NTU 107-1 MATH1201 | CALCULUS A-05 EXERCISE SET 3 | 9TH NOVEMBER, 2018

Please write down your solutions on a separate sheet of paper and submit it to your TA or instructor.
Submit your solutions to Problems (1) $\sim$ (5) on 16**th** **November, 2018**.
Submit your solutions to Problems (6) $\sim$ (9) on 21**th** **November, 2018**.
The rest are left for your self-revision.

1. (6 pts) Evaluate $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}\sqrt{n+1}}+\frac{1}{\sqrt{n}\sqrt{n+2}}+...+\frac{1}{\sqrt{n}\sqrt{n+n}}\right)$.

2. (5 pts) Evaluate the integral by interpreting it in terms of areas.
$$\int_{-3}^{0}(1+\sqrt{9-x^2})dx$$

3. Evaluate the integral.
   (a) (4 pts) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\csc^2\theta d\theta$
   (b) (4 pts) $\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}\frac{4}{\sqrt{1-x^2}}dx$
   (c) (5 pts) $\int_{e}^{e^4}\frac{dx}{x\sqrt{\ln x}}$

4. Find the following values.
   (a) (7 pts) If $x\sin(\pi x)=\int_0^{x^2}f(t)dt$, where f is a continuous function, find $f(4)$.
   (b) (4 pts) If $f(x)=\int_0^x x^2\sin(t^2)dt$, find $f'(x)$. (There is no misprint here.)
   (c) (5 pts) If $\int_0^4 e^{(x-2)^4}dx=k$, find the value of $\int_0^4 xe^{(x-2)^4}dx$.
   (Hint: consider the transformation $u:=x-2$.)

5. Find the general indefinite integral.
   (a) (5 pts) $\int\frac{1+x}{1+x^2}dx$
   (b) (4 pts) $\int(2+\tan^2\theta)d\theta$
   (c) (5 pts) $\int\frac{\cos(\ln t)}{t}dt$

6. Find the area of the regions bounded by the given curves.
   (a) (7 pts) $y=\sqrt{x}$, $y=-\sqrt[3]{x}$, $y=x-2$.
   (b) (7 pts) $y=1/x$, $y=x^2$, $y=0$, $x=e$.

(c) (5 pts) $y = \sqrt{x}$, $y = x^2$, $x = 2$.

7. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

   (a) (6 pts) $x = 1 + y^2$, $y = x - 3$; about the y-axis.

   (b) (4 pts) $x = 0$, $x = 9 - y^2$; about $x = -1$.

   (c) (7 pts) $x^2 - y^2 = a^2$, $x = a + h$ (where $a > 0$, $h > 0$); about the y-axis.

8. (a) (4 pts) Find the average value of the function $f(x) = 1/\sqrt{x}$ on the interval $[1, 4]$.

   (b) (3 pts) Find the value c guaranteed by the Mean Value Theorem for Integrals such that $f_{ave} = f(c)$, where $f(x) = 1/\sqrt{x}$.

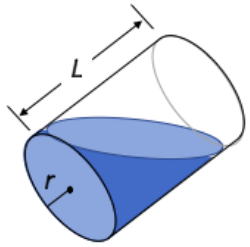
   (c) (5 pts) If f is a continuous function, what is the limit as $h \to 0$ of the average value of f on the interval $[x, x + h]$?

9. A cylindrical glass of radius r and height L is fully filled with water. It is then tilted to let the water flow out until the water remaining in the glass exactly covers its base.

   (a) (8 pts) Find the volume of the water in the glass.

   (b) (3 pts) Find the volume of the water in the glass from purely geometric consideration.

   (c) (7 pts) Suppose the glass is tilted until the water exactly covers half the base. Find the volume of the water in the glass.



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