Lab 4A: Asteroids

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Review

The Root-finding Problem

- The goal of this problem is to find zeros of a function f(x), that is

$$\begin{aligned}&\text{given } f : \mathcal{I} = (a, b) \subseteq \mathbb{R} \to \mathbb{R},\\&\text{find } \alpha \in \mathbb{C} \text{ such that } f(\alpha) = 0.\end{aligned}$$

Finding Root of Nonlinear Function in MATLAB

- The function `fzero` attempts to find the zero of a function near a given starting point or a given interval.
- **The starting point or interval should be carefully chosen.**

Hands On: Lab 4A Asteroids

Three Forms of Orbits

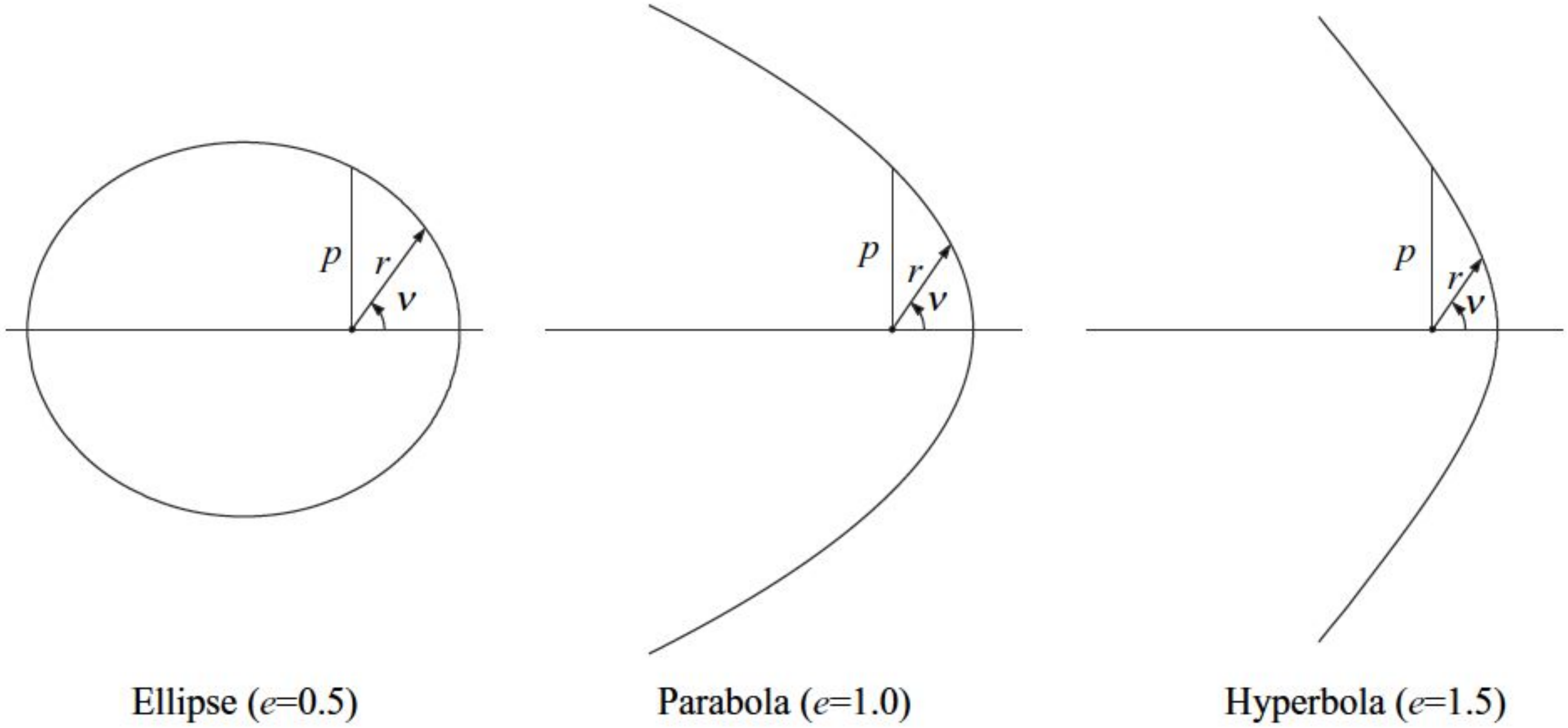
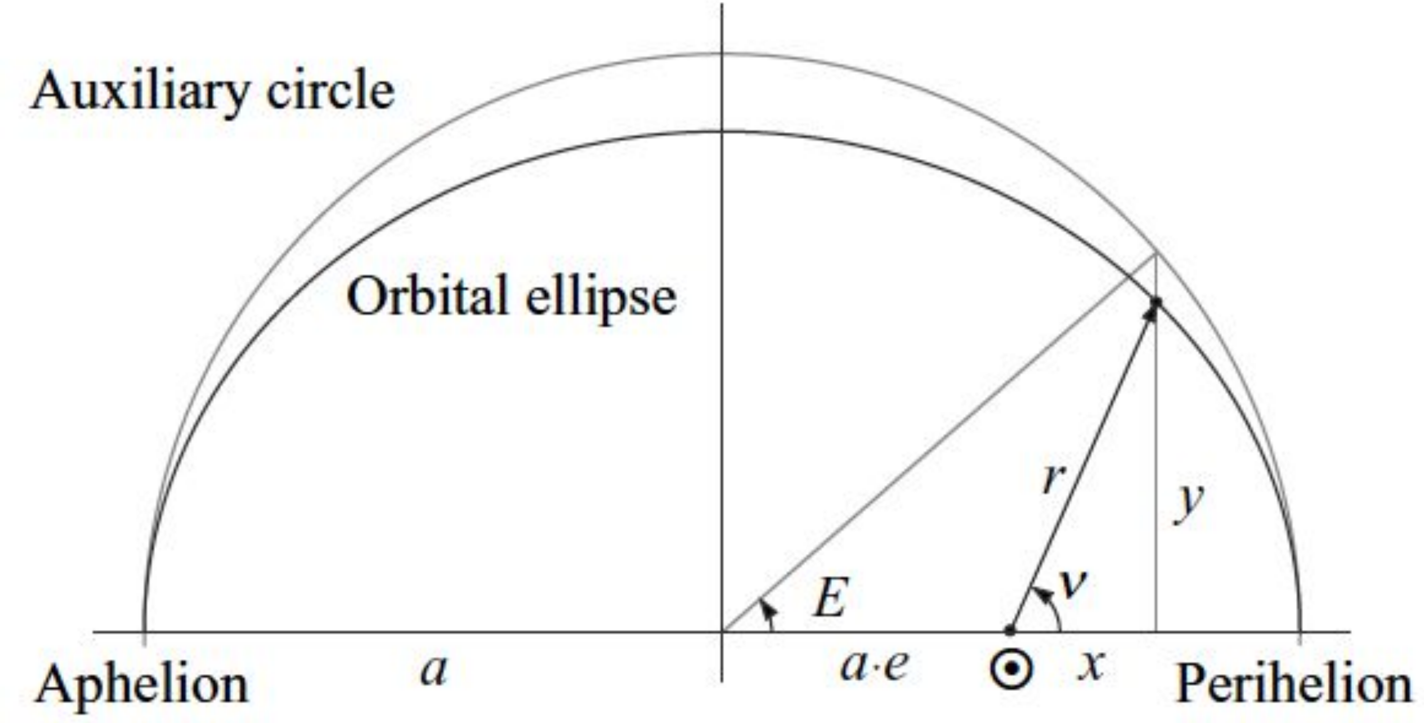


Fig. 4.2. Conic sections with eccentricities $e = 0.5$, $e = 1.0$ and $e = 1.5$ with the same orbital parameter p

