Robust Neuro-adaptive Asymptotic Consensus for a Class of Uncertain Multi-agent Systems: An Edge-based Paradigm

Dongdong Yue[1] Qi Li[1] Jinde Cao[1,2] Xuegang Tan[1]

[1]School of Automation, Southeast University

[2]School of Mathematics, Southeast University

Presenting author: Xuegang Tan

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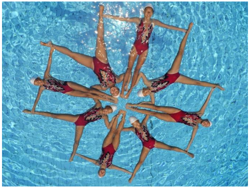
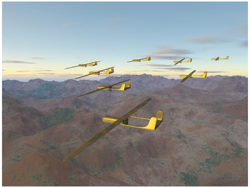
OUTLINE

Typical Multi-agent Systems

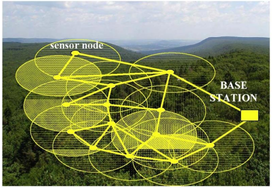
- Agents (Able to compute, send/receive data, make moves)
- Network (Communication topologies)
- Distributed control (Interact locally and act globally)



(a) Synchronised swimming



(b) Formation flying



(c) Sensor network

(The pictures were taken from un-copyrighted websites with thanks)

TOPOLOGY ASSUMPTION

Assumption 1

The communication topology $\mathcal{G}$ among the N agents is undirected and connected.

Lemma 1

Under Assumption 1, the laplacian matrix $\mathcal{L}$ has a simple eigenvalue 0 with $\mathbf{1}$ as a corresponding right eigenvector, and all the other eigenvalues have positive real parts.

W. Ren and R. W. Beard, *Distributed consensus in multi-vehicle cooperative control*. Springer, 2008 (Lemma 2.4 P. 28).

SYSTEM DYNAMICS

$$\dot{x}_i(t) = Ax_i(t) + B(u_i(t) + f_i(x_i(t)) + \omega_i(t)) \quad i = 1, 2, \cdots, N \quad (1)$$

$x_i(t) \in \mathbb{R}^n$: state vectors; $u_i(t) \in \mathbb{R}^m$: control inputs;
(A, B): compatible and stabilizable pair of matrices;
$f_i(x_i)$: unknown but smooth matching nonlinearity;
$\omega_i(t)$: unknown matching disturbance.

Assumption 2

For each agent i, there exists constant $\omega_{iM} > 0$, such that $\|\omega_i(t)\|_\infty \leq \omega_{iM}$, $i = 1, \cdots, N$.

Control goal:

$\lim_{t\to\infty} \|x_i - x_j\| = 0, \forall i, j \in \mathcal{V}$, despite of the unmodelled dynamics and disturbances.

Approximate $f_i(\cdot)$ using NN

Standard Setups:

$$f_i(x_i) = W_i^T S_i(x_i) + \epsilon_i, \quad \forall x_i \in \Omega_i \tag{2}$$

W_i: an ideal weight matrix with $\|W_i\|_F \leq W_{iM}$;
$S_i(x_i)$: a vector collection of basis functions with $\|S_i\| \leq S_{iM}$;
ϵ_i: the approximation error vector satisfying $\|\epsilon_i\| \leq \epsilon_{iM}$;
Ω_i: a sufficiently large compact set.

A. Das and F. L. Lewis, "Distributed adaptive control for synchronization of unknown nonlinear networked systems," *Automatica*, vol. 46, no. 12, pp. 2014–2021, 2010.

Pain Point:
UUB consensus (synchronization, tracking, $\cdots$) errors.

CONTROLLER DESIGN

$$
\begin{aligned}
u_i &= \sum_{j\in\mathcal{N}_i} c_{ij}(t)F(x_i - x_j) + \sum_{j\in\mathcal{N}_i} d_{ij}(t)sgn(F(x_i - x_j)) - \hat{W}_i^T(t)S_i(x_i) \\
\dot{c}_{ij}(t) &= \kappa_{ij}(x_i - x_j)^T\Gamma(x_i - x_j) \quad j\in\mathcal{N}_i \\
\dot{d}_{ij}(t) &= \nu_{ij}\|F(x_i - x_j)\|_1 \qquad j\in\mathcal{N}_i \\
\dot{\hat{W}}_i &= \tau_i[S_i(x_i)e_i^TP^{-1}B - \sigma_i(\hat{W}_i - \overline{W}_i(t))] \\
\dot{\overline{W}}_i &= \sigma_i\pi_i(\hat{W}_i - \overline{W}_i)
\end{aligned} \tag{3}
$$

$e_i = x_i - \frac{1}{N}\sum_{j=1}^N x_j$: consensus error of agent i;
$c_{ij}(t), d_{ij}(t), (i,j)\in\mathcal{E}$: dynamic coupling strengths;
$\hat{W}_i(t)$: an estimation of W_i;
$\overline{W}_i(t)$: a pseudo ideal weight matrix introduced for $f_i(\cdot)$;
F, Γ: feedback gain matrices; $\kappa_{ij}, \nu_{ij}, \tau_i, \sigma_i, \pi_i \in \mathcal{R}^+$.

