A Directed Spanning Tree Adaptive Control Framework for Time-Varying Formations

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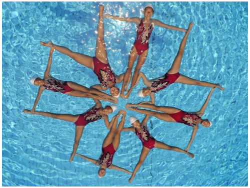
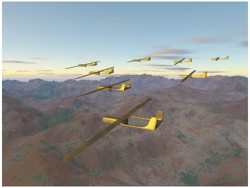
OUTLINE

MULTI-AGENT SYSTEM

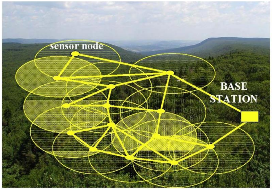
- Agents (Able to send/receive data, take actions, $\cdots$)
- Communications (Among agents, controller&actuator, $\cdots$)
- Group Behaviors (Consensus, formation, $\cdots$)



(a) Synchronised swimming



(b) Formation vehicles



(c) Sensor network

(The pictures were taken from un-copyrighted websites with thanks)

Synchronization→Formation

- W. Yu, J. Lv, X. Yu, and G. Chen, “Distributed adaptive control for synchronization in directed complex networks,” *SIAM J. Control Optim.*, vol. 53, no. 5, pp. 2980–3005, 2015.
- X. Dong and G. Hu, “Time-varying formation tracking for linear multiagent systems with multiple leaders,” *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3658–3664, 2017.

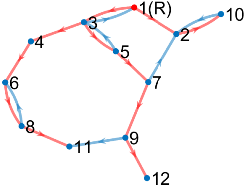


Figure 1: A digraph with a Directed Spanning Tree (DST)

Time-Varying Formation Control

$$\dot{x}_i = Ax_i + Bu_i \quad i \in \mathcal{I}_N \triangleq \{1, 2, \cdots, N\} \tag{1}$$

$x_i(t) \in \mathbb{R}^n$: state; $u_i(t) \in \mathbb{R}^m$: controller;
(A, B): compatible and stabilizable pair of matrices.

Definition 1 (TVF)

The multi-agent system (1) is said to achieve the time-varying formation defined by the time-varying vector $h(t) = (h_1^T(t), h_2^T(t), \cdots, h_N^T(t))^T$ if, for any initial states, there holds

$$\lim_{t\to\infty} ((x_i - h_i) - (x_j - h_j)) = 0, \ \forall i, j \in \mathcal{I}_N. \tag{2}$$

Assumption 1 (DST)

The weighted digraph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ has at least one DST $\bar{\mathcal{G}}(\mathcal{V}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$.

TIME-VARYING FORMATION TRACKING

$$\begin{aligned} \dot{x}_l &= Ax_l, & l &\in \mathcal{I}_M, \\ \dot{x}_i &= Ax_i + Bu_i, & i &\in \mathcal{I}_N \setminus \mathcal{I}_M. \end{aligned} \tag{3}$$

Definition 2 (TVFT)

The multi-agent system (3) is said to achieve the time-varying formation tracking defined by $h^F(t) = \mathrm{col}(h_{M+1}(t), h_{M+2}(t), \cdots, h_N(t))$ and by positive constants β_l, $l \in \mathcal{I}_M$, satisfying $\sum_{l=1}^{M} \beta_l = 1$ if, for any initial states, there holds

$$\lim_{t\to\infty} \Big(x_i - h_i - \sum_{l=1}^{M} \beta_l x_l\Big) = 0, \ \forall i \in \mathcal{I}_N \setminus \mathcal{I}_M. \tag{4}$$

For the special case $M = 1$, (4) becomes

$$\lim_{t\to\infty} \big(x_i - h_i - x_1\big) = 0, \quad i = 2, \cdots, N. \tag{5}$$

TVF CONTROLLER DESIGN

$$u_i = K_0 x_i + K_1 d_i + K_2 \sum_{j \in \mathcal{N}_1(i)} \alpha_{ij}(t)(d_i - d_j) \tag{6}$$

$$\alpha_{ij}(t) = \begin{cases} a_{ij}, & \text{if} \quad e_{ji} \in \mathcal{E} \setminus \bar{\mathcal{E}}, \\ \bar{a}_{k+1,i_k}(t), & \text{if} \quad e_{ji} \in \bar{\mathcal{E}} \end{cases} \tag{7}$$

$$\dot{\bar{a}}_{k+1,i_k} = \rho_{k+1,i_k} \Big((d_{i_k} - d_{k+1}) - \sum_{j \in \bar{\mathcal{N}}_2(k+1)} (d_{k+1} - d_j) \Big)^T \Gamma (d_{i_k} - d_{k+1}) \tag{8}$$

