Lab 2E: Terms and Conditions

2018/10/17

王嘉澤

Important Time

1. Lab 2E: 2018/10/19 06:00:00 a.m.
2. Homework 3: 2018/10/23 15:30:00
3. Group Latex Ch1 and Ch2: 2018/10/23 15:30:00
4. Quiz 2: 2018/10/31 Code
5. Midterm: 2018/11/07

Review

Blurring an Image

- Let X be an $m \times n$ pixel intensity matrix of an image. Then the following equation applies local blurring k times to X in each direction:

$$Z = VXH$$

where $V = (B_m)^k$ and $H = (B_n)^k$.

Blur: Matrix-vector Product Form

- Express this in matrix-vector product form gives,

$$z = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & & & \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & & \\ & \ddots & \ddots & \ddots & \\ & & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ & & & \frac{1}{4} & \frac{1}{2} \end{bmatrix} x = B_m x$$

Deblur: Restoring a Blurred Image

- The process of restoring a blurred image can be expressed using matrix inverses:

$$X = V^{-k} Z H^{-k}$$

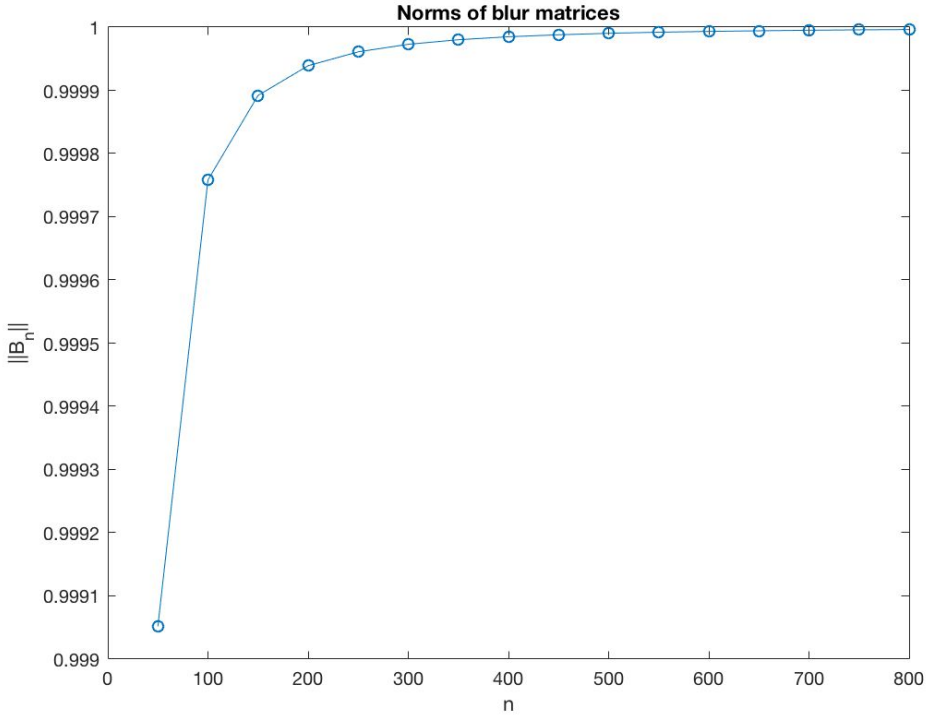
- In computational practice, the above equation is computed by solving linear systems.

Hands On

Step 1

- The condition number of a matrix has two factors, $\|\boldsymbol{A}\|$ and $\|\boldsymbol{A}^{-1}\|$.
- Show that the term $\|\boldsymbol{A}\|$ makes no trouble.
- For `n = 50, 100, 150, …, 800,` plot $\|\boldsymbol{B}_n\|$ versus `n.`

