Number Systems

When we type some letters or words, the computer translates them in numbers as computers can understand only numbers. A computer can understand positional number system where there are only a few symbols called digits and these symbols represent different values depending on the position they occupy in the number.

A value of each digit in a number can be determined using

- The digit
- The position of the digit in the number
- The base of the number system (where base is defined as the total number of digits available in the number system).

Decimal Number System

The number system that we use in our day-to-day life is the decimal number system. Decimal number system has base 10 as it uses 10 digits from 0 to 9. In decimal number system, the successive positions to the left of the decimal point represent units, tens, hundreds, thousands and so on.

Each position represents a specific power of the base (10). For example, the decimal number 1234 consists of the digit 4 in the units position, 3 in the tens position, 2 in the hundreds position, and 1 in the thousands position, and its value can be written as

$$(1x1000)+ (2x100) + (3x10) + (4x1)$$

$$(1x10^3)+ (2x10^2) + (3x10^1) + (4x10^0)$$

$$1000 + 200 + 30 + 4$$

$$1234$$

As a computer programmer or an IT professional, you should understand the following number systems which are frequently used in computers.

S.N.	Number System and Description
1	**Binary Number System** Base 2. Digits used : 0, 1
2	**Octal Number System** Base 8. Digits used : 0 to 7
3	**Hexa Decimal Number System** Base 16. Digits used : 0 to 9, Letters used : A- F

Binary Number System

Characteristics of binary number system are as follows:

- Uses two digits, 0 and 1.
- Also called base 2 number system
- Each position in a binary number represents a 0 power of the base (2). Example 2^0
- Last position in a binary number represents a x power of the base (2). Example 2^x where x represents the last position - 1.

Example

Binary Number : 10101_2

Calculating Decimal Equivalent:

Step	Binary Number	Decimal Number
Step 1	10101_2	$((1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	10101_2	$(16 + 0 + 4 + 0 + 1)_{10}$
Step 3	10101_2	21_{10}

Note : 10101_2 is normally written as 10101.

Hexadecimal Number System

Characteristics of hexadecimal number system are as follows:

- Uses 10 digits and 6 letters, 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
- Letters represents numbers starting from 10. A = 10. B = 11, C = 12, D = 13, E = 14, F = 15.
- Also called base 16 number system
- Each position in a hexadecimal number represents a 0 power of the base (16). Example 16^0
- Last position in a hexadecimal number represents a x power of the base (16). Example 16^x where x represents the last position - 1.

Example

Hexadecimal Number: $19FDE_{16}$

Calculating Decimal Equivalent:

Step	Binary Number	Decimal Number
Step 1	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (F \times 16^2) + (D \times 16^1) + (E \times 16^0))_{10}$
Step 2	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (15 \times 16^2) + (13 \times 16^1) + (14 \times 16^0))_{10}$
Step 3	$19FDE_{16}$	$(65536+ 36864 + 3840 + 208 + 14)_{10}$
Step 4	$19FDE_{16}$	106462_{10}

Note : $19FDE_{16}$ is normally written as 19FDE.

Decimal to Other Base System

- **Step 1 -** Divide the decimal number to be converted by the value of the new base.
- **Step 2 -** Get the remainder from Step 1 as the rightmost digit (least significant digit) of new base number.
- **Step 3 -** Divide the quotient of the previous divide by the new base.
- **Step 4 -** Record the remainder from Step 3 as the next digit (to the left) of the new base number.

Repeat Steps 3 and 4, getting remainders from right to left, until the quotient becomes zero in Step 3.

The last remainder thus obtained will be the most significant digit (MSD) of the new base number.

Example

Decimal Number: 29_{10}

Calculating Binary Equivalent:

Step	Operation	Result	Remainder
Step 1	29 / 2	14	1
Step 2	14 / 2	7	0
Step 3	7 / 2	3	1
Step 4	3 / 2	1	1
Step 5	1 / 2	0	1

As mentioned in Steps 2 and 4, the remainders have to be arranged in the reverse order so that the first remainder becomes the least significant digit (LSD) and the last remainder becomes the most significant digit (MSD).

Decimal Number : 29_{10} = Binary Number : $11101_{2.}$

Other base system to Decimal System

- **Step 1 -** Determine the column (positional) value of each digit (this depends on the position of the digit and the base of the number system).
- **Step 2 -** Multiply the obtained column values (in Step 1) by the digits in the corresponding columns.
- **Step 3 -** Sum the products calculated in Step 2. The total is the equivalent value in decimal.

Example

Binary Number : 11101_2

Calculating Decimal Equivalent:

Step	Binary Number	Decimal Number
Step 1	11101_2	$((1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	11101_2	$(16 + 8 + 4 + 0 + 1)_{10}$
Step 3	11101_2	29_{10}

Binary Number: 11101_2 = Decimal Number : 29_{10}

Shortcut method - Binary to Hexadecimal

Steps

- **Step 1 -** Divide the binary digits into groups of four (starting from the right).
- **Step 2 -** Convert each group of four binary digits to one hexadecimal symbol.

Example

Binary Number : 10101_2

Calculating hexadecimal Equivalent:

Step	Binary Number	Hexadecimal Number
Step 1	10101_2	0001 0101
Step 2	10101_2	1_{10} 5_{10}
Step 3	10101_2	15_{16}

Binary Number : 10101_2 = Hexadecimal Number : 15_{16}

Shortcut method - Hexadecimal to Binary

steps

- **Step 1 -** Convert each hexadecimal digit to a 4 digit binary number (the hexadecimal digits may be treated as decimal for this conversion).
- **Step 2 -** Combine all the resulting binary groups (of 4 digits each) into a single binary number.

Example

Hexadecimal Number : 15_{16}

Calculating Binary Equivalent:

Step	Hexadecimal Number	Binary Number
Step 1	15_{16}	1_{10} 5_{10}
Step 2	15_{16}	0001_2 0101_2
Step 3	15_{16}	00010101_2

Hexadecimal Number: 15_{16} = Binary Number: 10101_2