To facilitate the application of the phase-type method in Wang et al. (2023), we provide a general-purpose `R` function `phase_type_estimation` which can be used to estimate the lifetime distribution for general warranty datasets with two-layer censoring. The detailed description of `phase_type_estimation` is shown as follows.

Usage

```
phase_type_estimation(ts, lags, ncensor, end_of_study, warranty_length, m1 = NULL, m2
= NULL, epsilon = 5e-3, maximum_order = 5, GoF_plot = TRUE)
```

Input

- `ts`: a vector of the observed product lifetimes;
- `lags`: a vector of the observed sales lags, and it should be of the same length with `ts`;
- `ncensor`: number of censored products;
- `end_of_study`: end-of-study censoring time;
- `warranty_length`: length of the warranty period;
- `m1`: optional; the order of the phase-type distribution for the product lifetime T; if not specified, `m1` will be selected via cross-validation;
- `m2`: optional; the order of the phase-type distribution for the sales lag X; if not specified, `m2` will be selected via cross-validation;
- `epsilon`: the convergence threshold for the EM algorithm; the EM algorithm is terminated when the increment in log-likelihood is smaller than `epsilon`;
- `maximum_order`: this argument will be used if `m1` and `m2` are not provided; in this case, we would select `m1` and `m2` from $\{1, 2, 3, \ldots,$ `maximum_order`$\}$ using 5-fold cross-validation;
- `GoF_plot`: logical; if `GoF_plot = TRUE`, then a plot will be displayed to check the goodness-of-fit of the phase-type model.

Output

Given the required input, `phase_type_estimation` generates a list of nine components:

- `m1`: the order of the phase-type distributions for the product lifetime T;
- `pi_T`: ML estimate for the initial probability vector $\boldsymbol{\pi}$ of the phase-type distribution for T;
- `xi_T`: ML estimate for the exit vector $\boldsymbol{\xi}$ of the phase-type distribution for T;
- `Lambda_T`: ML estimate for the intensity matrix $\boldsymbol{\Lambda}$ of the phase-type distribution for T;
- `m2`: the order of the phase-type distributions for the sales lag X;

- `pi_X`: ML estimate for the initial probability vector $\widetilde{\boldsymbol{\pi}}$ of the phase-type distribution for X;
- `xi_X`: ML estimate for the exit vector $\widetilde{\boldsymbol{\xi}}$ of the phase-type distribution for X;
- `Lambda_X`: ML estimate for the intensity matrix $\widetilde{\boldsymbol{\Lambda}}$ of the phase-type distribution for X;
- `loglikelihood_max`: maximum of the log-likelihood function.

The `R` function `phase_type_estimation` is developed for general two-layer censored warranty datasets. Given a generic dataset, `phase_type_estimation` can yield the phase-type sieve estimator for the distributions of product lifetime and sales lag. Moreover, it can also generate a plot to check the goodness-of-fit of the phase-type model. Please see `toy example.R` for some examples,