Lab 1A: Conditioning

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Review

Condition

- To measure how much the output value of the function can change for a small change in the input argument.
- The problem is “well-conditioned” if small changes in input lead to small changes in the output.

$$\kappa = \frac{\text{relative change in output}}{\text{relative change in input}}$$

$$= \left|\frac{\Delta f / f}{\Delta x / x}\right| = \left|\frac{\Delta x f'(x) / f(x)}{\Delta x / x}\right| = \left|\frac{x f'(x)}{f(x)}\right|$$

Find $r = f(a)$ s.t. $p(r) = ar^2 + br + c = 0$

Then the condition number

$$\kappa = \left|\frac{xf'(x)}{f(x)}\right| = \left|\frac{af'(a)}{f(a)}\right| = \left|\frac{ar^2/p'(r)}{f(a)}\right| = \left|\frac{ar}{p'(r)}\right|$$

$$= \left|\frac{ar}{2ar+b}\right| = \left|\frac{ar}{\pm\sqrt{b^2-4ac}}\right| = \left|\frac{r}{r_1-r_2}\right|$$

Hands on

Rooting for accuracy

In this lab, we have a quadratic polynomial

$$p(x;s) = a(s)x^2 + b(s)x + c(s)$$

and its roots are $t_1(s)$ and $t_2(s)$.

Hence, its condition number is

$$\kappa = \left|\frac{t_k(s)}{t_1(s) - t_2(s)}\right|$$

Goal

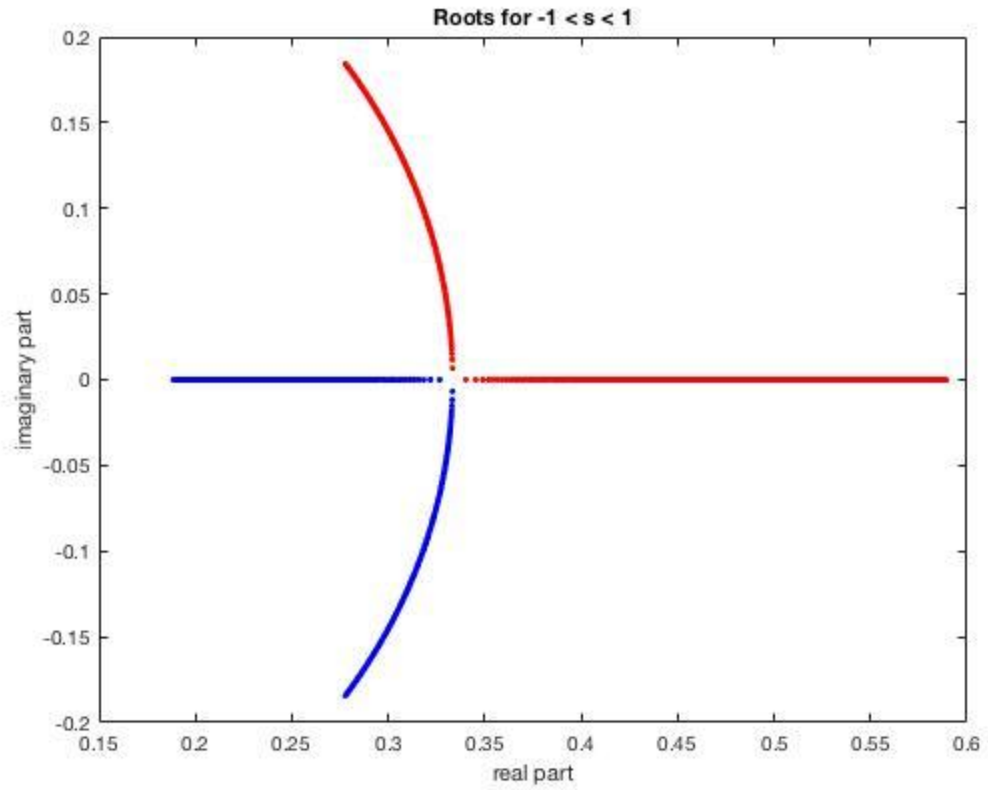
- To examine the sensitivity of the roots of a quadratic polynomial

$$p(x;s) = 9x^2 - (6+s)x + 1$$

whose roots come together, coalesce and separate again.

Step 1. Preparation

```
>> t1 = @(s) (6+s+sqrt((6+s).^2-36))/18;
t2 = @(s) (6+s-sqrt((6+s).^2-36))/18;
s = linspace(-1,1,800)';
plot(t1(s),'r.')
hold on
plot(t2(s),'b.')
xlabel('real part')
ylabel('imaginary part')
title('Roots for -1 < s < 1')
```


Roots for -1 < s < 1
imaginary part
real part
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6

Step 2.

