Princeton University

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ECO 518 – Econometric Theory II, Spring 2022

Problem Set 4, due February 22

1. Suppose $y_t = \mu + u_t$, and $u_t = \rho u_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim iid(0, \sigma^2)$ and $|\rho| < 1$.

(a) Derive the large sample distribution of $\sqrt{T}(\bar{y} - \mu)$, where $\bar{y} = T^{-1} \sum_{t=1}^{T} y_t$.

(b) Find the GLS estimator $\hat{\mu}_{GLS}$ of μ, and derive the large sample distribution of $\sqrt{T}(\hat{\mu}_{GLS} - \mu)$.

2. Continuation of question 7 from the last problem set: You are asked to construct a 95% confidence interval for the (population) mean of the interest rate spread based on:

(a) An estimator that averages the first 3 periodogram coordinates (not counting frequency zero), and a corresponding confidence interval that uses the student-t approximation discussed in the lecture notes (part (e) of PS 3).

(b) A feasible GLS estimator that models the spread as an AR(12) (and a constant).

3. Consider a stationary VMA(q) process, that is $y_t = \varepsilon_t + \Theta_1 \varepsilon_{t-1} + \cdots + \Theta_q \varepsilon_{t-q}$ with $\varepsilon_t \sim iid(0, \Sigma)$, and Σ and Θ_j are $n \times n$ matrices. Derive the autocovariance function $\Gamma(k) = E[y_t y'_{t-k}]$.

4. Show that a VAR(p) $y_t = \Phi_1 y_{t-1} + \cdots + y_{t-p} \Phi_p + \varepsilon_t$ with $\varepsilon_t \sim iid(0, \Sigma)$ and Σ, Φ_j $n \times n$ matrices can be written as $y_t = S' Z_t$, where $Z_t = A Z_{t-1} + e_t$ is a VAR(1) of suitable dimension and S is a selection matrix. [Hint: Recall the companion form of an AR(p)]. What condition on A is needed for the stationarity of the VAR(p)?

5. In the file IncomeMoneyGrowth.csv, you find monthly data for US real income growth and M3 growth from 1960:2 to 2019:12 (we exclude the upheaval associated with the Covid period). Consider a structural model that posits that there are two structural shocks, a money supply shock and a technology shock. Using specialized software (such as STATA, R, RATS, Eviews, etc), estimate a structural VAR with 12 lags (that includes an additional

constant for both variables), based on the identification that there is no contemporaneous reaction of income growth to the money supply shock.

(a) Graph the impulse response function for income of the two shocks for a horizon $h = 0, \cdots, 48$.

(b) What is the implication for the structural VAR parameters of an additional assumption that income does not react to the money supply shock for three months (=1 quarter)? Test this restriction in the data on the 5% level. What does your result imply of the plausibility of the "no contemporaneous correlation"-identification for quarterly data?

(c) Perform a 5% level hypothesis test of the restriction which would be implied by the long-run neutrality of money.