Lab 2D: Norms

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Review

Vector Norms

- 1-norm:
$$||x||_1 = \sum_{i=1}^{n} |x_i|$$
- 2-norm:
$$||x||_2 = \left(\sum_{i=1}^{n} |x_i|^2 \right)^{1/2} = \sqrt{\mathbf{x}^\mathbf{T}\mathbf{x}}$$
- ∞-norm:
$$||x||_\infty = \max_{i=1,\ldots,n} |x_i|$$
- p-norm:
$$||x||_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}$$

Matrix Norms

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

- 1-norm:

$$||A||_1 = \max_{1 \leq j \leq n} \sum_{i=1}^{m} |a_{ij}|$$ (The maximum of the column sum.)

- 2-norm:

$$||A||_2 = \max_{x \neq 0} \frac{||Ax||_2}{||x||_2}$$

- ∞-norm:

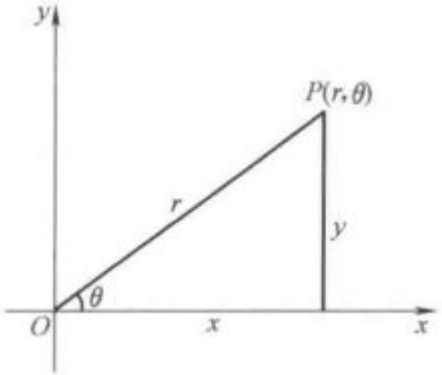
$$||A||_\infty = \max_{1 \leq i \leq m} \sum_{j=1}^{n} |a_{ij}|$$ (The maximum of the row sum.)

Recall

- Polar coordinate system:

$$P(x, y) \rightarrow P(r, \theta)$$

$$x \rightarrow r\cos(\theta)$$
$$y \rightarrow r\sin(\theta)$$

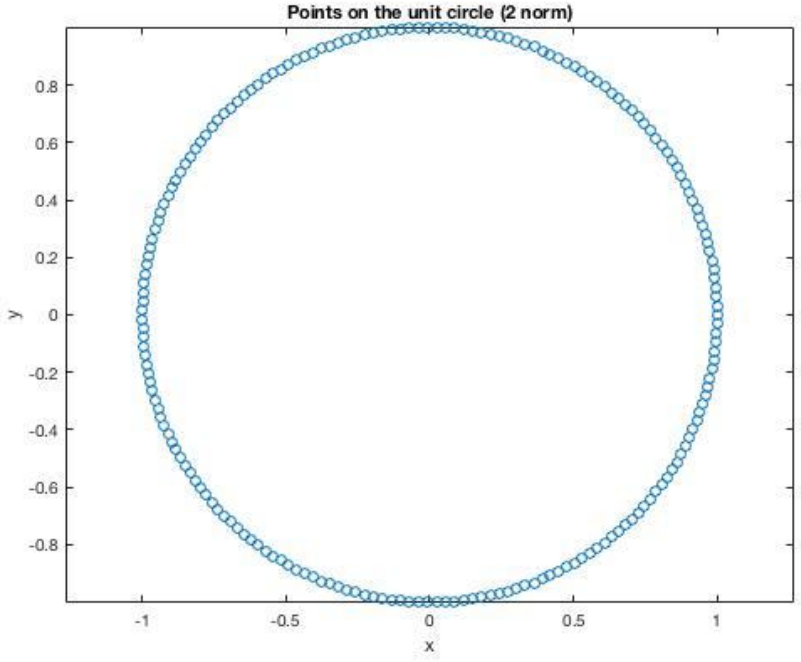


Hands On

Step 1

- Define a vector `theta` (θ) of 200 equally spaced values from 0 to π.
- Define the vector `x` and `y` as $\cos(\theta)$ and $\sin(\theta)$. (without using any loops)
- Plot `x` and `y`.

Step 1


Points on the unit circle (2 norm)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
y
-1
-0.5
0
0.5
1
x

