1081 Calculus 模組 07 Homework 5

Due Date: 11/21, 2019

Please hand in the following exercise of textbook in Part I and all problems in Part II. The rigorous and clear explanation is needed. An answer without process will get no point.

Part I:

1. (Ch. 5.1, Ex. 24; Ch. 5.2, Ex. 74; Ch. 5.3, Ex. 75; Problem Plus, Ex. 19)
   Evaluate the following limits.
   (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{3i}{n}}$ (b) $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \frac{1}{1+(i/n)^2}$ (c) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^4}{n^5}+\frac{i}{n^2}\right)$

   (d) $\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{n+1}}+\frac{1}{\sqrt{n+2}}+\cdots+\frac{1}{\sqrt{n+n}}\right)$

2. (Ch 5.3, Ex. 13, 17, 61, 78)
   Suppose f, g, h are continuous and g, h are differentiable. Use part 1 of the Fundamental Theorem of Calculus to show that
   $$\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)\,dt = f(g(x))g'(x) - f(h(x))h'(x). \tag{1}$$
   Apply (1) to find the derivatives of the following functions.
   (a) $f(x) = \int_1^{e^x} \ln t\,dt$ (b) $f(x) = \int_{\sqrt{x}}^{\pi/4} \theta\tan\theta\,d\theta$ (c) $f(x) = \int_x^{x^2} e^{t^2}\,dt$

3. (Ch. 5.4, Ex. 31, 37, 38, 43, 44; Ch. 5.5, Ex. 21, 25, 45, 46, 59, 69, 71)
   Evaluate the following definite and indefinite integrals.

   (a) $\int_0^1 x(\sqrt[3]{x}+\sqrt[4]{x})\,dx$ (b) $\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$ (c) $\int_0^{\pi/3} \frac{\sin\theta+\sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta$

   (d) $\int_0^{1/\sqrt{3}} \frac{t^2-1}{t^4-1}\,dt$ (e) $\int_0^2 |2x-1|\,dx$ (f) $\int \frac{(\ln x)^2}{x}\,dx$

   (g) $\int e^x\sqrt{1+e^x}\,dx$ (h) $\int \frac{1+x}{1+x^2}\,dx$ (i) $\int x^2\sqrt{2+x}\,dx$

   (j) $\int_1^2 \frac{e^{1/x}}{x^2}\,dx$ (k) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$ (l) $\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$

Part II:

1. (Ch. 5.3, Ex. 83; Review, Ex. 67)

   (a) If f is a continuous function such that

   $$\int_1^x f(t)\,dt = (x-1)e^{2x} + \int_1^x e^{-t} f(t)\,dt,\ \forall x.$$

   Find an explicit formula for $f(x)$.

   (b) Determine the continuous function f and constant a such that

   $$6 + \int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x},\ \forall x > 0.$$

   (Hint: Take the derivative both side.)

2. (Ch. 5.2, Ex. 56, 57)
   Use the properties of integrals to verify that

   $$1 \le \int_0^1 \sqrt{1+x^2}\,dx \le \int_0^1 \sqrt{1+x}\,dx. \tag{2}$$

   Apply (2) to show that

   $$2 \le \int_{-1}^1 \sqrt{1+x^2}\,dx \le \frac{4}{3}(2\sqrt{2}-1).$$