1081 Calculus2 模組 07 Quiz 1

Time: 17:50∼ 18:20 Date: Nov 28, 2019

DEPARTMENT: _____ ID NUMBER: _____ NAME: _____

It's necessary to explain all the reasons in detail and show all of your work on the answer sheet. Or you will NOT get any credits. If you used any theorems in textbook or proved in class, state it carefully and explicitly.

1. (20%) **True/False** (Write down **T** or **F** only. Any explanations are not required.)

   (a) If f and g are continuous on $[a, b]$, then
   $$\int_a^b (f(x)g(x))\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$$

   (b) For any given real number t, $\int_{-t}^{t} \sin tx\,dx = 0$

   (c) Suppose a and b are positive numbers, $\int_0^1 x^a(1-x)^b\,dx = \int_0^1 x^b(1-x)^a\,dx$

   (d) Suppose f is differentiable, then $\int (\frac{d}{dx}f(x))\,dx = f(x)$

   Answer: (a)_____ (b)_____ (c)_____ (d)_____

2. Evaluate the following integrals,summation, or the volume of the solid.

   (a)(10%) $\int (\tan^{\frac{1}{4}} x)(\sec^2 x)\,dx$

   (b)(10%) $\int \frac{\sin t}{\cos^2 t\sqrt{1+\sec t}}dt$

(c) (10%) $\lim_{n\to\infty}\sum_{i=1}^{n}(\frac{2i}{n})^2\sqrt{1+\left(\frac{i}{n}\right)^3}\frac{1}{n}$

(d)(10%) $\int_0^{\pi/3}\frac{\sin\theta+\sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta$

(e) (20%)Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other sphere.

(f) (20%)Find the volume of a cap of a sphere which is centered at $(R,0,0)$ with radius r and height h, rotating with z-axis