

Quantitative Methods 2015 CFA—级知识框架图



Framework

Time Value Calculation	R5 The Time Value of Money	
	R6 Discounted Cash Flow Applications	
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Time Value of Money

Interest rate

含义

Required rate of return

Discount rate

性质

Opportunity cost

- Nominal risk-free rate = real risk-free rate + expected inflation rate
- Required interest rate on a security = nominal risk-free rate + default risk premium + liquidity risk premium + maturity risk premium

构成

EAR ★





- 计算 $\left(1 + \frac{r}{m}\right)^m = 1 + EAR \rightarrow \lim_{n \to \infty} \left(1 + \frac{r}{m}\right)^m = e^r = 1 + EAR$
 - The greater the compounding frequency,
 - ✓ the greater the EAR will be in comparison to the stated rate
 - ✓ the greater the difference between EAR and the stated rate

- $FV=PV(1+r/m)^m$
- Annuity:
 - ordinary annuities
 - annuity due
- perpetuity

Discounted Cash Flow Applications

NPV & IRR

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Decision Rule

IRR的特点

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_N}{(1+r)^N} = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$$

$$NPV = 0 = CF_0 + \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \dots + \frac{CF_N}{(1+IRR)^N} = \sum_{t=0}^{N} \frac{CF_t}{(1+IRR)^t}$$

① Independent Projects:

- Accept it if NPV>0 Accept it if IRR>r (required rate of return)
- ② Mutually Exclusive Projects:
 - NPV method: Choose the one with higher NPV
 - IRR method: Choose the one with higher IRR
 - NPV and IRR methods may conflict with each other,以NPV判断为准

✓ When NPV= 0, the discount rate.

- ✓ IRR method assumes the project's cash flows will be reinvested at the IRR.
- ✓ Multiple solutions Problem of the IRR calculation (sign changes)

收益率计算

TWRR &MWRR ★★

	TWRR	MWRR	
		以现金流作为权重,现金流越多的一期,收益率给的 权重越大	
计算	① 找到每一期的HPR; ② n期HPR的几何平均	① 找到每一期的现金流; ② MWRR=计算IRR	
性质	✓ 不会受到现金流流入流出的影响;✓ 衡量基金经理业绩更准确	✓ 会受到现金流改变的影响,所以客户的决策对 MWRR会造成影响;✓ 如果客户投资的越来越多,相当于给后期的收益 率一个更高的权重	

Statistical Concepts and Market Return

Descriptive statistic: 描述一组数据的基本特征

Types of measurement scales

A
\boldsymbol{x}

	Nominal scales	Ordinal scales	Interval scales	Ratio scales
含义	定性区分	排序(> , <)	(>, <, +, -)	(>, <, +, -, *, /)
性质	只能求mode	Mode、median	没有绝对零点	Most refined

中心位置

Frequency	Frequency 1 Relative frequency 2 Cumulative frequency 3 Cumulative Relative Frequency			
Mean	计算: arithmetic mean、weighted mean、geometric mean、harmonic mean 注意: geometric mean=[(1+HPR ₁)(1+HPR _n)] ^{1/n}			
	结论: harmonic mean<= geometric mean<=arithmetic mean			
	定义: Quartile /Quintile/Deciles/Percentile			
Quantiles <u>*</u>	计算: L _y = (n+1)y/100			
	性质: 比如The third quartile > mean			

离散程度

Range = maximum value – minimum value

$$\sum_{i=1}^{N} |\mathbf{y}_i - \mathbf{\overline{y}}|$$

relative dispersion, free of scale (or free of unit)

$$MAD = \frac{\overline{i=1}^{N}}{n}$$
For population: $\sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}$

$$MAD < \sigma$$

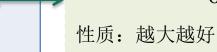


 $CV = \frac{S_x}{\overline{X}} \times 100\%$

Chebyshev inequality ★ 计算:

- 1. regardless of the shape of the distribution;
- the proportion of the values that lie within k standard deviations of the mean is at least $1 - 1/k^2$

