NTU 107-1 MATH1201 | CALCULUS A-05 EXERCISE SET 4 | 23TH NOVEMBER, 2018

> Please write down your solutions on a separate sheet of paper and submit it to your TA or instructor.
> Submit your solutions to Problems (1) ~ (2) on 30th **November, 2018**.
> Submit your solutions to Problems (3) ~ (6) on 12th **December, 2018**.

1. Evaluate the integral.
   (a) (4 pts) $\int (\cos x + \sin x)^2 \cos 2x \, dx$
   (b) (5 pts) $\int \frac{x^2}{\sqrt{9-x^2}} \, dx$, $|x| < 3$
   (c) (5 pts) $\int \frac{dx}{\sqrt{x^2+a^2}}$
   (d) (5 pts) $\int \frac{x^2+8x-3}{x^3+3x^2} \, dx$
   (e) (6 pts) $\int_0^a \frac{dx}{(x^2+a^2)^{3/2}}$, $a > 0$
   (f) (6 pts) $\int_0^x \sqrt{a^2 - t^2} \, dt$, $0 \leq x \leq a$
   (g) (7 pts) $\int e^{\sqrt[3]{x}} \, dx$

2. (7 pts) If $0 < a < b$, find

$$\lim_{t \to 0} \left\{ \int_0^1 [bx + a(1-x)]^t dx \right\}^{1/t}$$

3. Evaluate the integral or show that it is divergent.
   (a) (4 pts) $\int_{-3}^{3} \frac{x}{1+|x|} \, dx$
   (b) (5 pts) $\int_1^{\infty} \frac{\tan^{-1} x}{x^2} \, dx$
   (c) (7 pts) $\int_{-1}^{1} \frac{dx}{x^2-2x}$
   (d) (7 pts) $\int_{-1}^{\infty} \left(\frac{x^4}{1+x^6} \right)^2 dx$
   (Hint: $\left(\frac{x^4}{1+x^6} \right)^2$ can be written as $x^3 \cdot f(x)$)

4. We can extend our definition of average value of a continuous function to an infinite interval by defining the average value of f on the interval $[a, \infty)$ to be

$$\lim_{t \to \infty} \frac{1}{t-a} \int_a^t f(x) dx$$

(a) (6 pts) Find the average value of $f(x) = \tan^{-1} x$ on the interval $[0, \infty)$.

(b) (5 pts) If $f(x) \geq 0$ and $\int_a^\infty f(x)dx$ is divergent, show that the average value of f on the interval $[a, \infty)$ is $\lim_{x\to\infty} f(x)$, if this limit exists.

(c) (5 pts) If $\int_a^\infty f(x)dx$ is convergent, what is the average value of f on the interval $[a, \infty)$?

(d) (3 pts) Find the average value of $f(x) = \sin x$ on the interval $[0, \infty)$.

5. Find the exact length of the curve.

   (a) (5 pts) $y = 1 + 6x^{3/2}$, $0 \leq x \leq 1$.

   (b) (5 pts) $y = \ln(\sec x)$, $0 \leq x \leq \pi/4$.

   (c) (7 pts) $y = \sqrt{x - x^2} + \sin^{-1}(\sqrt{x})$ on the whole domain.

   (d) (7 pts) $y = \int_1^x \sqrt{t^3 - 1}\, dt$, $1 \leq x \leq 4$.

6. Find the exact area of the surface of revolution.

   (a) (5 pts) The curve $y = x^3$, $0 \leq x \leq 2$, rotated about the x-axis.

   (b) (9 pts) The curve $y = e^{-x}$, $x \geq 0$, rotated about the x-axis.

   (c) (5 pts) The curve $x = \sqrt{a^2 - y^2}$, $0 \leq y \leq a/2$, rotated about the y-axis.

   (d) (9 pts) The curve $x^2 + y^2 = r^2$, rotated about the line $y = r$.

The End.