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Model Reference Adaptive Stabilizing Control for Leader-following Consensus

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***Abstract*—Complex networks, neuroscience, and other applications have shown examples of multi-agent adaptive systems that must follow (over possibly short times) reference dynamics that are neither Hurwitz nor neutrally stable. However, such leader-following behavior would be impossible with existing adaptive consensus methods, e.g., based on model reference adaptive control (MRAC), since the stability of the reference dynamics is required. To fill this gap, we propose a novel model reference adaptive stabilizing control (MRASC) framework for leader-following consensus of multi-agent systems with unknown and heterogeneous dynamics. Differently from several approaches in the leader-following consensus literature, the proposed framework is free of any extra distributed observer layer for the leader's signal, as the reconstruction of such signals is intrinsic in the adaptive laws. Besides, the framework does not require Hurwitz or neutral stability of the leader and generalizes existing acyclic requirements on the communication graph among the follower. Starting from any weakly connected communication digraph, the proposed method allows to derive a lower bound, useful from the network design point of view, for the minimum number of followers that should be pinned by the leader.**

***Index Terms*—Model reference adaptive control, leader-following consensus, heterogeneous multi-agent systems.**

I. Introduction

Consensus of multi-agent systems serves as a building block for several complex problems over networks, e.g., graph signal processing [1], formation control [2], distributed optimization [3], and so on. Leader-following consensus can be regarded as a multi-agent system version of model reference tracking, to control multiple agents (followers) towards a desired reference goal (leader) [4]–[9]. The problem is particularly interesting when not all followers have direct access to the leader's signals, in which case the tracking should be achieved in a distributed fashion by sharing information with neighbors.

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Despite many reported results on leader-following consensus, heterogeneity and uncertainty of the agent dynamics remain challenging issues to address. It is common in the literature to assume completely known and homogeneous dynamics among all agents, including the leader [4]–[9], which can be restrictive in several applications where the agents' dynamics actually have modelling uncertainties. Recent years have witnessed some progress on addressing heterogeneity or uncertainty in leader-following consensus [10]–[19], either in the robust sense as in robust output regulation [10]–[12], or in the adaptive sense as in model reference adaptive control (MRAC) [13]–[19]. The latter framework of MRAC is especially flexible as it allows to consider scenarios where the leader is not autonomous as in output regulation, but it has external inputs that can be manipulated by the designer. In such scenarios, it is convenient to take fully advantage of the communication network by allowing the followers to exchange, along with their states, also their control inputs: such a pattern arises in several engineering problems such as cooperative adaptive cruise control for connected autonomous vehicles [16]–[18]. Note that discontinuous control methods have been used to address external input of the leader [20], [21]: however, such results are based on non-adaptive sliding mode control and it is not clear whether adaptive versions still apply with uncertainty in the agent dynamics.

Despite the promising framework, open problems still exist in MRAC-based leader-following consensus of heterogeneous and uncertain/unknown agents. Notably, complex networks, neuroscience, and other applications have shown examples of adaptive systems that must follow (over possibly short times) reference dynamics that are neither Hurwitz nor neutrally stable. For example, K. S. Narendra and co-workers have shown unstable dynamics playing the role of each other's adaptive reference [22], while R. Sepulchre and co-workers have shown spikes and impulsive patterns generated via unstable dynamics [23]. Indeed, as pointed out in [22], *"the existence of a stable reference model is not explicitly assured in all control situations"*. However, the leader dynamics in current MRAC-based consensus designs is restricted to be Hurwitz stable ([15]–[18]) or neutrally stable [13], a requirement stemming from traditional MRAC [24]. Such a gap in the leader dynamics of current MRAC-based methods is accentuated by the fact that the internal stability of the leader is not a necessary condition for consensus in general, as reported in [5]. In addition to this open problem, another one is the simultaneous treatment of external inputs of the leader and more general directed communication graphs among the followers: existing methods

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rely on acyclic graphs [16], [17], or only special classes of cyclic graphs addressable via projection techniques [18].

Bearing in mind the open problems of current MRAC-based designs discussed above, a novel framework named model reference adaptive stabilizing control (MRASC) is established for leader-following consensus of heterogeneous unknown linear multi-agent systems. The framework relies on a hierarchal adaptation methodology, which consists of a MRASC law for the followers directly pinned by the leader and a virtual MRASC (v-MRASC) law for the rest followers. The main contributions with respect to existing results on leader-following consensus are summarized as follows.

- Differently from existing MRAC-based literature, the leader dynamics is not required to be Hurwitz or neutrally stable as in [13], [15]–[18]. Instead, the leader is only required to be stabilizable, i.e., the same property as in classical leader-following consensus results [5].
- Instead of an autonomous leader [10], [11], [13], [14], external inputs of the leader are allowed. Moreover, no extra distributed observer layer (resp. estimator layer) for the leader's state (resp. leader's input), e.g., as proposed in [10], [11], [15], is required.
- A novel analysis for feasibility and uniqueness of the leader-following consensus manifold is presented, which extends the approaches in [16]–[18]. Besides, starting from any weakly connected digraph, the proposed method allows to derive a lower bound, useful from the network design point of view, for the number of followers that should be pinned by the leader. This allows to relax the acyclic graph assumption in [16]–[18].

It is worth mentioning that the developed tools go beyond a mere extension of MRASC [25] from leaderless to leader-follower: in fact, by locally constructing appropriate reference signals for the follower agents, we are able to overcome globally known signals required in [25]. In addition, feasibility and uniqueness of the leader-following consensus manifold and the bound on pinning nodes require new dedicated analysis, since these aspects are absent in a leaderless case like [25].

II. Preliminaries

A. Notations and Graph Theory

Denote $\mathbb{R}$ as the space of real numbers. Let I be the identity matrix and $\mathbf{1}$ be the vector with each element being 1, where the dimensions are omitted when clear from the context. For a matrix A, $A > 0$ means that A is positive definite, which we also denote as $\mathrm{sgn}(A) = 1$; $\mathrm{vec}(A)$ is the column vectorization operator, i.e., $\mathrm{vec}(A) = (a_1^T, \ldots, a_n^T)^T$ in which a_i is the i-th column of matrix A. For two sets M and N, $M \setminus N$ denotes the set difference operator, i.e., $M \setminus N = \{x \in M | x \notin N\}$; $|M|$ denotes the cardinality of M.

