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1. $\begin{pmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{pmatrix} \begin{pmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{pmatrix} = \begin{pmatrix} 0.8(0.8)+0.6(0.6) & 0.8(0.6)+0.6(-0.8) \\ 0.6(0.8)+(-0.8)(0.6) & 0.6(0.6)+(-0.8)(-0.8) \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

4. $(2A-I)(2A-I) = 4AA - 2A - 2A + I$ since A idempotent $AA = A$

$= 4A - 4A + I$

$= I$

5. $A' = -A$ so $a_{ji} = -a_{ij}$

$|A| = \sum_{j=1}^{p} a_{ij} c_{ij}$ where $c_{ij} = (-1)^{i+j} |m_{ij}|$

$|A'| = |-A| = (-1)^p |A|$

but $|A'| = |A|$ also

so $|A| = (-1)^p |A|$

$(1-(-1)^p)|A| = 0$

so when p is odd $(-1)^p = -1$ so $(1-(-1)^p)|A| = 0$

7a) i) $AA^-AA^- = AA^-$ since $AA^-A = A$ $\Rightarrow AA^-$ idempotent

$A^-AA^-A = A^-A$ since $AA^-A = A$ $\Rightarrow A^-A$ idempotent

ii) $rank(AA^-) \le \min(rank(A), rank(A^-))$

$\Rightarrow rank(AA^-) = rank(A)$ or

$rank(AA^-) \le rank(A^-) = rank(A^{-1}) = rank(A)$

when A nonsingular so $rank(AA^-) = rank(A)$

iii) suppose A nonsingular then $A^- = A^{-1}$

and $rank(A) = rank(A^{-1}) = rank(A^-)$

9.

- Def: A p-dimensional random vector X is said to have a multivariate Normal distribution with mean vector μ and covariance matrix Σ if its p.d.f. is given by
- $f(x) = \underline{(2\pi)^{-p/2}|\Sigma|}^{-1/2} exp\{-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu)\}, x \in R^p$
- We write $X \sim N_p(\mu, \Sigma)$
- When $p = 2$, $N_2(\mu, \Sigma)$ is called the bivariate Normal distribution.

9. (8 points) The random vector $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ has density function

$$f(x_1, x_2) = \frac{1}{2\pi} \exp\{-\frac{1}{2}(2x_1^2 + x_2^2 + 2x_1x_2 - 22x_1 - 14x_2 + 65)\}, (x_1, x_2)' \in R^2$$

Calculate $E(X)$ and $cov(X)$.

$p=2$

$$\left(\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} - \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}\right)' \Sigma^{-1} \left(\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} - \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}\right)$$

$$= (x_1 - \mu_1, X_2 - \mu_2) \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \begin{pmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \end{pmatrix}$$

$$= (\Sigma_{11}(x_1 - \mu_1) + \Sigma_{21}(x_2 - \mu_2), \Sigma_{12}(x_1 - \mu_1) + \Sigma_{22}(x_2 - \mu_2)) \begin{pmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \end{pmatrix}$$

$$= (\Sigma_{11}(x_1 - \mu_1) + \Sigma_{21}(x_2 - \mu_2))(x_1 - \mu_1) + (\Sigma_{12}(x_1 - \mu_1) + \Sigma_{22}(x_2 - \mu_2))(x_2 - \mu_2)$$

$$= \Sigma_{11}X_1^2 - 2\Sigma_{11}\mu_1 X_1 + \Sigma_{11}\mu_1^2 + \Sigma_{21}X_1X_2 - \Sigma_{21}\mu_2 x_1 - \Sigma_{21}\mu_1 X_2 + \Sigma_{21}\mu_2\mu_1$$
$$+ \Sigma_{12}x_1x_2 - \Sigma_{12}\mu_1 x_2 - \Sigma_{12}\mu_2 x_1 + \Sigma_{12}\mu_1\mu_2 + \Sigma_{22}x_2^2 - 2\Sigma_{22}\mu_2 x_2 + \Sigma_{22}\mu_2^2$$

from question $\Sigma_{11}X_1^2 = 2X_1^2 \Rightarrow \Sigma_{11} = 2$ $\quad (\Sigma_{21} + \Sigma_{12}) x_1 X_2 = 2x_1x_2 \Rightarrow \Sigma_{21} + \Sigma_{12} = 2$ (1)

$\Rightarrow -2\Sigma_{11}\mu_1 X_1 = -4\mu_1 X_1$ $\quad \Sigma_{22}X_2^2 = 1X_2^2 \Rightarrow \Sigma_{22} = 1$ $\quad \Sigma_{21} = \Sigma_{12} = 1$

$\Sigma_{11}\mu_1^2 = 2\mu_1^2 = 2(4)^2 = 32$ from (4)

$x_1(-2\Sigma_{11}\mu_1 - \Sigma_{21}\mu_2 - \Sigma_{12}\mu_2) = -22x_1 \Rightarrow 2\Sigma_{11}\mu_1 + \Sigma_{21}\mu_2 + \Sigma_{12}\mu_2 = 22$

since $\Sigma_{11} = 2$ $\quad \Rightarrow 4\mu_1 + (\Sigma_{21} + \Sigma_{12})\mu_2 = 22$ and from (1)

$\Rightarrow 4\mu_1 + 2\mu_2 = 22$

$\Rightarrow 2\mu_1 + \mu_2 = 11$ (2)

$x_2(-\Sigma_{21}\mu_1 - \Sigma_{12}\mu_1 - 2\Sigma_{22}\mu_2) = -14x_2 \Rightarrow \Sigma_{21}\mu_1 + \Sigma_{12}\mu_1 + 2\Sigma_{22}\mu_2 = 14$

from (1) and $\Sigma_{22} = 1$ $\quad (\Sigma_{21} + \Sigma_{12})\mu_1 + 2\Sigma_{22}\mu_2 = 2\mu_1 + 2\mu_2 = 14$

$\Rightarrow \mu_1 + \mu_2 = 7$ (3)

(2) − (3) $\Rightarrow \mu_1 = 11 - 7 = 4$ (4)

from (3) $\quad \mu_2 = 7 - \mu_1 = 7 - 4 = 3$ (5)

$\therefore cov(X) = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ $\quad E(X) = \mu = \begin{pmatrix} \mu_1 = 4 \\ \mu_2 = 3 \end{pmatrix}$