Let `s = -0.1`.

1. Use `roots` to find the roots of `p(x;s)` as a vector `r`.

2. Use `polyval` to see value of `p(r;s)` as a vector `yval`.

3. Compute its error by means of

```
err1 = min( abs(r(1)-t1(s)), abs(r(1)-t2(s)) )
```

4. If `k = 1`, try to find

$$\kappa = \left| \frac{t_k(s)}{t_1(s) - t_2(s)} \right|$$

```
r =

   0.3278 + 0.0606i
   0.3278 - 0.0606i


yval =

   1.0e-15 *

   0.0000 - 0.1110i
   0.0000 + 0.1110i


err1 =

   5.9286e-17


kappa =

    2.7501
```

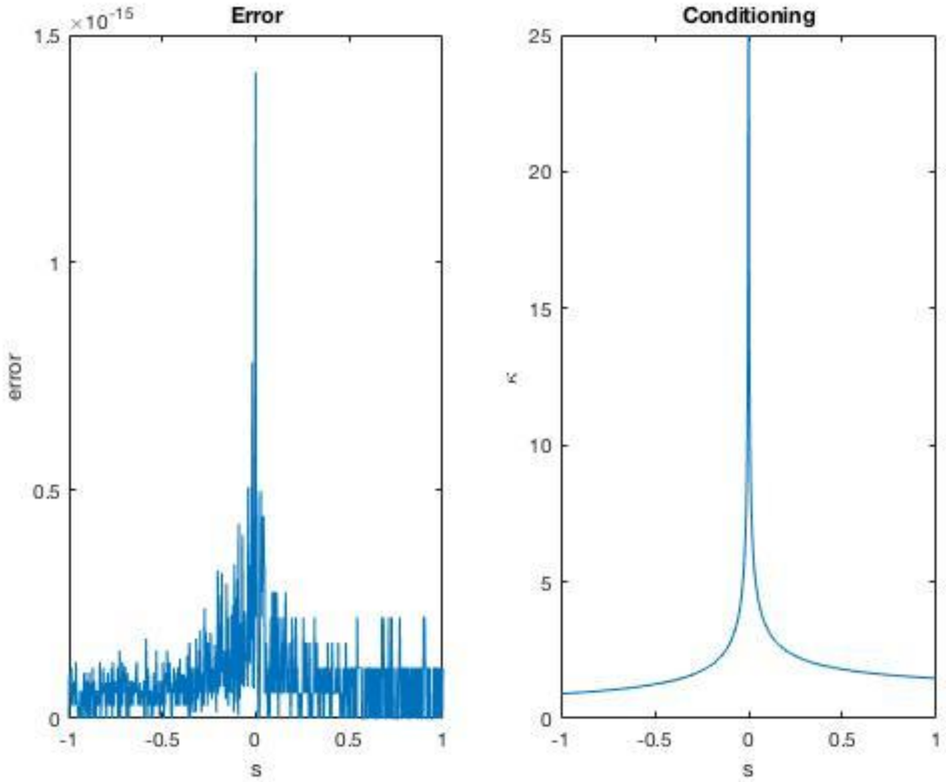
Step 3.

Define the vector

```
s = lnispace(-1,1,800)';
```

In a loop over the entries vector s,

1. Find the error, κ and store these results in vectors.
2. Plot the results.


Error
×10^-15
1.5
1
0.5
0
error
-1
-0.5
0
0.5
1
s
Conditioning
25
20
15
10
5
0
κ
-1
-0.5
0
0.5
1
s

