1. (10%) Solve for the unknown y in the following system of linear equations.

$$\begin{aligned} 4x + y + z + w &= 6 \\ 3x + 7y - z + w &= 1 \\ 7x + 3y - 5z + 8w &= -3 \\ x + y + z + 2w &= 3 \end{aligned}$$

2. (10%) Find the area of the triangle that has the vertices $P_1(2, 2, 0)$, $P_2(-1, 0, 2)$, and $P_3(0, 4, 3)$.

3. (10%) Find the coordinate vector of $\mathbf{v} = (5, -12, 3)$ relative to the basis $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for R^3, where $\mathbf{v}_1 = (1, 2, 3)$, $\mathbf{v}_2 = (-4, 5, 6)$, and $\mathbf{v}_3 = (7, -8, 9)$.

4. (10%) Find the eigenvalues and corresponding eigenvectors of

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}.$$

5. (10%) Let W be the intersection of the two planes

$$x - y - z = 0 \quad \text{and} \quad x + 2y + z = 0$$

in R^3. Find an equation for the orthogonal complement of W.

6. Find a counterexample, if possible, to these universally quantified statements, where the universe of discourse for all variables of all real numbers.
   a) (3%) $\forall x(x^2 \neq x)$
   b) (3%) $\forall x(x^2 \neq 2)$
   c) (3%) $\forall x(|x| > 0)$

7. (6%) Show that at least 3 of any 25 days chosen must fall in the same month of the year.

8. (10%) How many numbers must be selected from the first 15 positive integers to guarantee that at least two pairs of these numbers add up to 15?

9. Answer the following question about graph: (2% for each blank)
   a) List all positive integers n such that cycle C_n is bipartite: ______.
   b) The incidence matrix for wheel W_n has ______rows and ______columns.
   c) List all positive integers m and n such that complete bipartite graph $K_{m,n}$ has a Hamilton circuit: ______.
   d) There are ______ non-isomorphic simple graphs with 3 vertices.

10. For the statement that a postage of n cents can be formed using just 4-cent and 11-cent stamps for $n \geq 30$,
    a) (4%) prove by **principle of mathematical induction**.
    b) (4%) prove by **strong induction**.

11. Consider the recurrence relation $a_n = 2a_{n-1} + 3n$.
    a) (1%) Write the associated homogeneous recurrence relation.
    b) (1%) Find the general solution to the associated homogeneous recurrence relation.
    c) (2%) Find a particular solution to the given recurrence relation a_n.
    d) (1%) Write the general solution to the given recurrence relation a_n.
    e) (2%) Find the particular solution to the given recurrence relation a_n when $a_0 = 1$.