The average unit cost, *U*, of a satisfactory product, can be calculated as:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where, *M* is the number of satisfactory products assembled. *CT* is the total cost, which includes the costs incurred in manufacturing and assembling all the components and managing the scrap. *CT* is defined as:

|  |  |  |
| --- | --- | --- |
|  |  |  |

The component processing cost, *CB*, can be modeled as a function of production rate, *ri*:

|  |  |  |
| --- | --- | --- |
|  |  |  |

The process control cost, *C*control, can be modeled as a function of *ε*:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where, *Q* is the number of components of each type being produced.

Similarly, the value of *σ* for a component can be modeled as a function of *r*:

|  |  |  |
| --- | --- | --- |
|  |  |  |



Because of Law of total variance [1], the variance of the loaf-like distribution is:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Derivative

|  |  |  |
| --- | --- | --- |
|  |  |  |

|  |  |  |
| --- | --- | --- |
|  |  |  |

|  |  |  |
| --- | --- | --- |
|  |  |  |

Because of the new term, i.e., control cost, the derivative is updated as:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where *z* is given by:

|  |  |  |
| --- | --- | --- |
|  |  |  |

And similarly, the derivative of *U* with respect to *ε* can be computed as:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where *z* is given by:

|  |  |  |
| --- | --- | --- |
|  |  |  |

The probability of the standard normal distribution can be computed using the Gaussian error function [2].

|  |  |  |
| --- | --- | --- |
|  |  |  |

**TO DO**

1. Use Monte Carlo to validate the analysis
2. Asymmetrical case

**References**

[1] “Law of total expectation and Law of total variance.” [Online]. Available: https://stats.stackexchange.com/questions/260860/normal-distribution-with-uniform-mean.

[2] “Normal distribution.” [Online]. Available: https://mathworld.wolfram.com/NormalDistributionFunction.html.

**Appendix**

|  |  |  |
| --- | --- | --- |
|  |  |  |

Let

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| --- | --- | --- |
|  |  |  |

Let

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| --- | --- | --- |
|  |  |  |

Then

|  |  |  |
| --- | --- | --- |
|  |  |  |

**Others**

Irwin–Hall distribution