Monetary Economics with OLG (Part Deux: Inflation Control and Expectations)

The Big Picture. Previously we have only considered monetary policy being perfectly anticipated. What happens in a similar economy if there are monetary surprises? We study a monetary economy in which agents have imperfect information regarding: (i) the nature of specific markets (e.g. the random supply side of each market); and (ii) the nature of monetary policy (e.g. the random money supply process). Furthermore, agents face random trading opportunities in different markets that are informationally separated. In this simple parable, young agents produce on the spot goods that are market (location) specific. If it turns out that they must continue their old age in another market (location), they cannot carry their output with them. Since there is no other means of storage, fiat money will be the only asset that serves this purpose. This was Neil Wallace's (1980) simplified version of the classic Lucas parable, on expectations and the neutrality of money. This paper provided a counterexample to Keynesian macroeconomic doctrines which emphasized demand-side macroeconomic policies.

What we saw was that, under imperfect information regarding specific market fundamentals and monetary policy, a monetary equilibrium outcome resulted in what is empirically observed as a Phillips curve. The Phillips curve is a crown jewel in many models with Keynesian ("Old" or "New") features. A version of the Phillips curve hypothesis says that there is a positive correlation between output and inflation. In the Lucas parable, the Phillips curve arose as a result of agents not being able to separate the true signals driving changes in relative prices in each market (location).

However, the model predicts the following outcomes. First, if we increase money supply proportionally (e.g. if we double it) every period, then there is no effect on relative prices in all markets. If so, then there is no effect on agents' optimal decisions on labor and production.[1] In other words, we say that the model is money neutral. Second, the model also says the following. If money supply growth becomes predictable (and for simplicity if it becomes perfectly anticipated), then the so-called Phillips curve flips. That is, a naïve policymaker's persistent exploitation of the supposedly stable Phillips curve trade-off, will only result in agents anticipating such a policy. As a result, the agents' signal extraction problem goes away; and the equilibrium will produce a negative relation between aggregate output and inflation.

The two policy regimes, qualitatively, can provide an account of what happened before and after the 1970's stagflation. When the recession during the Oil Crisis hit, many central banks were persistently increasing money supply in the hope to reduce the recession. In many policymaker's minds, there was the stable Phillips curve hypothesis that have guided them in monetary policy quite successfully prior to the 1970's. However, the persistent demand-side policy of expanding money supply, led to more declines in real economic activity which were accompanied by high inflation.

Now that the metaphor is established, we shall refer to these informationally separated markets or locations as "islands". In the literature, models with such features have since been termed as Lucas-island-type models. In this tutorial we do a few exercises revisiting the framework that we have studied in class.

Please prepare you answers before attending tutorials. You will be expected to contribute to class discussions.

Exercise. Consider Wallace's example economy with two "islands", $i \in \{A, B\}$. Across both islands, in total, there are N number of young agents. Young agents deterministically become old in one period and then they exit the scene afterwards. Each young agent, on each island, is endowed with $y > 0$ units of time. Let l_t^i be the amount of each young agent's time spent working on island i. This produces l_t^i units of output on island i. Each young agent can consume $y - l_t^i$ units of leisure. Each young agents' output on island i can only be sold on island i. Denote c_{t+1}^{ij} as the old-age consumption of the same island-i born agent, where the consumption is a function of the random island j. Denote the price of each date- and island-specific good as p_t^i. The timing of events and actions are as follows:

[1] This is in stark contrast to many Keynesian-type models. In the latter, money is not neutral by construction, because *by assumption* prices cannot adjust instantaneously, so that there is a real, or relative price effect of money supply changes.

- Each period $t \in \mathbb{N}$ begins.

- Nature draws the population of newborns on island A, N_t^A, from the probability distribution $(\frac{1}{2}, \frac{1}{2})$ defined over the set $\{\alpha N, (1-\alpha)N\}$, where $\alpha \in (0, \frac{1}{2})$. So then island B must have $N - N_t^A$ number of young agents in the same period.

- Each island-i young agent does not observe N_t^i, and observes only the price of the island-i good, p_t^i. Young agents on $i \in \{A, B\}$ decide on labor supply $l_t^i \in [0, y]$.

- The stock of money supply M_t is determined as $M_t = \gamma_t M_{t-1}$, with M_0 known, and $1 \leq \gamma_t < +\infty$. (Assume that $|M_t| < +\infty$ for all t.) New money is transferred to each old agents as a lump sum $(\gamma_t - 1)M_{t-1}/N$.

- Nature moves again. For each young agent on island i, Nature draws a destination j from a probability distribution $(\frac{1}{2}, \frac{1}{2})$ with support $\{A, B\}$, so then the agent must go to island j next period.

- Period t closes and period $t+1$ begins. Old agents on each island $j \in \{A, B\}$ each consume c_{t+1}^{ij} and then they exit the economy.

Observe that since output on each island is not portable, and there is a future liquidity risk faced by each island-i young agent, they would want to exchange the fruits of their labor for real money balances, which is the only portable medium of exchange. In turn, each existing old agent, on each island, will expend their holdings of real money balances on consuming the output of the respective island, since they do not exist thereafter.

Each generation's expected lifetime payoff function is given by

$$U(y - l_t^i) + \beta \mathbb{E}\left\{U(c_{t+1}^{ij})\right\},$$

where $\beta \in (0, 1)$, and $\mathbb{E}$ is the mathematical expectations operator.

1. Write down the individual-state-contingent old-age budget constraint for an agent currently young in period $t \geq 1$.

2. Write out explicitly the expected utility function for the same agent.

3. Characterize the agent's optimal labor supply decision. Let the optimal labor supply function, of each date-t young agent on each island $i \in \{A, B\}$, be denoted by $L(p_t^i) \equiv L(p_t^i; y)$. What is each young agent's optimal demand for real money balances?

4. Assume U to be such that for each given p_{t+1}^j, the income effect from a variation in the relative price p_t^i/p_{t+1}^j, is dominated by its substitution effect. Explain in words what these two effects are, and what this assumption implies.

5. In equilibrium all markets must clear on each island. What are these markets and what are the market clearing conditions?

6. Suppose $\gamma_t = \gamma$ for all t, and this is common knowledge. Prove that given M_{t-1} and p_t^i, a higher γ_t is related to a higher return to money. What does this imply for the aggregate relation between inflation and output?

7. Now, suppose instead that γ_t is randomly distributed according to the fixed probability distribution $(\theta, 1-\theta)$ on $\{\gamma_L, \gamma_H\}$, $\theta \in (0, 1)$, and $\gamma_L = (1-\alpha)\gamma_H/\alpha < \gamma_H$— i.e. assume $\alpha > (1-\alpha)$.

(a) Define an agent-specific “state of the world” in this model economy. As the modeller, objectively, how many of such states of the world can be realized in this problem?

(b) Let’s modify the timing of events and actions a little. How many distinct states of the world can each island-i agent observe if the realization of γ_t were observable at the start of period t? Enumerate those states. Tabulate your results.

(c) Now let’s go back to the model’s timing of events and actions, where γ_t is not observable until l_t^i is chosen. Show that there are at most two distinct underlying states of the world that can be identified by agents upon observing the data on island-specific prices. Tabulate your results again.

What would an econometrician, someone who does not observe the data generating process in this economy, conclude about the data generated by an equilibrium in this economy? Explain, step by step, how you obtain the model’s equilibrium relationship.

8. In view of the results above, discuss what lessons can be learnt from this model.