NTU 107-1 MATH1201 — CALCULUS A-05 — EXERCISE SET 5 — 12TH DECEMBER, 2018

Please write your solutions to **Problems (1) to (4)** on a separate sheet of A4 paper and submit it to your TA **on or before 21th December, 2018**. The rest of the problems are for your self-revision.

1. (13pts)

   (a) Sketch the curve $r = 3 - 4\sin^2 \dfrac{\theta}{2}$

   (b) Compute the area of the region that is inside the larger loop of the curve $r = 1 + 2\cos\theta$ and outside the smaller loop of the curve $r = 1 + 2\cos\theta$

2. (20pts) Find the area of the region that lies inside the curve $r = 1+\cos\theta$ but outside the curves $r = 2\cos\theta$ and $r = -\cos\theta$

3. (12pts) Find the arc length of the curve. $x = \cos t + \ln\tan\frac{t}{2}, y = \sin t, \frac{\pi}{4} \leq t \leq \frac{3\pi}{4}$

4. (25 pts) A curve called the **folium of Descartes** is defined by the parametric equations

$$x = \frac{3t}{1+t^3} \qquad y = \frac{3t^2}{1+t^3}$$

   (a) Show that if (a, b) lies on the curve, then so does (b, a); that is, the curve is symmetric with respect to the line $y = x$. Where does the curve intersect this line?

   (b) Find the points on the curve where the tangenet lines are horizontal or vertical.

   (c) Show that the line $y = -x - 1$ is a slant asymptote.

   (d) Sketch the curve.

   (e) Show that a Cartesian equation of this curve is $x^3 + y^3 = 3xy$.

   (f) Show that the polor equation can be written in the form

$$r = \frac{3\sec\theta\tan\theta}{1+\tan^3\theta}$$

   (g) Find the area enclosed by the loop of this curve.

   (h) (Optional) Show that the area of the loop is the same as the area that lies between the asymptote and the infinite branches of the curve.

5. (16pts) Solve the differential equation.

   (a) $\dfrac{\mathrm{d}u}{\mathrm{d}t} = \dfrac{1+t^4}{ut^2 + u^4t^2}$.

   (b) $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{t\sec\theta}{\theta e^{t^2}}$.

6. (24pts) Solve the initial-value problem.

   (a) $xy' = y + x^2\sin x,\ y(\pi) = 0$.

   (b) $(x^2+1)\dfrac{\mathrm{d}y}{\mathrm{d}x} + 3x(y-1) = 0,\ y(0) = 2$.