hw3

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q2

```
Vj <- function(tau, sigma, z, nj)
    (sigma^2) * diag(nj) + (tau^2) * z %*% t(z)

Aj <- function(tau, sigma, beta, x, y, z, q, nj, Vj)
  sum(diag(t(z) %*% z %*% ((tau^2)*diag(q) - (tau^4)*t(z) %*% solve(Vj(tau, sigma, z, nj))%*%z))) + (tau
  t(z) %*% z %*% t(z) %*% solve(Vj(tau, sigma, z, nj)) %*% (y-x %*% beta)

Cj <- function(tau, sigma, beta, x, y, z, q, nj, Vj)
  (tau^4)*t(y-x %*% beta) %*% solve(Vj(tau, sigma, z, nj)) %*%
  z %*% t(z) %*% solve(Vj(tau, sigma, z, nj)) %*% (y-x %*% beta) +
  sum(diag((tau^2)*diag(q)-(tau^4)*t(z) %*% solve(Vj(tau, sigma, z, nj)) %*% z))

Dj <- function(tau, sigma, beta, x, y, z, nj, Vj)
  (tau^2)*z %*% t(z) %*% solve(Vj(tau, sigma, z, nj)) %*% (y-x %*% beta)


em <- function(beta0, tau0, sigma0, X, Y, Z, n, p, q, tol = 1e-5){
  beta_new=beta0
  tau_new=tau0
  sigma_new=sigma0
  N = cumsum(n)
  m <- length(n)

  z = list()
  y = list()
  x = list()
  for (j in 1:m) {
    zs=matrix(nrow = n[j],ncol = q)
    xs=matrix(nrow = n[j],ncol = p)
    ys = matrix(nrow = n[j],ncol = 1)
    if(j==1) {
      for (i in 1:n[1])
        for (k in 1:q)
          zs[i,k]=Z[i,k]
      for (i in 1:n[1])
        for (k in 1:p)
          xs[i,k]=X[i,k]
      for (i in 1:n[1])
          ys[i,1]=Y[i,1]
    }
```

```
  if(j>1) {
    for (i in 1:n[j])
      for (k in 1:q)
        zs[i,k]=Z[(N[j-1]+i),(q*(j-1))+k]
    for (i in 1:n[j])
      for (k in 1:p)
        xs[i,k]=X[(N[j-1]+i),k]
    for (i in 1:n[j])
        ys[i,1]=Y[(N[j-1]+i),1]
  }
  x[[j]] = xs
  z[[j]] = zs
  y[[j]] = ys
}

######E-Step
A=matrix(nrow = m,ncol = 1)
C=matrix(nrow = m,ncol = 1)
for (j in 1:m) {
  A[j] = Aj(tau_new, sigma_new[j], beta_new, x[[j]], y[[j]], z[[j]], q, n[j], Vj)
  C[j] = Cj(tau_new, sigma_new[j], beta_new, x[[j]], y[[j]], z[[j]], q, n[j], Vj)
}
siginv = diag(rep((1/sigma_new)^2,n))
tmp = c()
for (j in 1:m)
  tmp=c(tmp, Dj(tau_new,sigma_new[j],beta_new,x[[j]],y[[j]],z[[j]], n[j], Vj))

######M-Step
beta_hat=solve(t(X)%*%siginv%*%X)%*%t(X)%*%siginv%*%(Y-tmp)
tau_hat=sqrt(sum(C)/(m*q))
sigma_hat=sigma_new
for (j in 1:m)
  sigma_hat[j] = sqrt((A[j]-2*t(y[[j]]-(x[[j]]%*%beta_new)) %*%
                       Dj(tau_new,sigma_new[j],beta_new,x[[j]],y[[j]],z[[j]], n[j], Vj)+t(y[[j]]-(x

while((abs(tau_new-tau_hat) >= tol) || (abs(sigma_new-sigma_hat) >= tol) || (abs(beta_new-beta_hat) >
  beta_new <- beta_hat
  sigma_new <- sigma_hat
  tau_new <- tau_hat
  ######E-Step
  for (j in 1:m) {
    A[j]<-Aj(tau_new, sigma_new[j], beta_new, x[[j]], y[[j]], z[[j]], q, n[j], Vj)
    C[j]<-Cj(tau_new, sigma_new[j], beta_new, x[[j]], y[[j]], z[[j]], q, n[j], Vj)
  }
  tmp = c()
  for (j in 1:m)
      tmp=c(tmp, Dj(tau_new,sigma_new[j],beta_new,x[[j]],y[[j]],z[[j]], n[j], Vj))

  siginv=diag(rep((1/sigma_new)^2,n))
  ######M-Step
  beta_hat <- solve(t(X)%*%siginv%*%X)%*%t(X)%*%siginv%*%(Y-tmp)
  for (j in 1:m)
    sigma_hat[j] = sqrt((A[j]-2*t(y[[j]]-(x[[j]]%*%beta_new))
```

```r
      %*% Dj(tau_new,sigma_new[j],beta_new,x[[j]],y[[j]],z[[j]], n[j], Vj) +t(y[[j]]-(x[[j]]%*%beta_new
    tau_hat=sqrt(sum(C)/(m*q))
  }

  par <- list(bate = beta_hat, tau = tau_hat, sigma = sigma_hat)
  return(par)
}
```

```r
n <- c(10, 10, 10)
p <- 2
q <- 2
m <- length(n)

X <- matrix(runif(sum(n) * p), nrow = sum(n), ncol = p, byrow = T)
Z <- bdiag(matrix(rnorm(n[1] * q), nrow = n[1], ncol = q, byrow = T),
          matrix(rnorm(n[2] * q), nrow = n[2] , ncol = q, byrow = T),
          matrix(rnorm(n[3] * q), nrow = n[3], ncol = q, byrow = T))
Z <- as.matrix(Z)
beta <- matrix(rnorm(p), nrow = p, ncol = 1, byrow = T)
sigma <- rnorm(m)
tau <- rnorm(1)
b <- matrix(rnorm(m * q, 0, tau^2), nrow = m * q, ncol = 1, byrow = T)
e <- matrix(c(rmvnorm(1, rep(0, n[1]), (sigma[1])^2*diag(n[1])),
            rmvnorm(1, rep(0, n[2]), (sigma[2])^2*diag(n[2])),
            rmvnorm(1, rep(0, n[3]), (sigma[3])^2*diag(n[3]))),
            nrow = sum(n), ncol = 1, byrow = T)
Y <- X %*% beta + Z %*% b + e
beta0 <- matrix(c(1, 1),nrow = p,ncol = 1,byrow = T)
tau0 <- 1
sigma0 <- c(1, 1, 1)
em(beta0, tau0, sigma0, X, Y, Z, n, q = q, p = p)
```

```
## $bate
##            [,1]
## [1,]  1.2873534
## [2,] -0.8636941
##
## $tau
## [1] 0.01362274
##
## $sigma
## [1] 0.007755035 0.664061092 0.652090124
```