

NTU 107-1 MATH1201 Calculus A-05

Quiz 1

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Work ONLY on the problems indicated by your teaching assistants or instructor for this quiz. The rest are left for your self-revision.

Pocket calculator is not very helpful. Internet is not allowed.

Time limit: 40 minutes

1. (a) Using the precise definition of limits, show that, for any function f , if $\lim_{x \rightarrow x_0} |f(x)| = 0$, then $\lim_{x \rightarrow x_0} f(x) = 0$. (6 points)
- (b) Give a counter-example to the statement “if a is a non-zero number and $\lim_{x \rightarrow x_0} |f(x)| = |a|$, then $\lim_{x \rightarrow x_0} f(x) = a$ or $\lim_{x \rightarrow x_0} f(x) = -a$ ”. Justify your answer. (6 points)
2. Evaluate the following limits or explain why they don't exist.

(a) $\lim_{x \rightarrow 0} \frac{|3x - 1| - |3x + 1|}{x}$. (5 points)

(b) $\lim_{x \rightarrow a^+} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x - a}}{\sqrt{x^2 - a^2}}$, where $a > 0$. (5 points)

(c) $\lim_{x \rightarrow \infty} (\sqrt{324x^2 + 73x + \pi} - \sqrt{324x^2 + 17\pi})$. (5 points)

(d) $\lim_{x \rightarrow 0} x^3 \cos\left(\frac{1}{x^5}\right)$. (6 points)

(e) $\lim_{x \rightarrow 0^+} \frac{x^{1+x}}{(1+x)^x}$. (6 points)

3. Let

$$f(x) = \begin{cases} x^2 - m & \text{if } x < 3, \\ 1 - mx & \text{if } x \geq 3. \end{cases}$$

If $f(x)$ is continuous for all x on the real line, find m . (10 points)

4. Find all asymptotes of the graph of $f(x) = \frac{5^x + 4^{-x}}{5^x - 4^{-x}}$. (20 points)