1081 Calculus 模組 07 Homework 7

Due Date: 12/12, 2019

Please hand in the following exercise of textbook in Part I and all problems in Part II. The rigorous and clear explanation is needed. An answer without process will get no point.

Part I:

1. (Ch. 7.4, Ex. 9, 11, 14, 21, 23, 29, 31, 39, 47, 51)
   Evaluate the following integrals.

$$\text{(a)} \int \frac{x-9}{(x+5)(x-2)}\,dx \quad \text{(b)} \int \frac{2}{2x^2+3x+1}\,dx \quad \text{(c)} \int \frac{x^3-4x-10}{x^2-x-6}\,dx \quad \text{(d)} \int \frac{dx}{(x^2-1)^2}$$

$$\text{(e)} \int \frac{10}{(x-1)(x^2+9)}\,dx \quad \text{(f)} \int \frac{x+4}{x^2+2x+5}\,dx \quad \text{(g)} \int \frac{dx}{x^3-1} \quad \text{(h)} \int \frac{dx}{x\sqrt{x-1}}$$

$$\text{(i)} \int \frac{e^{2x}}{e^{2x}+3e^x+2}\,dx \quad \text{(j)} \int \frac{dx}{1+e^x}$$

2. (Ch. 7.8, Ex. 5, 11, 13, 22, 32, 33, 37, 40)
   Determine whether each improper integral is convergent or divergent. Evaluate those that are convergent.

$$\text{(a)} \int_3^\infty \frac{dx}{(x-2)^{3/2}} \quad \text{(b)} \int_0^\infty \frac{x^2}{\sqrt{1+x^3}} \quad \text{(c)} \int_{-\infty}^\infty xe^{-x^2}\,dx \quad \text{(d)} \int_1^\infty \frac{\ln x}{x^2}\,dx$$

$$\text{(e)} \int_0^1 \frac{dx}{\sqrt{1-x^2}} \quad \text{(f)} \int_0^3 \frac{dx}{x^2-6x+5} \quad \text{(g)} \int_0^1 x\ln x\,dx \quad \text{(h)} \int_0^1 \frac{e^{1/x}}{x^3}$$

3. (Ch. 7.8, Ex. 49, 50, 52, 54)
   Use the Comparison Theorem to determine whether the improper integral is convergent or divergent.

$$\text{(a)} \int_0^\infty \frac{x}{x^3+1} \quad \text{(b)} \int_1^\infty \frac{1+\sin^2 x}{\sqrt{x}}\,dx \quad \text{(c)} \int_0^\infty \frac{\tan^{-1} x}{2+e^x}\,dx \quad \text{(d)} \int_0^\pi \frac{\sin^2 x}{\sqrt{x}}\,dx$$

Part II:

1. (Ch. 7.4, Ex. 59, 61)
   (a) For $-\pi < x < \pi$, let $t = \tan\frac{x}{2}$. Show that

$$\sin x = \frac{2t}{1+t^2}, \qquad \cos x = \frac{1-t^2}{1+t^2}, \qquad \text{and} \qquad \frac{dx}{dt} = \frac{2}{1+t^2}. \tag{1}$$

   Apply (1) to calculate $\displaystyle\int \frac{dx}{3\sin x - 4\cos x}$.

   (b) There is another way to calculate above integral. Consider

$$3\sin x - 4\cos x = a\cos(x+b).$$

   Determine a and b, and then compute $\displaystyle\int \frac{dx}{a\cos(x+b)}$. Verify the result is actually the same as previous one.

2. (Ch. 7.8, Ex. 57)
Prove that

$$\int_0^1 \frac{dx}{x^p} \text{ converges when } p < 1; \qquad \int_1^\infty \frac{dx}{x^p} \text{ converges when } p > 1.$$

Hence, show that $\displaystyle\int_0^\infty \frac{dx}{x^p}$ is always divergent, no matter how we choose p.

3. (Ch. 7.8, Ex. 61)
Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function.

   (a) Suppose $\displaystyle\int_{-\infty}^\infty f(x)\,dx$ is convergent. Prove that

$$\lim_{t\to\infty} \int_{-t}^t f(x)\,dx = \int_{-\infty}^\infty f(x)\,dx. \tag{2}$$

   (b) Conversely, suppose $\displaystyle\lim_{t\to\infty} \int_{-t}^t f(x)\,dx$ exists. Is (2) still true? What can you say about the equation (2)?

4. (Ch. 7.8, Ex. 79)
Find the value of the constant C for which the integral

$$\int_0^\infty \left(\frac{1}{\sqrt{x^2+4}} - \frac{C}{x+2} \right) dx$$

converges. Evaluate the integral for this value of C.