Lab 3B: Pressure Fit

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Review

Least Squares Problems

- In least squares problems, we consider a linear system of equations having n unknowns but $m > n$ eqautions.
- In matrix form, the goal is to find a vector $x \in \mathbb{R}^n$ that satisfies

$$Ax = b, \quad A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m$$

- In general, such a problem has no solution.

Least Squares Problems

- A rectangular system of equations with $m > n$ is called **overdetermined**.
- The residual vector r is defined to be

$$r = b - Ax \in \mathbb{R}^m$$

Least Squares Problems

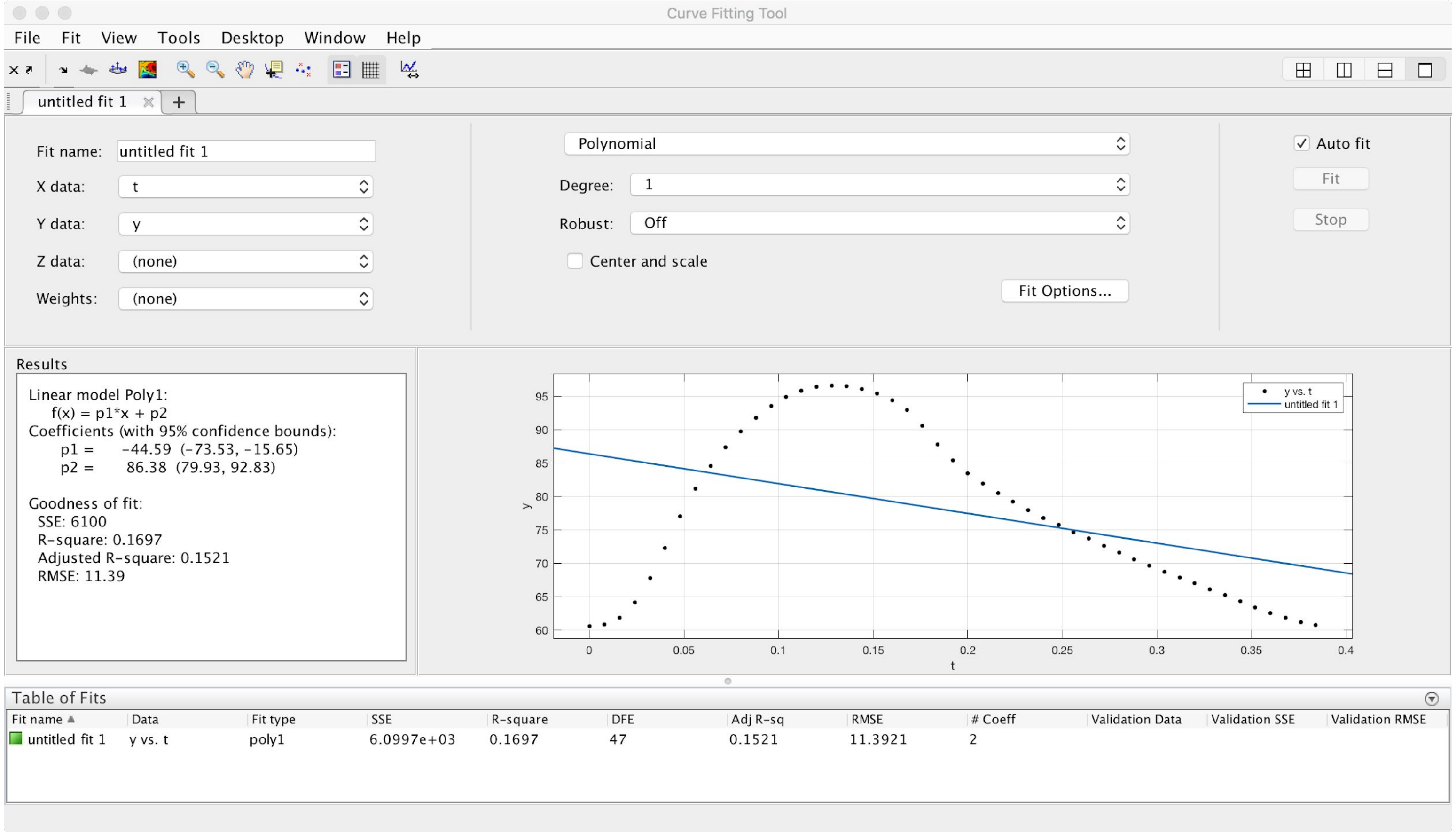
- The following is the formulation of the general least squares problem:

