A Directed Spanning Tree Adaptive Control Framework for Time-Varying Formations

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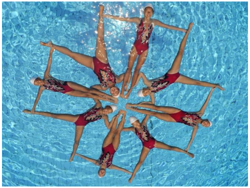
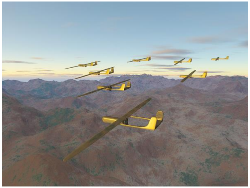
OUTLINE

MULTI-AGENT SYSTEM

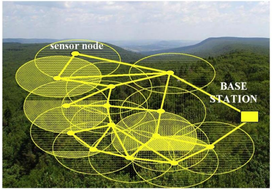
- Agents (Able to send/receive data, take actions, $\cdots$)
- Communications (Among agents, controller&actuator, $\cdots$)
- Group Behaviors (Consensus, formation, $\cdots$)



(a) Synchronised swimming



(b) Formation vehicles



(c) Sensor network

(The pictures were taken from un-copyrighted websites with thanks)

SYNCHRONIZATION→FORMATION

- W. Yu, J. Lv, X. Yu, and G. Chen, "Distributed adaptive control for synchronization in directed complex networks," *SIAM J. Control Optim.*, vol. 53, no. 5, pp. 2980–3005, 2015.
- X. Dong and G. Hu, "Time-varying formation tracking for linear multiagent systems with multiple leaders," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3658–3664, 2017.

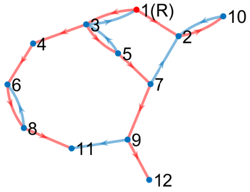


Figure 1: A digraph with a Directed Spanning Tree (DST)

TIME-VARYING FORMATION CONTROL

$$\dot{x}_i = Ax_i + Bu_i \quad i \in \mathcal{V} \triangleq \{1, 2, \cdots, N\} \tag{1}$$

$x_i(t) \in \mathbb{R}^n$: state; $u_i(t) \in \mathbb{R}^m$: controller;
(A, B): compatible and stabilizable pair of matrices.

Assumption 1

The weighted digraph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ has at least one DST $\bar{\mathcal{G}}(\mathcal{V}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$.

Definition 1

The multi-agent system (1) is said to achieve the time-varying formation (TVF) defined by the time-varying vector $h(t) = (h_1^T(t), h_2^T(t), \cdots, h_N^T(t))^T$ if, for any initial states, there holds

$$\lim_{t\to\infty} ((x_i(t) - h_i(t)) - (x_j(t) - h_j(t))) = 0, \forall i, j \in \mathcal{V}. \tag{2}$$

TVF CONTROLLER DESIGN

$$u_i = K_0 x_i + K_1 d_i + K_2 \sum_{j \in \mathcal{N}_1(i)} \alpha_{ij}(t)(d_i - d_j) \tag{3}$$

$$\alpha_{ij}(t) = \begin{cases} a_{ij}, & \text{if} \quad e_{ji} \in \mathcal{E} \setminus \bar{\mathcal{E}}, \\ \bar{a}_{k+1,i_k}(t), & \text{if} \quad e_{ji} \in \bar{\mathcal{E}} \end{cases} \tag{4}$$

$$\dot{\bar{a}}_{k+1,i_k} = \rho_{k+1,i_k} \Big((d_{i_k} - d_{k+1}) - \sum_{j \in \mathcal{N}_2(k+1)} (d_{k+1} - d_j) \Big)^T \Gamma (d_{i_k} - d_{k+1}) \tag{5}$$

$d_i = x_i - h_i$: local formation deviation of agent i;
$\mathcal{N}_1(i)$ ($\mathcal{N}_2(i)$): in-neighbor (out-neighbor) of i;
i_k: the unique in-neighbor of node $k+1$ in $\bar{\mathcal{G}}$, $k = 1, \cdots, N-1$;
K_0, K_1, K_2, Γ, ρ_{k+1,i_k}: feedback gains.

