Radiomics

A new application from established techniques

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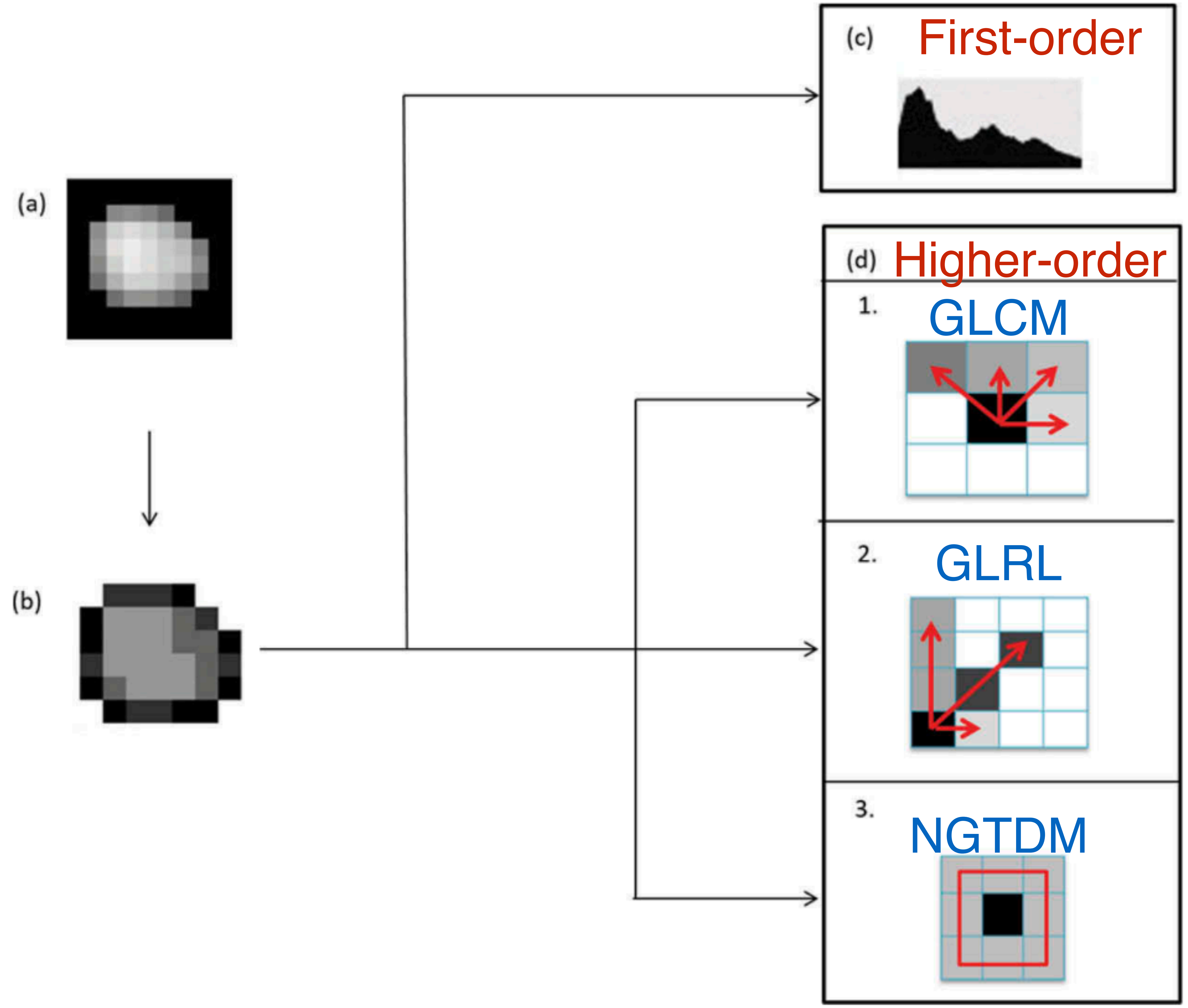
Outline

- **Radiomics Feature Extraction**
  - **Statistical Texture Features**
    - **First-order Texture Statistics**
    - **Higher order Texture Statistics**
  - **Filtering Approaches**
  - **Feature Selection**
  - **GLCM**
  - **GLRL**
  - **NGTDM**

Vishwa Parekh & Michael A. Jacobs (2016) Radiomics: a new application from established techniques, Expert Review of Precision Medicine and Drug Development, 1:2, 207-226

Statistical Texture Features

- The spatial distribution
- Two levels of statistical methods: first and higher order methods



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Statistical Texture Features

- First and higher order methods
- Moment Generating Function

$$M_X(t) := \mathbb{E}\left[e^{tX}\right], \quad t \in \mathbb{R}$$

$$m_n = E\left(X^n\right) = M_X^{(n)}(0) = \left.\frac{d^n M_X}{dt^n}\right|_{t=0}$$

  - n= 1, first order statistics : mean
  - n= 2, second order statistics : variance
  - n= 3, third order statistics : skewness
  - n= 4, first order statistics : kurtosis

https://en.wikipedia.org/wiki/Moment-generating_function

First-order Texture Statistics

- First-order histogram that describes distribution of voxel intensities
- For example:

$$H(i) = \frac{\text{No. of pixels with gray levels in } \{I \in B_i\}}{\sum \text{No. pixels in the image}}$$

First-order Texture Statistics

- Entropy measures the inherent randomness in the gray-level intensities of an image

$$\text{Entropy} = -K\sum_{i=1}^{B} H(i)\log H(i)$$

where K is a positive constant and is determined by the units of the application

$$\text{Entropy} = -\sum_{i=1}^{B} H(i)\log_2 H(i)$$

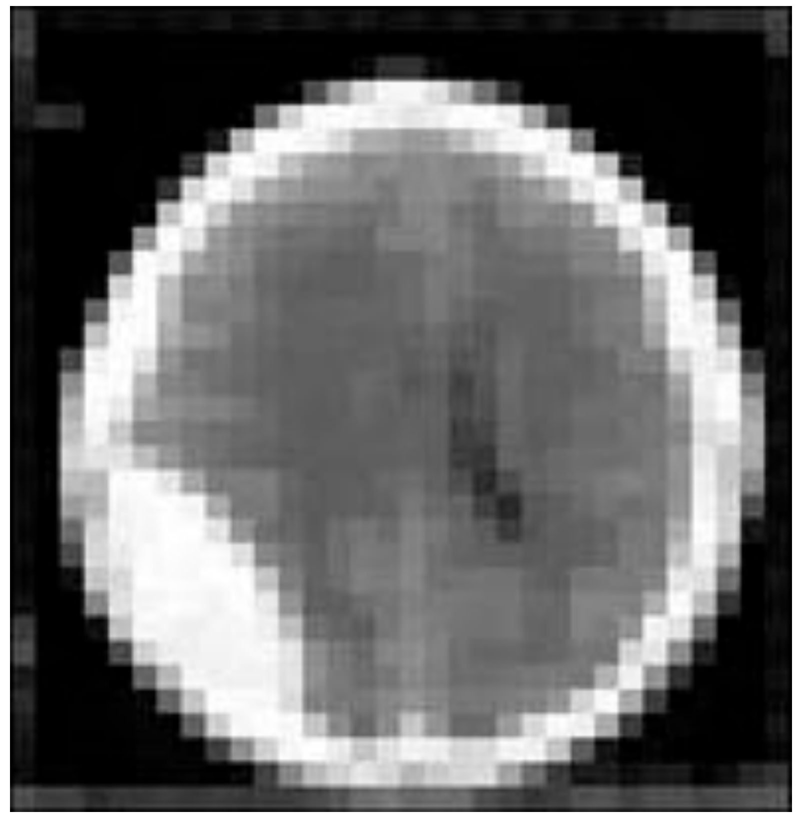
First-order Texture Statistics

- Uniformity measures the uniformity of gray-level intensities within an image or region of interest (ROI)

$$\text{Uniformity} = \sum_{i=1}^{B} H(i)^2$$

Limitation of First-order Texture Statistics

- Limitation of directly comparing the results between studies
- Another approach which can potentially overcome this limitation leads to an issue of dependence of first-order statistic features on ROI size
- Feature normalization is required



32x32 CT

256x256 CT

Max entropy : 5 8

http://ericlinhome.myweb.hinet.net/ct_read/01_introduction.html

Higher order Texture Statistics

- Information about the inter-voxel relationships within the image
- Three higher order texture statistics methods:
  - gray-level co-occurrence matrix (GLCM)
  - gray-level run length matrix (GLRL)
  - neighborhood gray-tone difference matrix (NGTDM)

Filtering Approaches

- Linear or non-linear transformation
- Spatial filtering techniques

Filtering Approaches

- Statistical filters: average filter, range filter, entropy filter, edge filters like Prewitt filter, Sobel filter, Laplacian filter, and Laplacian of Gaussian (LoG) filter
- Special kernels: fractal dimension filter

