1081 Calculus 模組 07 Homework 3

Due Date: Oct 24, 2019

Please hand in the following exercise of textbook in Part I and all problems in Part II. The rigorous and clear explanation is needed. An answer without process will get no point.

Part I:

1. (Ch. 3.10, Pb.3, 4)
   Find the linearization $L(x)$ of the function at a.
   (a) $f(x) = \sqrt{x},\ a = 4$
   (b) $f(x) = 2^x,\ a = 0$

2. (Ch. 3.10, Pb.12, 13)
   Find the differential of following functions.
   (a) $y = \dfrac{1+2x}{1+3x}$
   (b) $y = x^2 \sin(2x)$
   (c) $y = \tan(\sqrt{x})$
   (d) $y = \dfrac{1-x^2}{1+x^2}$

3. (Ch. 3.10, Pb.23, 28)
   Use a linear approximation (or differentials) to estimate the given number.
   (a) $(1.999)^4$
   (b) $\cos(29°)$

4. (Ch. 4.1, Pb.39, 40)
   Find the critical numbers of the function.
   (a) $f(x) = x^{4/5}(x-4)^2$
   (b) $g(x) = 4x - \tan x$

5. (Ch. 4.1, Pb.51, 59)
   Find the absolute maximum and absolute minimum values of f and g on the given interval.
   (a) $f(x) = 3x^4 - 4x^3 - 12x^2 + 1,\ [-2, 3]$
   (b) $g(x) = x^{-2}\ln x,\ [\frac{1}{2}, 4]$

6. (Ch.4.1, Pb.63)
   Let a and b be positive numbers. Find the maximum value of $f(x) = x^a(1-x)^b$ for $x \in [0, 1]$.

7. (Ch.4.1, Pb.77)
   Prove that the function
   $$f(x) = x^{101} + x^{51} + x + 1$$
   has neither a local maximum nor a local minimum.

8. (Ch.4.2, Pb.9)
Let $f(x) = 1 - x^{2/3}$. Show that $f(-1) = f(1)$ but there is no number c in $(-1, 1)$ such that $f'(c) = 0$. Why does this not contradict to Rolle's Theorem?

9. (Ch.4.2, Pb.17)
Let $f(x) = (x-3)^{-2}$. Show that there is no value of c in $(1, 4)$ such that $f(4) - f(1) = f'(c)(4-1)$. Why does this not contradict to Mean Value Theorem?

10. (Ch.4.2, Pb.11,14)
Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Find all numbers c that satisfy the conculsion of the Mean Value Theorem.

    (a) $f(x) = 2x^2 - 3x + 1$, $[0, 2]$

    (b) $f(x) = \dfrac{x}{x+2}$, $[1, 4]$

11. (Ch.4.2, Pb.28)
Suppose that f and g are continuous on $[a, b]$ and differentiable on (a, b). Suppose also that $f(a) = g(a)$ and $f'(x) < g'(x)$ for $a < x < b$. Prove that $f(b) < g(b)$.
(*Hint*: Apply the Mean Value Theorem to the function $h(x) = f(x) - g(x)$.)

Part II:

1. Show that the equation $x^3 + 3x - 1 = 0$ has exactly one real root.

2. Apply the Mean Value Theorem to prove the following statements:

    (a) $|\tan x - \tan y| \geq |x - y|$ for all $x, y \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

    (b) $\sqrt{1+x} < 1 + \frac{x}{2}$ for all $x > 0$.

3. Prove the Cauchy Mean Value Theorem: Suppose that f and g are both continuous on $[a, b]$ and differentiable on (a, b). Then there exists $c \in (a, b)$ such that

$$(f(b) - f(a))g'(c) = (g(b) - g(a))f'(c). \tag{1}$$

In particular, if $g(x) = x$, then (1) reduces to the Mean Value Theorem.
(*Hint*: Consider $h(x) = (f(b)-f(a))g(x)-(g(b)-g(a))f(x)$ and apply Rolle's Theorem.)

Part III: The following problems are **Supplementary Exercises**, and you **don't** need to hand in this part.
Ch.3.4, Pb.22, 34, 41, 44
Ch.4.1, Pb.6, 13, 41, 80
Ch.4.2, Pb.23, 35, 37, 38