A directed graph (or simply *digraph*) $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ consists of a node set $\mathcal{V} = \{1, 2, \ldots, N\}$, an edge set $\mathcal{E} = \{\mathcal{E}_{ij} | i \to j, i \neq j\}$, and an adjacency matrix $\mathcal{A} = (a_{ij}) \in \mathbb{R}^{N \times N}$ such that $a_{ij} > 0$ if $\mathcal{E}_{ji} \in \mathcal{E}$; and $a_{ij} = 0$ otherwise. For $\mathcal{E}_{ji} \in \mathcal{E}$, j is called an (in-)neighbor of i and i is an out-neighbor of j conversely, denoted as $j \in \mathcal{N}_i$ or $i \in \mathcal{N}_j^{\text{out}}$. The Laplacian matrix $\mathcal{L} = (l_{ij}) \in \mathbb{R}^{N \times N}$ of $\mathcal{G}$ contains $l_{ij} = -a_{ij}$ for $i \neq j$, and $l_{ii} = \sum_{j=1}^{N} a_{ij}$. A *path* is a sequence of directed edges connecting a pair of nodes. A *directed spanning tree* is a subgraph of $\mathcal{G}$ with the same node set, such that there exists a root (i.e., a node that has no neighbors) and one can find a unique path from the root to every other node. A *cycle* is a path with the same starting and ending node[1], and a digraph without cycles is called *acyclic*. A digraph $\mathcal{G}$ is *weakly connected* if there are no isolated nodes (i.e., without any in-neighbor or out-neighbor); it is *strongly connected* if there is a path between any pair of two distinct nodes. Any weakly connected digraph admits a unique strong component decomposition, i.e., a decomposition into disjoint maximal strongly connected subgraphs called *strongly connected components (SCCs)* [26]. By considering all nodes inside each SCC as a single node and any edges connecting different SCCs as a single edge, one can get the *condensation digraph* of $\mathcal{G}$. The condensation digraph of any weakly connected digraph $\mathcal{G}$ is acyclic [27, Lemma 3.2.2]. A SCC that has no in-neighbor in the condensation digraph is called a *leading SCC (LSCC)*. A LSCC that contains a single node is called a *singleton LSCC (s-LSCC)*.

B. Problem Formulation

Consider a leader-following network in which the leader and the followers have linear time-invariant dynamics described by

$$\dot{x}_0 = A_0 x_0 + B_0 r, \tag{1a}$$

$$\dot{x}_i = A_i x_i + B_i u_i, \quad i \in \mathcal{V}, \tag{1b}$$

where $x_0 \in \mathbb{R}^n$ is the leader state, $r \in \mathbb{R}^q$ is a vector-valued continuous function of t representing the leader's external input, $x_i \in \mathbb{R}^n$ is the follower state, and $u_i \in \mathbb{R}^q$ is the control input to be designed. The pairs (A_i, B_i) have dimension $A_i \in \mathbb{R}^{n \times n}$ and $B_i \in \mathbb{R}^{n \times q}$, $i = 0, 1, 2, \ldots, N$, and $\mathcal{G}$ is the digraph describing the interaction of the followers one another. Denote $\bar{\mathcal{G}}$ as the augmented communication graph with the leader 0 involved. In $\bar{\mathcal{G}}$, the leader has no in-neighbors, only out-neighbors: if a follower $i \in \mathcal{N}_0^{\text{out}}$, we say that the follower i is pinned by the leader.

The goal of the leader-following consensus problem is to design u_i such that

$$\lim_{t \to \infty} e_i = 0 \text{ where } e_i = x_i - x_0, \ \forall i \in \mathcal{V}. \tag{2}$$

In the following, we formulate the leader-following consensus problem in the framework of MRAC, a classical adaptive control approach covered in textbooks such as [24] and explored in various consensus literature [14]–[17]. In line with such literature, the leader pair (A_0, B_0) is taken to be known, whereas the follower pairs (A_i, B_i) are allowed to be *unknown* and *heterogeneous*.

The main idea to achieve MRAC-based consensus is that, under some structural requirements on the pairs (A_0, B_0) and (A_i, B_i), the follower agents (1b) are able to match the behavior of the leader (1a) via some ideal (yet unknown) control gains. Such structural requirements are formalized via the celebrated matching conditions in MRAC [24, Ch. 6.2].

[1]Simple digraphs are considered in this paper where self-loops or multiple edges are excluded. Then, the smallest cycle is of length 2, i.e., an undirected edge. Conversely, any undirected edge should be understood as a cycle.

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Assumption 1: For each $i \in \mathcal{V}$, there exist ideal gains $K_i^* \in \mathbb{R}^{q\times n}$ and $L_i^* \in \mathbb{R}^{q\times q}$ such that

$$A_i + B_i K_i^* = A_0, \quad B_i L_i^* = B_0. \tag{3}$$

Furthermore, assume that L_i^* is sign definite, i.e., it is either positive definite ($\mathrm{sgn}(L_i^*) = 1$) or negative definite ($\mathrm{sgn}(L_i^*) = -1$). The true values of K_i^* , L_i^* are unknown but $\mathrm{sgn}(L_i^*)$ is a priori known.

Remark 1: In the adaptive control literature, the single-input case is often considered. In this case, the matrices K_i^* and L_i^* reduce to a vector and a scalar respectively, so that sign definiteness of a scalar is trivially satisfied.

Assumption 1 allows to view the leader dynamics (1a) as reference dynamics to be matched by the followers with feedback control. Traditionally, in the MRAC and MRAC-based consensus literature, the leader matrix A_0 is Hurwitz stable or neurally stable. Motivated by the fact that in many practical cases the leader may have unstable internal dynamics [5], [22], [23], we are interested in extending MRAC-based consensus results when the matrix A_0 is not necessarily stable.

Assumption 2: The pair (A_0, B_0) is stabilizable.

Under Assumption 2, if follows from [28] that there exists a unique solution $P > 0$ of the algebraic Riccati equation:

$$A_0^T P + PA_0 - PB_0B_0^T P + I = 0. \tag{4}$$

Furthermore, upon defining

$$G = -B_0^T P, \tag{5}$$

the matrix $A_0 + B_0 G$ is asymptotically stable, i.e., G is a stabilizing gain matrix of (A_0, B_0).

