NTU 107-1 MATH1201 Calculus A-05
Exercise set 1

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Write your solutions to the following problems on a separate sheet of paper and submit it to your TA or instructor.
Submit your solutions to Problems *(4), (5), (6) and (7)* on **October 12**.
Submit your solutions to Problems *(8), (9) and (10)* on **October 17**.
The rest are left for your self-revision.

1. Given $g(2) = 4, f(2) = 2$ and $g'(x) = \sqrt{x^2+5}, f'(x) = \sqrt{x^3+1}$ for all $x > 0$, find the derivative of $g(f(x))$ at $x = 2$.

2. Let $f(x) = e^x \cdot \ln(2 + \sin x)$. Find $f'(x)$ and $f'(0)$.

3. If $y^4 + xy^2 - 2 = 0$, find y'.

4. Find the following limits or explain why they do not exist.

   (a) (5 points) $\lim\limits_{x\to 0} \dfrac{\sqrt{x \sin x}}{x}$

   (b) (5 points) $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{1 - \tan x}{\sin x - \cos x}$

   (c) (5 points) $\lim\limits_{x\to 0} \csc x \sin(\sin x)$

   (d) (5 points) $\lim\limits_{x\to 0} \dfrac{e^{\sin x} - 1}{x}$

5. (12 points) Find the n-th derivative of the function $f(x) = \dfrac{x^n}{1-x}$.

6. (18 points) Let $f(x) = x^r|x|$, where $r > 0$ is a positive number such that x^r is a well-defined function on $\mathbb{R}$ (e.g. r is a rational number $\frac{p}{q}$ with q being odd). Determine whether f is differentiable at 0 and find $f'(0)$ if it does.

7. (10 points) The figure shows a circular arc of length s and a chord of length d, both subtended by a central angle θ. Find $\lim_{\theta\to 0^+} \dfrac{s}{d}$.

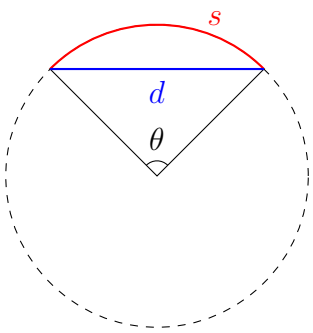

Figure of Problem 7

8. Find the derivatives of the following functions.

   (a) (5 points) $f(x) = \dfrac{\sin x}{1 + \cos x}$

   (b) (5 points) $f(x) = \log_2 \sqrt{x} + \tan^{-1}(x^3)$

   (c) (5 points) $f(x) = x^{\cos x}$

   (d) (5 points) $y = \dfrac{(2x+1)^5(x^2+1)^3}{(3x-2)^6(x^3+1)^4}$, find $y'(0)$.

9. Suppose that $f(x)$ is a twice differentiable function such that

$$\lim_{x\to 1} \frac{(f(x))^3 - 8}{x - 1} = 18 \quad \text{and} \quad \lim_{t\to 0} \frac{f'(1+t) - f'(1-3t)}{t} = 1\ .$$

   (a) (15 points) Find $f(1)$, $f'(1)$ and $f''(1)$.

   (b) (15 points) Suppose that $g(x) = f(e^{2x})$ is an one-to-one function and $h(x) = g^{-1}(x)$, the inverse function of $g(x)$. Find $h(2)$, $h'(2)$ and $h''(2)$.

10. (20 points) A lamp located 4 units to the right of the y-axis and a shadow created by the elliptical region $x^2 + 5y^2 \le 6$. If the point $(-6, 0)$ is on the edge of the shadow, how far above the x-axis is the lamp located?

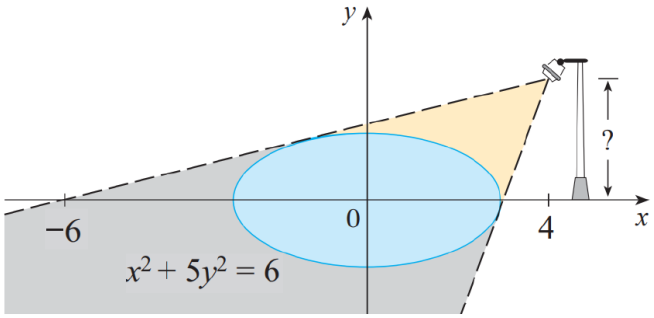

Figure of Problem 10