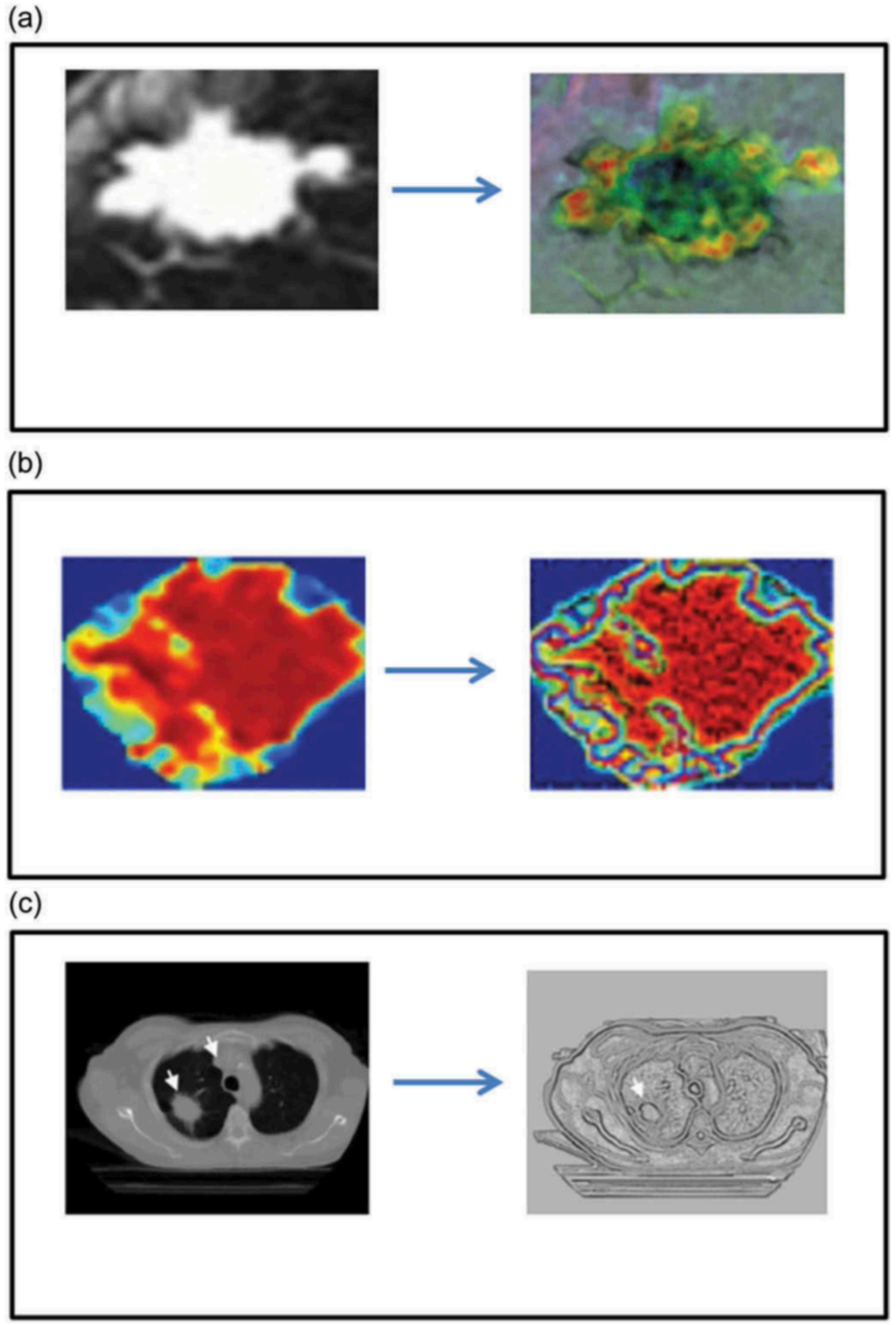
The word which is blue means the filter can be found in ITK

Filtering Approaches

Statistical Kernel
(median filter)

Edge Kernel
(Laplacian of Gaussian filter)

Special Kernel
(Fractal dimension filter)



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Feature Selection

- Subset of features that can characterize the tissue of interest
- A large number of GLCM features are extracted is the challenge
- Reduction is needed in order to avoid overfitting
- The procedure can be either supervised or unsupervised

Feature Selection

- Supervised learning :
  - final class label (e.g. benign or malignant)
  - filtering method:

    Fisher’s criterion, Wilcoxon rank-sum test, Student’s t-test, etc.
  - wrapper method:

    greedy forward selection and greedy backward elimination

Feature Selection

- Unsupervised learning :
  - linear dimension reduction:

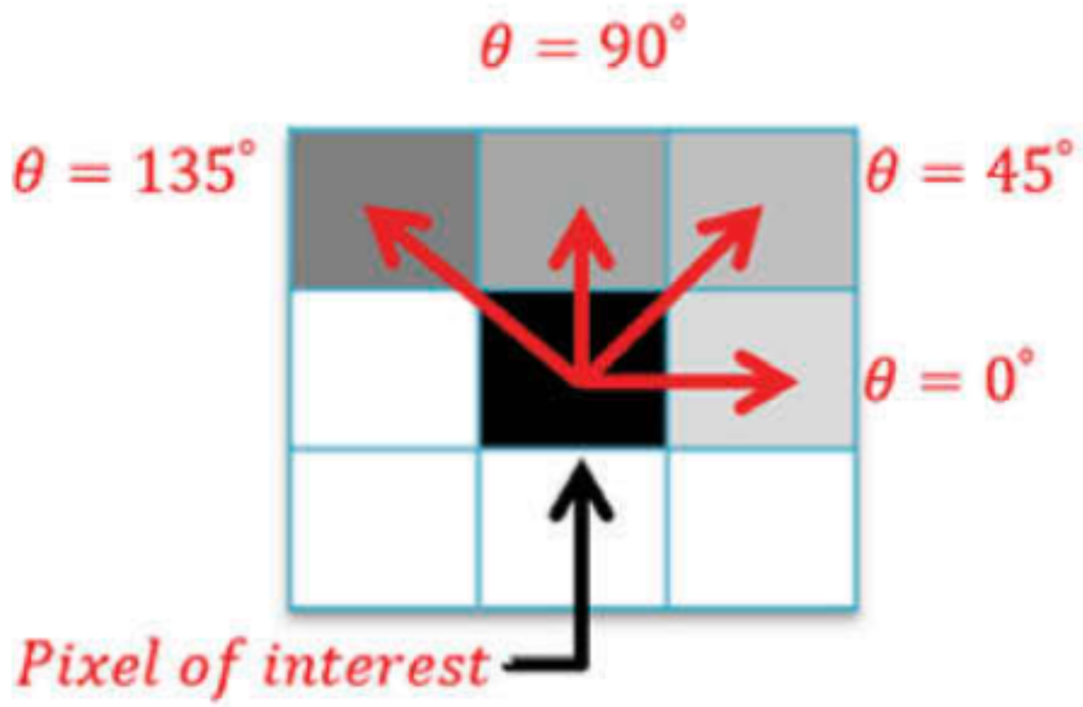
    principal component analysis (PCA), multidimensional scaling
  - nonlinear dimension reduction:

    isometric mapping, locally linear embedding, and diffusion map

Gray-Level Co-occurrence Matrix (GLCM)

- GLCM shows the relationship between voxel pair and the frequency of each intensity pairs within an image or a region of interest.

- These have two parameters which are the distance (d) and the angle (θ)



Gray-Level Co-occurrence Matrix (GLCM)

- Example :

The symmetric GLCM

Ng = 5, d = 1, θ = 0°

2	1	5	3	2
2	2	4	4	1
4	3	4	2	2
3	2	4	4	1
1	4	4	2	3

I	1	2	3	4	5
1	0	1	0	3	1
2	1	4	3	4	0
3	0	3	0	2	1
4	3	4	2	6	0
5	1	0	1	0	0

Gray-Level Co-occurrence Matrix (GLCM)

- The most commonly used features :

$$\mathrm{ASM} = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}(G_{\mathrm{norm}}(i,j))^2$$

$$\mathrm{Entropy} = -\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} G_{\mathrm{norm}}(i,j)\log_2(G_{\mathrm{norm}}(i,j))$$

$$\mathrm{Contrast} = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g} |i-j|^2 G_{\mathrm{norm}}(i,j)$$

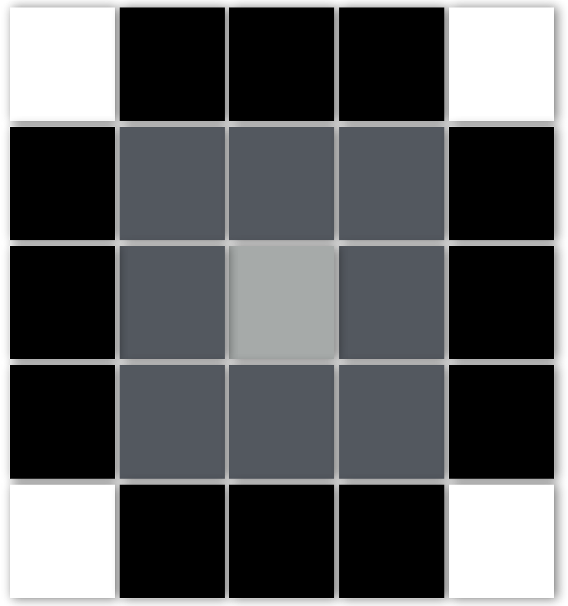
$$\mathrm{Correlation} = \frac{\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} ij\left(G_{\mathrm{norm}}(i,j) - \mu_x(i)\mu_y(j)\right)}{\sigma_x(i)\sigma_y(j)}$$

G_{norm} : Normalized GLCM

Gray-Level Co-occurrence Matrix (GLCM)

- Example :