Remark 2: The matrix P will be used for designing suitable adaptation law for all followers. This implies that the followers should know the state matrix of the leader, a common assumption in the literature [13], [14]. Differently from [13], [14], no global eigenvalue information of the Laplacian matrix is needed in this paper to solve the algebraic Riccati equation.

III. MRASC FOR LEADER-FOLLOWING CONSENSUS

In this section we propose and analyze a new approach for leader-following consensus based on model reference adaptive stabilizing control (MRASC). The challenge is that the leader's state and input are not available to all followers: this requires to reconstruct some appropriate reference signals locally for the agents not pinned by the leader.

The idea of the proposed solution is a hierarchical control strategy described as follows. For each agent $i \in \mathcal{N}_0^{\text{out}}$, the control law is designed to track the leader. For each agent $i \in \mathcal{V} \setminus \mathcal{N}_0^{\text{out}}$, as no leader's signal is available, the control law is designed to track a virtual reference arising from the weighted average of its neighbors: we denote such an approach as v-MRASC to distinguish it from the existing MRASC [25].

Assumption 3: The augmented graph $\bar{\mathcal{G}}$ contains a directed spanning tree with the leader being the root.

Assumption 4: The subgraph $\mathcal{G}_{\text{v-MRASC}}$ of $\mathcal{G}$ with node set $\mathcal{V}_{\text{v-MRASC}} = \mathcal{V}$ and edge set $\mathcal{E}_{\text{v-MRASC}} = \{\mathcal{E}_{ji} | i \in \mathcal{V} \setminus \mathcal{N}_0^{\text{out}}\}$, i.e., obtained by preserving all incoming edges to any node that is not pinned by the leader, is acyclic.

Remark 3: Assumption 3 guarantees the propagation of the leader dynamics across the network [5], and it serves as a structural foundation for the proposed hierarchical control strategy (see Proposition 2 in Sect. III-C). Assumption 4 guarantees the well-posedness of the proposed v-MRASC law, and paves the way for the derivation of the lower bound for the minimum number of pinned agents (see Sect. IV). Note that previous works on MRAC-based leader-following consensus require that $\mathcal{G}$ itself is acyclic [16]–[18].

A. *MRASC for agents $i \in \mathcal{N}_0^{out}$*

For each agent $i \in \mathcal{N}_0^{\text{out}}$, access to the leader's signals (x_0 and r) is possible, which can be used to track the leader without any communication with other neighbors from the follower group (if they exist). Define the tracking error as $e_i = x_i - x_0$. The following result holds.

Proposition 1: Under Assumptions 1-2, consider the following MRASC law for agent $i \in \mathcal{N}_0^{\text{out}}$:

$$\begin{aligned}
u_i &= \hat{K}_i x_i + \hat{L}_i r + \hat{S}_i e_i,\\
\dot{\hat{K}}_i &= -\gamma\,\mathrm{sgn}(L_i^*) B_0^T P e_i x_i^T,\\
\dot{\hat{L}}_i &= -\gamma\,\mathrm{sgn}(L_i^*) B_0^T P e_i r^T,\\
\dot{\hat{S}}_i &= -\gamma\,\mathrm{sgn}(L_i^*) B_0^T P e_i e_i^T,
\end{aligned} \tag{6}$$

where $\gamma \in \mathbb{R}^+$ is a constant gain, $\hat{K}_i, \hat{L}_i$ are the estimates of K_i^*, L_i^* in (3), respectively, and $\hat{S}_i$ is the estimate of $S_i^* \triangleq L_i^* G$ with G as in (5). Then, the tracking error e_i converges to zero asymptotically. Furthermore, the parameter estimation errors $\tilde{K}_i \triangleq \hat{K}_i - K_i^*$, $\tilde{L}_i \triangleq \hat{L}_i - L_i^*$, and $\tilde{S}_i \triangleq \hat{S}_i - S_i^*$ are globally uniformly bounded.

Proof. This result is a natural extension of [25, Th. 1] from the single-input to the multiple-input case, upon considering the leader (1a) to play the role of a reference model for any follower $i \in \mathcal{N}_0^{\text{out}}$. Nevertheless, in what follows let us sketch the main steps of the proof for self-containedness and for clarification on the fundamental properties (existence and uniqueness) of the solutions.

For any agent $i \in \mathcal{N}_0^{\text{out}}$, the closed-loop error dynamics is

$$\begin{aligned}
\dot{e}_i &= (A_0 + B_0 G) e_i + B_0 {L_i^*}^{-1} (\tilde{K}_i x_i + \tilde{L}_i r + \tilde{S}_i e_i),\\
\dot{\tilde{K}}_i &= -\gamma\,\mathrm{sgn}(L_i^*) B_0^T P e_i x_i^T,\\
\dot{\tilde{L}}_i &= -\gamma\,\mathrm{sgn}(L_i^*) B_0^T P e_i r^T,\\
\dot{\tilde{S}}_i &= -\gamma\,\mathrm{sgn}(L_i^*) B_0^T P e_i e_i^T.
\end{aligned} \tag{7}$$

It is crucial to note that the leader dynamics (1a) is linear, thus guaranteeing existence and uniqueness in $[t_0, \infty)$ [29, Th. 3.2]. Furthermore, the fact that x_i can be written as $x_i = x_0 + e_i$ enables (7) to be expressed in state-space form $\dot{\chi}_i = F_i(t, \chi_i)$ using only $\chi_i = \left(e_i^T, (\mathrm{vec}(\tilde{K}_i))^T, (\mathrm{vec}(\tilde{L}_i))^T, (\mathrm{vec}(\tilde{S}_i))^T\right)^T \in \mathbb{R}^{q^2+2qn+1}$ and inputs (x_0, r), without explicitly including x_i (or x_0) in χ_i. The system (7) exhibits quadratic nonlinearities and contains an equilibrium at the origin. Over any compact time interval $t \in [t_1, t_2]$, F_i and $\frac{\partial F_i}{\partial \chi_i}$ are continuous, implying that F_i is locally Lipschitz in χ_i in $[t_1, t_2] \times \mathbb{R}^{q^2+2qn+1}$ [29, Lem. 3.2].

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Since $[t_1, t_2]$ can be selected arbitrarily in $[t_0, \infty)$, F_i is locally Lipschitz in χ_i uniformly in t in $[t_0, \infty)$.

