NTU 107-1 MATH1201 | CALCULUS A-05 EXERCISE SET 2 | 17TH OCTOBER, 2018

Please write your solutions to **Problems (1) to (5)** on a separate sheet of A4 paper and submit it to your TA **on or before 24th October, 2018**. The rest of the problems are for your self-revision.

1. (15 pts) The area of a triangle with sides of lengths $a$ and $b$ and contained angle $\theta$ is $A = \frac{1}{2}ab\sin\theta$.
   (a) If $a = 2$ cm, $b = 3$ cm, and $\theta$ increases at a rate of 0.2 rad/min, how fast is the area increasing when $\theta = \pi/3$?
   (b) If $a = 2$ cm, $b$ increases at a rate of 1.5 cm/min, and $\theta$ increases at a rate of 0.2 rad/min, how fast is the area increasing when $b = 3$ cm and $\theta = \pi/3$?
   (c) If $a$ increases at a rate of 2.5 cm/min, $b$ increases at a rate of 1.5 cm/min, and $\theta$ increases at a rate of 0.2 rad/min, how fast is the area increasing when $a = 2$ cm, $b = 3$ cm and $\theta = \pi/3$?

2. (10pts) Let $f(x) = \dfrac{1+\cos x}{1+\sin x}$. Use a differential to estimate $f(44^\circ)$.

3. (15pts) Find the linear approximation of the function $g(x) = \sin^{-1}\left(\dfrac{x-1}{x+1}\right) - \tan^{-1}\left(\sqrt{x}\right)$ at the point $x = 3$.

4. (16pts) Find the absolute maximum and absolute minimum values of $f$ on the given interval.
   (a) $f(x) = \dfrac{\sqrt{x}}{1+x^2}$, $[0, 2]$.
   (b) $f(x) = x^{-2}\ln x$, $[\frac{1}{2}, 4]$.

5. (4pts) If a current $I$ passes through a resistor with resistance $R$, Ohm's Law states that the voltage drop is $V = RI$. If $V$ is constant and $R$ is measured with a certain error, use differentials to show that the relative error in calculating $I$ is approximately the same (in magnitude) as the relative error in $R$.

6. (8pts) Show that $\left|\tan\dfrac{x}{2} - \tan\dfrac{y}{2}\right| \geq \dfrac{|x-y|}{2}$ for $x, y \in (-\pi, \pi)$.

7. (12pts) Find the following limits if they exists.
   (a) $\lim\limits_{x\to\infty}\left[x + x^2\ln\left(1 - \dfrac{2}{x}\right)\right]$
   (b) $\lim\limits_{x\to 0^+}(1-\cos x)^{\frac{1}{\ln x}}$

8. (10pts) Sketch the curve $y = 1 + \dfrac{1}{x} + \dfrac{1}{x^2}$. (Hint: You should first find all the intercepts, asymptotes, local extema and inflection points of the given curve. Determine also the intervals on which the given function is increasing, decreasing, concave upward or concave downward.)

9. (10pts) Let $p$ and $q$ be two functions given by

$$p\colon (0,+\infty) \to \mathbb{R},\ x \mapsto e^x \quad \text{and} \quad q\colon (0,+\infty) \to \mathbb{R},\ x \mapsto x^e\,.$$

   Consider the function defined by $f(x) := \frac{p(x)}{q(x)}$ for all $x \in (0, +\infty)$.
   (a) Find all the (either global or local) extremal points of $f$ on $(0, +\infty)$ and determine whether $f$ attains its maximum or minimum there.
   (b) Which number is larger, $e^\pi$ or $\pi^e$?