Main Results

Theorem 1

Under the Assumptions 1 and 2, the asymptotic consensus problem of MAS (1) can be solved under dynamic neuro-adaptive protocol (3) with $F = -B^T P^{-1}$ *and* $\Gamma = P^{-1} B B^T P^{-1}$, *where* $P > 0$ *is a solution to the following linear matrix inequality (LMI):*

$$AP + PA^T - \eta BB^T + \theta P \leq 0 \tag{4}$$

for some scalars $\eta, \theta > 0$. *Moreover, each estimated NN weight matrix* $\hat{W}_i$ *converges to the corresponging pesudo ideal weight matrix* $\overline{W}_i$ *and each coupling gain* $c_{ij}(t)$ *as well as* $d_{ij}(t)$ *converges to some finite value.*

Main Results

Corollary 1

Under the Assumptions 1 and 2, the asymptotic consensus problem of MAS (1) can be solved under static neuro-adaptive protocol

$$\begin{aligned} u_i &= c \sum_{j \in \mathcal{N}_i} F(x_i - x_j) + d \sum_{j \in \mathcal{N}_i} sgn(F(x_i - x_j)) \\ &\quad - \hat{W}_i^T(t) S_i(x_i) \\ \dot{\hat{W}}_i &= \tau_i [S_i(x_i) e_i^T P^{-1} B - \sigma_i(\hat{W}_i - \overline{W}_i(t))] \\ \dot{\overline{W}}_i &= \sigma_i \pi_i (\hat{W}_i - \overline{W}_i) \end{aligned} \tag{5}$$

with $F = -B^T P^{-1}$, $c > \frac{\eta}{2\lambda_2}$ *and* $d > (\epsilon_M + \omega_M)(N-1)$, *where* P *and* η *are defined in (4).*

MAIN RESULTS

Remark 1

Note that the Lyapunov function V in Theorem 1 does not depend on any information of the underlying network $\mathcal{G}$. Thus, it is easy to extend the results to the MAS under switching communication topologies as long as each possible topology is connected and undirected.

$$\Big(V = e^T(\Xi \otimes P^{-1})e + \sum_{i=1}^{N} tr(\frac{1}{\tau_i}\tilde{W}_i^T\tilde{W}_i + \frac{1}{\pi_i}\tilde{\bar{W}}_i^T\tilde{\bar{W}}_i) + \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\frac{1}{2\kappa_{ij}}(c_{ij}(t)-\tilde{c})^2 + \frac{1}{2\nu_{ij}}(d_{ij}(t)-\tilde{d})^2 \Big)$$

A NUMERICAL EXAMPLE

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$, f_i(x_i) = {x_{i1}}^2 \cos(i + x_{i2})$ and $d_i(t) = 0.1 \sin(it)$

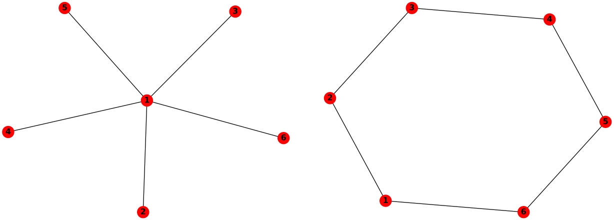


Figure: Two possible communication topologies.

A NUMERICAL EXAMPLE

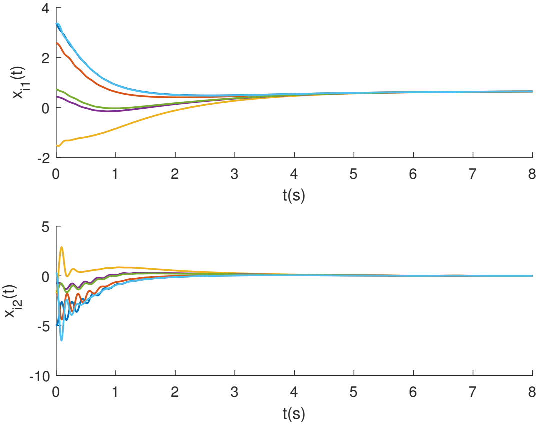


Figure: States of agents $x_i(t)$.

A NUMERICAL EXAMPLE

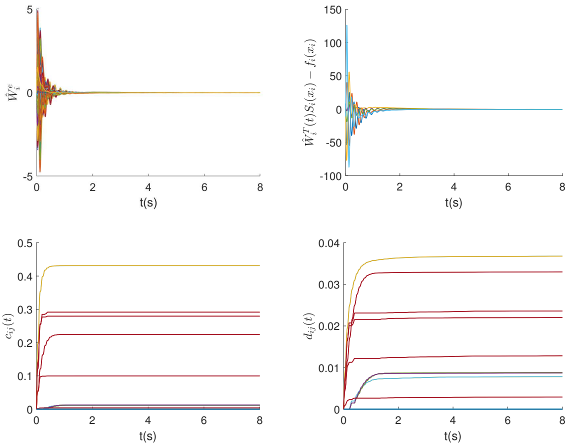


Figure: Pseudo converge error matrices $\hat{W}_i - \overline{W}_i$ (TL); Approximation errors $\hat{f}_i(x_i) - f_i(x_i)$ (TR); Coupling weights $c_{ij}(t)$ (BL) and $d_{ij}(t)$ (BR).

CONCLUSION

List of Contributions:

- An edge-based framework is constructed for controlling a class of heterogeneous uncertain MASs **WITHOUT** knowing the communication laplacian spectrum;
- Robust neuro-adaptive **ASYMPTOTIC** consensus is rigorously proved and numerically testified.

Thank you for listening!

Question?