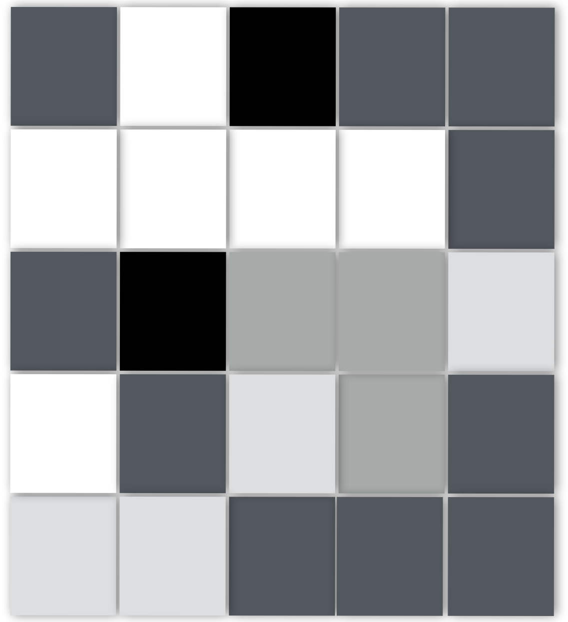
$$ASM = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \left(G_{norm}(i,j)\right)^2$$



$A_1 =$

1	5	5	5	1
5	4	4	4	5
5	4	3	4	5
5	4	4	4	5
1	5	5	5	1

$ASM = 0.15$



$A_2 =$

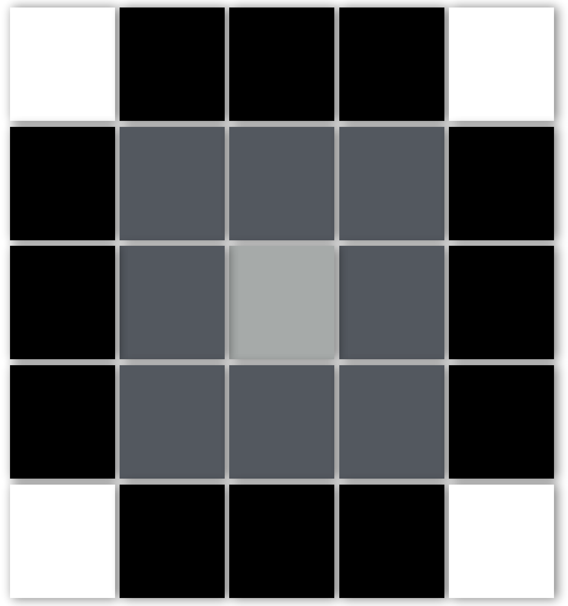
4	1	5	4	4
1	1	1	1	4
4	5	3	3	2
1	4	2	3	4
2	2	4	4	4

$ASM = 0.08$

Gray-Level Co-occurrence Matrix (GLCM)

- Example :

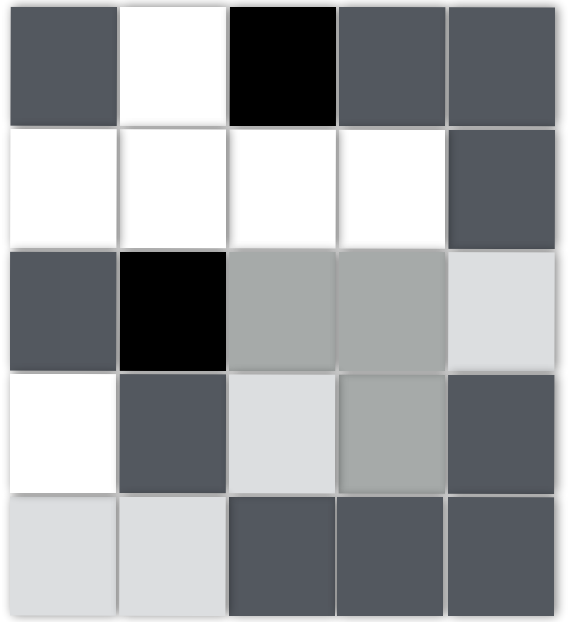
$$\text{Entropy} = -\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} G_{norm}(i,j)\log_2\left(G_{norm}(i,j)\right)$$



$A_1 =$

1	5	5	5	1
5	4	4	4	5
5	4	3	4	5
5	4	4	4	5
1	5	5	5	1

Entropy = 2.8464



$A_2 =$

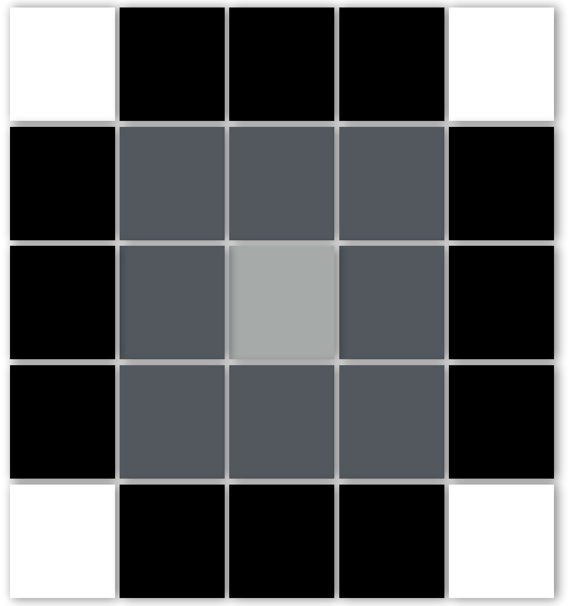
4	1	5	4	4
1	1	1	1	4
4	5	3	3	2
1	4	2	3	4
2	2	4	4	4

Entropy = 3.9087

Gray-Level Co-occurrence Matrix (GLCM)

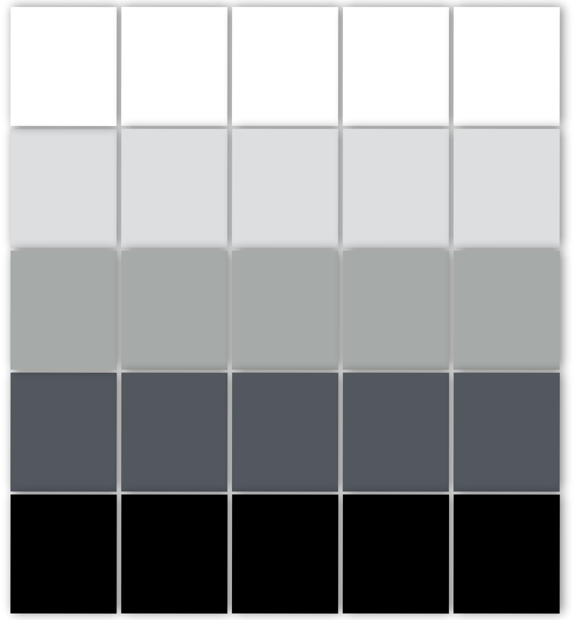
- Example :

$$\text{Contrast} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} |i-j|^2 G_{norm}(i,j)$$



$$A_1 = \begin{array}{|c|c|c|c|c|} \hline 1 & 5 & 5 & 5 & 1 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 5 & 4 & 3 & 4 & 5 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 1 & 5 & 5 & 5 & 1 \\ \hline \end{array}$$

Contrast = 3.6



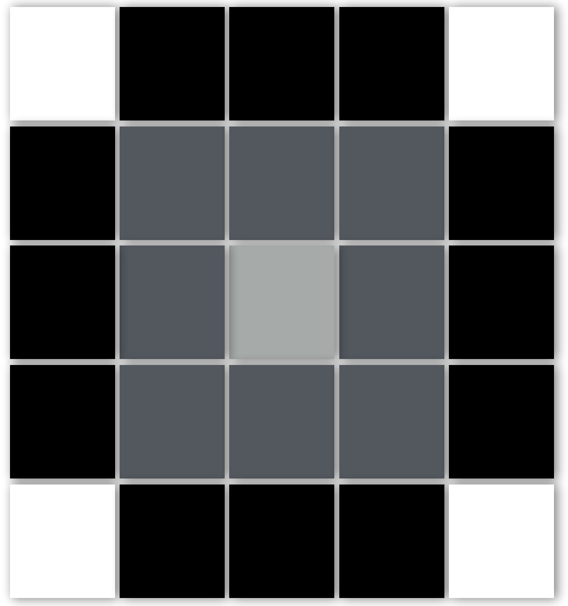
$$A_2 = \begin{array}{|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 \\ \hline 2 & 2 & 2 & 2 & 2 \\ \hline 3 & 3 & 3 & 3 & 3 \\ \hline 4 & 4 & 4 & 4 & 4 \\ \hline 5 & 5 & 5 & 5 & 5 \\ \hline \end{array}$$

Contrast = 0

Gray-Level Co-occurrence Matrix (GLCM)

- Example :

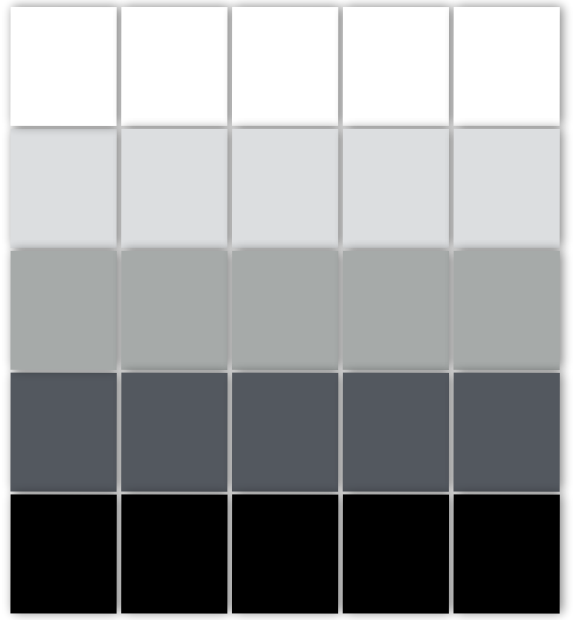
$$\text{Correlation} = \frac{\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} ij\left(G_{\text{norm}}(i,j) - \mu_x(i)\mu_y(j)\right)}{\sigma_x(i)\sigma_y(j)}$$



