Lab 1B: Stability

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Review

Stability

- **A stable algorithm gives nearly the right answer to nearly the right question.**

Stability

An algorithm $\tilde{f}$ for a problem f is **stable** if for each $x \in X$,

$$\frac{||\tilde{f}(x) - f(\tilde{x})||}{||f(\tilde{x})||} = \mathcal{O}(\epsilon_{\text{machine}})$$

for some $\tilde{x}$ with

$$\frac{||\tilde{x} - x||}{||x||} = \mathcal{O}(\epsilon_{\text{machine}})$$

Backward Stability

- **A backward stable algorithm gives exactly the right answer to nearly the right question.**

- **If an algorithm always produces small backward errors, then it is stable.**

Backward Stability

An algorithm $\tilde{f}$ for a problem f is **backward stable** if for each $x \in X$,

$$\tilde{f}(x) = f(\tilde{x})$$

for some $\tilde{x}$ with

$$\frac{||\tilde{x} - x||}{||x||} = \mathcal{O}(\epsilon_{\text{machine}})$$

Hands on

Lab 1B

Consider the function,

$$f(x) = \frac{e^x - 1}{x}$$

Question: How to compute f?

Lab 1B

Compare two ways to compute f:

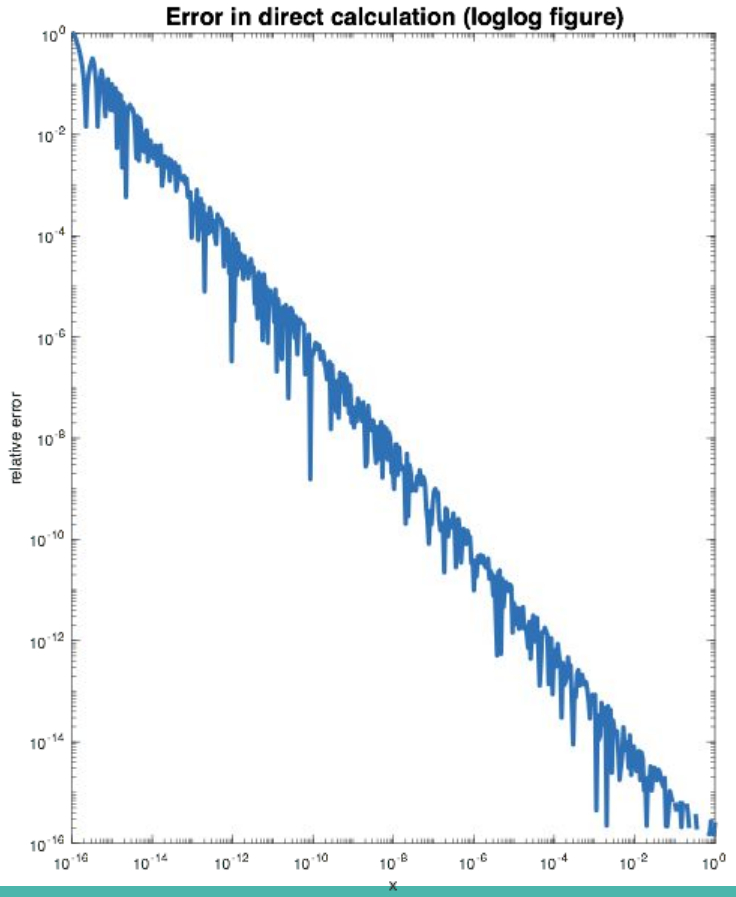
- Using the following sequence (direct calculation)

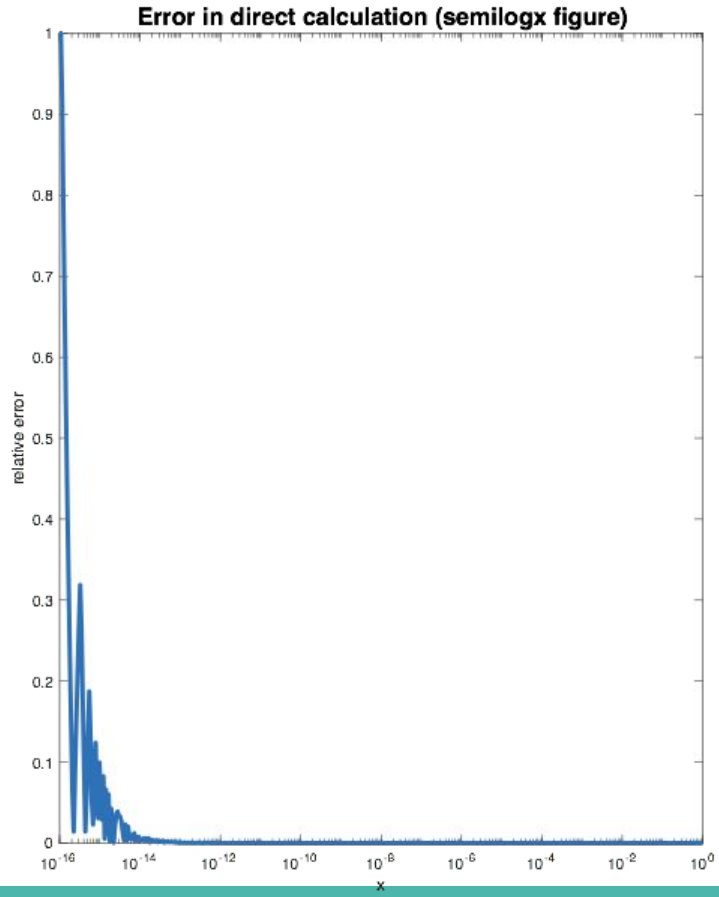
$$y_1 = e^x, \quad y_2 = y_1 - 1, \quad y_3 = y_2/x$$

- Using the following expansion (truncated series)

$$f(x) = 1 + \frac{1}{2!}x + \frac{1}{3!}x^2 + \cdots$$

Lab 1B Direct Calculation Error





Lab 1B Truncated Series Error