Notations

- τ: the period of the orbit
- ε: the eccentricity of the ellipse
- ν: true anomaly
- r: the distance from the body to the sun
- M: the mean anomaly
- ψ: the eccentric anomaly
- a: half of the (maximum) diameter of the ellipse
- μ: a gravitational parameter

Eccentric Anomaly and True Anomaly



Key Equations

$$\tan\frac{\nu}{2} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}}\tan\frac{\psi}{2} \tag{1}$$

$$M = \psi - \varepsilon\sin\psi \tag{2}$$

$$r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos\nu} \tag{3}$$

$$a^3 = \mu\left(\frac{\tau}{2\pi}\right)^2 \tag{4}$$

Explainations to the Key Equations

- The four key equations in the previous slide was proposed by Johannes Kepler. These equations describe the motion of celestial bodies in the solar system.
- The **First Kepler Law** states that celestial bodies in the solar system follow elliptical orbits with the sun at one focus
- Equation (2) is called the Kepler's Equation.
- Equation (4) is the Third Kepler Law.
- The gravitational parameter for the Sun is 39.47524 AU^3/yr^2.

Solving the Kepler's Equation

$$M = \psi - \varepsilon \sin \psi \qquad (2)$$

- Equation (2) implicitly defines ψ as a function of M.
- Equation (2) cannot be solved in closed form.

Therefore, given a value of M , rootfinding must be used to find ψ.

Problem

1. Write a function that solves the Kepler's equation and calculates other parameters in the key equations.

   (Link for detailed instructions)

Tips for Choosing the Starting Point

$$M = \psi - \varepsilon \sin \psi \tag{2}$$

- For $\varepsilon < 0.8$, choose $\psi_0 = M$.
- For $\varepsilon >= 0.8$, choose $\psi_0 = \pi$.

What We Learned?

Rootfinding for Nonlinear Equations

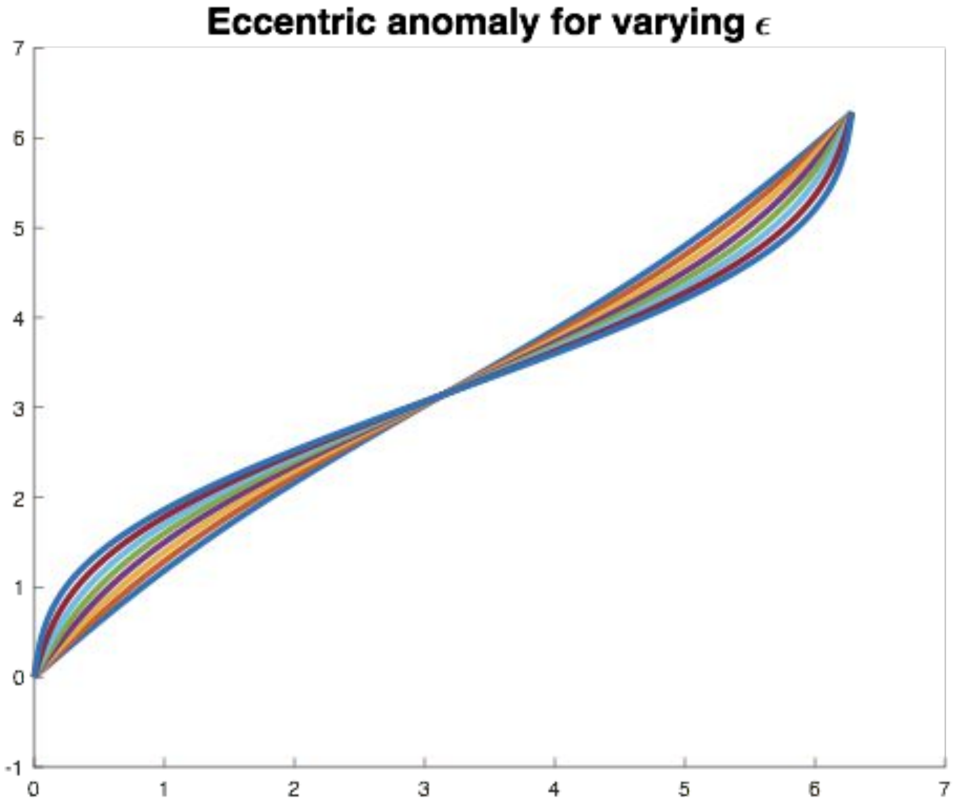
- The usage of the function `fzero`.