Given $A \in \mathbb{R}^{m \times n}$, $m \geq n$, $b \in \mathbb{R}^m$,
find $x \in \mathbb{R}^n$ such that $||b - Ax||_2 = ||r||_2$ is minimized.

Solving Least Squares Problem

- MATLAB: using the backslash operator \
- Classical: solving the normal equation
- “Modern Classical”: using QR factorization

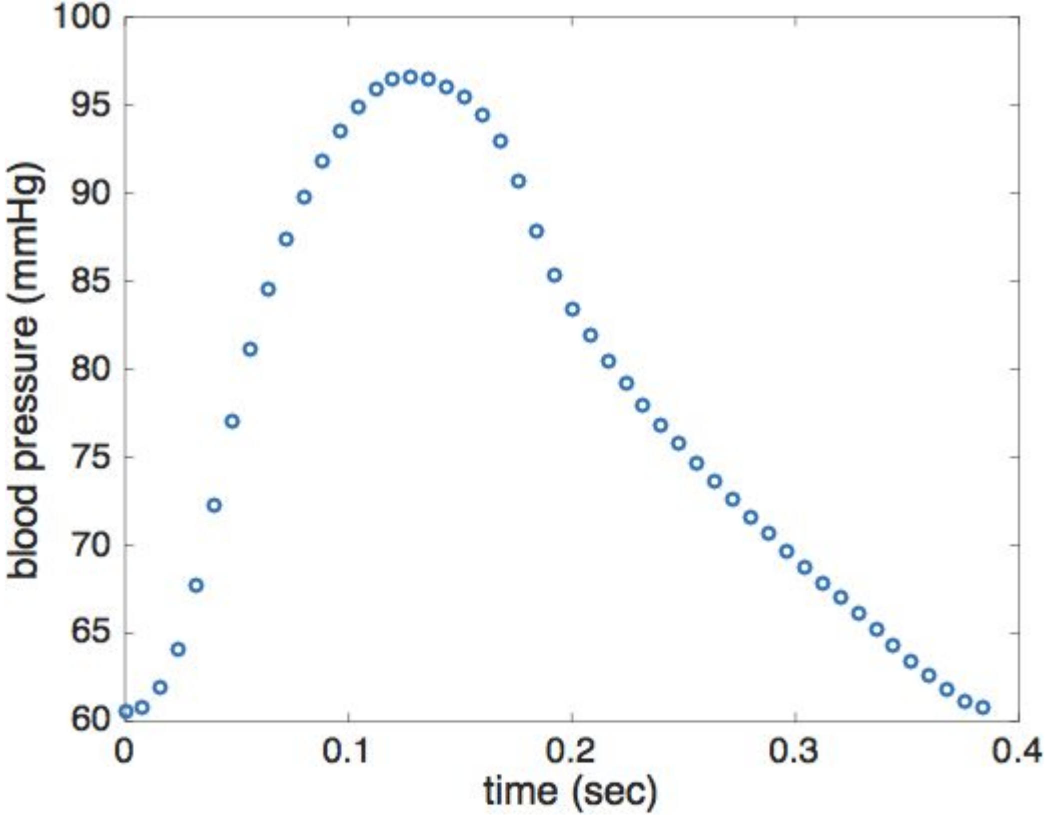
MATLAB Curve Fitting Tool



Hands On: Lab 3B Pressure Fit

The Blood Pressure Data

This figure represents arterial blood pressure collected at 8 ms intervals from an infant patient:



Least Squares Data-fitting

- Call the data points (t_i, y_i) for $i = 1, \cdots, m$
- These data can be fit using a function of the form

$$f(t) = c_1 f_1(t) + c_2 f_2(t) + \cdots + c_n f_n(t)$$

where the functions $f_1, \cdots, f_n$ are predetermined

and the coefficients $c_1, \ldots, c_n$ are to be chosen

to optimize the problem.

