3.

$$y_i \sim \text{Poisson}(\theta_i)$$

$$\theta_i = \exp(x_i^T \beta) \longrightarrow y_i \sim \text{Poisson}(\exp(x_i^T \beta))$$

$$L(\beta | y) = \prod_{i=1}^{n} P(y_i | \beta)$$

$$= \prod_{i=1}^{n} \frac{(\exp(x_i^T \beta))^{y_i} \exp(-\exp(x_i^T \beta))}{y_i!}$$

$$\ln(L(\beta|y)) = \sum_{i=1}^{n} (y_i \cdot x_i^T \beta - \exp(x_i^T \beta) - \ln(y_i!))$$

$$\max_{\beta} \ln(L(\beta|y)) = \max_{\beta} \sum_{i=1}^{n} (y_i \cdot x_i^T \beta - \exp(x_i^T \beta) - \underbrace{\ln(y_i!)}_{\text{Not dependent on } \beta})$$

$$= \max_{\beta} \sum_{i=1}^{n} (y_i \cdot x_i^T \beta - \exp(x_i^T \beta))$$

$$\boxed{= \min_{\beta} \sum_{i=1}^{n} (\exp(x_i^T \beta) - y_i \cdot x_i^T \beta)}$$

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4. Let $F(\beta) = \sum_{i=1}^{n} (\exp(x_i^T \beta) - y_i (x_i^T \beta))$,

Then,

$$F'(\beta) = \sum_{i=1}^{n} (x_i^T \cdot \exp(x_i^T \beta) - y_i \cdot x_i^T)$$

$$F''(\beta) = \sum_{i=1}^{n} (\exp(x_i^T \beta) + x_i^T \exp(x_i^T \beta) \cdot x_i)$$

$$F''(\beta) = (1 + \underbrace{\|x\|_2^2}_{\geq 0 \ \forall x}) \cdot \sum_{i=1}^{n} \underbrace{\exp(x_i^T \beta)}_{\geq 0}$$

$$\boxed{\begin{array}{l} F''(\beta) \geq 0 \quad \forall \beta \\ \therefore F(\beta) \text{ is convex} \end{array}}$$