1081 Calculus 模組 07 Homework 6

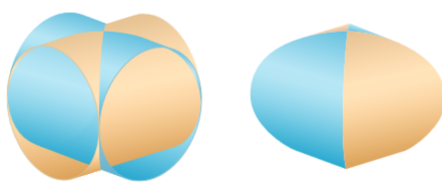
Due Date: 12/5, 2019

Please hand in the following exercise of textbook in Part I and all problems in Part II. The rigorous and clear explanation is needed. An answer without process will get no point.

Part I:

1. (Ch 6.2, Ex. 7,14; Ch 6.3, Ex. 15,41)
   Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.
   (a) $y = \ln x,\ y = 1,\ y = 2,\ x = 0$; rotating about the y-axis.
   (b) $y = \sin x,\ y = \cos x,\ 0 \le x \le \frac{\pi}{4}$; rotating about $y = -1$.
   (c) $y = x^3,\ y = 8,\ x = 0$; rotating about $x = 3$.
   (d) $x^2 + (y-1)^2 = 1$; rotating about $y = 1$.

2. (Ch 6.2, Ex. 66)
   Find the volume common to two circular cylinders, each with radius r, if the axes of the cylinders intersect at right angles.



3. (Ch 7.1, Ex. 9,29,44; Ch 7.2, Ex. 8,39; Ch 7.3, Ex. 12,16,25,28)
   Evaluate the following definite and indefinite integrals.

   (a) $\displaystyle\int \cos^{-1} x\, dx$ (b) $\displaystyle\int_0^{\pi} x \sin x \cos x\, dx$ (c) $\displaystyle\int x^{\frac{3}{2}} \ln x\, dx$

   (d) $\displaystyle\int \frac{\sin^3 \sqrt{x}}{\sqrt{x}}\, dx$ (e) $\displaystyle\int \csc x\, dx$ (f) $\displaystyle\int_0^2 \frac{dt}{\sqrt{4+t^2}}$

   (g) $\displaystyle\int_{\sqrt{2}/3}^{2/3} \frac{dx}{x^5\sqrt{9x^2-1}}$ (h) $\displaystyle\int x^2\sqrt{3+2x-x^2}\, dx$ (i) $\displaystyle\int \frac{x^2+1}{(x^2-2x+2)^2}\, dx$

4. (Ch 7.2, Ex. 67)
   Show that
$$\int_{-\pi}^{\pi} \sin mx \cos nx\, dx = 0, \text{ where } m, n \in \mathbb{N}$$

Part II:

1. Prove the following general mean value theorem for integral: Suppose f is continuous on $[a, b]$, g is integrable on $[a, b]$, and $g \geq 0$. Then there is $c \in [a, b]$ such that

$$\int_a^b f(x)g(x)\,dx = f(c)\int_a^b g(x)\,dx. \tag{1}$$

In particular, if $g(x) \equiv 1$, then (1) reduces to familiar mean value theorem for integral.
(Hint: Since f is continuous, there are m, M such that $m \leq f(x) \leq M$ for $x \in [a, b]$. Then we can obtain that

$$m\int_a^b g(x)\,dx \leq \int_a^b f(x)g(x)\,dx \leq M\int_a^b g(x)\,dx.$$

Apply intermediate value theorem for f to conclude the desired result.)

2. (Ch 7.1, Ex. 49, 50)
Use integration by part to show the following reduction formula

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \frac{n-1}{n}\int_0^{\frac{\pi}{2}} \sin^{n-2} x\,dx, \tag{2}$$

where $n \geq 2$ is an integer. Then apply (2) to show that, for $n \in \mathbb{N}$,

$$\int_0^{\frac{\pi}{2}} \sin^{2n} x\,dx = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots 2n}\frac{\pi}{2},$$
$$\int_0^{\frac{\pi}{2}} \sin^{2n+1} x\,dx = \frac{2\cdot 4\cdot 6\cdots 2n}{3\cdot 5\cdot 7\cdots(2n+1)}.$$

3. (Ch 7.1, Ex. 52)
Use integration by part to show the following reduction formula

$$\int x^n e^x\,dx = x^n e^x - n\int x^{(n-1)}e^x\,dx, \text{ where } n \in \mathbb{N}. \tag{3}$$

Apply (3) to derive that

$$\int x^n e^x\,dx = \left[\sum_{k=0}^{n}(-1)^{n-k}\frac{n!}{k!}x^k\right]e^x + C.$$