Lab 2A: Magic

李岳洲

2018/10/01

Magic Square

- A nxn Matrix whose entries are the integers from 1 to n.
- The sum of every row, every column, diagonal and anti-diagonal is the magic number $n(n^2+1)/2$.
- Example:
  Durer's magic square

$$\begin{bmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{bmatrix}$$

Review

- Inner product: $\begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = a+b+c+d$

$$M \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} m_1 & m_2 & m_3 & m_4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = m_1+m_2+m_3+m_4$$

Review

- In the following parts, we will use this formula

$$x = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}, \quad x^T A x = x^T (a_1 + \ldots + a_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij}$$

Hands on

Hands on: Lab 2A Magic

- Mathworks Grader Lab 2A: Magic

- Goal:

  Using MATLAB and some techniques in Linear Algebra to check the special properties of Durer's magic square.

$$\begin{bmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{bmatrix}$$

Problem 1

- Construct the matrix $\mathbb{M}$ be the Durer's magic square.

$$\mathbb{M}= \begin{bmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{bmatrix}$$

Problem 2

- Use the matrix-vector multiplication to compute the row sums of the matrix $\mathbb{M}$.

Problem 3

- Use a transpose and the matrix-vector multiplication to compute the column sums of the matrix $\mathbb{M}$.

Problem 4

- Compute the sum of the four 2x2 submatrices made by drawing two lines horizontally and vertically through the center of the matrix M.

$$M = \begin{bmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{bmatrix}$$

Problem 5

- The sum of another 2x2 submatrix made by four interior elements of the matrix M.

$$M = \begin{bmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{bmatrix}$$

Problem 6

- Compute the sum of the diagonal elements and the anti-diagonal elements of the matrix $\mathbb{M}$.

$$\mathbb{M} = \begin{bmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{bmatrix}$$

What We Learned

Advantage

- MATLAB optimizes on the matrix-matrix multiplication and the matrix-vector multiplication.

Questions Or Comments?