Lab 5A: Nonsmooth Finite Difference

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Review

Finite Difference Formula

The goal is to find an approximation to the first order derivative of a function $f(x)$.

One type of approximation is based on the following finite difference formula

$$f'(x) \approx \frac{1}{h} \sum_{k=-p}^{q} a_k f(x + kh)$$

where p, q are integers, and a_k are weights of the formula.

Weights for Central Finite Difference

Order of accuracy	Node location $-4h$	$-3h$	$-2h$	$-h$	0	h	$2h$	$3h$	$4h$
2				$-\frac{1}{2}$	0	$\frac{1}{2}$			
4			$\frac{1}{12}$	$-\frac{2}{3}$	0	$\frac{2}{3}$	$-\frac{1}{12}$		
6		$-\frac{1}{60}$	$\frac{3}{20}$	$-\frac{3}{4}$	0	$\frac{3}{4}$	$-\frac{3}{20}$	$\frac{1}{60}$	
8	$\frac{1}{280}$	$-\frac{4}{105}$	$\frac{1}{5}$	$-\frac{4}{5}$	0	$\frac{4}{5}$	$-\frac{1}{5}$	$\frac{4}{105}$	$-\frac{1}{280}$

Weights for Forward Finite Difference

Order of accuracy	Node location 0	h	$2h$	$3h$	$4h$
1	-1	1			
2	$-\frac{3}{2}$	2	$-\frac{1}{2}$		
3	$-\frac{11}{6}$	3	$-\frac{3}{2}$	$\frac{1}{3}$	
4	$-\frac{25}{12}$	4	-3	$\frac{4}{3}$	$-\frac{1}{4}$

Hands On: Lab 5A NonsmoothFD

Three Finite Difference Formulas

- First order

$$W_1(h) = \frac{f(h) - f(0)}{h} = f'(0) + O(h)$$

- Second order

$$W_2(h) = \frac{f(h) - f(-h)}{2h} = f'(0) + O(h^2)$$

- Fourth order

$$W_4(h) = \frac{-f(2h) + 8f(h) - 8f(-h) + f(-2h)}{12h} = f'(0) + O(h^4)$$

Smooth Function

- This is the “control” case of a function with infinitely many derivatives everywhere.

$$f(x) = e^{\sin(x+1)}$$

Nonsmooth Functions

- The following two functions lacks smoothness at the point x=0:

$$g_1(x) = \begin{cases} 5x + 1, & x \leq 0 \\ (x+1)^5, & x > 0 \end{cases}$$

$$g_2(x) = \begin{cases} 10x^2 + 5\sin(x) + 1, & x \leq 0 \\ (x+1)^5, & x > 0 \end{cases}$$

Problem

1. Write a function to compute the three finite difference approximations for a given function.

   (Link for detailed instructions)

What We Learned?

Order of Accuracy of FD: Smooth vs Nonsmooth

- The effects on the order of accuracy.