Polynomial Fits

- Typical choices for the functions f_j are monomials,

$$f_j(t) = t^{j-1}, \quad \text{for } j = 1, \cdots, n.$$

- These lead to polynomial fits. A typical case is n = 2 which is a straight line.

Periodic Fits

Fact: The data are taken over one heartbeat.

- These data are roughly periodic in time.
- This fact can be used to exploit a fit. Let $T = t_m - t_1$ be the period of the beat. Then one periodic fit is

$$f(t) = c_1 + c_2 \cos\left(\frac{2\pi t}{T}\right) + c_3 \sin\left(\frac{2\pi t}{T}\right) + c_4 \cos\left(\frac{4\pi t}{T}\right) + c_5 \sin\left(\frac{4\pi t}{T}\right)$$

The Trigonometric Polynomial

- A trigonometrical polynomial of order n is defined by

$$S_n(x) = \frac{1}{2}a_0 + \sum_{v=1}^{n}(a_v \cos vx + b_v \sin vx)$$

- A trigonometrical polynomial becomes a Fourier series when $n \to \infty$,

$$f(x) = \frac{1}{2}a_0 + \sum_{v=1}^{\infty}(a_v \cos vx + b_v \sin vx)$$

Problem

1. Write a function to perform least squares data-fitting for different types of fits.

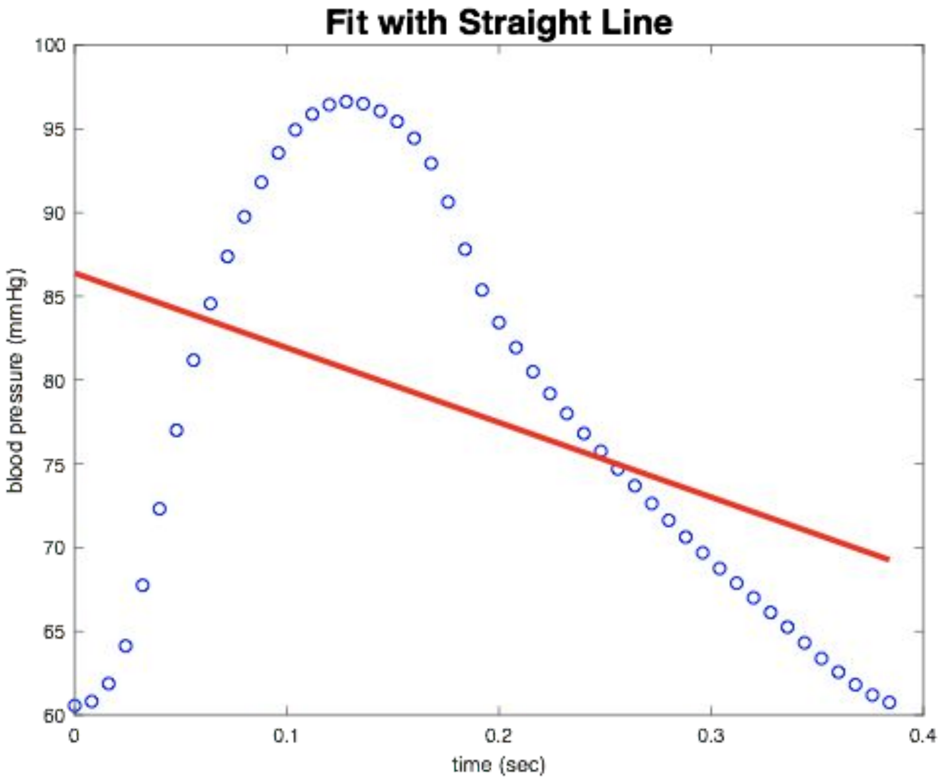
   (Link for detailed instructions)

What We Learned?

