MATH 1B :)

MIDTERM #1 REVIEW

- INTEGRATION BY PARTS:
  ↳ $\int u\,dv = uv - \int v\,du$

- REDUCTION FORMULA:
  ↳ $\int \sin^n x\,dx = -\dfrac{\cos x \sin^{n-1} x}{n} + \dfrac{n-1}{n}\int \sin^{n-2} dx$

- SIN/COS TRIG INTEGRALS:
  ↳ sin/cos like being even, if both even ⇒ use ½ angle

- TAN/SEC TRIG INTERALS:
  ↳ tan = ~~even~~ odd? separate $\sec\theta\tan\theta$
  ↳ both = even? separate $\sec^2\theta$

  $\dfrac{\sin x}{\cos x}$ → convert numerator to same terms then separate fractn

- TRIG SUBSTITUTION:
  ↳ $\sqrt{a^2-x^2} \Rightarrow x = a\sin\theta$ $\quad -\pi/2 \le \theta \le \pi/2$
  ↳ $\sqrt{a^2+x^2} \Rightarrow x = a\tan\theta$
  ↳ $\sqrt{x^2-a^2} \Rightarrow x = a\sec\theta$

- PARTIAL FRACTIONS:
  ↳ yay! Don't forget: long division + AX+B stuff

- MIDPOINT RULE:
  ↳ $\int_a^b f(x)\,dx \approx M_n = \left(\dfrac{b-a}{n}\right)\left[f\left(\dfrac{x_0+x_1}{2}\right) + f\left(\dfrac{x_1+x_2}{2}\right) \dots f\left(\dfrac{x_{n-1}+x_n}{2}\right)\right]$

- MIDPOINT ERROR
  ↳ $|E_M| \le \dfrac{k(b-a)^3}{24n^2}$

- TRAPEZOIDAL RULE:
  ↳ $\int_a^b f(x)\,dx \approx T_n = \left(\frac{b-a}{2n}\right)\left[f(x_0)+2f(x_1)+2f(x_2)+\cdots f(x_n)\right]$

- TRAPEZOIDAL ERROR:
  ↳ $|E_T| \leq \frac{k(b-a)^3}{12n^2}$

- SIMPSON'S RULE:
  ↳ $\int_a^b f(x)\,dx \approx S_n = \left(\frac{b-a}{3n}\right)\left[f(x_0)+4f(x_1)+2f(x_2)\cdots 4f(x_{n-1})+f(x_n)\right]$

- SIMPSON'S ERROR:
  ↳ $|E_s| \leq \frac{k(b-a)^5}{180n^4}$

- ARC LENGTH:
  ↳ $\int_a^b \sqrt{1+(dy/dx)^2}\,dx$

- SURFACE AREA OF A ROTATION:
  ↳ ↷ x axis ⇒ $S = \int 2\pi y\,ds = \int 2\pi y \sqrt{1+(dx/dy)^2}\,dy$
  ↳ ↷ y axis ⇒ $S = \int 2\pi x\,ds = \int 2\pi x \sqrt{1+(dy/dx)^2}\,dx$

- HYDROSTATIC PRESSURE:
  ↳ $F = \rho g A h$

- MOMENT:
  ↳ not needed? just used to find centroid

- CENTROID: $x[f(x)-g(x)]$
  ↳ $\bar{x} = 1/A \int_a^b x f(x)\,dx$
  ↳ $\bar{y} = 1/A \int_a^b 1/2\,(f(x))^2\,dx$ $\quad 1/2\left[f(x)^2 - g(x)^2\right]$

- BOUNDED + MONOTONIC SEQUENCE (INC/DEC) ⇒ CONVERGES!