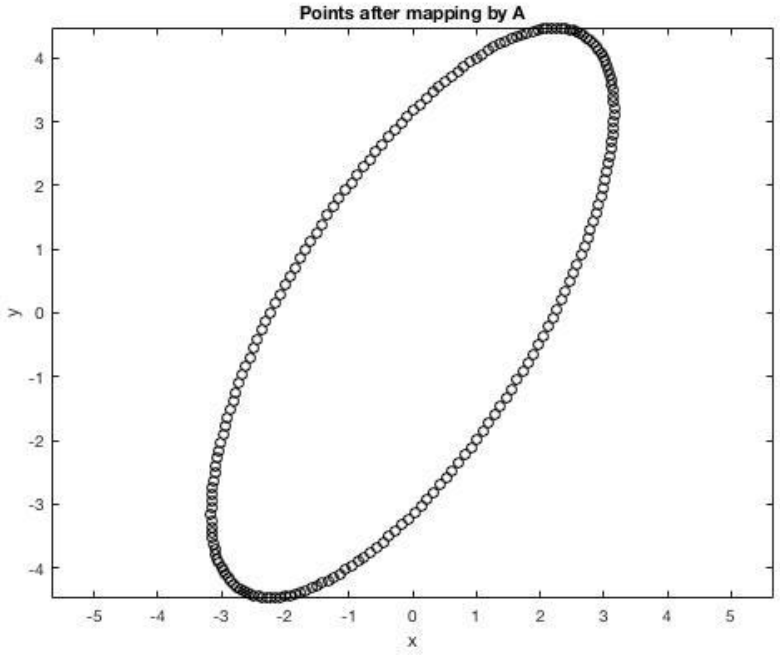
Step 2

- Let `A = magic(2)`.
- For each `j`, define the vector $v = [\, x_j \quad y_j \,]^T$, and compute `u = Av`.
- Plot all `u`.
- Store the vector $||u||_2$ for all `j`.

  Recall:

$$||x||_2 = \left(\sum_{i=1}^{n} |x_i|^2 \right)^{1/2} = \sqrt{\mathbf{x^T x}}$$

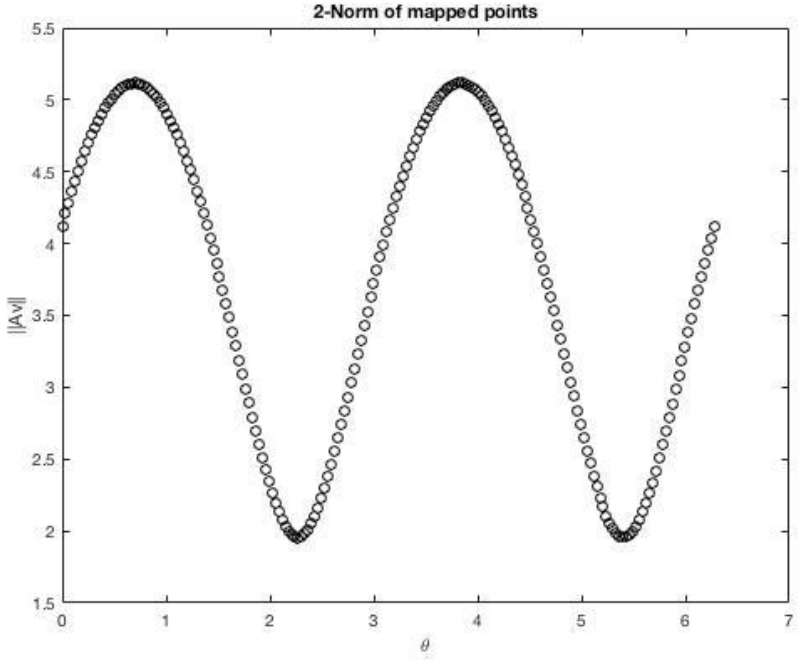
Step 2



Step 3

- Plot $||u||_2$.
- Set x-axis as θ and y-axis as $||u||_2$ or $||Av||_2$.

Step 3



Step 4

- Use `format long` and `max` to compute the maximum value of $||u||_2$.
- Compute `estimate` as the norm $||A||_2$.
- Use `norm(A,2)` to compute `actual` as the actual value of the norm $||A||_2$.

  Recall:

$$||A||_2 = \max_{x \neq 0} \frac{||Ax||_2}{||x||_2}$$

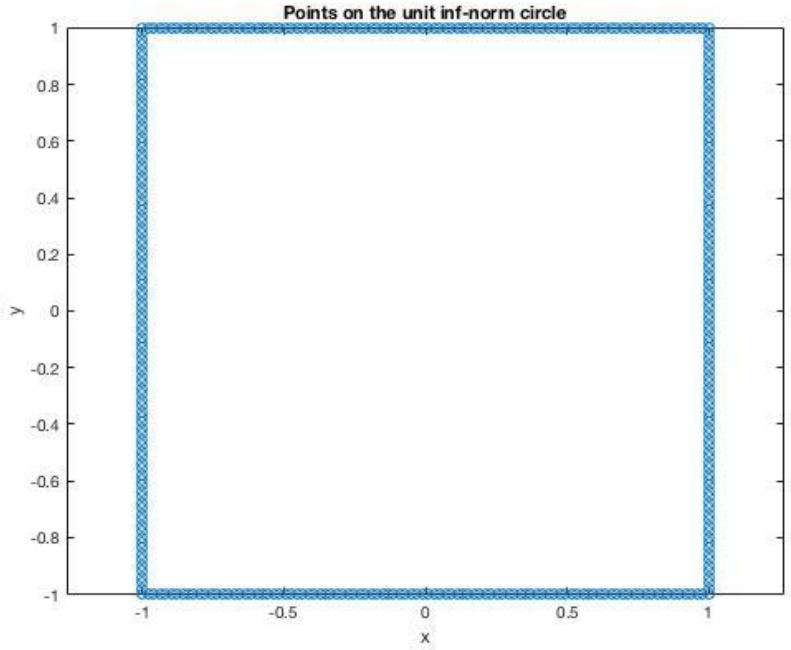
Step 4

```
estimate =

   5.116671165042550


actual =

   5.116672736016928
```

Step 5

- Define `x` and `y` be the unit vectors in the ∞-norm.
- Plot `x` and `y`.
- Set x-label, y-label and title.