Denote $\Gamma_i = L_i^{*-1} \operatorname{sgn}(L_i^*)$. Along the trajectory of (7), consider the candidate Lyapunov function

$$V_i = e_i^T P e_i + \gamma^{-1} \mathrm{tr}\big(\tilde{K}_i^T \Gamma_i \tilde{K}_i + \tilde{L}_i^T \Gamma_i \tilde{L}_i + \tilde{S}_i^T \Gamma_i \tilde{S}_i\big).$$

Using the adaptive law (6), The time derivative of V_i results in (details are omitted due to space limits)

$$\dot{V}_i = -e_i{}^T (I + P B_0 B_0^T P) e_i \leq 0,$$

which guarantees that all solutions of χ_i remain bounded as time evolves. Then, the existence of χ_i in $[t_0, \infty)$ is assured, and uniqueness follows from [29, Th. 3.3]. Finally, the asymptotic convergence of e_i is guaranteed by LaSalle-Yoshizawa Theorem [30, Th. 2.1]. ■

B. v-MRASC for agents $i \in \mathcal{V} \setminus \mathcal{N}_0^{out}$

For each agent $i \in \mathcal{V} \setminus \mathcal{N}_0^{\text{out}}$, direct access to the leader's signals is not possible: thus, the idea is to track a virtual reference constructed from the neighbors' signals (x_j and u_j, $j \in \mathcal{N}_i$). Such a virtual reference is constructed as the weighted average of its neighbors denoted by $x_{i,j}^a = \frac{\sum_{j \in \mathcal{N}_i} a_{ij} x_j}{\sum_{j \in \mathcal{N}_i} a_{ij}}$. Define the tracking error as $e_i' = x_i - x_{i,j}^a$. It is clear that $e_i' = \frac{\sum_{j \in \mathcal{N}_i} a_{ij} e_{ij}}{\sum_{j \in \mathcal{N}_i} a_{ij}}$ where $e_{ij} = x_i - x_j$.

Lemma 1: Under Assumption 1, $\forall i \in \mathcal{V} \setminus \mathcal{N}_0^{\text{out}}$ and $\forall j \in \mathcal{N}_i$, define $K_{ij}^* = K_i^* - L_i^* L_j^{*-1} K_j^*$, $L_{ij}^* = L_i^* L_j^{*-1}$ and $S_i^* = K_i^* + L_i^* G$ where G is defined in (5). Then, the controller

$$u_i^* \triangleq \frac{\sum_{j \in \mathcal{N}_i} a_{ij} u_{ij}^*}{\sum_{j \in \mathcal{N}_i} a_{ij}}, \tag{8}$$

where $u_{ij}^* \triangleq K_{ij}^* x_j + L_{ij}^* u_j + S_i^* e_{ij}$, when applied to agent i, results in the following stable dynamics for e_i'

$$\dot{e}_i' = (A_0 + B_0 G) e_i'. \tag{9}$$

Proof. Under Assumption 1, it is easy to verify that, for any $i \in \mathcal{V} \setminus \mathcal{N}_0^{\text{out}}$ and any $j \in \mathcal{N}_i$, the dynamics of i and j is matched through

$$A_i + B_i K_{ij}^* = A_j, \quad B_i L_{ij}^* = B_j. \tag{10}$$

Then, substituting u_i^* into (1b) in place of u_i leads to

$$\begin{aligned}
\dot{e}_i' &= \frac{\sum_{j \in \mathcal{N}_i} a_{ij} \dot{e}_{ij}}{\sum_{j \in \mathcal{N}_i} a_{ij}} \\
&= \frac{\sum_{j \in \mathcal{N}_i} a_{ij} (A_i x_i + B_i \frac{\sum_{p \in \mathcal{N}_i} a_{ip} u_{ip}^*}{\sum_{p \in \mathcal{N}_i} a_{ip}} - A_j x_j - B_j u_j)}{\sum_{j \in \mathcal{N}_i} a_{ij}} \\
&= \frac{\sum_{j \in \mathcal{N}_i} a_{ij} (A_i x_i - A_j x_j - B_j u_j) + B_i \sum_{p \in \mathcal{N}_i} a_{ip} u_{ip}^*}{\sum_{j \in \mathcal{N}_i} a_{ij}} \\
&= \frac{\sum_{j \in \mathcal{N}_i} a_{ij} (A_i x_i + B_i u_{ij}^* - A_j x_j - B_j u_j)}{\sum_{j \in \mathcal{N}_i} a_{ij}} \\
&= \frac{\sum_{j \in \mathcal{N}_i} a_{ij} (A_i + B_i S_i^*) e_{ij}}{\sum_{j \in \mathcal{N}_i} a_{ij}} \\
&= \frac{\sum_{j \in \mathcal{N}_i} a_{ij} (A_0 + B_0 G) e_{ij}}{\sum_{j \in \mathcal{N}_i} a_{ij}} = (A_0 + B_0 G) e_i',
\end{aligned} \tag{11}$$

where we have used (3), (10) and the definition of S_i^*. It is clear that the dynamics of e_i' is stable due to (5). ■

Note that the ideal controller u_i^* in (8) cannot be directly implemented in (1b) since it depends on $K_{ij}^*, L_{ij}^*, S_i^*$, which eventually depends on the unknown matching gains K_i^*, L_i^* and K_j^*, L_j^* of all its neighbors. Therefore, some distributed adaptation mechanisms must be devised.

The main result of this section can now be stated as follows.

Theorem 1: Under Assumptions 1-4, consider the following v-MRASC (12) law for agent $i \in \mathcal{V} \setminus \mathcal{N}_0^{\text{out}}$:

$$\begin{aligned}
u_i &= \frac{1}{\sum_{j \in \mathcal{N}_i} a_{ij}} \Big(\sum_{j \in \mathcal{N}_i} a_{ij} (\hat{K}_{ij} x_j + \hat{L}_{ij} u_j + \hat{S}_i e_{ij}) \Big), \\
\dot{\hat{K}}_{ij} &= -\gamma \operatorname{sgn}(L_i^*) B_0^T P e_i' x_j^T, \qquad j \in \mathcal{N}_i, \\
\dot{\hat{L}}_{ij} &= -\gamma \operatorname{sgn}(L_i^*) B_0^T P e_i' u_j^T, \qquad j \in \mathcal{N}_i, \\
\dot{\hat{S}}_i &= -\gamma \operatorname{sgn}(L_i^*) B_0^T P e_i' e_i'^T,
\end{aligned} \tag{12}$$

where $\hat{K}_{ij}, \hat{L}_{ij}, \hat{S}_i$ are the estimates of $K_{ij}^*, K_{ij}^*, S_i^*$ defined in Lemma 1, respectively. Then, the tracking error e_i' (to the weighted average of its neighbors) converges to zero asymptotically. Furthermore, the parameter estimation errors $\tilde{K}_{ij} \triangleq \hat{K}_{ij} - K_{ij}^*$, $\tilde{L}_{ij} \triangleq \hat{L}_{ij} - L_{ij}^*$, $\tilde{S}_i \triangleq \hat{S}_i - S_i^*$, $\forall j \in \mathcal{N}_i$, are globally uniformly bounded.