For sample: $s^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}$



Sharp ratio= $\frac{R_p - R_f}{R_p}$

Skewness



Positive skewed	✓ Mode <median<mean (mean="0时)" a="" and="" extreme="" fat="" few="" frequent="" gains="" losses="" positive="" right="" skewness<="" small="" tail="" th="" ✓="" 投资者更加prefer=""></median<mean>
Negative skewed	 ✓ Mode>media>mean ✓ left fat tail ✓ frequent small gains and a few extreme losses. (mean=0时)

kurtosis

t分布:和z分布比,低峰肥尾。所以t分布更加分散一些,σ更大。



Leptokurtic	✓ Sample kurtosis>3; Excess kurtosis>0 ✓ 相同σ,尖峰肥尾(more frequent extremely large deviations from the mean than a normal distribution.)
Platykurtic	Sample kurtosis<3; Excess kurtosis<0
Normal distribution	Sample kurtosis=3; Excess kurtosis=0

Probability Concepts

概率和事件(event)

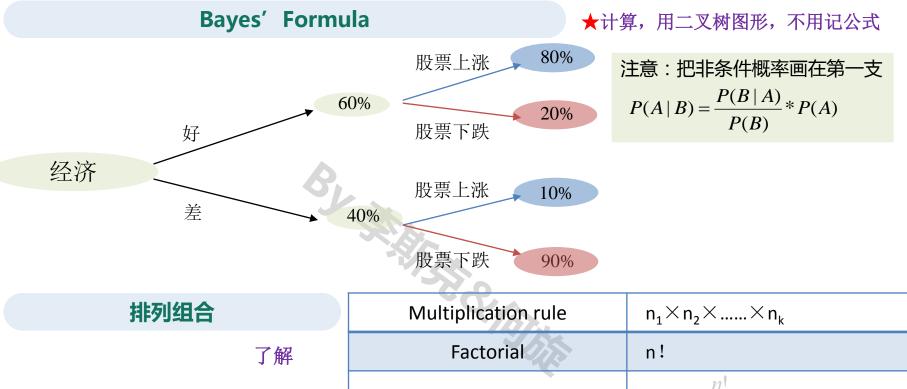
Properties of Probability	 ✓ 0≤ P(E) ≤ 1 ✓ P(E₁)+ P(E₂)++ P(Eₙ)=1 • E₁Eₙ: Mutually exclusive and Exhaustive Empirical probability: 分析过去,得到将来 Priori probability: 分析过去,得到过去的推理 Subjective probability: 主观 		
概率分类			
	Mutually exclusive Exhaustive events	P(AB)=P(A B)=P(B A)=0 include all possible outcomes	
Event分类	Independence ★★	✓ P(AB)=P(A)×P(B) ✓ If exclusive, must not independence ✓ Independence →ρ=0; 反之不对	

概率计算	\star
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	Odds	Odds for an event: P(E)/(1-P(E))
		Odds against an event: (1-P(E))/P(E)
	Joint probability	$P(AB)=P(A B)\times P(B)=P(B A)\times P(A)$
	Addition rule	at least one of two events will occur P(A or B)=P(A)+P(B)-P(AB)

Covariance & Correlation ★★

	计算		性质
Covariance	COV = E[(X-E(X))(Y-E(Y))]	✓	how one random variable moves with another random variable The covariance of X with itself is equal to the variance of X Covariance ranges from negative infinity to positive infinity
Correlation	$\rho_{XY} = \frac{\text{COV}(X,Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$	✓	Correlation measures the linear relationship between two random variables



了解 Factorial n! Labeling (or Multinomial) $\frac{n!}{n_1 \times n_2 \times \ldots \times n_k!}$ Combination $n \in \mathbb{R}$ $n \in \mathbb{R}$ n

Common Probability Distributions

Probability Distribution分类

分类	定义	性质
Discrete	the number of outcomes is counted	measurable and positive probability
Continuous	the number of outcomes is infinite	P (x)=0 even though x can occur
注意: Cumulative probability function F(x)=P(X<=x)		