Step 5


Points on the unit inf-norm circle
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
y
-1
-0.5
0
0.5
1
x

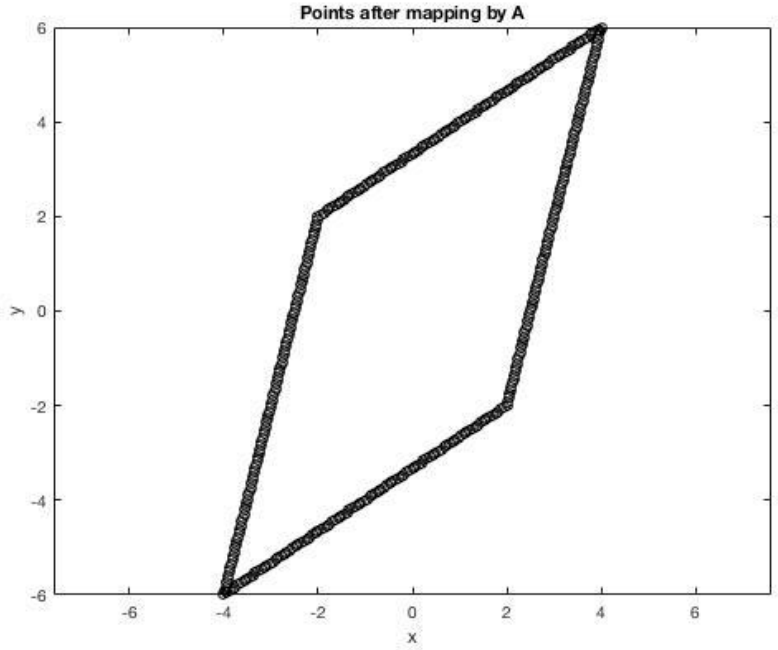
Step 6

- Repeat Step 2.
- Let `A = magic(2)`.
- For each `j`, define the vector $v = [\, x_j \quad y_j \,]^T$, and compute `u = Av`.
- Plot all `u`.
- Store the vector $||u||_\infty$ for all `j`.

Recall:

$$||x||_\infty = \max_{i=1,\ldots,n} |x_i|$$

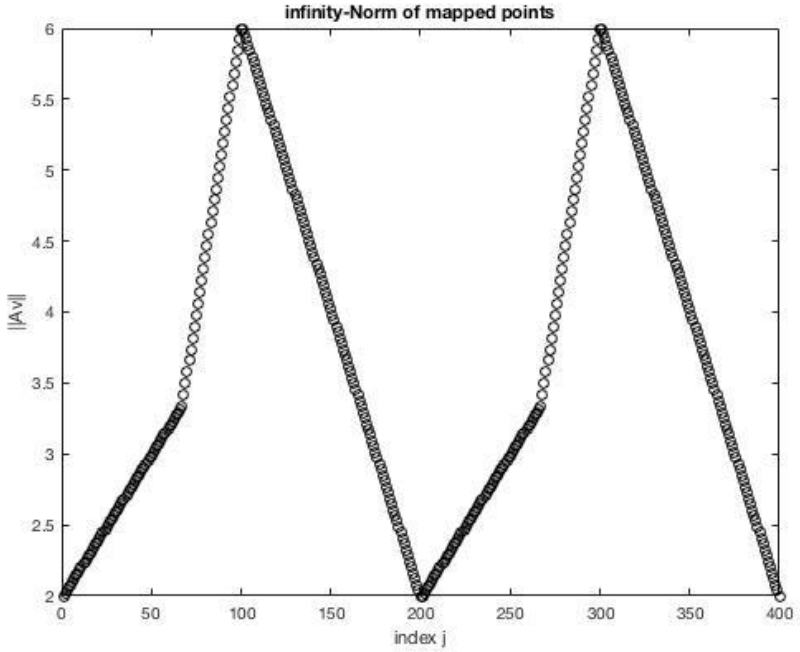
Step 6



Step 7

- Plot $||u||_\infty$ for all `j`.
- Set x-axis as the index `j` and y-axis as $||u||_\infty$ or $||Av||_\infty$.

Step 7


infinity-Norm of mapped points
||Av||
index j
6
5.5
5
4.5
4
3.5
3
2.5
2
0
50
100
150
200
250
300
350
400

Step 8

- Compute `estimate_inf` as the norm $||A||_\infty$.
- Compute `actual_inf` as the actual value of the norm $||A||_\infty$.

  Recall:

  $$||A||_\infty = \max_{1 \le i \le m} \sum_{j=1}^{n} |a_{ij}|$$

Step 8

```
estimate_inf =

     6


actual_inf =

     6
```

What We Learned?

What We Leanered?

- How to compute the vector norm and the matrix norm in MATLAB.

Questions or comments?