Lab 3A: Eye See You

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2018/10/22

Review

Least-Squares Fitting (Linear)

- If we have n points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ and need to find the approach linear function to fit these n points, we can use "least-squares fitting" to solve.

Least-Squares Fitting (Linear)

- Suppose these linear function is `y = ax + b` and fit $(x_1, y_1), \ldots, (x_n, y_n)$. The method of least-squares fitting is try to solve the following equation.

$$min_{a,b} \left\| \begin{pmatrix} 1 & x_1 \\ . & . \\ . & . \\ . & . \\ 1 & x_n \end{pmatrix} \begin{pmatrix} b \\ a \end{pmatrix} - \begin{pmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{pmatrix} \right\|_2 = min_b || Ab - Y ||_2$$

Hands On

Goal (Non-Linear)

- You will get the several points `(x,y)` from your own image.

$$x = f(t) \qquad y = g(t)$$

- You will also use the following periodic functions for the least-squares fitting.

$$f(t) = b_1 + b_2\cos(2\pi t) + b_3\cos(4\pi t) + b_4\cos(6\pi t) + b_5\sin(2\pi t) + b_6\sin(4\pi t) + b_7\sin(6\pi t)$$

$$g(t) = c_1 + c_2\cos(2\pi t) + c_3\cos(4\pi t) + c_4\cos(6\pi t) + c_5\sin(2\pi t) + c_6\sin(4\pi t) + c_7\sin(6\pi t)$$

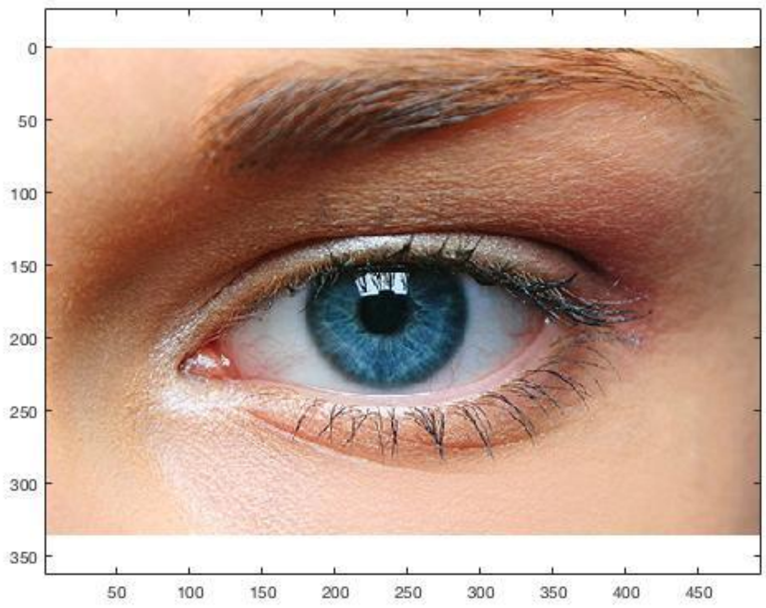
Preparation

- Please download the file “Lab3A_Eye_See_You.zip” in the folder “Homeworks” (Google Drive).
- Use the file “EyeSeeYou.m” to do the following steps.

Step 1

- Using your phone, take a picture of an open eye.
- Load the image into MATLAB using imread and display this image.

