SICP Exercise 1.14

Yuchong Pan

August 3, 2016

Since the interpreter keeps track of a chain of deferred operations, and since either the amount or the number of the types of the coins is decremented by a constant each time, the space used by the process grows as $\Theta(n)$.

Since the value of (cc a 1) equals the value of (cc ($-$ a 1) 1) plus the value of (cc a 0), and since evaluating (cc a 0) requires $\Theta(1)$ steps, then we have

$$R_1(n) = R_1(n-1) + \Theta(1)$$

which yields

$$R_1(n) = \Theta(n)$$

Similarly, we have that the value of (cc a 2) equals the value of (cc ($-$ a 5) 2) plus the value of (cc a 1) and that evaluating (cc a 1) requires $\Theta(n)$ steps. Hence, we have

$$R_2(n) = R_2(n-5) + \Theta(n)$$

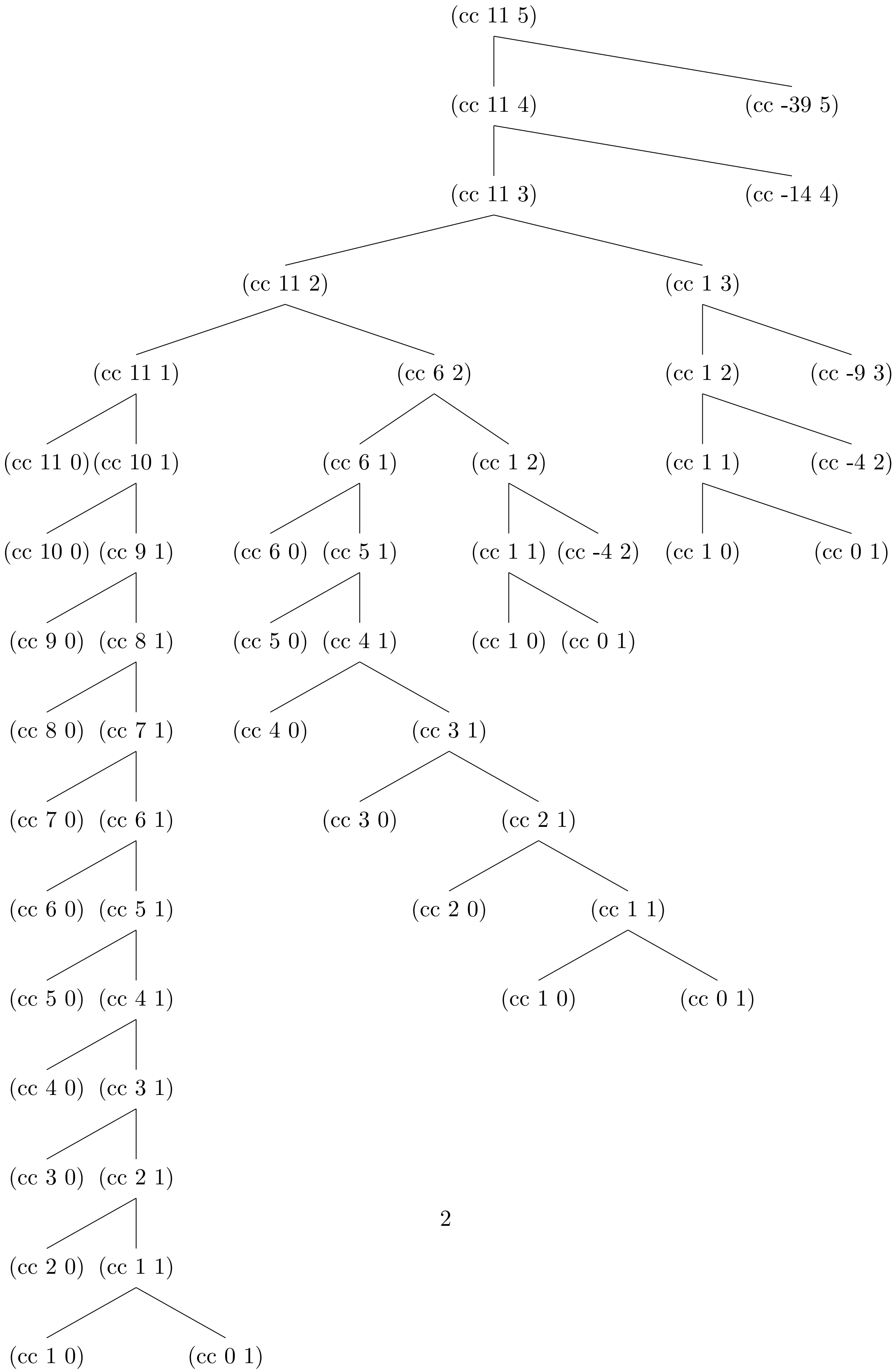
which yields

$$R_2(n) = \Theta(n^2)$$

By analogy, we can deduce that

$$R_5(n) = \Theta(n^5)$$

In a nutshell, the process requires $\Theta(n^5)$ steps and space $\Theta(n)$.


(cc 11 5)
(cc 11 4)
(cc -39 5)
(cc 11 3)
(cc -14 4)
(cc 11 2)
(cc 1 3)
(cc 11 1)
(cc 6 2)
(cc 1 2)
(cc -9 3)
(cc 11 0)
(cc 10 1)
(cc 6 1)
(cc 1 2)
(cc 1 1)
(cc -4 2)
(cc 10 0)
(cc 9 1)
(cc 6 0)
(cc 5 1)
(cc 1 1)
(cc -4 2)
(cc 1 0)
(cc 0 1)
(cc 9 0)
(cc 8 1)
(cc 5 0)
(cc 4 1)
(cc 1 0)
(cc 0 1)
(cc 8 0)
(cc 7 1)
(cc 4 0)
(cc 3 1)
(cc 7 0)
(cc 6 1)
(cc 3 0)
(cc 2 1)
(cc 6 0)
(cc 5 1)
(cc 2 0)
(cc 1 1)
(cc 5 0)
(cc 4 1)
(cc 1 0)
(cc 0 1)
(cc 4 0)
(cc 3 1)
(cc 3 0)
(cc 2 1)
(cc 2 0)
(cc 1 1)
(cc 1 0)
(cc 0 1)