NTU 107-1 MATH1201 | CALCULUS A-05 QUIZ 2 | 26TH OCTOBER, 2018

Work out **ALL** questions below. Provide sufficient justification to every step of your arguments.
Write your solutions as well as your ID number clearly on A4-sized paper and submit them to *Instructor's office* **before 4pm (GMT +8) on 29th October, 2018**.
Recommended time limit: 150 minutes.

1. Let a and b be two real numbers and

$$f(x) := \begin{cases} \dfrac{e^{-x^{-2}} - a}{x} & \text{for } x \neq 0\,, \\ b & \text{for } x = 0\,. \end{cases}$$

   (a) (8 points) If f is continuous everywhere on $\mathbb{R}$, what are the values of a and b?

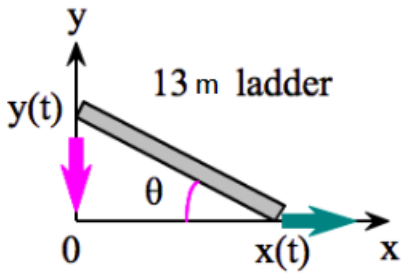
   (b) (8 points) If f is continuous everywhere on $\mathbb{R}$, determine whether $f'(0)$ and $f''(0)$ exist. Find their values if they do.

2. (a) (6 points) Let $f(x) = \dfrac{x(x-1)(x-2)\cdots(x-n)}{(x+1)(x+2)\cdots(x+n)}$. Find $f'(0)$.

   (b) (8 points) Suppose that the function $f\colon \mathbb{R} \to \mathbb{R}$ satisfies $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$. If $k := f'(0)$, show that $f'(x) = kf(x)$.
   (Hint: you may have to consider the cases $f(0) = 0$ and $f(0) \neq 0$ separately.)

3. (8 points) Find $\dfrac{dy}{dx}$ if $\tan^{-1}\left(\frac{y}{x}\right) = \ln\left(\sqrt{x^2+y^2}\right)$.

4. (12 points) A ladder 13 metres long is leaning against a wall when its base starts to slide away. By the time the base is 12 metres from the wall, the base is moving at the rate of 0.5 m/s. At what rate is the area of the triangle formed by the ladder, the wall and the ground changing at that moment?



5. (a) (10 points) Evaluate $\lim\limits_{x\to\infty}(\sin((x+2)^{\frac{3}{4}}) - \sin(x^{\frac{3}{4}}))$ using the Mean Value Theorem.

(b) (7 points) Applying the Mean Value Theorem to show that $\dfrac{1}{1+x} < \dfrac{\ln(1+x)}{x} < 1$ for $x > 0$.

6. Determine if the following limits exist or not. Evaluate them if they do.

(a) (5 points) $\lim\limits_{x\to 0} \dfrac{\sin^{-1} x}{x}$

(b) (8 points) $\lim\limits_{x\to 0} \left(\dfrac{\sin^{-1} x}{x}\right)^{\frac{1}{x^2}}$

(Hint: avoid differentiating quotients whenever you want to apply l'Hospital's rule.)

(c) (8 points) $\lim\limits_{x\to\infty} x\left(\left(1+\dfrac{1}{x}\right)^{x} - e\right)$

7. Let

$$f(x) := \frac{\sqrt{|x|}\,(x-2)}{\sqrt{x+1}}\,.$$

(a) (1 point) What is the domain of f ?

(b) (3 points) Find all vertical asymptotes of the graph of f.

(c) (6 points) Evaluate $\lim\limits_{x\to\infty} (f(x) - x)$. Thus find all slant asymptotes of the graph of f.

(d) (6 points) Find all critical points of f (in its domain). Identify also the intervals on which f is increasing or decreasing.

(e) (6 points) Identify all the intervals on which the graph of f is concave upward or downward. Is there any inflection point?

(f) (4 points) Sketch the graph of f using the results above. Label all local extrema and inflection points with their coordinates.

The End.