$d_i = x_i - h_i$: local formation deviation of agent i;
$\mathcal{N}_1(i)$ ($\mathcal{N}_2(i)$): in-neighbor (out-neighbor) of i;
i_k: the unique in-neighbor of node $k+1$ in $\bar{\mathcal{G}}$, $k = 1, \cdots, N-1$;
K_0, K_1, K_2, Γ, ρ_{k+1,i_k}: feedback gains.

TVF CONTROLLER DESIGN

Algorithm 1

1. Find a K_0 such that the formation feasibility conditions

$$(A+BK_0)(h_{i_k}(t)-h_{k+1}(t))-(\dot{h}_{i_k}(t)-\dot{h}_{k+1}(t))=0 \tag{9}$$

hold, $\forall k \in \{1,\cdots,N-1\}$, for any DST $\bar{\mathcal{G}}$. If such K_0 exists, continue; else, the algorithm terminates without solutions;

2. Choose K_1 such that $(A+BK_0+BK_1, B)$ is stabilizable. For some η, $\theta \in \mathbb{R}^+$, solve the following LMI:

$$(A+BK_0+BK_1)P+P(A+BK_0+BK_1)^T-\eta BB^T+\theta P \leq 0$$

to get a $P>0$;

3. Set $K_2=-B^TP^{-1}$, $\Gamma=P^{-1}BB^TP^{-1}$ and choose $\rho_{k+1,i_k} \in \mathbb{R}^+$.

MAIN RESULT FOR TVF

Theorem 1

Under Assumption 1, and feasibility condition (9), the TVF problem in Definition 1 is solved by controller (6) with adaptive coupling weights (7)-(8), along the designs in Algorithm 1.

Remark 1

In state-of-the-art TVF, the number of feasibility conditions is of the order $\frac{N(N-1)}{2}$:

$$(A + BK_0)(h_i(t) - h_j(t)) - (\dot{h}_i(t) - \dot{h}_j(t)) = 0, \ \forall i \to j.$$

The proposed number of feasibility conditions in (9) is $N - 1$, i.e., exploiting the DST structure leads to the minimum number of conditions.

EXAMPLE: TVF

Agents: $N = 12$, $A = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$;

Communication topology: Figure 1;

Required formation: a pair of nested hexagons.

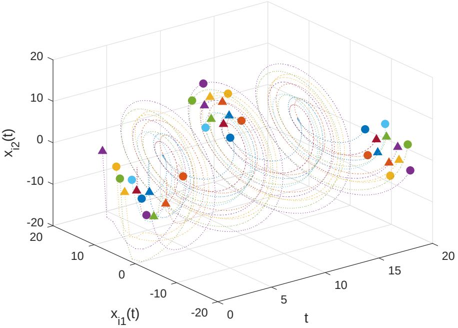


Figure 2: Trajectories of the agents $x_i(t)$ with snapshots at $t = 0, 10, 20$.

Example: TVF

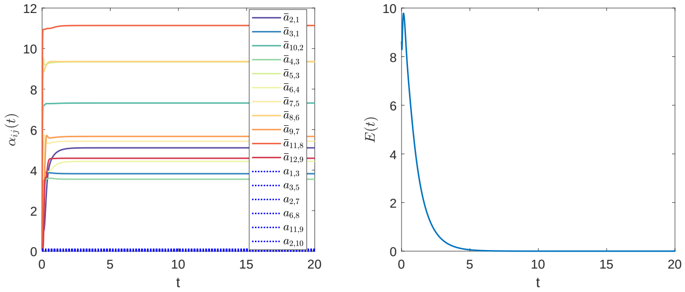


Figure 3: Coupling weights $\alpha_{ij}(t)$ and global formation error $E(t)$ with proposed adaptive method ($\rho_{k+1,i_k} = 0.1$).