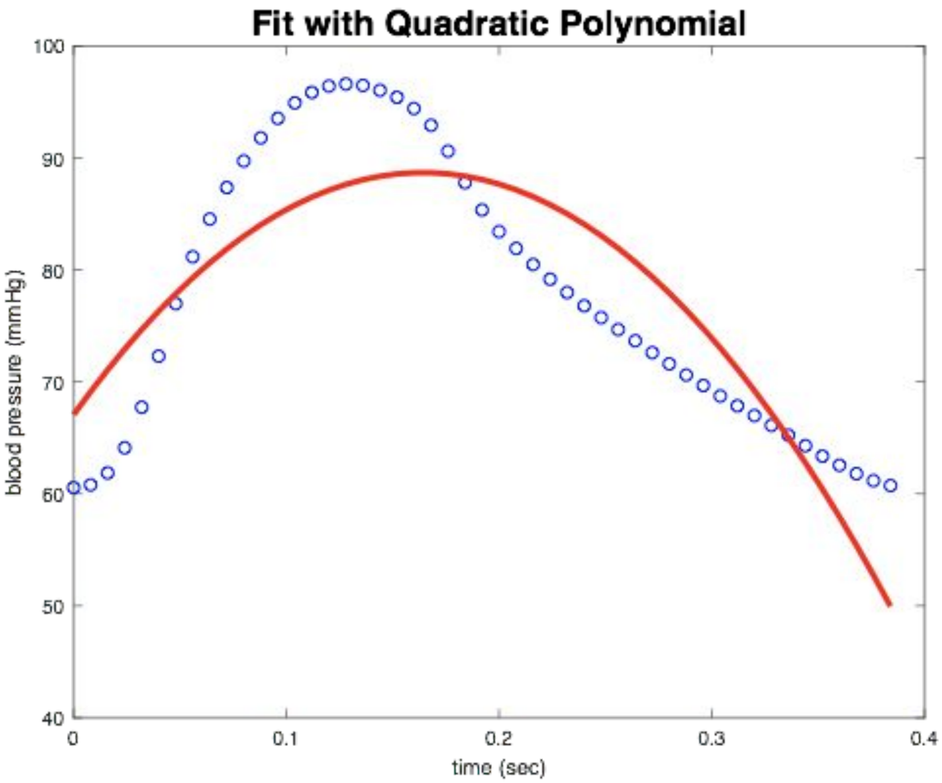
Two Types of Data-fitting

- Polynomial fits
- Periodic fits

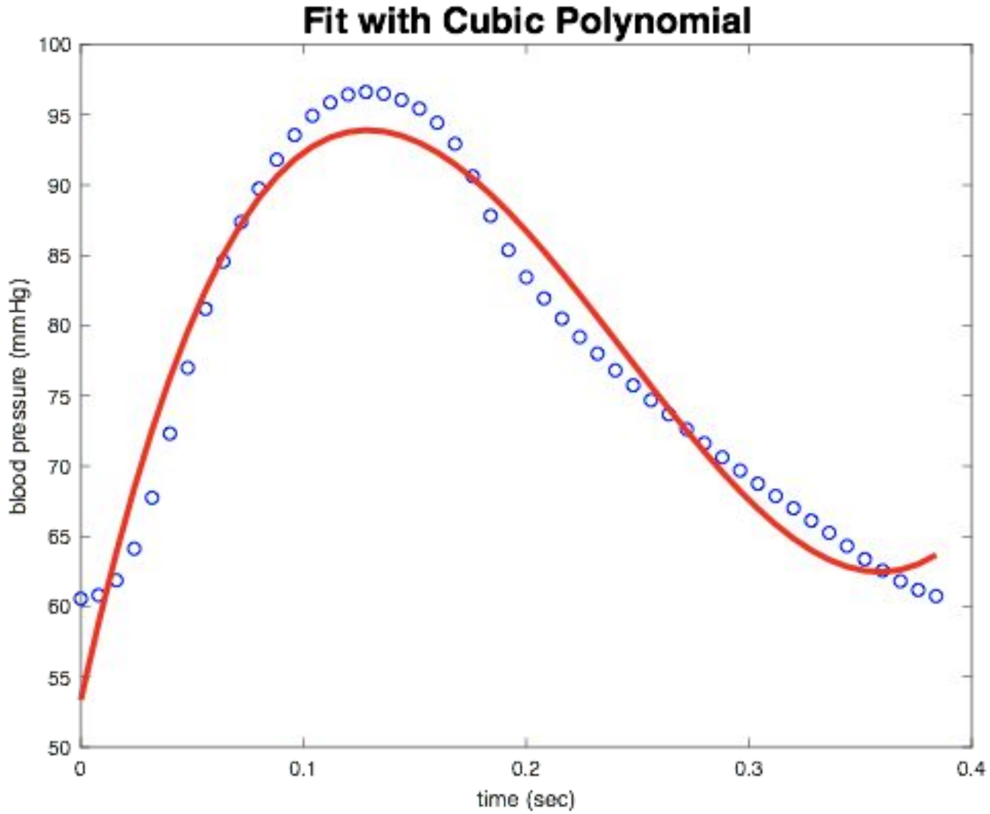
Polynomial Fits: Straight Line