A_1 =

1	5	5	5	1
5	4	4	4	5
5	4	3	4	5
5	4	4	4	5
1	5	5	5	1

Correlation = -0.2950



A_2 =

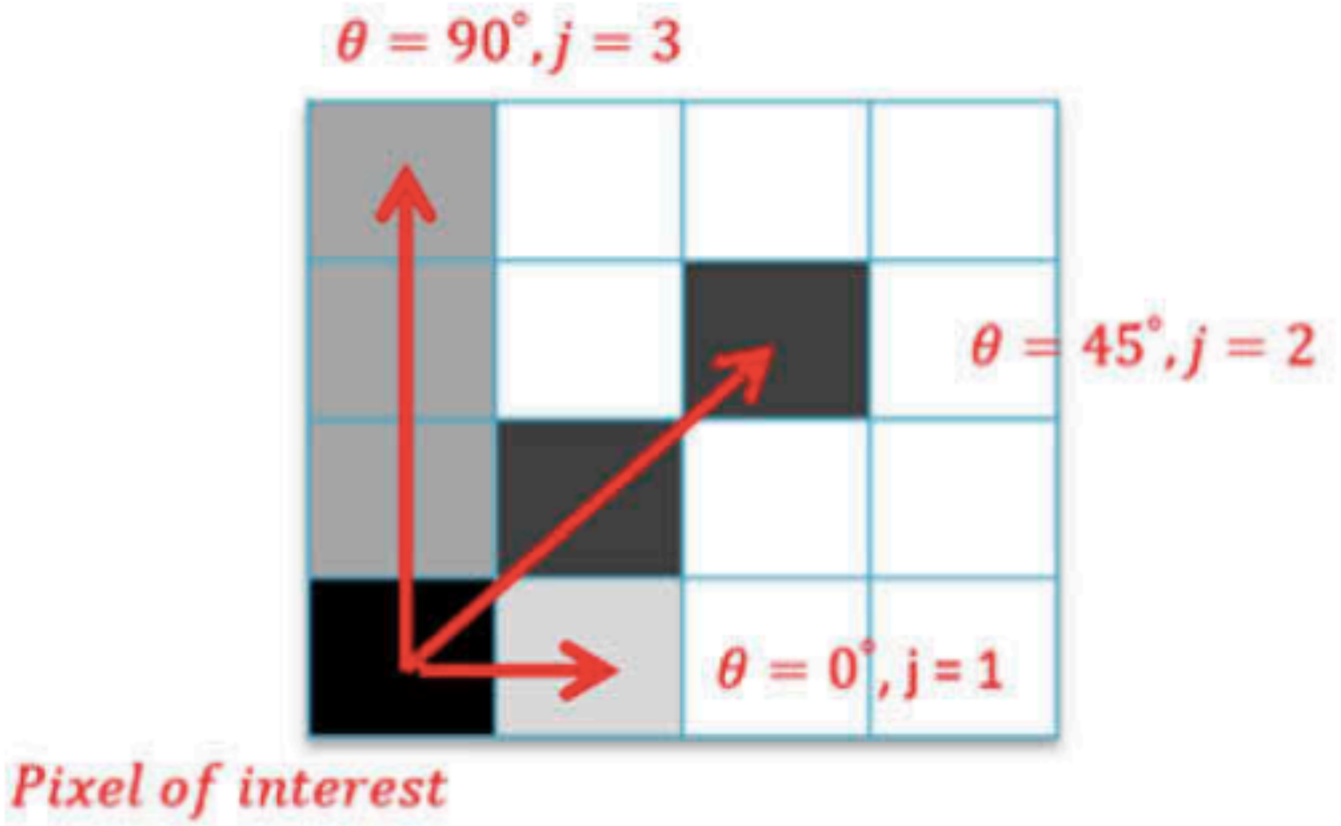
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5

Correlation = 1.0000

Gray-Level Run Length Matrix (GLRL)

- GLRL is defined as the number of contiguous voxels that have the same gray-level value and it characterizes the gray-level run lengths of different gray-level intensities in any direction.

- Based on the direction angle (θ), different GLRL matrices can be constructed.



Gray-Level Run Length Matrix (GLRL)

- Example :

2D - GLRL

i is gray-level value, j is the number of times

$\theta = 0°$

2	1	5	3	2
2	2	4	4	1
4	3	4	2	2
3	2	4	4	1
1	4	4	2	3

I\j	j=1	j=2	j=3	j=4	j=5
I=1	4	0	0	0	0
I=2	4	2	0	0	0
I=3	4	0	0	0	0
I=4	2	3	0	0	0
I=5	1	0	0	0	0

Gray-Level Run Length Matrix (GLRL)

- The most commonly used features :

$$SRE = \frac{\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} GLRL(i,j)/j^2}{\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} GLRL(i,j)}$$

$$LRE = \frac{\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} j^2 GLRL(i,j)}{\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} GLRL(i,j)}$$

$$GLN = \frac{\sum_{i=1}^{N_g} \left(\sum_{j=1}^{N_r} GLRL(i,j) \right)^2}{\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} GLRL(i,j)}$$

$$RLN = \frac{\sum_{j=1}^{N_r} \left(\sum_{i=1}^{N_g} GLRL(i,j) \right)^2}{\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} GLRL(i,j)}$$

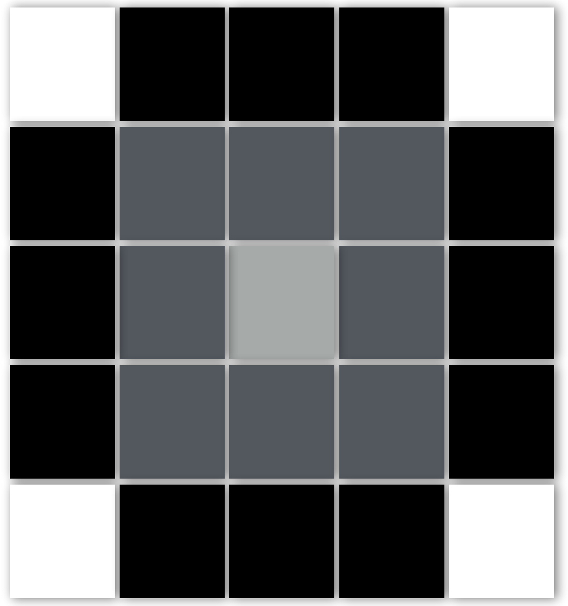
$$RP = \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} \frac{GLRL(i,j)}{P}$$

N_g is gray-level , N_r is the number of different run lengths

Gray-Level Run Length Matrix (GLRL)

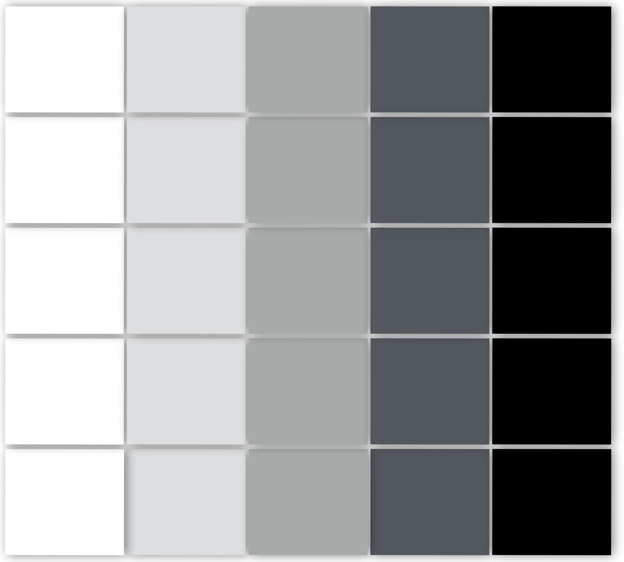
- Example :

$$\mathrm{SRE} = \frac{\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} \mathrm{GLRL}(i,j)/j^2}{\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} \mathrm{GLRL}(i,j)}$$



$$A_1 = \begin{array}{|c|c|c|c|c|} \hline 1 & 5 & 5 & 5 & 1 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 5 & 4 & 3 & 4 & 5 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 1 & 5 & 5 & 5 & 1 \\ \hline \end{array}$$

SRE = 0.7908



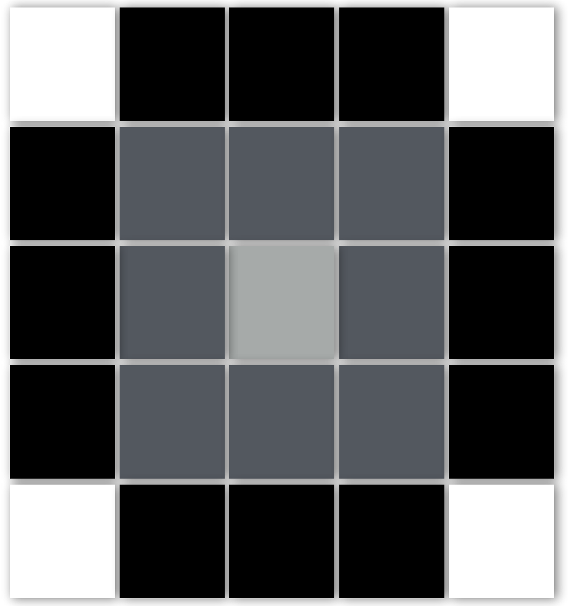
$$A_2 = \begin{array}{|c|c|c|c|c|} \hline 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2 & 3 & 4 & 5 \\ \hline \end{array}$$

SRE = 1.0000

Gray-Level Run Length Matrix (GLRL)