- USE $e^{\ln(\text{blah})}$ W/ UGLY EXPONENTS

- PROBABILITY $\quad$ cdf $= \int$ pdf
  ↳ Probability density function: $P(a \le X \le b) = \int_a^b f(x)\,dx$
  ↳ $\int_{-\infty}^{\infty} f(x)\,dx = 1$
  ↳ Mean $= \mu = \int_{-\infty}^{\infty} x f(x)\,dx$
  ↳ Median $= \int_m^{\infty} f(x)\,dx = 1/2$

- NORMAL DISTRIBUTION:
  ↳ $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$

- HOW TO FIND FORMULA FOR SERIES:
  ↳ ① Make subtraction tree
  ② Takes $1 \Rightarrow an+b$ terms to get constant
  $2 \Rightarrow an^2+bn+c$
  $3 \Rightarrow an^3+bn^2+cn+d$

- HARMONIC SERIES
  ↳ $1/n \rightarrow$ diverges

- $(A^3+B^3) = (A-B)(A^2+AB+B^2)$

TESTS FOR CON/DIV:

- TEST FOR DIVERGENCE:
  ↳ $\lim_{n\to\infty} a_n \neq 0$ diverges!

- GEOMETRIC SERIES:
  ↳ ar^n $|r|<1$ converges, $|r| \geq 1$ diverges, sum $= \frac{a}{1-r}$

- TELESCOPING SERIES:
  ↳ $b_n - b_{n+1}$ converges! or $\to \infty$, diverges!

- P-SERIES:
  ↳ $1/n^p$ $p>1$ converges, $p \leq 1$ diverges

- ALTERNATING SERIES:
  ↳ $(-1)^n a_n$, decreasing, +/-, lim = 0 $|R_N| \leq a_{N+1}$

- INTEGRAL TEST (CONT, ⊕, dec)
  ↳ integrate → converges or diverges

- ROOT TEST:
  ↳ $\wedge 1/n$ $\lim n \to \infty$, <1 conv, >1 div

- RATIO TEST:
  ↳ $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$ <1 conv, >1 div

- DIRECT COMP.
  ↳ smaller than a converger = converges!
  ↳ bigger than a diverger = diverges!

- LIMIT COMP.
  ↳ $\frac{a_n}{b_n} = L$ $L>0$ conv (b_n conv) $L>0$ div (b_n div)

  xPT $\cos\frac{1}{n} \to \frac{1}{n^2}$
  if you see $\frac{1}{n}$, use it to compare!

- expon > poly > log
- $\frac{n!}{n^n} \leq \frac{1}{n}$
- $\lim_{n \to \infty} \left(1 + \frac{a}{n}\right)^n = e^a$
- $\lim_{n \to \infty} \frac{n^c}{a^n} = \infty$ (when $\overset{+}{a} < 1$)
- always keep in mind parameters of Σ / $\int$
- $n^n > n! > a^n > n^c > \ln n$

MATH IB

MIDTERM #2 REVIEW

- INTERVAL OF CONVERGENCE FOR POWER SERIES:
  - ↳ use ratio test!

$$\left|\frac{a_{n+1}}{a_n}\right| < 1$$

[but ✓ endpoints!]

  - if centered @ $x=a$, radius $= \pm R$

- STANDARD FORM:
  - ↳ $f(x) = \sum_{n=0}^{\infty} c_n$

- VARIOUS MACLAURIN SERIES:
  - $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$
  - $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
  - $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
  - $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
  - $\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$
  - $\ln(1+x) = \sum_{n=0}^{\infty} (-1)^{n-1} \frac{x^n}{n}$
  - $(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$
    - ↳ $1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3$

- EASY I.O.C:
  - ↳ get into form $\frac{1}{1-x}$ then $|\text{blah}| < 1$
  - $f'(x), f(x), \int f(x) =$ same radius of convergence

- TAYLOR SERIES REPRESENTATION:
  - ↳ $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

[Maclaurin @ $a=0$]

$|x-a| \leq d$ where $d = x$ you want to find

- TAYLOR'S INEQUALITY:

$$|R_{(n)}(x)| \leq \frac{|x-a|^{n+1}}{(n+1)!} \circ f^{(n+1)}(max)$$

$|x-a| \leq d$

$T_n(x) - R_n(x) = f(x)$

- ALTERNATING SERIES REMAINDER:

↳ S_n = sum of 1^{st} n terms

$|error| = |S - S_n| \leq b_{n+1}$ (aka the next term)

- ORTHAGONAL TRAJECTORIES

↳ $\frac{dy}{dx} \Rightarrow -\frac{dx}{dy}$ ∟

- EULER'S METHOD:

↳ x, y, dy/dx, equation!