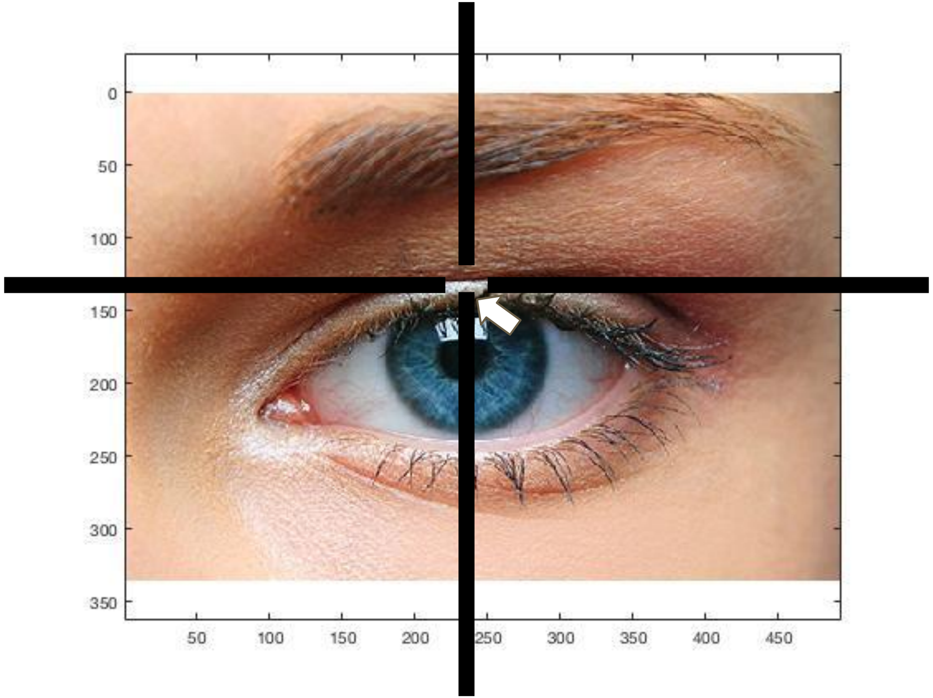
Step 1



Step 2

- Using the command `ginput(10)`, collect the upper 10 points of your eye **from right to left**.

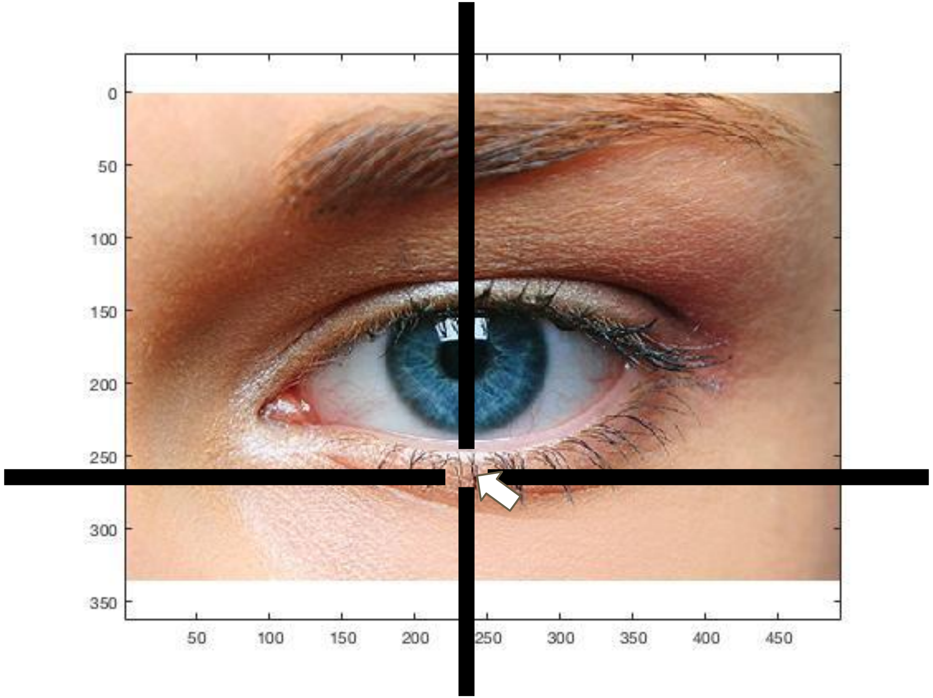
Step 2



Step 3

- Repeat step 2, using the command `ginput(10)`, collect the lower **10** points of your eye **from left to right**.

