**Tolerance Allocation Based on a Rate-Sigma-Cost Model**

Nomenclature

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| *Ci* | Unit cost of the *i*-th process |  | *Fi* | Constant of *i*-th process |
| *Ai* | Constant of the *i*-th process |  | *r* | Manufacturing rate |
| *Bi* | Constant of *i*-th process |  | *Gi* | Constant |
| *Ei* | Constant of *i*-th process |  | *σ* | Standard deviation |
| *Vi* | Total cost of component type *i* |  | *CT* | Total cost of products |
| *U* | Average cost of a product |  | *Q* | Number of satisfactory components |
| *Ni* | Number of manufactured component type *i* |  | *M* | Number of satisfactory products |
| *m* | Number of different components in a product |  | *Xi* | Dimension of component *i* |
| *Y* | Dimension of assembly |  |  |  |

To model the relationship between tolerance and manufacturing cost, we build two models: a cost-rate model and a *σ*-rate model. For the *i*-th component, the two models are given by Eq. and Eq. .

*r* is the rate, which can be computed by the volume affected divided by operation time.

cost-rate model for the *i*th component:

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*σ*-rate model for the *i*th component:

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To compare different processes, we use rate as a metric to evaluate the precision and cost relationship, as shown in Fig. 1.



**Fig. 1**

Using Eq., and Eq. , we can get the relationship between cost and *σ*, as shown in Eq. .

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Equation (3) is a reciprocal powered model, which is used in some literature.

We need to find constants to build the cost-rate model and the *σ*-rate model.

**Minimize unit cost of products by optimizing manufacturing rate of components**

Total cost of component type *i*:

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*Ni* is the number of manufactured component type *i* (including unsatisfactory components)

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*αi* is the satisfaction rate of component type *i*.

Lower specification limit:

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Upper specification limit:

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New lower specification limit:

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New upper specification limit:

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Even function integration,

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in which, erf(*x*) is the error function. This equation has been verified by an example.

Total cost of products

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Average cost of a product

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Number of satisfactory products

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in which, *βi* is the satisfaction rate of products.

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Put Eq. and Eq. into Eq. , we can have the unit cost represented as a function of *ri*, *αi*, and *βi*, as shown below:

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The product characteristic value, *Y*, is computed from the component characteristic values, *Xi*, by the design function *f*:

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Variables *X1*, *X2*, …, *Xm* follow normal distributions with standard deviation *σi*. We use first order Taylor series to approximate variable Y. And the distribution of Y is approximated using a normal distribution with a standard deviation *σy.* We assume the assembly is unbiased.

Taylor series in several variables:

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Variance of *Y*, *σ2Y*

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in which, *Di* is an intermediate variable, which can be computed by:

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Equations to estimate the satisfactory rate of products:

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We assume that the assembly process is under statistical process control, i.e., the mean of the assembly equals the nominal value of *Y*, thus *β* can be simplified:

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in which, *US’* is the upper specification (constant), and can be computed using the following equation:

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The specification limits for components can either be fixed or be optimized. The specification limits of products are fixed (given by design).

**Scenario One: fix specification**:

When specification limits for components are fixed at *µ*±3*σi*, then *αi* is a constant.

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Average cost of a product, *U*, is a function of manufacturing rates, *r*1, *r*2, …, *rm* of components. We optimize these manufacturing rates to minimize *U*. This is an unconstrained multivariate optimization problem. The problem can be solved using gradient based optimization algorithms or using TensorFlow.

**Scenario Two: optimize specification**:

When specification limits for components are defined at *µ*±*kiσi*, then *αi* should be computed

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The objective function, Eq. is a function of *r*1, *r*2,…*rm*, and *k*1, *k*2,…, *km*.

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The error function, erf(*x*) can be approximated using power series expansion (we use a seventh order approximation):

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**Case study**

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