Proof. Note that u_i is always well-posed under Assumptions 3-4, more details can be found in [16, Sect. V-B], [18] and [31]. Then, following similar steps as in (11) but substituting u_i instead of u_i^* gives the closed-loop dynamics of e_i' as

$$\begin{aligned}
\dot{e}_i' &= \frac{\sum_{j \in \mathcal{N}_i} a_{ij} (A_i x_i + B_i \hat{u}_{ij} - A_j x_j - B_j u_j)}{\sum_{j \in \mathcal{N}_i} a_{ij}} \\
&= \frac{\sum_{j \in \mathcal{N}_i} a_{ij} (A_i x_i + B_i u_{ij}^* - A_j x_j - B_j u_j)}{\sum_{j \in \mathcal{N}_i} a_{ij}} \\
&\quad + B_i \frac{\sum_{j \in \mathcal{N}_i} a_{ij} \tilde{u}_{ij}}{\sum_{j \in \mathcal{N}_i} a_{ij}} \\
&= (A_0 + B_0 G) e_i' + B_i \frac{\sum_{j \in \mathcal{N}_i} a_{ij} \tilde{u}_{ij}}{\sum_{j \in \mathcal{N}_i} a_{ij}},
\end{aligned} \tag{13}$$

where $\hat{u}_{ij} = \hat{K}_{ij} x_j + \hat{L}_{ij} u_j + \hat{S}_i e_{ij}$ and $\tilde{u}_{ij} = \hat{u}_{ij} - u_{ij}^* = \tilde{K}_{ij} x_j + \tilde{L}_{ij} u_j + \tilde{S}_i e_{ij}$.

For each agent $i \in \mathcal{V} \setminus \mathcal{N}_0^{\text{out}}$, consider the candidate Lyapunov function

$$V_i' = e_i'^T P e_i' + \frac{\sum_{j \in \mathcal{N}_i} a_{ij} \mathrm{tr}(\tilde{K}_{ij}^T \Gamma_i \tilde{K}_{ij} + \tilde{L}_{ij}^T \Gamma_i \tilde{L}_{ij} + \tilde{S}_i^T \Gamma_i \tilde{S}_i)}{\gamma \sum_{j \in \mathcal{N}_i} a_{ij}}, \tag{14}$$

where $\Gamma_i = L_i^{*-1} \operatorname{sgn}(L_i^*)$ and P is defined in (4). The time derivative of V_i' along the trajectory of (12) and (13) is

$$\begin{aligned}
\dot{V}_i' =& 2 e_i'^T P (A_0 + B_0 G) e_i' + 2 e_i'^T P B_i \frac{\sum_{j \in \mathcal{N}_i} a_{ij} \tilde{u}_{ij}}{\sum_{j \in \mathcal{N}_i} a_{ij}} \\
&+ \frac{2 \sum_{j \in \mathcal{N}_i} a_{ij} \mathrm{tr}(\tilde{K}_{ij}^T \Gamma_i \dot{\hat{K}}_{ij} + \tilde{L}_{ij}^T \Gamma_i \dot{\hat{L}}_{ij})}{\gamma \sum_{j \in \mathcal{N}_i} a_{ij}} + \frac{2 \mathrm{tr}(\tilde{S}_i^T \Gamma_i \dot{\hat{S}}_i)}{\gamma}.
\end{aligned} \tag{15}$$

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Note that

$$\begin{aligned}\operatorname{tr}(\tilde{K}_{ij}^T\Gamma_i\dot{\hat{K}}_{ij}) &= -\gamma\operatorname{tr}(\tilde{K}_{ij}^T L_i^{*-1}B_0^T P e_i' x_j^T)\\ &= -\gamma e_i'^T P B_0 L_i^{*-1}\tilde{K}_{ij}x_j\\ &= -\gamma e_i'^T P B_i \tilde{K}_{ij} x_j, \end{aligned}\tag{16}$$

where we have used the property that the trace operator is invariant under cyclic permutations. Similarly, one has

$$\begin{aligned}\operatorname{tr}(\tilde{L}_{ij}^T\Gamma_i\dot{\hat{L}}_{ij}) &= -\gamma e_i'^T P B_i \tilde{L}_{ij}u_j,\\ \operatorname{tr}(\tilde{S}_{i}^T\Gamma_i\dot{\hat{S}}_{i}) &= -\gamma e_i'^T P B_i \tilde{S}_{i}e_i'.\end{aligned}\tag{17}$$

Then, it follows from (15)-(17), the algebraic Riccati equation (4), and the definition of e_i' that

$$\begin{aligned}\dot{V}_i' =& {e_i'}^T(A_0^TP + PA_0 - 2PB_0B_0^TP)e_i'\\ &+ 2e_i'^T PB_i\frac{\sum_{j\in\mathcal{N}_i} a_{ij}\tilde{u}_{ij}}{\sum_{j\in\mathcal{N}_i} a_{ij}} - 2e_i'^T PB_i\tilde{S}_i e_i'\\ &- \frac{2}{\sum_{j\in\mathcal{N}_i} a_{ij}}\Big(\sum_{j\in\mathcal{N}_i} a_{ij}e_i'^T PB_i(\tilde{K}_{ij}x_j + \tilde{L}_{ij}u_j)\Big)\\ =& -{e_i'}^T(I + PB_0B_0^TP)e_i' \le 0.\end{aligned}\tag{18}$$