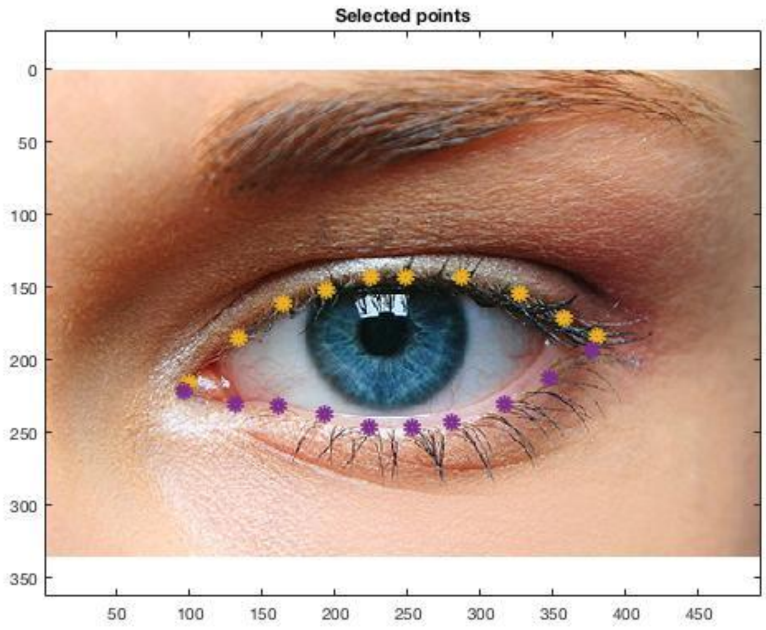
Step 3



Step 4

- Stack `xup` and `xlo` into a vector `x`, and stack `yup` and `ylo` into a vector `y`.
- Plot the points using `'o'` markers.

Step 4



Selected points
0
50
100
150
200
250
300
350
50
100
150
200
250
300
350
400
450

Step 5

- Now let `t` be a vector where $t_i = (i-1)/20$ for $i = 1, \ldots, 20$. Referring back to equation, create a 20 x 7 matrix `A` whose columns are the values of the functions $1, cos(2\pi t)$, and so on, through $\sin(6\pi t)$.

(2):

$$f(t) = b_1 + b_2 \cos(2\pi t) + b_3 \cos(4\pi t) + b_4 \cos(6\pi t) + b_5 \sin(2\pi t) + b_6 \sin(4\pi t) + b_7 \sin(6\pi t)$$

(3):

$$g(t) = c_1 + c_2 \cos(2\pi t) + c_3 \cos(4\pi t) + c_4 \cos(6\pi t) + c_5 \sin(2\pi t) + c_6 \sin(4\pi t) + c_7 \sin(6\pi t)$$

Step 5

```
t =

         0
    0.0500
    0.1000
    0.1500
    0.2000
    0.2500
    0.3000
    0.3500
    0.4000
    0.4500
    0.5000
    0.5500
    0.6000
    0.6500
    0.7000
    0.7500
    0.8000
    0.8500
    0.9000
    0.9500
```

```
A =

    1.0000    1.0000    1.0000    1.0000         0         0         0
    1.0000    0.9511    0.8090    0.5878    0.3090    0.5878    0.8090
    1.0000    0.8090    0.3090   -0.3090    0.5878    0.9511    0.9511
    1.0000    0.5878   -0.3090   -0.9511    0.8090    0.9511    0.3090
    1.0000    0.3090   -0.8090   -0.8090    0.9511    0.5878   -0.5878
    1.0000    0.0000   -1.0000   -0.0000    1.0000    0.0000   -1.0000
    1.0000   -0.3090   -0.8090    0.8090    0.9511   -0.5878   -0.5878
    1.0000   -0.5878   -0.3090    0.9511    0.8090   -0.9511    0.3090
    1.0000   -0.8090    0.3090    0.3090    0.5878   -0.9511    0.9511
    1.0000   -0.9511    0.8090   -0.5878    0.3090   -0.5878    0.8090
    1.0000   -1.0000    1.0000   -1.0000    0.0000   -0.0000    0.0000
    1.0000   -0.9511    0.8090   -0.5878   -0.3090    0.5878   -0.8090
    1.0000   -0.8090    0.3090    0.3090   -0.5878    0.9511   -0.9511
    1.0000   -0.5878   -0.3090    0.9511   -0.8090    0.9511   -0.3090
    1.0000   -0.3090   -0.8090    0.8090   -0.9511    0.5878    0.5878
    1.0000   -0.0000   -1.0000    0.0000   -1.0000    0.0000    1.0000
    1.0000    0.3090   -0.8090   -0.8090   -0.9511   -0.5878    0.5878
    1.0000    0.5878   -0.3090   -0.9511   -0.8090   -0.9511   -0.3090
    1.0000    0.8090    0.3090   -0.3090   -0.5878   -0.9511   -0.9511
    1.0000    0.9511    0.8090    0.5878   -0.3090   -0.5878   -0.8090
```

