Lab3

Grace Cui

11/8/2017

Problem1

Read data

```
markov <- read.table('/Users/yuecui/Desktop/Everything Starts with Data/Week8/markov100.txt')
library(expm)
```

```
P<-as.matrix(markov)
```

Problem 1(a)

```
a <- c(1,rep(0,99)) ###initial
a1<-a%*%(P%^%10)
a1[5]
```

```
## [1] 0.045091
```

The probability of being in State 5 after 10 transitions is 0.045091.

Problem 1(b)

```
a <- c(1,rep(0,99))
b<- c(0,1,rep(0,98))
c<- c(0,0,1,rep(0,97))

a1<-a%*%(P%^%10)
b1<-b%*%(P%^%10)
c1<-c%*%(P%^%10)

(a1[10]+b1[10]+c1[10]) / 3
```

```
## [1] 0.08268901
```

Problem 1(c)

```
Q=t(P) - diag(100)
Q[100,]=rep(1,100)
rhs = c(rep(0,99),1)
Pi=solve(Q)%*%rhs
Pi[1]
```

```
## [1] 0.01256589
```

Problem 1(d)

```
B =P[1:99,1:99]
Q =diag(99) - B
e = rep(1,99)
```

```r
m = solve(Q) %*% e
m[1]
```

```
## [1] 254.9395
```

Problem 2

Read data

```r
traffic <- read.table('/Users/yuecui/Desktop/Everything Starts with Data/Week8/webtraffic.txt',
                      header =T)
```

Problem 2(a)

```r
M <-as.matrix(traffic)
M2<-colSums(M)
Traffic<-matrix(M2,nrow = 9, ncol = 9,byrow=T)
Traffic
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0  447  553    0    0    0    0    0    0
##  [2,]    0   23  230  321    0    0    0    0   63
##  [3,]    0  167   43  520    0    0    0    0   96
##  [4,]    0    0    0   44  158  312  247    0  124
##  [5,]    0    0    0    0   22   52   90  127  218
##  [6,]    0    0    0    0   67   21    0  294   97
##  [7,]    0    0    0    0    0   94    7  185   58
##  [8,]    0    0    0    0  262    0    0   30  344
##  [9,]    0    0    0    0    0    0    0    0    0
```

Problem 2(b)

```r
Traffic[9,1] <- 1000
Traffic
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0  447  553    0    0    0    0    0    0
##  [2,]    0   23  230  321    0    0    0    0   63
##  [3,]    0  167   43  520    0    0    0    0   96
##  [4,]    0    0    0   44  158  312  247    0  124
##  [5,]    0    0    0    0   22   52   90  127  218
##  [6,]    0    0    0    0   67   21    0  294   97
##  [7,]    0    0    0    0    0   94    7  185   58
##  [8,]    0    0    0    0  262    0    0   30  344
##  [9,] 1000    0    0    0    0    0    0    0    0
```

```r
P<-prop.table(Traffic,1)
P
```

```
##       [,1]       [,2]       [,3]       [,4]      [,5]       [,6]
##  [1,]    0 0.44700000 0.55300000 0.00000000 0.0000000 0.00000000
##  [2,]    0 0.03610675 0.36106750 0.50392465 0.0000000 0.00000000
##  [3,]    0 0.20217918 0.05205811 0.62953995 0.0000000 0.00000000
##  [4,]    0 0.00000000 0.00000000 0.04971751 0.1785311 0.35254237
##  [5,]    0 0.00000000 0.00000000 0.00000000 0.0432220 0.10216110
```

```
## [6,]    0 0.00000000 0.00000000 0.00000000 0.1398747 0.04384134
## [7,]    0 0.00000000 0.00000000 0.00000000 0.0000000 0.27325581
## [8,]    0 0.00000000 0.00000000 0.00000000 0.4119497 0.00000000
## [9,]    1 0.00000000 0.00000000 0.00000000 0.0000000 0.00000000
##             [,7]       [,8]      [,9]
## [1,] 0.00000000 0.00000000 0.0000000
## [2,] 0.00000000 0.00000000 0.0989011
## [3,] 0.00000000 0.00000000 0.1162228
## [4,] 0.27909605 0.00000000 0.1401130
## [5,] 0.17681729 0.24950884 0.4282908
## [6,] 0.00000000 0.61377871 0.2025052
## [7,] 0.02034884 0.53779070 0.1686047
## [8,] 0.00000000 0.04716981 0.5408805
## [9,] 0.00000000 0.00000000 0.0000000
```