TVF CONTROLLER DESIGN

Algorithm 1

1. Find a K_0 such that the formation feasibility conditions

$$(A + BK_0)(h_{i_k}(t) - h_{k+1}(t)) - (\dot{h}_{i_k}(t) - \dot{h}_{k+1}(t)) = 0 \quad (6)$$

hold, $\forall k \in \{1, \cdots, N-1\}$, for any DST $\bar{\mathcal{G}}$. If such K_0 exists, continue; else, the algorithm terminates without solutions;

2. Choose K_1 such that $(A + BK_0 + BK_1, B)$ is stabilizable. For some $\eta, \theta \in \mathbb{R}^+$, solve the following LMI:

$$(A + BK_0 + BK_1)P + P(A + BK_0 + BK_1)^T - \eta BB^T + \theta P \leq 0$$

to get a $P > 0$;

3. Set $K_2 = -B^T P^{-1}$, $\Gamma = P^{-1}BB^T P^{-1}$ and choose $\rho_{k+1,i_k} \in \mathbb{R}^+$.

MAIN RESULT FOR TVF

Theorem 1

Under Assumption 1, and feasibility condition (6), the TVF problem in Definition 1 is solved by controller (3) with adaptive coupling weights (4)-(5), along the designs in Algorithm 1.

Remark 1

In state-of-the-art TVF, the number of the feasibility conditions is of the order N^2:

$$(A + BK_0)(h_i(t) - h_j(t)) - (\dot{h}_i(t) - \dot{h}_j(t)) = 0, \ \forall i \to j.$$

The number of the proposed feasibility conditions in (6) is exactly $N - 1$, which is the minimum for distributed TVF control of N agents.

TVF Example

Agents: $N = 12$, $A = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$;

Communication topology: Figure 1;

Required formation: a pair of nested hexagons.

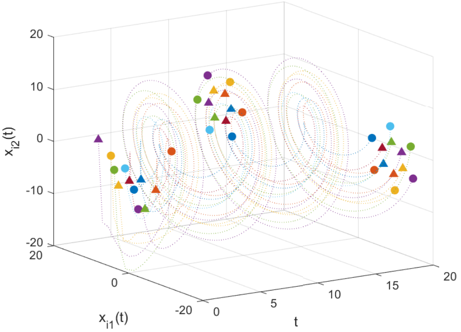


Figure 2: Trajectories of the agents $x_i(t)$ with snapshots at $t = 0, 10, 20$.

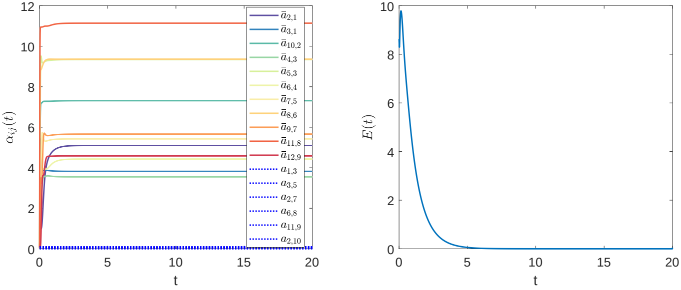


Figure 3: Coupling weights $\alpha_{ij}(t)$ and global formation error $E(t)$ with proposed adaptive method ($\rho_{k+1,i_k} = 0.1$).

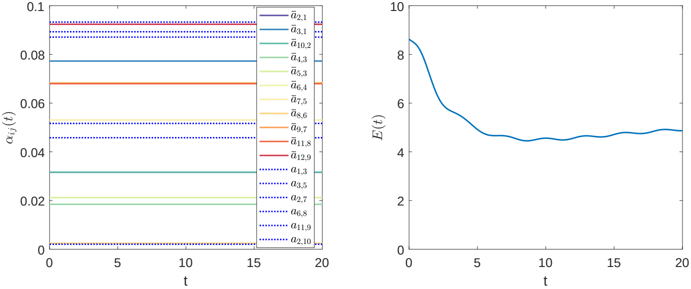


Figure 4: Coupling weights $\alpha_{ij}(t)$ and global formation error $E(t)$ with nonadaptive control ($\rho_{k+1,i_k} = 0$).

CONCLUDING REMARKS

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- **MINIMUM** number of distributed formation feasibility conditions are derived;
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- Future works?

Thank you for listening!

Question?