ST202/206 Extra Question - Class 3

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We study an interesting property of probability distributions: **memorylessness**. The meaning of the property is that the distribution of a 'waiting time' until a certain event is independent of how much time has already elapsed.

Definition 1 (discrete memorylessness)**.** *Suppose X is a discrete random variable with support $\{1, 2, \ldots\}$. Then the distribution of X is memoryless if for any s and t in $\{0, 1, 2, \ldots\}$, we have*

$$\mathbb{P}(X > s + t \mid X > s) = \mathbb{P}(X > t).$$

Definition 2 (continuous memorylessness)**.** *Suppose X is a continous random variable with support $[0, +\infty)$. Then the distribution of X is memoryless if for any $s, t \geq 0$, we have*

$$\mathbb{P}(X > s + t \mid X > s) = \mathbb{P}(X > t).$$

We recall the geometric distribution with success rate p and support $\{1, 2, \ldots\}$ from the lecture. This version of the geometric random variable counts the number of trials required to have the first success, and has PMF:

$$F_Y(y) = p(1-p)^{y-1} \qquad \text{for } y = 1, 2, \ldots$$

Question. (a) Show that geometric distributions with support $\{1, 2, \ldots\}$ are memoryless and they are indeed the **only** discrete memoryless distributions.
(b) What continuous distributions with support $[0, +\infty)$ are memoryless? Justify your answer.
(c) If you walk around Covent Garden on a busy afternoon, what is the the distribution of the time until you next run into a friend likely to be? Why?