Step 6

- Apply linear least squares (using backslash) to solve for the coefficients b_j in (2) using the `x` data, and to solve for the coefficients c_j in (3) using the `y` data.

(2):

$$f(t) = b_1 + b_2 \cos(2\pi t) + b_3 \cos(4\pi t) + b_4 \cos(6\pi t) + b_5 \sin(2\pi t) + b_6 \sin(4\pi t) + b_7 \sin(6\pi t)$$

(3):

$$g(t) = c_1 + c_2 \cos(2\pi t) + c_3 \cos(4\pi t) + c_4 \cos(6\pi t) + c_5 \sin(2\pi t) + c_6 \sin(4\pi t) + c_7 \sin(6\pi t)$$

Step 7

- Evaluate the functions in (2) and (3) at 500 equally spaced values of t between 0 and 1.
- On top of the axes showing the eye image and the selected points, and using the coefficients from the previous step, plot the curve defined by equation (1).

(1):

$$x = f(t) \qquad y = g(t)$$

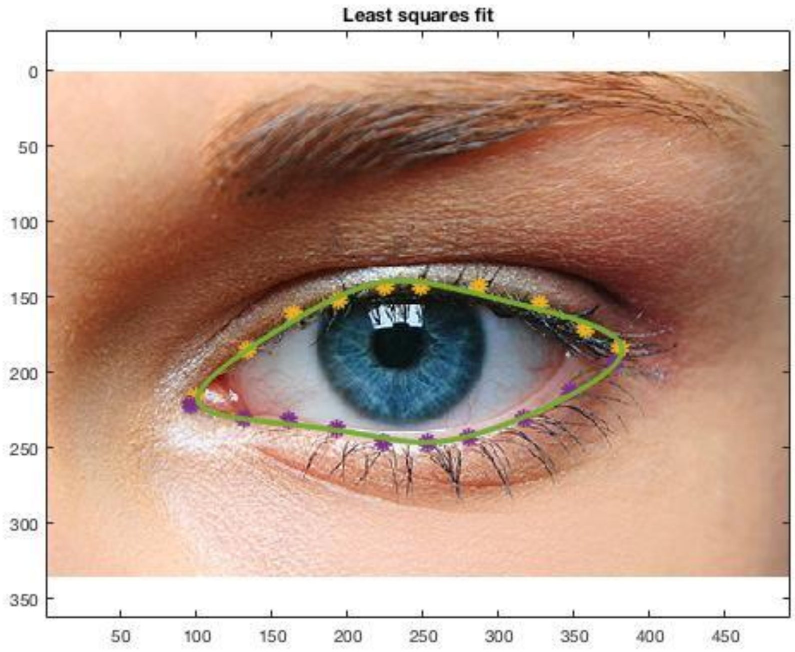
(2):

$$f(t) = b_1 + b_2 \cos(2\pi t) + b_3 \cos(4\pi t) + b_4 \cos(6\pi t) + b_5 \sin(2\pi t) + b_6 \sin(4\pi t) + b_7 \sin(6\pi t)$$

(3):

$$g(t) = c_1 + c_2 \cos(2\pi t) + c_3 \cos(4\pi t) + c_4 \cos(6\pi t) + c_5 \sin(2\pi t) + c_6 \sin(4\pi t) + c_7 \sin(6\pi t)$$

Step 7



Least squares fit
0
50
100
150
200
250
300
350
50
100
150
200
250
300
350
400
450

Homework

- Please write the report on A4 pages and hand in at 4:30 pm on 2018/10/29 (Monday).

What We Learned?

What We Leanered?

- How to use MATLAB to solve the “nonlinear least-squares fitting” problem from a practical example.

Questions or comments?