Let us compactly denote the closed-loop error system as $\dot{\chi}_i' = \mathcal{F}_i(t, \chi_i')$, where $\chi_i' \in \mathbb{R}^{n+qn+|\mathcal{N}_i|(qn+q^2)}$ consists of e_i', $\operatorname{vec}(\tilde{S}_i)$, $\operatorname{vec}(\tilde{K}_{ij})$ and $\operatorname{vec}(\tilde{L}_{ij})$ for all $j \in \mathcal{N}_i$. Over any compact interval $t \in [a, b]$, $\mathcal{F}_i$ is locally Lipschitz in χ_i' uniformly in t since $\mathcal{F}_i$ and $\frac{\partial\mathcal{F}_i}{\partial\chi_i'}$ are continuous. Therefore, $\mathcal{F}_i$ is locally Lipschitz in χ_i' uniformly in t for $t \ge t_0$. Following similar procedures in the proof of Proposition 1, the existence and uniqueness of the solutions χ_i' can be verified. In fact, since the existence and uniqueness of x_0 and e_i for all $i \in \mathcal{N}_0^{\text{out}}$ has been proved in Proposition 1, x_i and u_i are well-defined in $[t_0, \infty)$, $i \in \mathcal{N}_0^{\text{out}}$. As such, the existence and uniqueness of all χ_i' can be verified layer by layer. Applying LaSalle-Yoshizawa Theorem [30, Th. 2.1], it is guaranteed that all the solutions χ_i' are globally uniformly bounded and

$$\lim_{t\to\infty} {e_i'}^T(I + PB_0B_0^TP)e_i' = 0.\tag{19}$$

Equation (19), together with the fact that the matrix $I + PB_0B_0^TP$ is positive definite, implies that $\lim_{t\to\infty} e_i' = 0$. This completes the proof. ■

C. Leader-following consensus

Before presenting the main result on leader-following consensus, we present a novel lemma on linear algebra over graphs, the proof of which, interestingly, is inspired by a well-known result ([32, Lem. 2.4]) in algebraic graph theory.

Lemma 2: Under Assumption 3, consider a system of linear equations (with variables x_i, $i \in \mathcal{V}$):

$$\begin{cases} x_i = c, & i \in \mathcal{N}_0^{\text{out}},\\ x_i = \frac{\sum_{j\in\mathcal{N}_i} a_{ij}x_j}{\sum_{j\in\mathcal{N}_i} a_{ij}}, & i \in \mathcal{V}\setminus\mathcal{N}_0^{\text{out}}, \end{cases}\tag{21}$$

where $c \in \mathcal{R}^n$ is a constant vector. Then, the system of equations (21) has a unique solution $x_i = c$, $\forall i \in \mathcal{V}$.

Proof. In the proof, we assume $n = 1$. The case with $n > 1$ can be proved in a similar way involving Kronecker product.

Without loss of generality, let us order the agents in such a way that the first s agents are pinned by the leader, i.e., $x_s \in \mathcal{N}_0^{\text{out}}$, $1 \le s < N$. Note that $s = N$ would be a trivial case. After subtracting the first row from the second to the s-th rows (this does not influence the solution), one gets the augmented matrix of the system of linear equations as in (20).

It is observed that the second to the N-th rows of (20), along with a virtual first row of all zero, completely describes a new system of linear equations $\mathcal{L}'x = 0$, where $x = (x_1, x_2 \ldots, x_N)^T$ and $\mathcal{L}'$ is a Laplacian matrix associated with a new graph $\mathcal{G}'$ in which node 1 has no neighbors. Under Assumption 3, this new graph $\mathcal{G}'$ has a directed spanning tree with root node 1: in fact, there is an edge from node 1 to each node $i \in \{2, \ldots, s\}$ (if $s > 1$), and the edges pointing to node $i \in \{s+1, \ldots, N\}$ in $\mathcal{G}$ is preserved. Then, by [32, Lem. 2.4], the solution of the second to the N-th rows of (20) is given by $x = c_0\mathbf{1}_N$ for some c_0. Combined with the first row of (20), the solution of (21) is given by $x = c\mathbf{1}_N$. ■

Theorem 2: Under Assumptions 1-4, consider the hierarchical control design with MRASC (6) law applied to agent $i \in \mathcal{N}_0^{\text{out}}$ and v-MRASC (12) law applied to agent $i \in \mathcal{V}\setminus\mathcal{N}_0^{\text{out}}$. Then, for each $i \in \mathcal{V}$, the tracking error e_i converges to zero asymptotically, and all the parameter estimation errors in (6) and (12) are globally uniformly bounded.

Proof. According to Proposition 1 and Theorem 1, $x_i \to x_0$ for $i \in \mathcal{N}_0^{\text{out}}$ and $x_i \to \frac{\sum_{j\in\mathcal{N}_i} a_{ij}x_j}{\sum_{j\in\mathcal{N}_i} a_{ij}}$ for $i \in \mathcal{V}\setminus\mathcal{N}_0^{\text{out}}$; all the parameter estimation errors are globally uniformly bounded. Then, by Lemma 2, there exists a unique leader-following consensus manifold, i.e., $x_i \to x_0$, $\forall i \in \mathcal{V}$. ■

$$\left(\begin{array}{cccc:cccc|c} 1 & & & & & & & & c\\ -1 & 1 & & & & & 0 & & 0\\ -1 & & \ddots & & & & & & \vdots\\ -1 & & & 1 & & & & & 0\\ \hdashline -a_{s+1,1} & \ldots & \ldots & -a_{s+1,s} & \sum_{j\in\mathcal{N}_{s+1}} a_{s+1,j} & & \ldots & -a_{s+1,N} & 0\\ \ldots & \ldots & \ldots & \ldots & \ldots & & \ddots & \ldots & \vdots\\ -a_{N,1} & \ldots & \ldots & -a_{N,s} & -a_{N,s+1} & & \ldots & \sum_{j\in\mathcal{N}_N} a_{N,j} & 0 \end{array}\right)\tag{20}$$

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Remark 4: In line with standard adaptive control literature [24], [30], even in our leader-following scenario the tracking errors are shown to converge and the estimated gain parameters may not converge to their ideal values. In a word, the tracking objective can be achieved without requiring exact parameter estimation.

Remark 5: Previous works based on MRAC [16]–[18] require the leader dynamics A_0 to be Hurwitz, not required in Theorem 2. Relaxing the leader dynamics allows the agents to track richer dynamics such as spikes, impulses and other patterns generated by unstable dynamics [5], [22], [23].