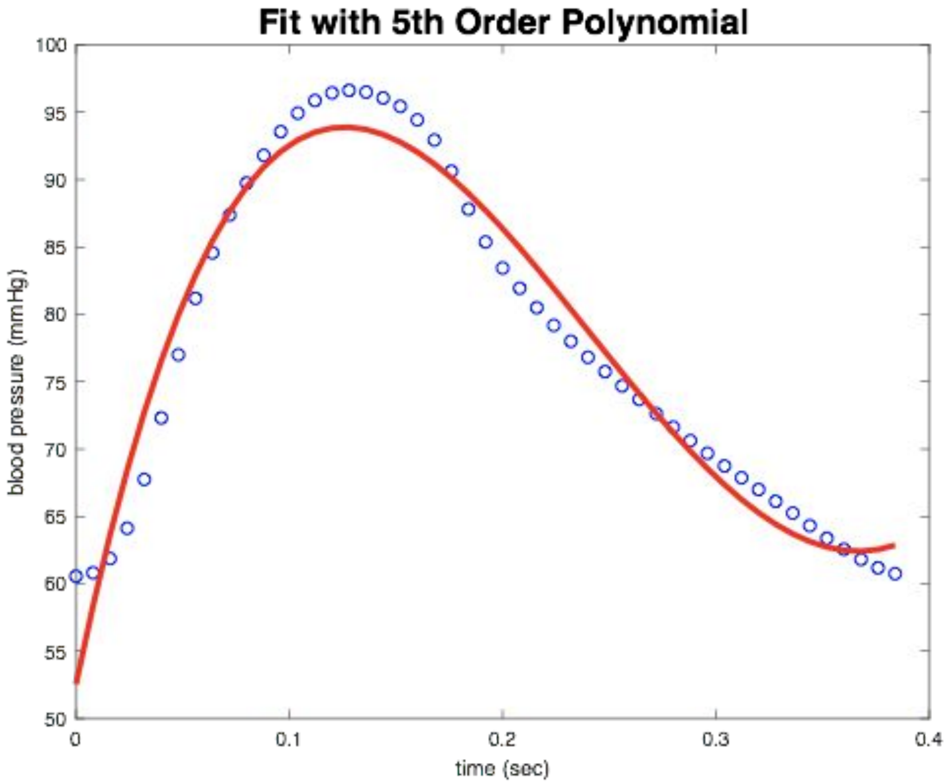
Polynomial Fits: Quadratic Polynomial



Polynomial Fits: Cubic Polynomial



Polynomial Fits: High-Order Polynomial



Periodic Fits