- The function `fzero` uses a combination of bisection, secant, and inverse quadratic interpolation methods to find the desired root.

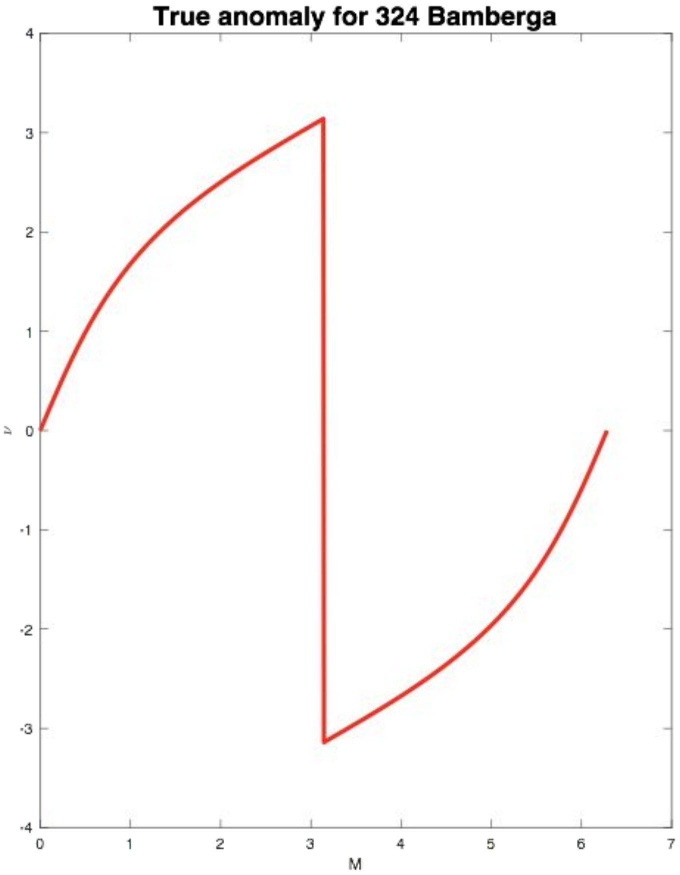
Kepler's Laws of Planetary Motion

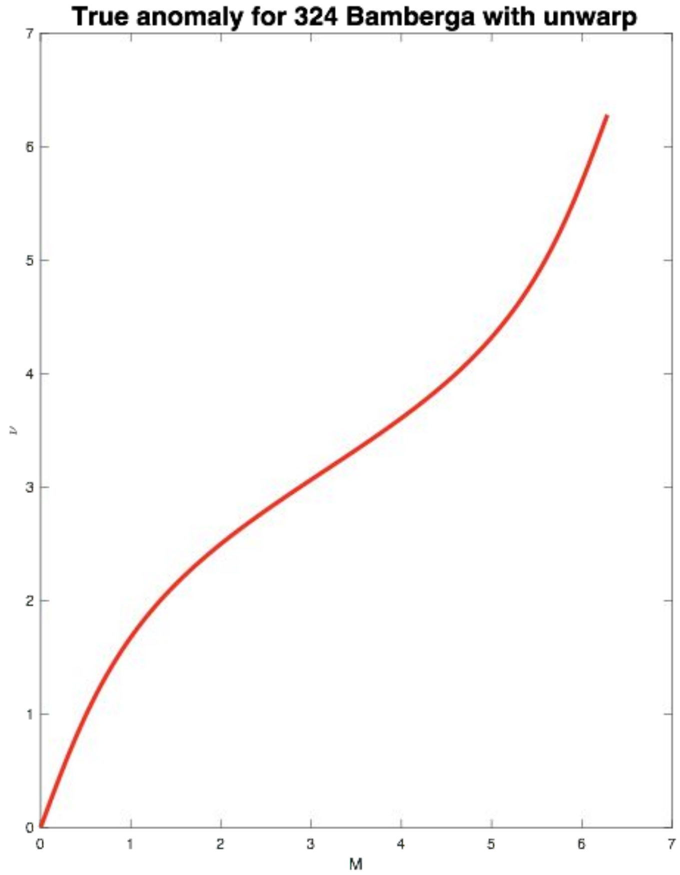
1. The orbit of a planet is an ellipse with the Sun at one of the two foci.
2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.
3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

