Introduction to Computational Mathematics
Quiz 1 Solution
8th October, 2018

1. (a) (2 pts) Find the condition number of the function $f(x) = \log x$.
   (b) (2 pts) Using the above result, show that $\log x$ is ill-conditioned near $x = 1$.
   (c) (2 pts) Show that the function $f(x) = \sqrt{x}$ for $x > 0$ is well-condition by computing the condition number.

   (a)
   $$\kappa_f(x) = \left|\frac{x \cdot f'(x)}{f(x)}\right| = \left|\frac{x \cdot \frac{1}{x}}{\log x}\right| = \left|\frac{1}{\log x}\right|$$
   (Process + Answer : 1 + 1 points)

   (b) Taking the limit as x goes to 1, we will have
   $$\lim_{x\to 1} \kappa_f(x) = \lim_{x\to 1} \left|\frac{1}{\log x}\right| = \infty$$
   (Compute the limit: 1 point)
   The condition number $\kappa_f(x)$ will be very large for x near 1, hence it's ill-conditioned.
   . (Conclusion: 1 point)

   (c)
   $$\kappa_f(x) = \left|\frac{x \cdot f'(x)}{f(x)}\right| = \left|\frac{x \cdot \frac{1}{2\sqrt{x}}}{\sqrt{x}}\right| = \frac{1}{2}$$
   (Compute the condition number $\kappa_f(x)$: 1 point)
   The condition number $\kappa_f(x)$ will be $\frac{1}{2}$ for every x, hence it's well-conditioned.
   . (Conclusion: 1 point)

2. (4 pts) For the matrix A, please write the pseudo code and use twice "for loops" to perform an LU factorization.
   $$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ -1 & -3 & 0 \end{bmatrix}$$
   .

```
function [L,U] = lufact(A)

n = length(A);
L = eye(n);

for j = 1:n-1
.     for i = j+1:n
.         L(i,j) = A(i,j) / A(j,j);
.         A(i,j:n) = A(i,j:n) - L(i,j)*A(j,j:n);
.     end
end
U = triu(A);
```

   . (1-st For Loop + 2-nd For Loop + Compute L + Compute U: 1 + 1 + 1 + 1 points)