TAOCP Section 1.1 Exercises

Yuchong Pan

December 16, 2017

1. $t \leftarrow a,\ a \leftarrow b,\ b \leftarrow c,\ c \leftarrow d,\ d \leftarrow t.$

2.

Proof. Consider 2 cases, $m > n$ and $m \leq n$ for original inputs.
For the first case (i.e., $m > n$ originally), we have $0 \leq r < n$ after step E1.
If $r = 0$, then the algorithm terminates in step E2.
Otherwise, since $r < n$, we have $n < m$, or $m > n$, after step E3.
For the second case (i.e., $m \leq n$ originally), we have $0 \leq r \leq m$ after E1.
If $r = 0$, then $m = n$ and the algorithm terminates in step E2.
Otherwise, $r = m < n$, so we have $n < m$, or $m > n$, after step E3.
Therefore, step E3 always gives $m > n$, so m is always n at the beginning of step E1, except possibly the first time this step occurs. □

3. Algorithm F. Given two positive integers m and n, find their greatest common divisor, i.e., the largest positive integer that evenly divides both m and n.
F1. [Find remainder.] Divide m by n and let r be the remainder. Set $m \leftarrow r$. (We will have $0 \leq m < n$.)
F2. [Does m equal zero?] If $m = 0$, the algorithm terminates; n is the answer.
F3. [Exchange.] Exchange $m \leftrightarrow n$. (We will have $m > n > 0$.) ■

4. Let $m = 2166$, and let $n = 6099$.

m	n	r
2166	6099	2166
6099	2166	1767
2166	1767	399
1767	399	171
399	171	57
171	57	0

Therefore, 57 is the greatest common divisor of 2166 and 6099.

5. The ”Procedure for Reading This Set of Books” fails to meet the following three features of algorithms.

- Finiteness: as shown in the flow chart, the procedure never terminates because readers will return to Chapter 1 after they finish all of the 12 chapters.
- Output: the procedure does not have an output that is returned to readers.

- Effectiveness: some steps of the procedure are not effective; e.g., "work exercises" is not effective to most readers because they may be stuck in some exercise problems (forever).

6. As stated in the text, only the remainder of m after division by n is relevant.
Therefore, we will find T_5 by trying the algorithm for $m = 1, 2, 3, 4, 5$.

- $m = 1$.

#	m	n	r
1	1	5	1
2	5	1	0

- $m = 2$.

#	m	n	r
1	2	5	2
2	5	2	1
3	2	1	0

- $m = 3$.

#	m	n	r
1	3	5	3
2	5	3	2
3	3	2	1
4	2	1	0

- $m = 4$.

#	m	n	r
1	4	5	4
2	5	4	1
3	4	1	0

- $m = 5$.

#	m	n	r
1	5	5	0

Therefore, $T_5 = \frac{2+3+4+3+1}{5} = \frac{13}{5}$.

7. Consider $m < n$.
Since m is fixed, the contributions of the cases for which $m \geq n$ are negligible to U_m.
Let r be the remainder of m after division by n, and we have $r = m$.
After the first execution of Algorithm E, we set $m' \leftarrow n,\ n' \leftarrow r = m$.
Let r' be the remainder of m' after division by n'.
Since $m' = n$ and $n' = m$, then r' is the remainder of n after division by m.
After the second execution of Algorithm E, we set $m'' \leftarrow n' = m,\ n'' \leftarrow r'$.
Therefore, after the first two executions of Algorithm E, only the remainder of n after division by m is relevant, so we can find U_m by trying the algorithm for $n = 1, 2, \ldots, m$.
Hence, U_m is well defined.
Recall that after the first execution of Algorithm E, we set $m' \leftarrow n,\ n' \leftarrow m$.
Since m is known and n is allowed to range over all positive integers, then n' is known and m' is allowed to range over all positive integers.
Therefore, we have $U_m = T_{n'} + 1 = T_m + 1$.

8. Let $N = 2$.
Define $\theta_j, \phi_j, a_j, b_j$ for $0 \leq j < N$ as follows.

j	θ_j	ϕ_j	a_j	b_j
0	$a^m b^{m+1}$	$a^m b$	1	0
1	$a^{n+1} b^n$	ab^n	2	0

9. Let $C_1 = (Q_1, I_1, \Omega_1, f_1)$ and $C_2 = (Q_2, I_2, \Omega_2, f_2)$ be computational methods.
Suppose that there exists a surjective function $F\colon\ Q_2 \to Q_1$ such that for all $q_1, q_2 \in Q_2$ with $f_2(q_1) = q_2$, we have either $F(q_1) = F(q_2)$ or $f_1(F(q_1)) = F(q_2)$.
Suppose furthermore that for all $i \in I_2$, $F(i) \in I_1$, and for all $\omega \in \Omega_2$, $F(\omega) \in \Omega_1$.
Then we say that "C_2 is a representation of C_1" or "C_2 simulates C_1".