**Nomenclature**

*xi*0: nominal dimension of the ith component

*xi*: the dimension of the ith component

*μi*: the mean of the dimension of the ith component

*y*0: the nominal value of product

*μ*0: the mean of the dimension of the product

*f*(*x*1, *x*2, …, *xm*): the design function

*l*: Control cost function

# Symmetrical case

The average unit cost, *U*, of a satisfactory product, can be calculated as:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where, *M* is the number of satisfactory products assembled. *CT* is the total cost, which includes the costs incurred in manufacturing and assembling all the components and managing the scrap. *CT* is defined as:

|  |  |  |
| --- | --- | --- |
|  |  |  |

*C*control may be the cost of maintenance. The more frequent maintenance is carried out, the larger the *C*control, but the more precise the machine will be (the less deviation of the means of the process, as measured by the variable *ε* below).

The component processing cost, *CB*, can be modeled as a function of production rate, *ri*:

|  |  |  |
| --- | --- | --- |
|  |  |  |

The process control cost, *C*control, can be modeled as a function of *ε*:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where, *Q* is the number of components of each type being produced.

The value of *σ* for a component can be modeled as a function of *r*:

|  |  |  |
| --- | --- | --- |
|  |  |  |



Because of the law of total variance [1], the variance of the loaf-like distribution is:

|  |  |  |
| --- | --- | --- |
|  |  |  |

The unit cost, *U*, can be computed as:

|  |  |  |
| --- | --- | --- |
|  |  |  |

## Optimization

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Because of the new term, i.e., control cost, the derivative is updated as:

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| --- | --- | --- |
|  |  |  |

where *z* is given by:

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| --- | --- | --- |
|  |  |  |

and

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And similarly, the derivative of *U* with respect to *ε* can be computed as:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where *z* is given by:

|  |  |  |
| --- | --- | --- |
|  |  |  |

and

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| --- | --- | --- |
|  |  |  |

# Asymmetrical case



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The process control cost, *C*control, can be modeled as a function of *εL* and *εR*:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where, *Q* is the number of components of each type being produced.



*y* follows a normal distribution, i.e., *y*~*N*(*μy*, *σy*), then the product pass rate, *β*, can be computed:

|  |  |  |
| --- | --- | --- |
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The mean of products, *μy*, can be computed by the mean of individual components:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where *μ*1, *μ*2, … *μm* are the means of the individual components. See appendix for proof.

And the standard deviation of products, *σy*, can be computed by the following equation:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
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The probability of the standard normal distribution can be computed using the Gaussian error function [2].

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## Optimization

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where,

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| --- | --- | --- |
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where *z*1 is given by:

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| --- | --- | --- |
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and *z*2 is given by:

|  |  |  |
| --- | --- | --- |
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and

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and

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and

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Similarly,

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where,

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| --- | --- | --- |
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where *z*1 is given by:

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| --- | --- | --- |
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and *z*2 is given by:

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| --- | --- | --- |
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and

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and

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where

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Similarly,

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where

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|  |  |  |

in which, *q*1 and *q*2 are given by Eq. and Eq. , and

|  |  |  |
| --- | --- | --- |
|  |  |  |

and

|  |  |  |
| --- | --- | --- |
|  |  |  |

where

|  |  |  |
| --- | --- | --- |
|  |  |  |

# To do

# References

[1] “Law of total expectation and Law of total variance.” [Online]. Available: https://stats.stackexchange.com/questions/260860/normal-distribution-with-uniform-mean.

[2] “Normal distribution.” [Online]. Available: https://mathworld.wolfram.com/NormalDistributionFunction.html.

# Appendix

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| --- | --- | --- |
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Let

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| --- | --- | --- |
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Let

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Then

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Assume *m*=2, i.e., there are two types of components. The result can be extended to cases when *m*>2.

|  |  |  |
| --- | --- | --- |
|  |  |  |

where *h*(*x*1, *x*2) is the design function, *f*(*x*1, *x*2) is the density function. Because *x*1 and *x*2 are independent, *f*(*x*1, *x*2) = *fX*1(*x*1)*fX*2(*x*2), *μ*1 and *μ*2 are the mean of *x*1 and *x*2.

# Others

Irwin–Hall distribution