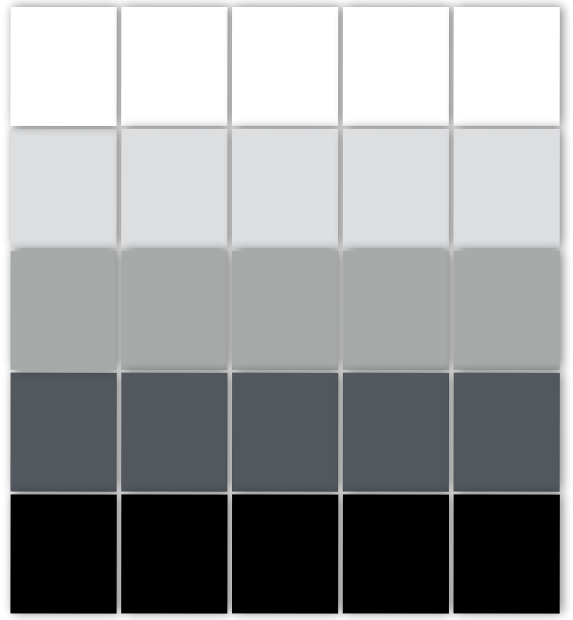
- Example :

$$\mathrm{LRE} = \frac{\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} j^2 \mathrm{GLRL}(i,j)}{\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} \mathrm{GLRL}(i,j)}$$



$$A_1 = \begin{array}{|c|c|c|c|c|} \hline 1 & 5 & 5 & 5 & 1 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 5 & 4 & 3 & 4 & 5 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 1 & 5 & 5 & 5 & 1 \\ \hline \end{array}$$

LRE = 49



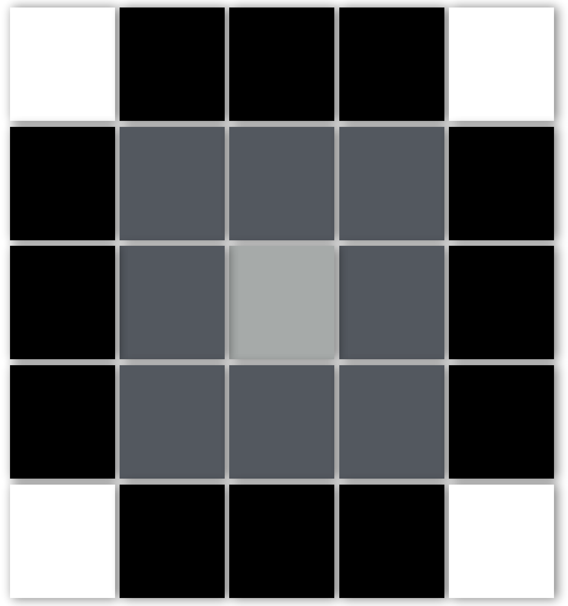
$$A_2 = \begin{array}{|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 \\ \hline 2 & 2 & 2 & 2 & 2 \\ \hline 3 & 3 & 3 & 3 & 3 \\ \hline 4 & 4 & 4 & 4 & 4 \\ \hline 5 & 5 & 5 & 5 & 5 \\ \hline \end{array}$$

LRE = 125

Gray-Level Run Length Matrix (GLRL)

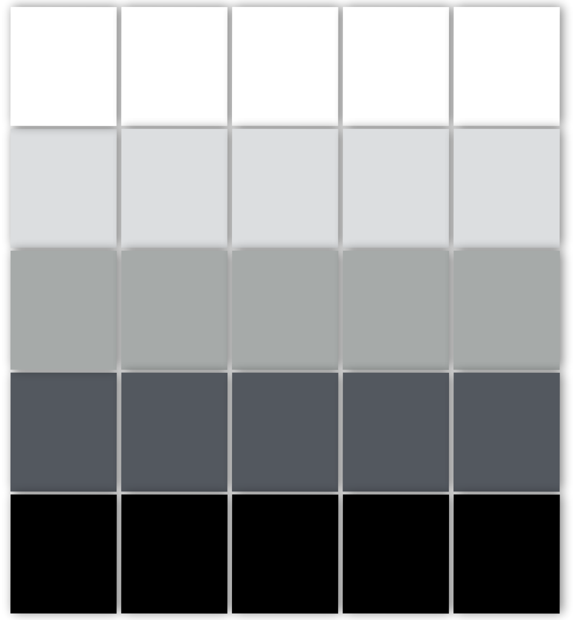
- Example :

$$\mathrm{GLN} = \frac{\sum_{i=1}^{N_g} \left(\sum_{j=1}^{N_r} \mathrm{GLRL}(i,j) \right)^2}{\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} \mathrm{GLRL}(i,j)}$$



$$A_1 = \begin{array}{|c|c|c|c|c|} \hline 1 & 5 & 5 & 5 & 1 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 5 & 4 & 3 & 4 & 5 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 1 & 5 & 5 & 5 & 1 \\ \hline \end{array}$$

GLN = 5.7059



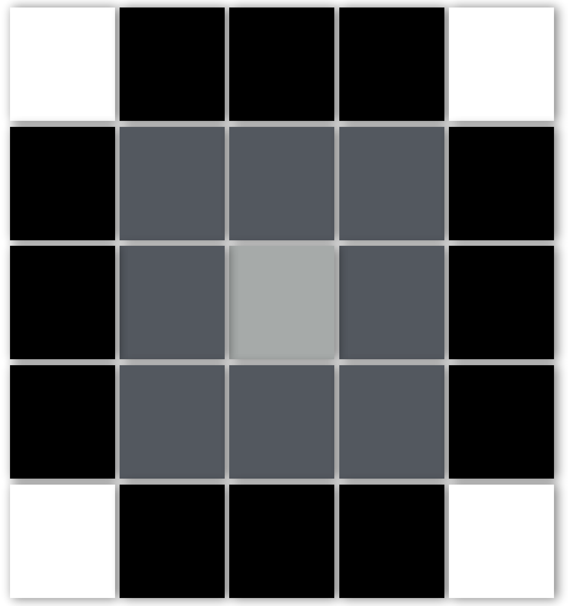
$$A_2 = \begin{array}{|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 \\ \hline 2 & 2 & 2 & 2 & 2 \\ \hline 3 & 3 & 3 & 3 & 3 \\ \hline 4 & 4 & 4 & 4 & 4 \\ \hline 5 & 5 & 5 & 5 & 5 \\ \hline \end{array}$$

GLN = 1.0000

Gray-Level Run Length Matrix (GLRL)

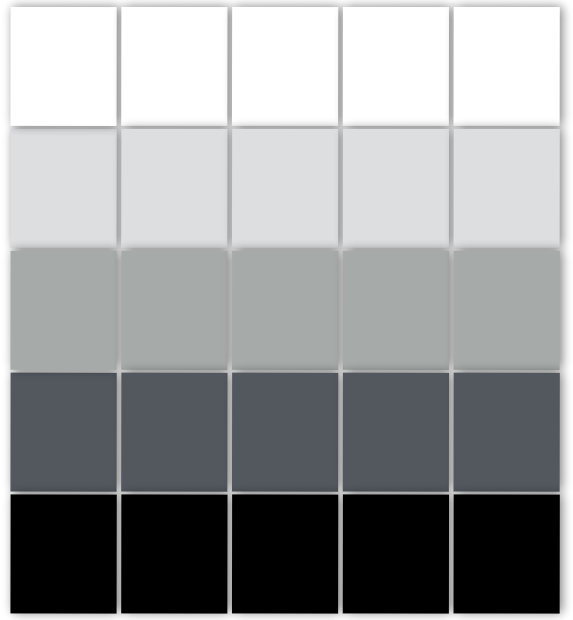
- Example :

$$\mathrm{RLN} = \frac{\sum_{j=1}^{N_r}\left(\sum_{i=1}^{N_g}\mathrm{GLRL}(i,j)\right)^2}{\sum_{i=1}^{N_g}\sum_{j=1}^{N_r}\mathrm{GLRL}(i,j)}$$



$$A_1 = \begin{array}{|c|c|c|c|c|}\hline 1 & 5 & 5 & 5 & 1 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 5 & 4 & 3 & 4 & 5 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 1 & 5 & 5 & 5 & 1 \\ \hline \end{array}$$

RLN = 10.8824



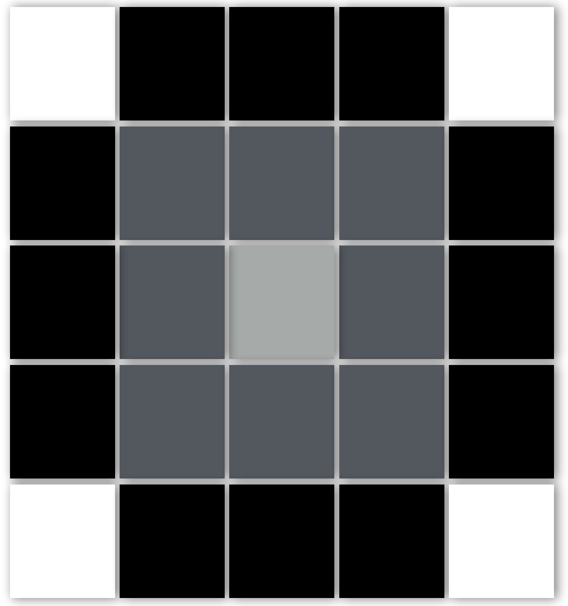
$$A_2 = \begin{array}{|c|c|c|c|c|}\hline 1 & 1 & 1 & 1 & 1 \\ \hline 2 & 2 & 2 & 2 & 2 \\ \hline 3 & 3 & 3 & 3 & 3 \\ \hline 4 & 4 & 4 & 4 & 4 \\ \hline 5 & 5 & 5 & 5 & 5 \\ \hline \end{array}$$

