Introduction to Computational Mathematics
Quiz 3

December 12, 2018

1. (3 pts) Find a sequence $\{x_k\}$ such that $\lim_{k \to \infty} (x_k - x_{k-1}) = 0$ but $\{x_k\}$ diverges.

2. (3 pts) Consider the equations

$$u^2 \log u + v \log v = -0.2$$
$$u^4 + v^2 u = 1.$$

   Write the intersection of these curves in the form $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ for two-dimensional $\mathbf{f}$ and $\mathbf{x}$, and compute the Jacobian matrix of $\mathbf{f}$.

3. (4 pts) The iteration formula of the secant method is

$$x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}.$$

   Let $\epsilon_k = r - x_k$ be the errors in successive root approximations. By using Taylor's expansion, show that

$$\epsilon_{k+1} \approx -\frac{1}{2} \frac{f''(r)}{f'(r)} \epsilon_k \epsilon_{k-1}$$