Please write down your solutions on a separate sheet of paper and submit it to your TA or instructor on 26^{th} **December, 2019**.

Recommended time limit: 150 minutes.

1. Evaluate the following integrals.

   (a) $\int \sin(3x)\cos(5x)\,dx$.

   (b) $\int \frac{3x+1}{x^2(x^2+25)}\,dx$.

2. Suppose that f is a continuous and positive function on $[0,5]$, and the area between the graph of $y = f(x)$ and the x-axis for $0 \le x \le 5$ is 8. Let $A(c)$ denote the area between the graph of $y = f(x)$ and the x-axis for $0 \le x \le c$, and let $B(c)$ denote the area between the graph of $y = f(x)$ and the x-axis for $c \le x \le 5$. Let $R(c) = A(c)/B(c)$. If $R(3) = 1$ and $\left.\dfrac{dR}{dc}\right|_{c=3} = 7$, find $f(3)$.

3. Compute the area of the region enclosed by the graphs of the equations $y = \tan x$, $y = x$ and $y = 3$.

4. Sketch the solid obtained by rotating the region bounded by $y = 0$ and $y = \cos x$ for $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ about the y-axis and find its volume.

5. (a) Determine whether $\int_{-1}^{1} \frac{x+1}{\sqrt[3]{x}}\,dx$ converges or diverges. Evaluate the value if it converges.

   (b) Determine whether $\int_{2}^{\infty} \frac{1+\cos^2 x}{\sqrt{x}[2-\sin^4 x]}\,dx$ converges or diverges. Evaluate the value if it converges.

6. Water is run at a constant rate of 1 ft^3/min to fill a cylindrical tank of radius 3 ft and height 5 ft. Assuming that the tank is initially empty, make a conjecture about the average weight of the water in the tank over the time period required to fill it, and check your conjecture by integrating. [Take the weight density of water to be 62.4 lb/ft^3].

7. Solve the following differential equations.

   (a) $x \ln x = y(1 + \sqrt{3+y^2})y', \quad y(1) = 1.$

   (b) $y' \tan x = a + y, \quad y(\pi/3) = a, \quad 0 < x < \pi/2.$