RLN = 5.0000

Gray-Level Run Length Matrix (GLRL)

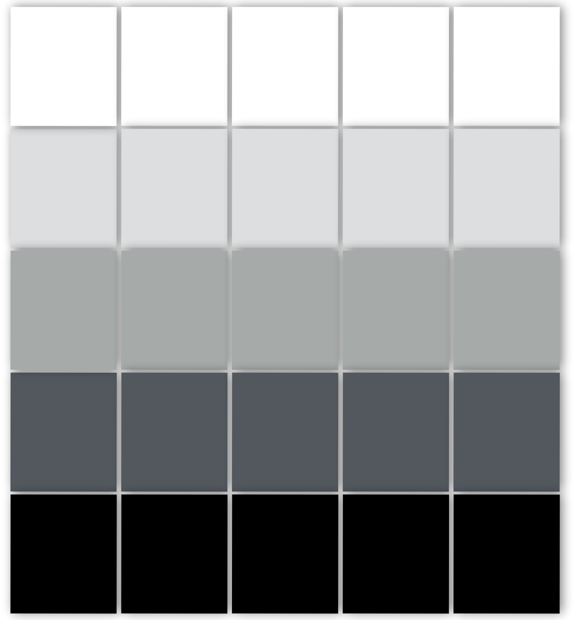
- Example :

$$RP = \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} \frac{GLRL(i,j)}{P}$$



$$A_1 = \begin{array}{|c|c|c|c|c|} \hline 1 & 5 & 5 & 5 & 1 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 5 & 4 & 3 & 4 & 5 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 1 & 5 & 5 & 5 & 1 \\ \hline \end{array}$$

RP = 0.6800



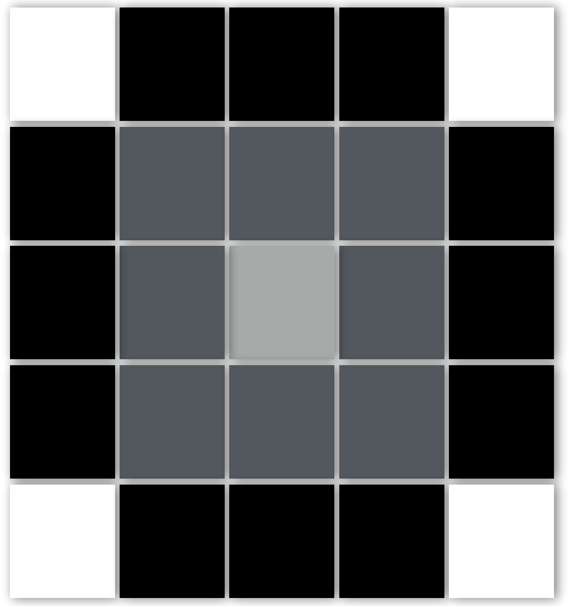
$$A_2 = \begin{array}{|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 \\ \hline 2 & 2 & 2 & 2 & 2 \\ \hline 3 & 3 & 3 & 3 & 3 \\ \hline 4 & 4 & 4 & 4 & 4 \\ \hline 5 & 5 & 5 & 5 & 5 \\ \hline \end{array}$$

RP = 0.2000

Gray-Level Run Length Matrix (GLRL)

- Example :

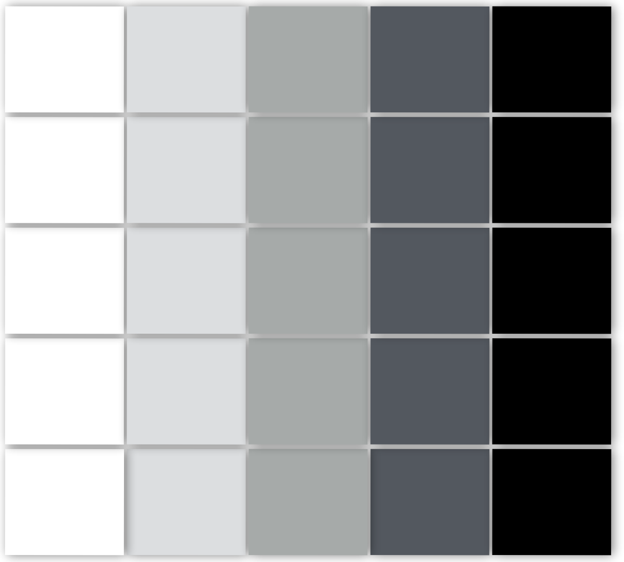
$$RP = \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} \frac{GLRL(i,j)}{P}$$



$A_1 =$

1	5	5	5	1
5	4	4	4	5
5	4	3	4	5
5	4	4	4	5
1	5	5	5	1

RP = 0.6800



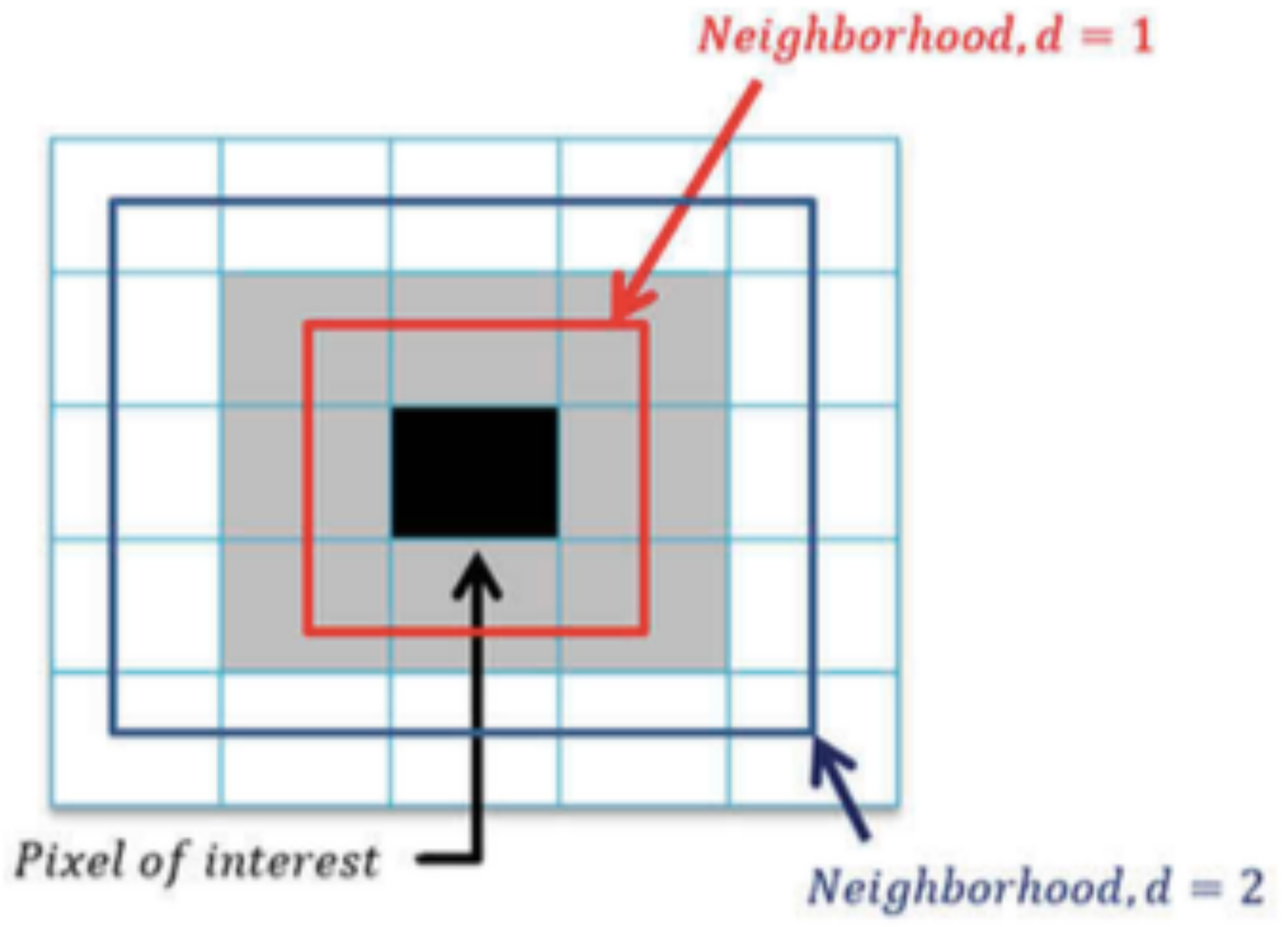
$A_2 =$

1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5

RP = 1.0000

Neighborhood Gray-Tone Difference Matrix (NGTDM)

- NGTDM is a texture analysis method based on the visual properties of an image.
- The each gray-level value in NGTDM is defined as *gt*.
- The size of the neighborhood is defined by the user. ($W = (2d+1)^2$)



Neighborhood Gray-Tone Difference Matrix (NGTDM)

- NGTDM of an image ($N_x \times N_y$) with N_g gray levels is given using the following set of equations :

$$\mathrm{ANGT}(i,j) = \frac{1}{W-1}\left(\sum_{ik=-d}^{d}\sum_{jk=-d}^{d} I(i+ik, j+jk)\right), \quad (ik, jk) \neq (0,0)$$

$$\forall i \in \{1,2,3,\ldots,N_x\} \text{ and } j \in \{1,2,3,\ldots,N_y\}$$

$$\mathrm{NGTDM}(gt) = \sum_{(i,j)|I(i,j)=gt} |gt - \mathrm{ANGT}(i,j)| \forall gt \in \{1,2,\ldots,N_g\}$$