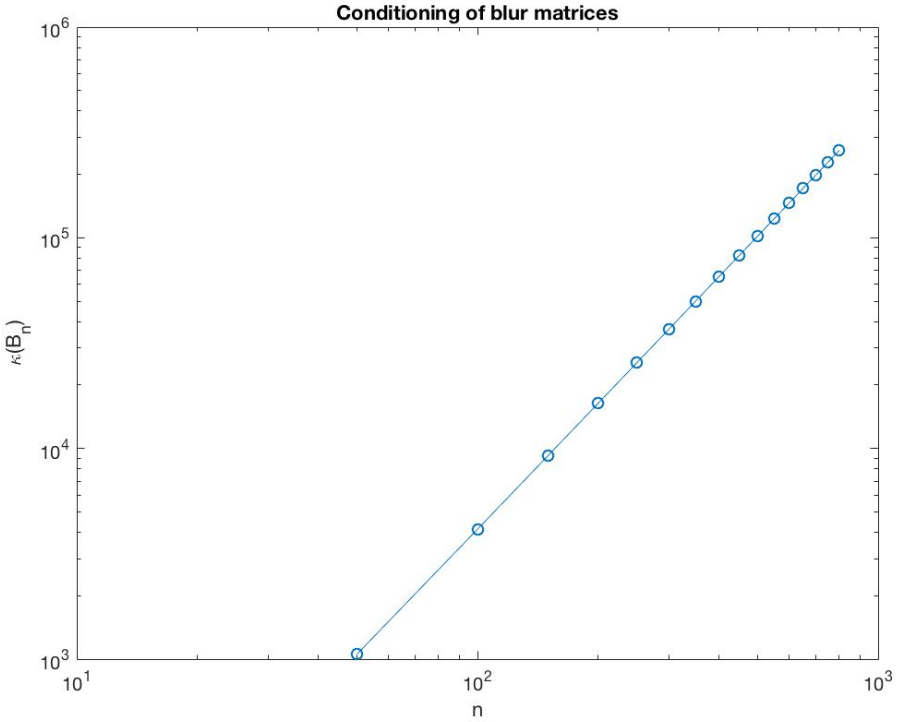
Step 1


Norms of blur matrices
1
0.9999
0.9998
0.9997
0.9996
0.9995
0.9994
0.9993
0.9992
0.9991
0.999
||B_n||
0
100
200
300
400
500
600
700
800
n

Step 2

- For the same `n` as step 1, plot $\kappa(\boldsymbol{B}_n)$ versus `n`.
- Use a log-log scale for this graph and get essentially a straight line.
- This implies that $\log\kappa \approx a\log n + b$ or $\kappa \approx Cn^p$.

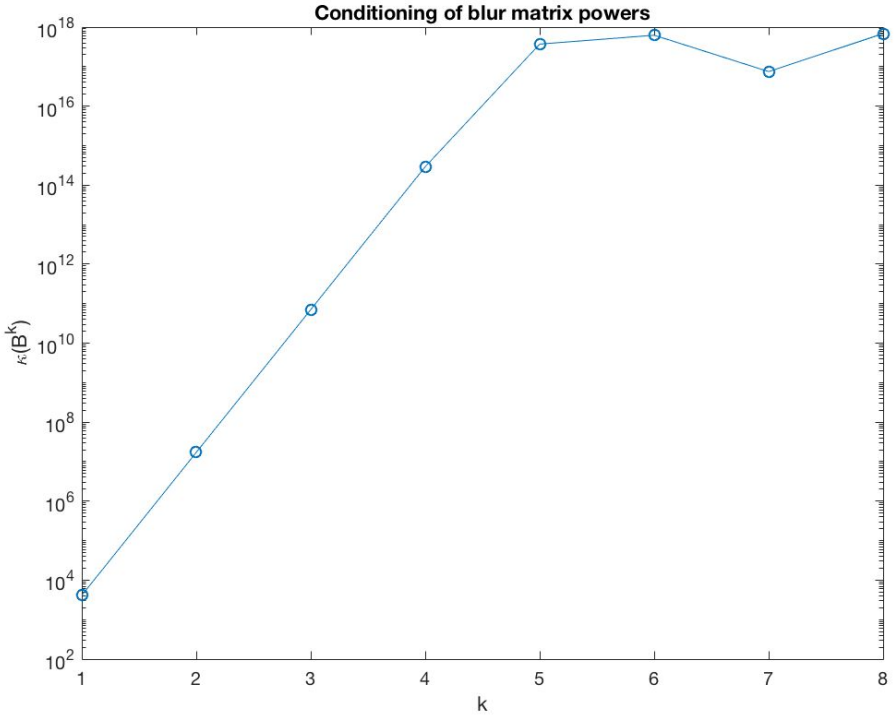
Step 2


Conditioning of blur matrices
10^6
10^5
10^4
10^3
$\kappa(B_n)$
10^1
10^2
10^3
n

Step 3

- Let $\boldsymbol{V} = \boldsymbol{B}_{100}$.
- For `k = 1, 2, 3, …, 8`, plot $\kappa(\boldsymbol{V}^k)$ as a function of **`k`**.
- The graph is straight on a semi-log scale, which implies $\kappa \approx C q^k$

Step 3


Conditioning of blur matrix powers
10^{18}
10^{16}
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
$\kappa(B^k)$
1
2
3
4
5
6
7
8
k

Step 4

- Let `x` be a random vector of a length `100`.
- For `k = 1, 2, 3, …, 8`, let **`z = Vx`** and then solve **`Vy = z`** for **`y`**.
- Record the relative error in the result.
- Then make a table showing both sides of the following inequality.

$$\frac{\|\boldsymbol{y}-\boldsymbol{x}\|}{\|\boldsymbol{x}\|} \leq \kappa(\boldsymbol{V})\varepsilon_{\text{mach}}$$

Step 4

k	upper_bound	error
1	9.1785e-13	6.0833e-14
2	3.7941e-09	5.92e-11
3	1.5683e-05	1.2162e-07
4	0.064886	0.000535
5	82.585	1.388
6	138.48	1.5526
7	16.644	1.7561
8	151.99	8.8404

What We Learned?

What We Leanered?

- Compute condition numbers of blur matrix powers and compare them to the errors of repeated blur/deblur operations.

- Deblur has poor performance since its condition number is large i.e. inverse of blur matrix condition number is large.

Questions or comments?