Discrete Probability Distribution

类型	性质&计算
Discrete uniform	例: X={1,2,3,4,5}, p(x)=0.2
Bernoulli random variable	P(Y=1)=p, P(Y=0)=1-p Expectation=p, Variance=p(1-p)
Binomial random variable ★★	$p(x) = P(X = x) = {}_{n}C_{x}p^{x}(1 - p)^{n-x}$ Expectation=np, Variance=np(1-p)

Continuous Probability Distribution

类型		性质&计算
Uniform Distribution	取值区间: (a, b) $P(x_1 \le X \le x_2) = ($	$(x_2-x_1)/(b-a)$
	Properties	 ✓ X~N(µ, σ²); 取值区间: (-∞, +∞) ✓ Symmetrical distribution: skewness=0; kurtosis=3 ✓ A linear combination of normally distributed is also normally distributed. ✓ The tails get thin and go to zero but extend infinitely.
Normal distribution ★★	confidence intervals	68% confidence interval is $[\mu-\sigma,\mu+\sigma]$ 90% confidence interval is $[\mu-1.65\sigma,\mu+1.65\sigma]$ 95% confidence interval is $[\mu-1.96\sigma,\mu+1.96\sigma]$ 99% confidence interval is $[\mu-2.58\sigma,\mu+2.58\sigma]$
	Standard, Z分布	$Z = \frac{X - \mu}{\sigma}$
	safety-first Ratio	$[E(R_{\scriptscriptstyle P}) - R_{\scriptscriptstyle L}]/\sigma_{\scriptscriptstyle P}$; Maximize SFR <=> Minimize P (Rp< R_L)
Lognormal distribution	✓ Lognormal →t	, then X is lognormal. he price of asset; normal →the return of asset Bounded from below by zero (取值不能小于0)

Monte Carlo simulation & Historical simulation

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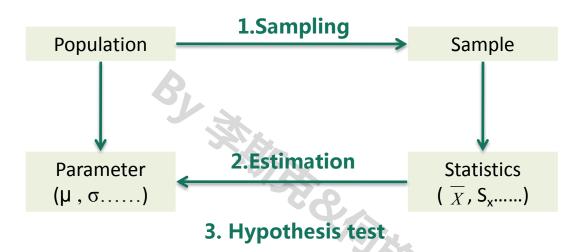
Monte Carlo simulation	 ✓ based on their assumed distributions, to produce a distribution of possible security values; ✓ It is fairly complex and will assume a parameter distribution; ✓ It is not an analytic method but a statistical one.
Historical simulation	 ✓ Selected historical data to generate a distribution; ✓ the past can not indicate the future; ✓ historical simulation cannot address the sort of "what if" questions that Monte Carlo simulation can.

Reading 10 & 11

Sampling and Estimation、Hypothesis Testing

难点

Framework



1. Sampling

抽样方法	✓ Simple random sampling✓ Stratified random sampling: to separate the population into smaller groups	
Data	✓ Time-series data: data taken over a period of time✓ Cross-sectional data: data taken at a single point of time	
Sample statistic特点 ★★	✓ sampling error of the mean= sample mean- population mean ✓ The sample statistic itself is a random variable ✓ Central Limit Theory: $n>=30 \rightarrow sample mean \sim N(\mu, \sigma^2/n)$ ✓ Standard error $=S/\sqrt{n}$ or s/\sqrt{n}	

Less peaked than a normal distribution ("fatter tails")

2. Estimation		
Desirable properties ★★	 ✓ Unbiasedness: expected value of the estimator is equal to parameter that estimate ✓ Efficiency: dispersion is smaller ✓ Consistency: the accuracy increases as n increases. 	
Estimation	1. Point estimate: 2. Confidence interval estimate $\bigstar \star \frac{1}{x} \pm z_{a/2} \frac{S}{\sqrt{n}}$ or $x \pm t_{a/2} \frac{S}{\sqrt{n}}$	
选择哪一个分布?	✓ 方差已知用z,方差未知用t,非正态总体小样本不可估计; ✓ 如果n>=30,都可以用z.	