Neighborhood Gray-Tone Difference Matrix (NGTDM)

- Example :

$$gt = 3,\ d = 1,\ W = 9$$

$$ANGT(3,2) = (2+2+4+4+4+2+3+4)\ /\ (9-1) = 25\ /\ 8$$

$$NGTDM(3) = |\ 3 - 25\ /\ 8\ | = 1\ /\ 8 = 0.125$$

2	1	5	3	2
2	2	4	4	1
4	3	4	2	2
3	2	4	4	1
1	4	4	2	3

I	NGTDM
1	0
2	3.500
3	0.125
4	5.125
5	0

Neighborhood Gray-Tone Difference Matrix (NGTDM)

- There are five features derived from the NGTDM :

$$\text{Coarseness} = \left[\epsilon + \sum_{i=1}^{N_g} P_i \text{NGTDM}(i)\right]^{-1}$$

$$\text{Contrast} = \left[\frac{1}{N_t(N_t-1)}\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} P_i P_j (i-j)^2\right]\left[\frac{1}{n^2}\sum_{i=1}^{N_g}\text{NGTDM}(i)\right]$$

$$\text{Busyness} = \frac{\left[\sum_{i=1}^{N_g} P_i \text{NGTDM}(i)\right]}{\left[\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} iP_i - jP_j\right]}, \qquad P_i \neq 0, \quad P_j \neq 0$$

$$\text{Complexity} = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\left\{\frac{|i-j|}{n^2(P_i+P_j)}\right\}\{P_i \text{NGTDM}(i) + P_j \text{NGTDM}(j)\}, \quad P_i \neq 0, \quad P_j \neq 0$$

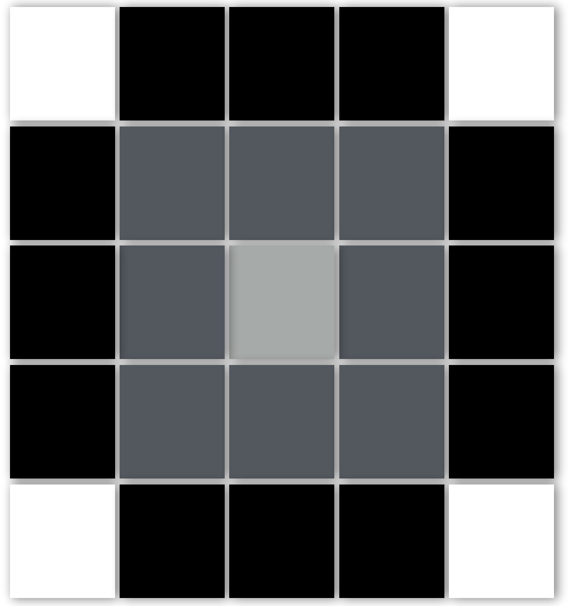
$$\text{Texture strength} = \frac{\left[\sum_{i=1}^{N_g}\sum_{j=1}^{N_g}(P_i+P_j)(i-j)^2\right]}{\left[\epsilon + \sum_{i=1}^{N_g}\text{NGTDM}(i)\right]}$$

N_t : total number of different gray levels present in the image
N_g : gray level

Neighborhood Gray-Tone Difference Matrix (NGTDM)

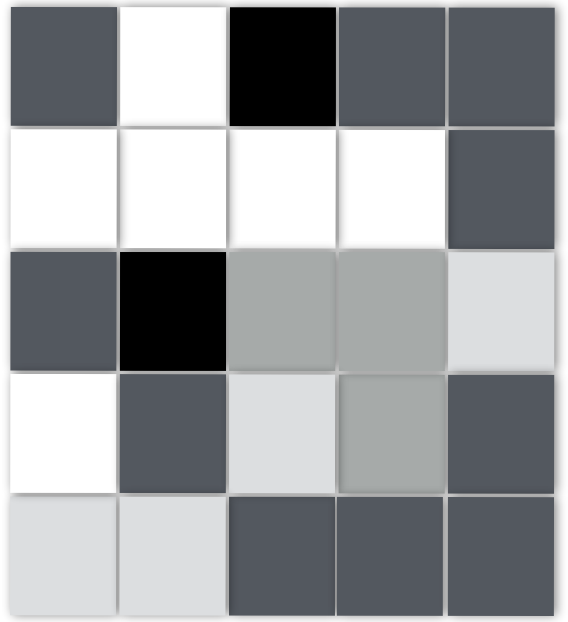
- Example :

$$\text{Coarseness} = \left[\epsilon + \sum_{i=1}^{N_g} P_i \text{NGTDM}(i)\right]^{-1}$$



$$A_1 = \begin{array}{|c|c|c|c|c|}\hline 1 & 5 & 5 & 5 & 1 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 5 & 4 & 3 & 4 & 5 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 1 & 5 & 5 & 5 & 1 \\ \hline \end{array}$$

Coarseness = 2.7778



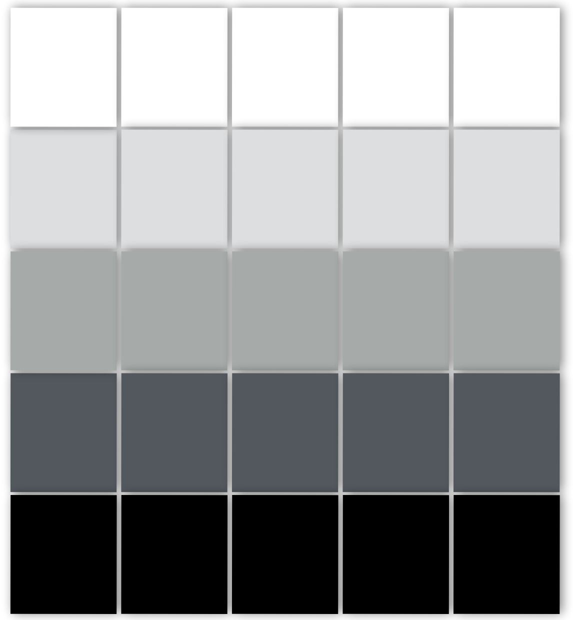
$$A_2 = \begin{array}{|c|c|c|c|c|}\hline 4 & 1 & 5 & 4 & 4 \\ \hline 1 & 1 & 1 & 1 & 4 \\ \hline 4 & 5 & 3 & 3 & 2 \\ \hline 1 & 4 & 2 & 3 & 4 \\ \hline 2 & 2 & 4 & 4 & 4 \\ \hline \end{array}$$

Coarseness = 0.3937

Neighborhood Gray-Tone Difference Matrix (NGTDM)

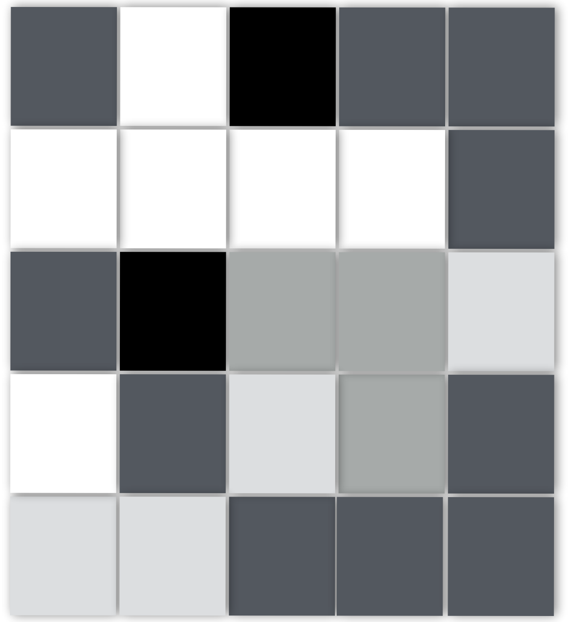
- Example :

$$\text{Contrast} = \left[\frac{1}{N_t(N_t-1)}\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} P_i P_j (i-j)^2\right]\left[\frac{1}{n^2}\sum_{i=1}^{N_g}\text{NGTDM}(i)\right]$$



$$A_1 = \begin{array}{|c|c|c|c|c|}\hline 1 & 1 & 1 & 1 & 1 \\ \hline 2 & 2 & 2 & 2 & 2 \\ \hline 3 & 3 & 3 & 3 & 3 \\ \hline 4 & 4 & 4 & 4 & 4 \\ \hline 5 & 5 & 5 & 5 & 5 \\ \hline \end{array}$$

Contrast = 0.0000



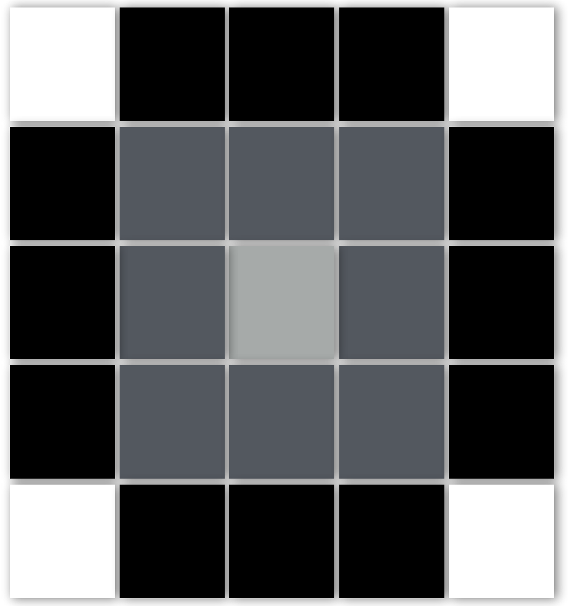
$$A_2 = \begin{array}{|c|c|c|c|c|}\hline 4 & 1 & 5 & 4 & 4 \\ \hline 1 & 1 & 1 & 1 & 4 \\ \hline 4 & 5 & 3 & 3 & 2 \\ \hline 1 & 4 & 2 & 3 & 4 \\ \hline 2 & 2 & 4 & 4 & 4 \\ \hline \end{array}$$