- POPULATION GROWTH

↳ $\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$ [equilibrium @ P=0 + P=M]

- MIXING PROBLEMS:

↳ $\frac{dy}{dt}$ = rate in − rate out

- NATURAL GROWTH:

↳ $\frac{dP}{dt} = kP$

$P(0) = P_0$

$P(t) = P_0 e^{kt}$

- LOGISTIC EQUATION:

↳ $\frac{M}{1+Ae^{-kt}} = P(t)$

$A = \frac{M-P_0}{P_0}$

- CONVERT $f(x)$ TO A POWER SERIES
  1. substitute x!
  2. multiply by stuff!
  3. integrate!
  4. differentiate!

MATH 1B ☺

FINAL REVIEW

- 1^{st} ORDER + SEPARABLE D.E.
  ↳ just split up and do separate integrals

- 1^{st} ORDER + LINEAR D.E.
  ↳ $y' + P(x)y = Q(x)$
  ⇒ multiply everything by $I(x) = e^{\int P(x)\,dx}$

- 2^{nd} ORDER + LINEAR + HOMOGENOUS D.E.
  ↳ $ay'' + by' + cy = 0$
  ⇒ use $ar^2 + br + c = 0$
  ↳ r = 2 answers: $y = C_1 e^{r_1 t} + C_2 e^{r_2 t}$
  ↳ r = 1 answer: $y = C_1 e^{r_1 t} + C_2 t e^{r_1 t}$
  ↳ r = 2 complex: $y = C_1 e^{\alpha t} \cos \beta t + C_2 e^{\alpha t} \sin \beta t$

- 2^{nd} ORDER + LINEAR + NONHOMOGENEOUS D.E.
  - UNDETERMINED COEFFICIENTS ↳ $ay'' + by' + cy = G(x)$
    - after solving for homo ⇒ y_c
    - find $y = \square$ that matches the form of $G(x)$
      ↳ ae^x, $a\cos x + a\sin x$, split up if needed
    - find y, y', y'' and sub into original equation + solve for the constants ⇒ y_p
    - final answer ⇒ $y = y_c + y_p$ (check for repetition)

  - VARIATION OF PARAMETERS
    - solve for homo ⇒ y_c
    - $y_p = u_1 y_1 + u_2 y_2$
    - solve the system of equations for:
      - $u_1' y_1 + u_2' y_2 = 0$
      - $u_1' y_1' + u_2' y_2' = G/a$
    - final answer ⇒ $y = y_c + y_p$ (✓ for repetition)

- DE w/ series solution (1st + 2nd ORDER)
  - $y = \sum_{n=0}^{\infty} C_n x^n$
  - $y' = \sum_{n=0}^{\infty} C_{n+1}(n+1)x^n$
  - $y'' = \sum_{n=0}^{\infty} C_{n+2}(n+2)(n+1)x^n$

  ↳ Plug into original equation
  - make everything "x^n"
  - same starting point $\sum$
  - solve for C_0, C_1, C_2, etc.
  - find pattern

- MASS-SPRING

  ↳ $\underset{\text{(massss)}}{my''} + \underset{\text{(damp)}}{cy'} + \underset{\text{(hooke)}}{ky} = \underset{\text{(ext. force)}}{F(t)}$

- ELECTRICAL CIRCUITS

  ↳ $\underset{\text{(inductance)}}{LQ''} + \underset{\text{(resistance)}}{RQ'} + \underset{\text{(elastance)}}{\frac{1}{C}Q} = \underset{\text{(electromotive force)}}{E(t)}$

- PREDATOR-PREY

  ↳ $R' = kR + aRW$, $W' = -rW + bRW$
  - equilibrium solutions @ $dR/dt = dW/dt = 0$

- MIXING PROBLEMS