The Result of ψ for Different Values of ε

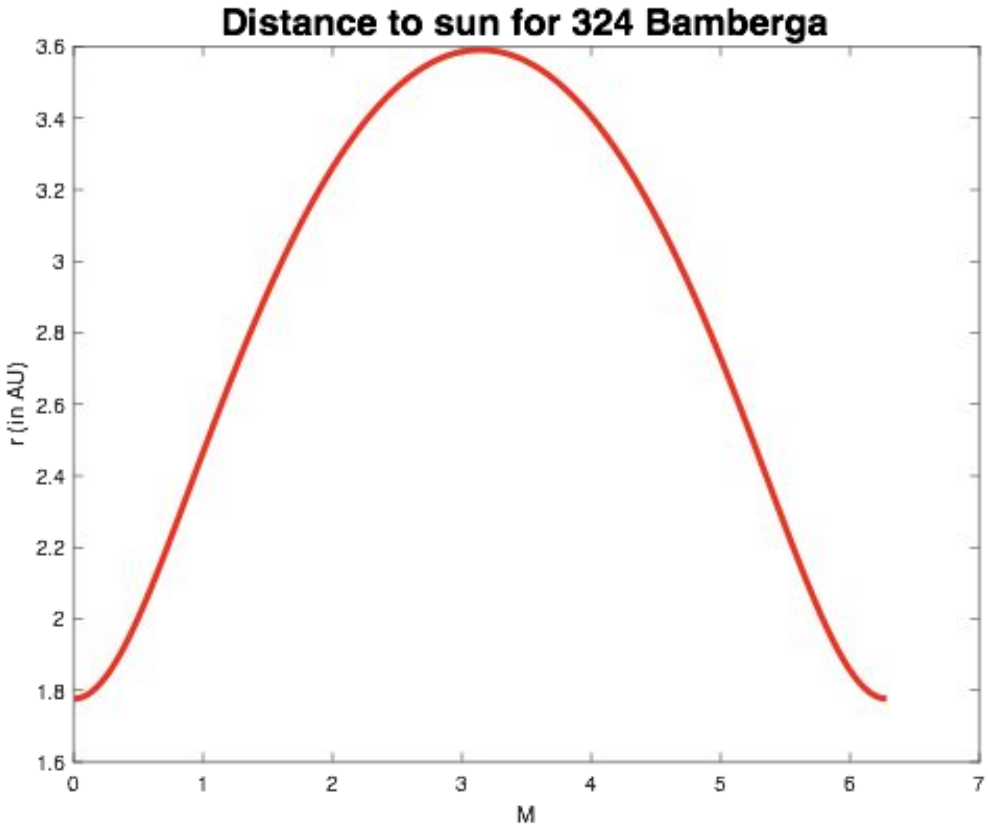


Results for 324 Bamberga: the v(M) Plot

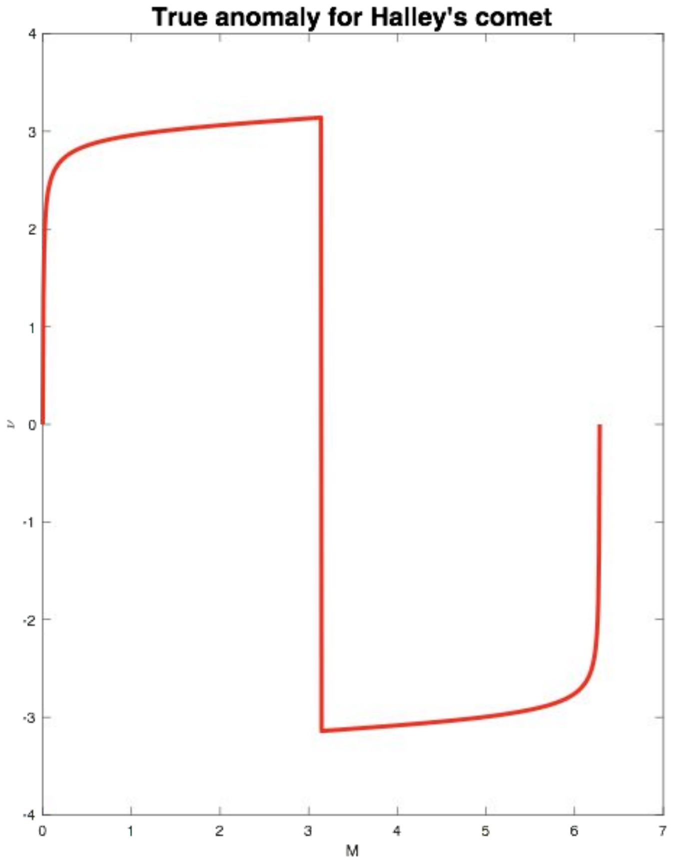