EXAMPLE: TVF

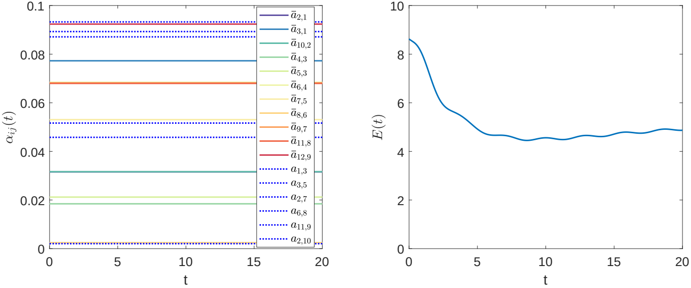


Figure 4: Coupling weights α_{ij} and global formation error $E(t)$ with nonadaptive control ($\rho_{k+1,i_k} = 0$, same initial α_{ij} as in Fig. 3).

Example: TVFT with a single leader

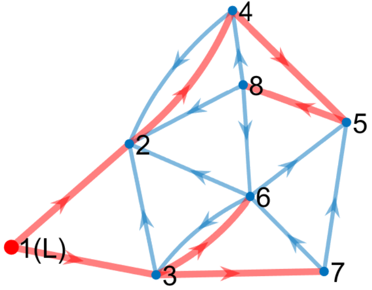


Figure 5: A digraph with a DST rooting at the leader.

Example: TVFT with a single leader

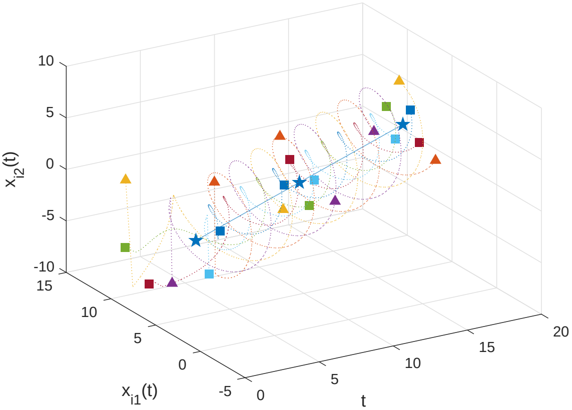
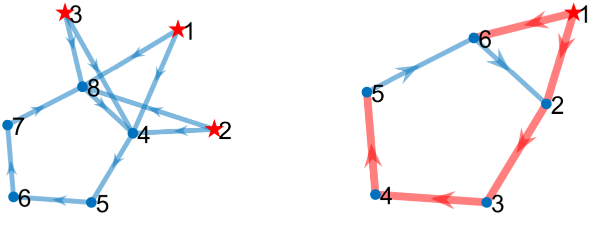


Figure 6: Trajectories of the agents $x_i(t)$ with snapshots at $t = 0, 10, 20$, where the pentagram is the leader.

Example: TVFT with three leaders



(a) $\mathcal{G}$ (b) $\mathcal{G}'$

Figure 7: A digraph with a generalized DST rooting at the leadership, and the induced graph $\mathcal{G}'$ with a single leader.

Example: TVFT with three leaders

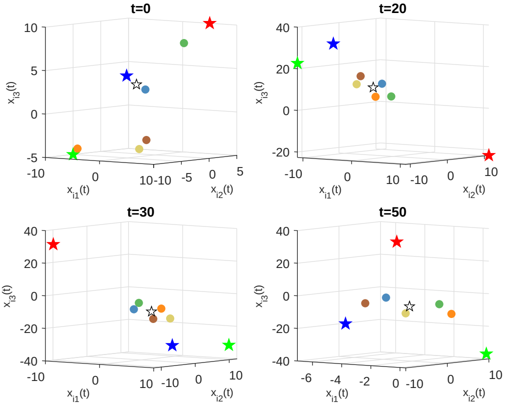


Figure 8: Trajectories of the agents $x_i(t)$ with snapshots at $t = 0, 20, 30, 50$, where three filled pentagrams and an unfilled pentagram are used to mark the leaders and their average.

Concluding Remarks

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- A natural unifying framework for the DST adaptive method in the presence of one or more leaders;

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- Future works?

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- **MINIMUM** number of distributed formation feasibility conditions are derived;
- A natural unifying framework for the DST adaptive method in the presence of one or more leaders;
- Future works?
- Further reading: `https://arxiv.org/abs/2005.01349`.

Thank you for listening!

Question?