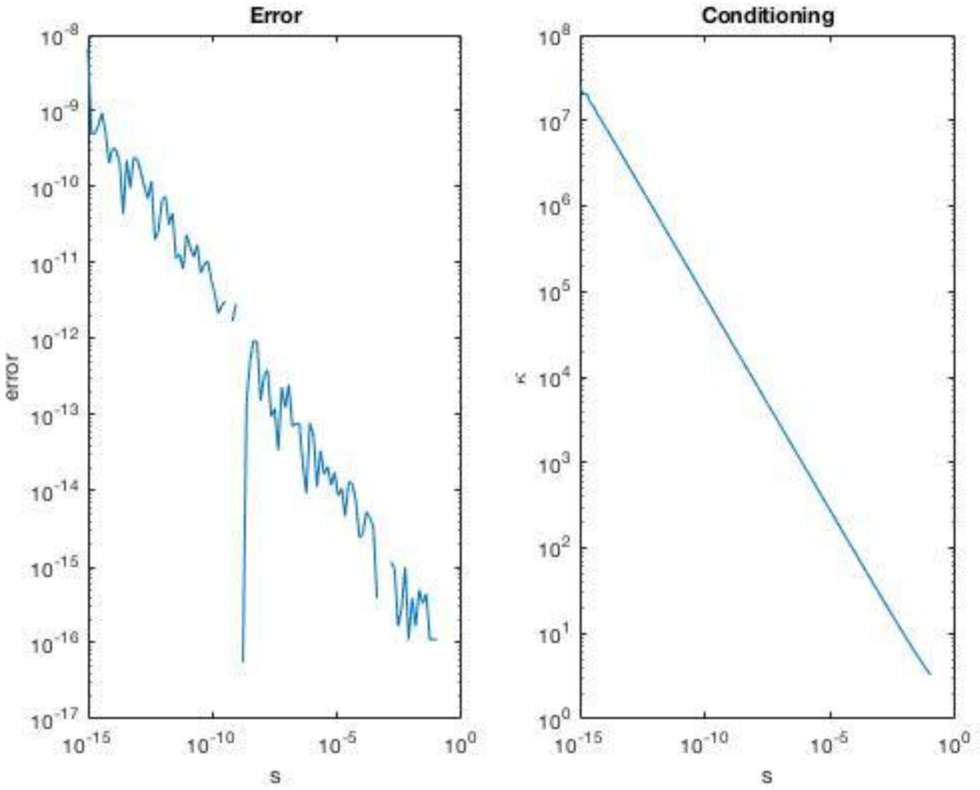
Step 4. Repeat step 3.

Define the vector

```
s = logspace(-15,-1,100)';
```

In a loop over the entries vector s,

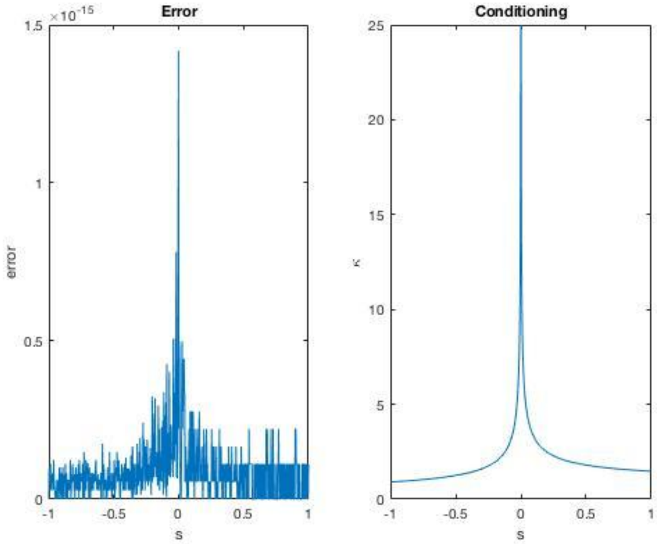
1. Find the error, κ and store these results in vectors.
2. Plot the results.


Error
error
s
Conditioning
κ
s

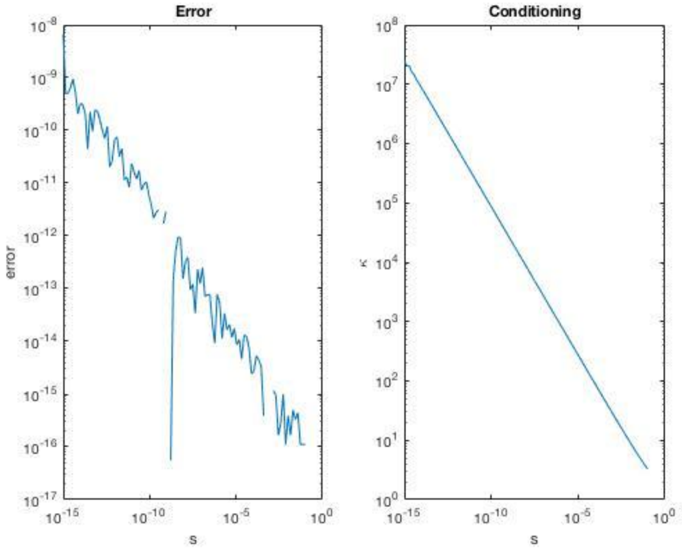
- Question:

  When we approach very close to 0, is the results are strikingly different?

1. `s = lnispace(-1,1,800)';`



2. `s = logspace(-15,-1,100)';`



Questions

Or

Comments?