Remark 6: It was pointed out in [16]–[18] that although existing MRAC-based leader-following methods may also work in the presence of cycles, the stability analysis requires the graph to be acyclic and a rigorous proof for general graphs with cycles remained an open problem. Here, by separating Assumption 4 from Assumption 3, we can handle this problem from a network design perspective, i.e., by selecting the pinned followers $i \in \mathcal{N}_0^{\text{out}}$ such that Assumptions 3-4 hold, as discussed in the following section.

IV. On the Minimum Number of Pinned Agents

Note that Assumptions 3-4 can always be satisfied in the trivial case where $i \in \mathcal{N}_0^{\text{out}}$ for all $i \in \mathcal{V}$, resulting in a centralized MRASC law. Motivated by the pinning control method [33]–[35], we are interested in a possibly small number of agents to be pinned by the leader such that Assumptions 3-4 hold. Without loss of generality, the original digraph among the followers is assumed to be weakly connected, or else each weakly connected components could be analyzed separately.

The fulfillment of Assumptions 3-4 naturally leads to the notion of singleton leading strongly connected components (s-LSCCs, see Sect. II-A) and of independent cycles. Since the notion of independent cycles appeared only in undirected graphs historically [36], [37], let us present the following definition for the directed case.

Definition 1 (Independent Cycles): In a weakly connected digraph $\mathcal{G}$, a cycle is called independent, if it does not share a node with any other cycle.

For Assumptions 3-4 to be satisfied, it is necessary that every s-LSCC is pinned and every independent cycle contains a pinned node. More specifically, the following result provides a lower bound for the minimum number of pinned agents.

Theorem 3: Starting from any weakly connected digraph $\mathcal{G}$ among the followers, the minimum number of followers pinned by the leader such that Assumptions 3-4 hold, denoted by $N_{\text{min\_pin}}$, satisfies

$$N_{\text{min\_pin}} \geq N_{\text{s-LSCCs}} + N_{\text{max\_ICs}}, \tag{22}$$

where $N_{\text{s-LSCCs}}$ is the number of s-LSCCs of $\mathcal{G}$, and $N_{\text{max\_ICs}}$ is the maximum number of independent cycles of $\mathcal{G}$.

Proof. On one hand, to fulfill Assumption 3, all s-LSCCs should be pinned by the leader. On the other hand, to fulfill Assumption 4, one needs to break all independent cycles by pinning at least one node in each independent cycle. Then, it would be sufficient for Assumption 3 to hold since all LSCCs would be pinned by the leader, while it may not be sufficient for Assumption 4 to hold. Thus, inequality (22) holds. ■

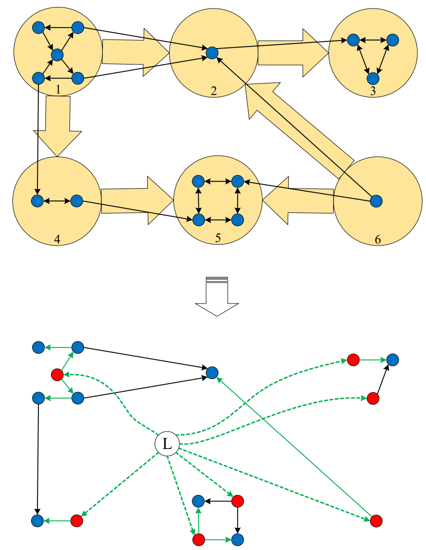

Fig. 1. An illustration example of Theorem 3. At the top is the digraph $\mathcal{G}$ among the followers and its condensation digraph. The digraph $\mathcal{G}$ has $N_{\text{s-LSCCs}} = 1$ (SCC 6) and $N_{\text{max\_ICs}} = 5$ (one for SCC 1, 3, 4 and two for SCC 5). At the bottom is the augmented digraph $\bar{\mathcal{G}}$ after pinning $N_{\text{min\_pin}} = 7$ followers (in red) by the leader (L). It can be easily verified that Assumptions 3-4 hold where a DST is highlighted in green and the resulting $\mathcal{G}_{\text{v-MRASC}}$ is acyclic. The strict inequality in (22) holds due to SCC 3, where one extra follower is pinned to break all the cycles in the 3-order complete digraph.

Remark 7: Theorem 3 not only provides a lower bound for "how many" agents should be pinned, but also provides instructions on "which ones" should be pinned first: each s-LSCC and at least one node in each independent cycle must be pinned. In fact, these are two critical problems consistently pursued in the pinning control literature. Existing results on these crucial problems mainly focus on undirected graphs by resorting to node degree distribution and controllability analysis [33]–[35]. Differently, the proposed MRASC framework applies to digraphs, and Theorem 3 provides a dedicated pinning solution from the structural perspective.

Fig. 1 shows an illustrative example of Theorem 3, where strict inequality in (22) holds due to a SCC that is a 3-order complete graph. Roughly speaking, for a given digraph with a fixed number of nodes, strict inequality in (22) may arise if the digraph contains SCCs that have dense edges. It can be easily verified that equality in (22) holds in several examples, including general acyclic digraphs (line, star, tree, etc.), directed ring graph, undirected line and undirected ring with even number of nodes.

V. Simulation

Consider a network of $N = 4$ heterogeneous unknown second-order agents (1b). Their true parameters (unknown for control design) are, for $i = 1, \ldots, 4$,

$$A_i = \begin{pmatrix} 0 & 1 \\ i & i \end{pmatrix}, \; B_i = \begin{pmatrix} 0 \\ (-1)^i i \end{pmatrix}. \tag{23}$$

The leader dynamics follows (1a) with stabilizable pair (but unstable A_0)

$$A_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0.1 \end{pmatrix}, \; B_0 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \tag{24}$$

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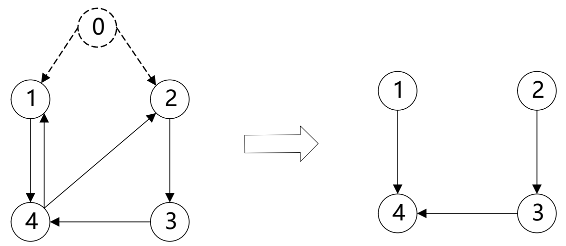

Fig. 2. Case 1: The leader-following communication digraph $\bar{\mathcal{G}}^1$ (left) and the resulting $\mathcal{G}^1_{\text{v-MRASC}}$ (right).