Contrast = 0.9440

Neighborhood Gray-Tone Difference Matrix (NGTDM)

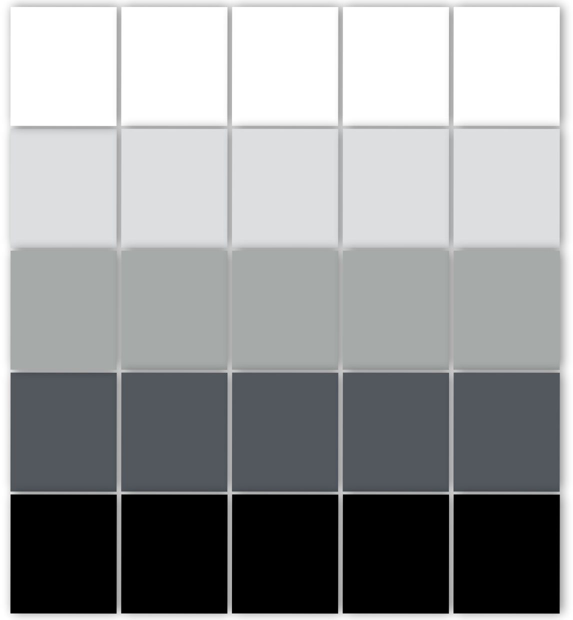
- Example :

$$\text{Busyness} = \frac{\left[\sum_{i=1}^{N_g} P_i \text{NGTDM}(i)\right]}{\left[\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} iP_i - jP_j\right]}, \qquad P_i \neq 0, \quad P_j \neq 0$$



$$A_1 = \begin{bmatrix} 1 & 5 & 5 & 5 & 1 \\ 5 & 4 & 4 & 4 & 5 \\ 5 & 4 & 3 & 4 & 5 \\ 5 & 4 & 4 & 4 & 5 \\ 1 & 5 & 5 & 5 & 1 \end{bmatrix}$$

$$\text{Busyness} = 1.6213 \times 10^{15}$$



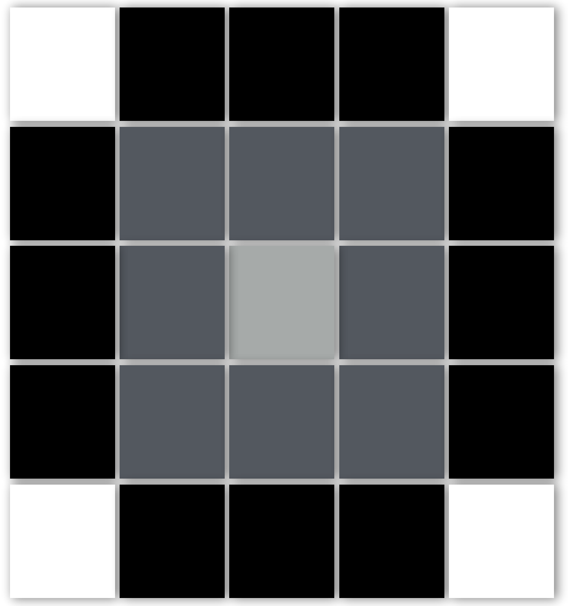
$$A_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix}$$

$$\text{Busyness} = 0.0000$$

Neighborhood Gray-Tone Difference Matrix (NGTDM)

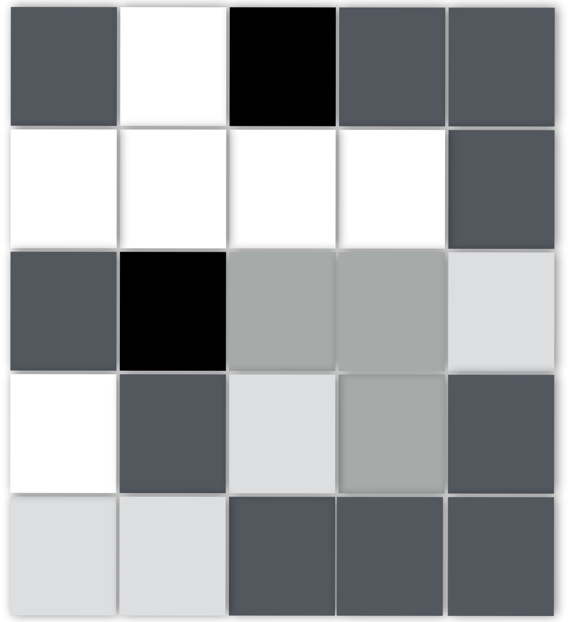
- Example :

$$\text{Complexity} = \sum_{\substack{i=1 \\ P_i \neq 0}}^{N_g} \sum_{\substack{j=1 \\ P_j \neq 0}}^{N_g} \left\{ \frac{|i-j|}{n^2 (P_i + P_j)} \right\} \{ P_i \mathrm{NGTDM}(i) + P_j \mathrm{NGTDM}(j) \}$$



$$A_1 = \begin{array}{|c|c|c|c|c|} \hline 1 & 5 & 5 & 5 & 1 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 5 & 4 & 3 & 4 & 5 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 1 & 5 & 5 & 5 & 1 \\ \hline \end{array}$$

Complexity = 0.3163



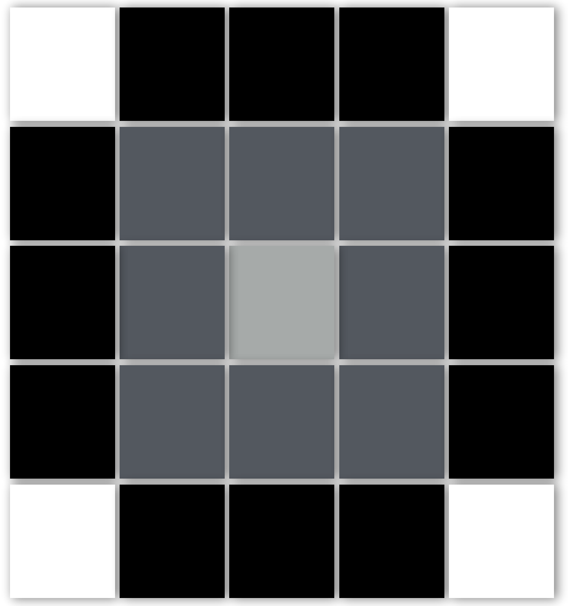
$$A_2 = \begin{array}{|c|c|c|c|c|} \hline 4 & 1 & 5 & 4 & 4 \\ \hline 1 & 1 & 1 & 1 & 4 \\ \hline 4 & 5 & 3 & 3 & 2 \\ \hline 1 & 4 & 2 & 3 & 4 \\ \hline 2 & 2 & 4 & 4 & 4 \\ \hline \end{array}$$

Complexity = 4.7702

Neighborhood Gray-Tone Difference Matrix (NGTDM)

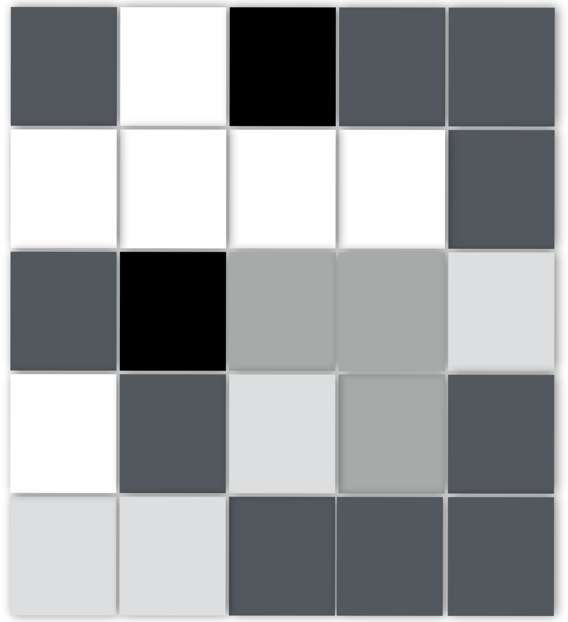
- Example :

$$\text{Texture strength} = \frac{\left[\sum_{i=1}^{N_g}\sum_{j=1}^{N_g}(P_i + P_j)(i-j)^2\right]}{\left[\epsilon + \sum_{i=1}^{N_g}\text{NGTDM}(i)\right]}$$



$$A_1 = \begin{array}{|c|c|c|c|c|} \hline 1 & 5 & 5 & 5 & 1 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 5 & 4 & 3 & 4 & 5 \\ \hline 5 & 4 & 4 & 4 & 5 \\ \hline 1 & 5 & 5 & 5 & 1 \\ \hline \end{array}$$

Texture Strength = 24.4000



$$A_2 = \begin{array}{|c|c|c|c|c|} \hline 4 & 1 & 5 & 4 & 4 \\ \hline 1 & 1 & 1 & 1 & 4 \\ \hline 4 & 5 & 3 & 3 & 2 \\ \hline 1 & 4 & 2 & 3 & 4 \\ \hline 2 & 2 & 4 & 4 & 4 \\ \hline \end{array}$$

Texture Strength = 2.9825

Thank You