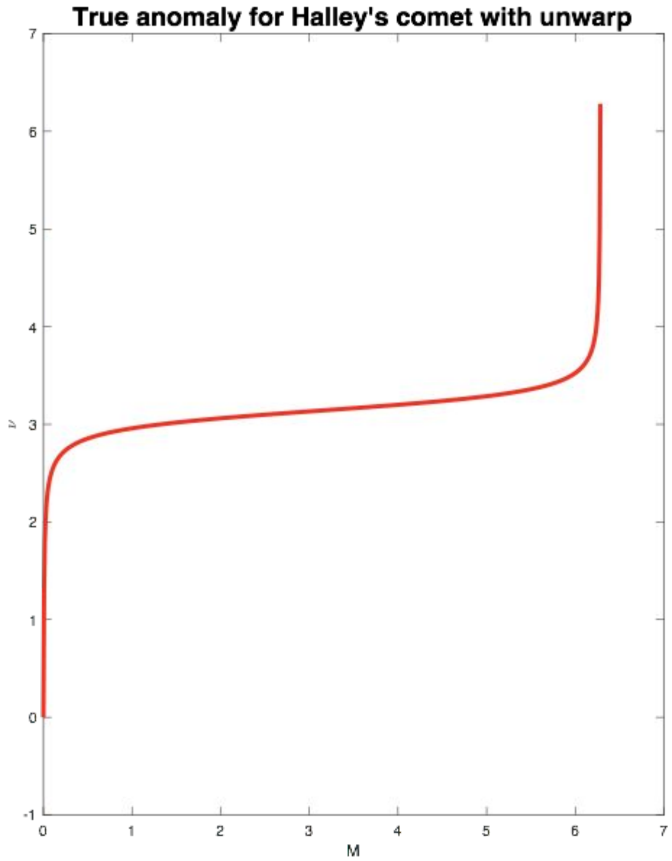


Results for 324 Bamberga: the r(M) Plot



Results for Halley's Comet: the ν(M) Plot





Results for Halley's Comet: the r(M) Plot