Problem 2(c)

```r
Q=t(P) - diag(9)
Q[9,]=rep(1,9)
rhs = c(rep(0,8),1)
Pi=solve(Q)%*%rhs
Pi
```

```
##             [,1]
## [1,] 0.15832806
## [2,] 0.10085497
## [3,] 0.13077897
## [4,] 0.14012033
## [5,] 0.08058898
## [6,] 0.07583914
## [7,] 0.05446485
## [8,] 0.10069664
## [9,] 0.15832806
```

Problem 2(d)

```r
vector <- c(0.1,2,3,5,5,3,3,2,0)
sum(vector * Pi) * 8
```

```
## [1] 18.44585
```

Problem 2(e)

```r
Traffic
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
## [1,]    0  447  553    0    0    0    0    0    0
## [2,]    0   23  230  321    0    0    0    0   63
## [3,]    0  167   43  520    0    0    0    0   96
## [4,]    0    0    0   44  158  312  247    0  124
## [5,]    0    0    0    0   22   52   90  127  218
## [6,]    0    0    0    0   67   21    0  294   97
## [7,]    0    0    0    0    0   94    7  185   58
## [8,]    0    0    0    0  262    0    0   30  344
## [9,] 1000    0    0    0    0    0    0    0    0
```

```r
Traffic[2,6]<-Traffic[2,3]*0.3
Traffic[2,3]<-Traffic[2,3]*0.7

Traffic[2,7]<-Traffic[2,4]*0.2
Traffic[2,4]<-Traffic[2,4]*0.8
```

```r
P<-prop.table(Traffic,1)
P
```

```
##       [,1]       [,2]       [,3]       [,4]      [,5]       [,6]
##  [1,]    0 0.44700000 0.55300000 0.00000000 0.0000000 0.00000000
##  [2,]    0 0.03610675 0.25274725 0.40313972 0.0000000 0.10832025
##  [3,]    0 0.20217918 0.05205811 0.62953995 0.0000000 0.00000000
##  [4,]    0 0.00000000 0.00000000 0.04971751 0.1785311 0.35254237
##  [5,]    0 0.00000000 0.00000000 0.00000000 0.0432220 0.10216110
##  [6,]    0 0.00000000 0.00000000 0.00000000 0.1398747 0.04384134
##  [7,]    0 0.00000000 0.00000000 0.00000000 0.0000000 0.27325581
##  [8,]    0 0.00000000 0.00000000 0.00000000 0.4119497 0.00000000
##  [9,]    1 0.00000000 0.00000000 0.00000000 0.0000000 0.00000000
##             [,7]       [,8]      [,9]
##  [1,] 0.00000000 0.00000000 0.0000000
##  [2,] 0.10078493 0.00000000 0.0989011
##  [3,] 0.00000000 0.00000000 0.1162228
##  [4,] 0.27909605 0.00000000 0.1401130
##  [5,] 0.17681729 0.24950884 0.4282908
##  [6,] 0.00000000 0.61377871 0.2025052
##  [7,] 0.02034884 0.53779070 0.1686047
##  [8,] 0.00000000 0.04716981 0.5408805
##  [9,] 0.00000000 0.00000000 0.0000000
```

```r
Q=t(P) - diag(9)
Q[9,]=rep(1,9)
rhs = c(rep(0,8),1)
Pi2=solve(Q)%*%rhs
Pi2
```

```
##             [,1]
##  [1,] 0.16162840
##  [2,] 0.10034341
##  [3,] 0.12104331
##  [4,] 0.12275720
##  [5,] 0.08164613
##  [6,] 0.08250884
##  [7,] 0.06003218
##  [8,] 0.10841213
##  [9,] 0.16162840
```

```r
var(Pi)
```

```
##             [,1]
## [1,] 0.001410675
```

```r
var(Pi2)
```

```
##             [,1]
## [1,] 0.001219604
```

Variance of Pi2 decreased slightly from the variance of Pi, therefore the link helped balancing the traffic slightly.