NTU 107-1 MATH1201

CALCULUS A-05
QUIZ 3

28TH DECEMBER, 2018

Work out **ALL** questions below. Provide sufficient justification to every step of your arguments. Write your solutions as well as your ID number clearly on A4-sized paper and submit them to *Instructor's office* **before 6pm (GMT +8)** on **2nd January, 2019**. Recommended time limit: 150 minutes.

1. Let

$$f(x) = \int_0^{\frac{1}{x}} \frac{t^2}{t^4+1}\,dt + \int_0^x \frac{1}{t^4+1}\,dt, \qquad x \neq 0$$

   (a) (4 pts) Find $f'(x)$.

   (b) (5 pts) Find $f(1) + f(-1)$.

   (c) (4 pts) Using the above results, find $f(3) + f(-2)$.

2. (a) (8 pts) Evaluate the integral $\int_0^{\frac{\pi}{2}} \left|\cos^2 x - 3\sin^2 x\right| dx$.

   (b) (8 pts) Compute $\int \frac{1}{e^{2x}+e^x+1} dx$.

3. (a) (5 pts) Determine whether the improper integral

$$\int_0^1 \frac{\cos t}{t^{4/3}}\,dt$$

   is convergent or divergent?

   (b) (5 pts) Evaluate the following limit

$$\lim_{x\to 0^+} x^{1/6} \int_{\sqrt{x}}^1 \frac{\cos t}{t^{4/3}}\,dt$$

4. Figure 1 shows a curve C with the property that, for every point P on the middle curve $y = 2x^2$, a vertical line through P bounded a region A between the curves $y = 2x^2$ and $y = x^2$ while a horizontal line through P bounded a region B between the curves $y = 2x^2$ and C, and the area of B is twice the area of A.

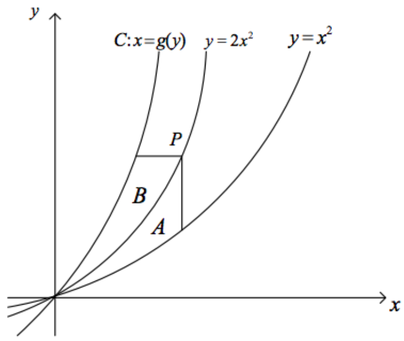

Figure 1: The three curves $C : x = g(y)$, $y = 2x^2$ and $y = x^2$.

(a) (9 pts) Find an equation $x = g(y)$ for C.
(Hint: Compute the areas of A and B.)

(b) (6 pts) Let R be the region bounded by the curve C, $y = x^2$, $x = 2$ and $y = 8$. Find the volume of the solid obtained by rotating R about the x-axis.

5. Consider the plane curve $3ay^2 = x(a - x)^2$ where $a > 0$ is a constant.

(a) (6 pts) Find the arc length of the loop defined by the curve.

(b) (4 pts) Find the surface area of the surface obtained by rotating the loop around x-axis.

6. (a) (7 pts) Find all points of intersection of the two polar curves $r = \sqrt{2}\sin\theta$ and $r^2 = \cos 2\theta$.

(b) (6 pts) Find the area of the shaded region in Figure 2.

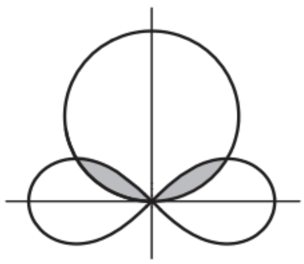

Figure 2: The two curves $r = \sqrt{2}\sin\theta$ and $r^2 = \cos 2\theta$.

7. (a) (7 pts) Solve the differential equation $x\frac{dy}{dx} - 2y = x^3 \cdot \tan x \cdot \sec x$, $x > 0$ and $y(\pi/3) = 0$.

   (b) (5 pts) Find the orthogonal trajectories of the family of curves $y = \frac{k}{x+1}$, where k is an arbitrary constant.

8. (a) (7 pts) Slove the initial value problem:

$$\begin{cases} 2x(x+3)y' + (4x+3)y = 2x^{\frac{1}{2}}(x+3)^{\frac{1}{2}} \\ y(1) = \frac{1}{2}, \quad x > 0 \end{cases}$$

   (b) (4 pts) Find $\lim_{x\to\infty} y(x)$ and $\lim_{x\to 0^+} y(x)$.

The End.