Symmetrical

✓ Degrees of freedom (df): n-1

Data-mining bias

Survivorship bias Look-ahead bias

Time-period bias

估计的bias

Sample selection bias

standard normal distribution

As the degrees of freedom gets larger, the shape of t-distribution approaches

3. Hypothesis test

步骤:检验μ

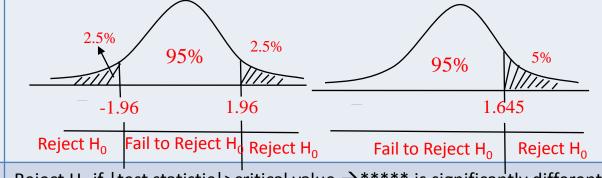


	Two-tailed	$H_0: m = m_0 H_a: m^1 m_0$
1. 提出假设	One-tailed	$H_{0}: \textit{M} ^{3} \textit{M}_{0} H_{a}: \textit{M} < \textit{M}_{0} \textit{or}, H_{0}: \textit{M} ^{3} \textit{M}_{0} H_{a}: \textit{M} < \textit{M}_{0}$

H₀ is what we want to reject

2. 计算test statistic
$$= \frac{X - \mu_0}{s / \sqrt{n}}$$

3. 画分布找到 critical value



A. 判断 Reject H_0 if |test statistic|>critical value \rightarrow ***** is significantly different from ******
Fail to reject H_0 if |test statistic|<critical value \rightarrow ***is not significantly different from ***



Type I error and Type II error

Decision	True c	ondition
Decision	H ₀ ✓	H _o ₩
Do not reject H ₀	Correct Decision	Incorrect Decision Type II error
Reject H ₀	Incorrect Decision Significance level =P (Type I error)	Correct Decision Power of test = 1- P (Type II error)

- 1. Type I error $\uparrow \rightarrow$ Type II error \checkmark
- 2. Increase the Sample Size → Type I error & Type II error •

其他检验 ★★

Test type	Assumptions	H _o	Test-statistic
	Independent populations	$\mu_1 - \mu_2 = 0$	t
Mean	Paired comparisons test	μ _d =0	$t = \frac{\overline{d}}{s_{\overline{d}}}$
Variance	Normally distributed population	$\sigma^2 = \sigma_0^2$	C^2
Variance	Two independent populations	$\sigma_{1}^{2} = \sigma_{2}^{2}$	F test

Nonparametric tests

了解

Parametric tests	specific to population parameters
Nonparametric tests	Nonparametric tests are used: ✓ The assumptions that support a parametric test are not met. ✓ When data are ranks (ordinal measurement scale) rather than values. ✓ The hypothesis does not involve the parameters of the distribution, such as testing whether a variable is normally distributed.

Technical Analysis

了解

·	3. Price and volume reflect the collective behavior of buyers and sellers.
Assumptions	 Investor behavior is reflected in trends and patterns that trend to repeat. Efficient markets hypothesis dose not hold.
和基本面分 析的区别	Fundamentalists believe that prices react quickly to changing stock values, while technicians believe that the reaction is slow.
Advantages	 Actual price and volume data are observable. Technical analysis itself is objective, while much of the data used in fundamental analysis is subject to assumptions or restatements. It can be applied to the prices of assets that do not produce future cash flows, such as commodities. It can also be useful when financial statement fraud occurs.
Disadvantage	illiquid markets不能用
重要图形和 指标	 ✓ Head-and-shoulders pattern: Price target = Neckline – (Head – Neckline) ✓ Inverse head and shoulders: price target = neckline + (neckline – head) ✓ Prevailing up trend: upward moves in prices consist of 5 waves and downward moves occur in 3 waves ✓ Prevailing down trend: downward moves in prices consist of 5 waves and upward moves occur in 3 waves

1. Prices are determined by the interaction of supply and demand.

2. Only participants who actually trade affect prices.

Principles



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