
- Which of the following numbers is prime?  
(a)  $2^6 - 1$  (b)  $2^7 - 1$  (c)  $2^8 - 1$  (d)  $2^9 - 1$
- What is the value of the postfix expression " $7\ 2\ 3\ * - 4\ ^ 9\ 3\ / +$ ", where  $^$  stands for exponentiation?  
(a) 3 (b) 4 (c) 8 (d) 12
- Let  $T$  be a full  $m$ -ary tree, which a node has either 0 or  $m$  child nodes, with  $n$  vertices. If  $m=3$  and  $n=100$ , which of the following statements about this tree is correct?  
(a) there are 98 edges (b) there are 33 internal vertices (c) the height of  $T$  is 4 (d) there are 66 leaf nodes
- Which of following sets is countable?  
(a)  $(0,1)$  (b)  $Z^+$  (c)  $\{N, Z, Q\}$  (d)  $P(N)$ , the power set of natural numbers
- Let  $P(n)$  be a propositional function. Which of the following statements is enough to verify  $P(n)$  is true for all positive integers  $n$ ?  
(a)  $P(k)$  is true for large  $k$  (b)  $P(1) \wedge [\forall k > 1 (P(k) \rightarrow P(k+1))]$  is true (c)  $P(1) \wedge [\forall k > 1 (P(1) \wedge \dots \wedge P(k) \rightarrow P(k+1))]$  is true (d)  $P(1) \wedge P(n)$  is true for some  $n > 1$
- Which of the following graphs is planar?  
(a) complete,  $K_4$  (b) 3-cube,  $Q_3$  (c) complete bipartite,  $K_{3,3}$  (d) wheel,  $W_4$

## 二. 是非題 (每題二分, 答錯倒扣一分)

- The incidence matrix for representation of any simple graph is a symmetric matrix.
- The cardinality of  $Q$  is the same as the cardinality of  $Z$ .
- Among 100 people there are at least 9 who were born in the same month.
- $(P(S), \subseteq)$  is a partially ordered set, where  $P(S)$  is a power set of  $S=\{1,2,4\}$ .
- " $\neg p \rightarrow q$ " is logically equivalent to " $\neg(q \leftrightarrow p)$ ", where  $\neg$  stands for "not".
- There are 81 ways to put 4 distinguishable balls into 3 different boxes.
- Traveling salesman problem is the problem to find a Euler circuit of least cost.
- There are 1250 positive integers less than 100000 having the sum of their digits equal to 12.
- The least number of colors needed for coloring a planar graph is no longer than 5.
- A simple weighted graph is connected if and only if it has a minimum spanning tree.

## 三. 計算題 (每題五分)

- Over the set of  $\{1,2,3,4,5,6\}$ , what is the next permutation in lexicographic order after 326541?
- How many one-to-one functions are there from a set with 5 elements to a set with 7 elements?



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## PART II

一、是非題(每題二分，答錯倒扣一分)

1. A square matrix  $A$  is called skew-symmetric if  $A^t = -A$ . If  $B$  is a square matrix, then  $B - B^t$  is skew-symmetric.
2. If  $a, b, c, \dots, i \in \mathbb{R}$ , then  $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \neq \det \begin{bmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{bmatrix}$ .
3.  $p_1 = 6 - x^2$  and  $p_2 = 1 + x + 4x^2$  are linear independent in  $P_2$ .
4. The set of vectors  $(1, 6, 4)$ ,  $(2, 4, -1)$  and  $(-1, 2, 5)$  is a basis for a vector space  $\mathbb{R}^3$ .
5. Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the rotation of  $\mathbb{R}^2$  through the angle  $\pi/4$ .  $T$  has rank=2 and nullity=1.
6. If  $A$  and  $B$  are similar matrices, then  $\det(A) = \det(B)$ .
7. A square matrix  $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$  is diagonalizable.
8. If  $A$ ,  $B$  and  $C$  are three square matrices, then  $\text{Trace}(ABC) = \text{Trace}(ACB)$ .
9. If  $A$  is an  $n \times n$  matrix, then the sum of the multiplicities of the eigenvalues of  $A$  equals  $n$ .
10. The matrix  $A = \begin{bmatrix} 2 & -1 & -3 \\ -1 & 2 & 4 \\ -3 & 4 & 9 \end{bmatrix}$  is not positive definite.

二、選擇題(單選或多重選擇，每題四分)

1. Which of the following are elementary matrices?  
(a)  $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$  (b)  $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$  (c)  $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$  (d)  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$
2. Which of the following are linear combinations of  $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$  and  $C = \begin{bmatrix} 4 & -2 \\ 0 & -2 \end{bmatrix}$ ?  
(a)  $\begin{bmatrix} 6 & 3 \\ 0 & 8 \end{bmatrix}$  (b)  $\begin{bmatrix} -1 & 7 \\ 5 & 1 \end{bmatrix}$  (c)  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$  (d)  $\begin{bmatrix} 6 & -1 \\ -8 & -8 \end{bmatrix}$
3. Which of the following are Hermitian matrices?  
(a)  $\begin{bmatrix} 1 & 1+i \\ 1-i & -3 \end{bmatrix}$  (b)  $\begin{bmatrix} 0 & i \\ i & 2 \end{bmatrix}$  (c)  $\begin{bmatrix} i & i \\ -i & i \end{bmatrix}$  (d)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
4. Which of the following are the singular values of  $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ ?  
(a) 1 (b)  $\sqrt{2}$  (c)  $\sqrt{3}$  (d)  $\sqrt{6}$
5. Suppose that  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is a linear transformation such that  $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$  and  $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ .  
 $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) + T\left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right) = ?$   
(a)  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$  (b)  $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$  (c)  $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$  (d)  $\begin{bmatrix} 6 \\ 2 \end{bmatrix}$

三、計算題(十分)

1. Let  $A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$ , express  $A$  in the form  $A = LDU$ , where  $L$  is lower triangular with 1's along the main diagonal,  $U$  is upper triangular, and  $D$  is a diagonal matrix.