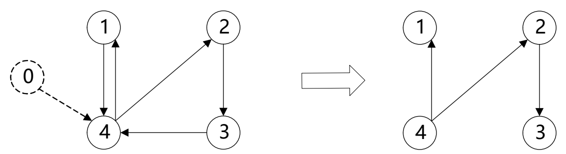

Fig. 3. Case 2: The leader-following communication digraph $\bar{\mathcal{G}}^2$ (left) and the resulting $\mathcal{G}^2_{\text{v-MRASC}}$ (right).

and with external reference input $r = \sin(t)$. Assumptions 1-2 hold with $\text{sgn}(L_i^*) = -1$ for $i = 1, 3$, and $\text{sgn}(L_i^*) = 1$ for $i = 2, 4$.

Solving the algebraic Riccati equation (4) with the `are` command in Matlab gives

$$P = \begin{pmatrix} 1.4483 & 0.3090 \\ 0.3090 & 0.6615 \end{pmatrix}.$$

We consider two cases of leader-following communication topologies as shown in Figs. 2-3. It can be easily verified that Assumptions 3-4 hold in both cases. The difference is that the minimum number of pinned nodes by Theorem 3 is used in Case 2: in fact, $N_{\text{s-LSCCs}} = 0$ and $N_{\text{max\_ICs}} = 1$ ($1 \to 4 \to 1$ or $2 \to 3 \to 4 \to 2$) in this case.

The initial states of the leader and the followers are randomly chosen according to Gaussian distribution, and all the initial adaptive gains are set as zero. After implementing the proposed methodology, the states of the agents along with the associated adaptive gains are shown in Figs. 4-5 for Case 1. Since the results for Case 2 exhibit similar overall transient behaviors, they are omitted due to the lack of space. Instead, provided in Fig. 6 (top and middle) are the tracking errors for both cases where, interestingly, Case 2 has slightly better transient performance than Case 1, even though there is only one single follower pinned by the leader. This indicates the significance of Theorem 3, which helps to identify the key node, agent 4: in Case 1, agent 4 has two neighbors to track that may cause extra oscillations.

For comparison purpose, we also apply the MRAC-based method in the previous works [17] in Case 2, where a crucial difference is the lack of the stabilizing term $\hat{S}_i e_i$ for agent $i \in \mathcal{N}_0^{\text{out}}$. The tracking errors are shown in Fig. 6 (bottom), where significant tracking errors remain. The simulation results are consistent with the theoretical results, and highlight the advantage of the proposed MRASC-based solution for leader-following consensus.

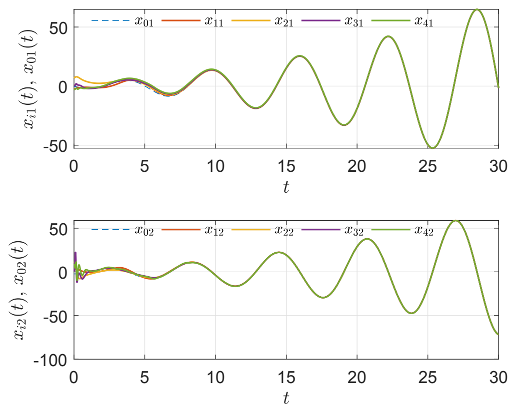

Fig. 4. The states of the leader and followers in Case 1.

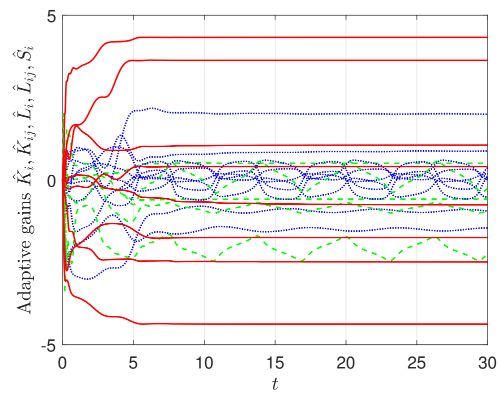

Fig. 5. The adaptive gains in Case 1 (the blue dashed curves stand for $\hat{K}_i$, $\hat{K}_{ij}$; the green dashed curves stand for $\hat{L}_i$, $\hat{L}_{ij}$; the red curves stand for $\hat{S}_i$).

VI. Conclusions

Leader-following consensus for a class of multi-agent systems with unknown and heterogeneous LTI dynamics over a directed graph has been considered. A hierarchical MRASC-based framework has been developed: the proposed framework can not only guarantee asymptotic consensus error while the leader dynamics is not necessarily stable, but also handle more general directed communication topologies than the state of the art. In particular, to handle directed cycles that are known to be harmful for consensus, a solution inspired by pinning control has been proposed. Future work may address exact parameter estimation and extensions to uncertain time-varying systems like switched systems [38].

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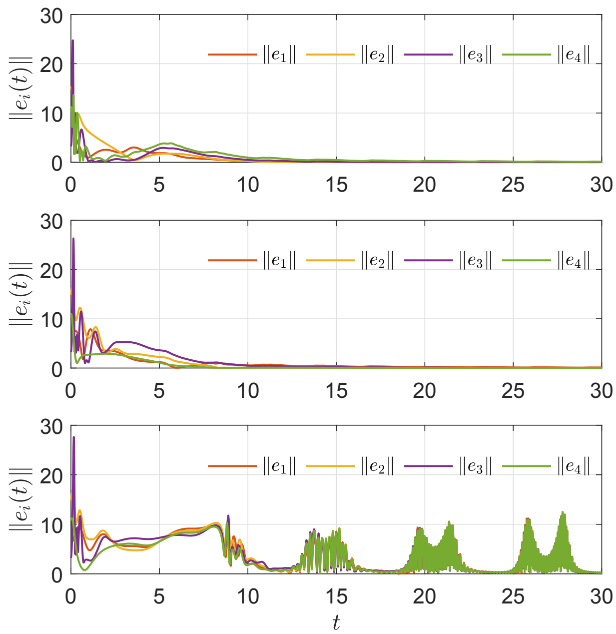

Fig. 6. The leader-following tracking errors in Case 1 (top), Case 2 (middle), and Case 2 with the method of [17] (bottom).

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