Lab 2B: Blur

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Review and Prerequisite

Basic Matrix Computation in MATLAB

- Matrix transpose

  `A'` or `A.'`

- Matrix-vector Multiplication
- Special matrices

  `ones, zeros, eye, rand,...`

Some More Tips

- How to create functions in MATLAB?

  (A template is given in Section Lab 2B of MATLAB coder)

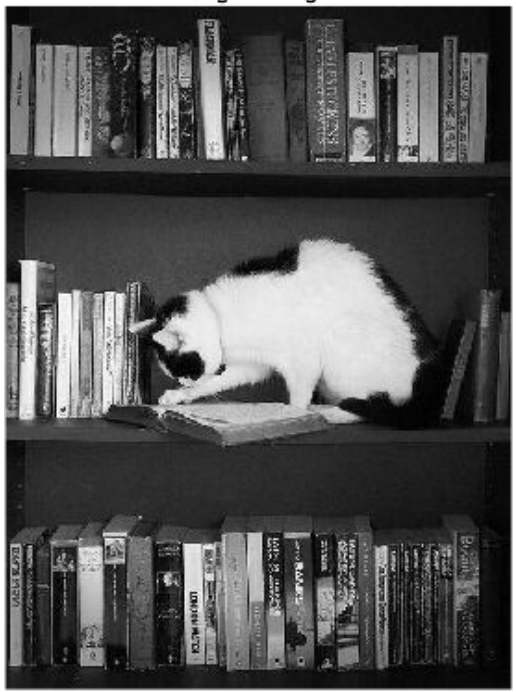
- How to use the function `diag` ?

- How to use the function `imshow` ?

Hands on: Lab 2B Blur

Blur: Example

Original image



Blurred vertically

Blurred in both directions

Blur: Pixel-by-pixel Version

- Consider an $m \times 1$ vector x of pixel intensities, mapped to a vector z of intensities after blurring.
- Each pixel in the new vector is modelled as a weighted average

$$z_i = \frac{1}{4}x_{i-1} + \frac{1}{2}x_i + \frac{1}{4}x_{i+1}$$

- Pixel values that are beyond the boundaries are taken to be zero. $(x_0 \text{ and } x_m)$

Blur: Matrix-vector Product Form

- Express this in matrix-vector product form gives,

$$z = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & & & \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & & \\ & \ddots & \ddots & \ddots & \\ & & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ & & & \frac{1}{4} & \frac{1}{2} \end{bmatrix} x = B_m x$$

Blur: Matrix-vector Product Form

- Express the elements of the vectors x and z in the matrix-vector product from previous slide gives,

$$\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & & & \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \frac{1}{4} \\ & & & \frac{1}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_m \end{bmatrix}$$

Blur: Real 2D Image

- Now consider actual 2D image.
- Let X be an $m \times n$ image and Z be a blurred version of it.
- Blur in the vertical direction

$$B_m X = \begin{bmatrix} B_m X_1 & B_m X_2 & \cdots & B_m X_n \end{bmatrix}$$

- Fully blurred image is given by

$$Z = \left[B_n (B_m X)^T \right]^T = B_m X B_n^T = B_m X B_n$$

Blur Multiple Times

- We can simulate a larger amount of blur by applying our simple blur multiple times,

$$(B_m)^k X (B_n)^k, \quad k \geq 1, k \in \mathbb{N}$$

- Let $V = (B_m)^k$ and $H = (B_n)^k$, the whole process of blurring an image can be written as

$$Z = VXH$$

Problems

1. Write a function which construct the blur matrix.

   (Link for detailed instructions)

2. Write a function that returns the blurred images for a given image.

   (Link for detailed instructions)

What We Learned

Matrix Operations in MATLAB

- Construct tridiagonal matrix
- Matrix-vector multiplication
- Matrix transpose
- Create special matrices